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Survey/review study

The Cooperative Output Regulation by the Distributed Observer Approach

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Abstract: The cooperative output regulation problem (CORP) is an extension of the leader-following consensus problem of multi-agent systems (MASs), and has been studied by two approaches, namely, the distributed observer (DO) approach and the distributed internal model (DIM) approach. The two approaches are, respectively, the extensions of the classical feedforward control approach and the classical internal model approach (for a single system) to the MASs. This paper overviews the CORP by the DO approach with the emphasis on linear MASs. After formulating the CORP, we present the evolution process of three types of DOs and the corresponding solutions to the CORP. Furthermore, some variants and extensions of the DO approach are also briefly surveyed for completeness.

Keywords: cooperative output regulation; multi-agent systems; distributed observer

1. Introduction

The output regulation problem (ORP) has been one of the central control problems in the control community for over a half century [1−3]. The ORP aims to design a feedback control law for a given plant such that the output of the plant asymptotically track a class of reference inputs in the presence of a class of disturbances while ensuring the internal stability of the closed-loop system. Both reference inputs and disturbances are generated by the so-called exosystem. The ORP was first studied for linear systems with both the reference input and disturbance being step functions [4−5], and then extended to various general cases in, for instance, [6−12]. Two approaches, namely, the feedforward control approach and the internal model (IM) approach have been developed, where the former makes use of the solution of the regulator equations to obtain the precise feedforward control quantity to cancel the tracking error incurred by the exogenous signal, while the latter converts the ORP of the given plant to a stabilization problem of the augmented system composed of the given plant and a well designed IM. An advantage of the IM approach is that it is able to deal with systems with parametric uncertainties.

Since 2000, the cooperative control of MASs has become a mainstream control problem following the publications of a few celebrated papers, say, [13−16]. The cooperative control of MASs was first studied for simple linear MASs where each agent is a single integrator [15−19], double integrator [20], or harmonic oscillator [21]. The consensus of general linear homogenous MASs was treated in [22] and the consensus problem of some weak nonlinear MASs was also investigated in [23] for Lipschitz nonlinear systems, in [24] for Euler-Lagrange systems, and in [25] for rigid-body systems. Since around 2010, the research on the cooperative control has focused on more complex MASs featuring strong nonlinearities, time-delays, large uncertainties, and jointly connected switching networks, and a more challenging objective (such as simultaneous tracking and disturbance rejection) has been attracting the attention.

The CORP is an extension of the leader-following consensus problem of MASs in the sense that it handles simultaneously the asymptotic tracking and disturbance rejection problems for heterogeneous and possibly uncertain MASs. The CORP is also a generalization of the conventional ORP for a single plant to a MAS with a leader by viewing the leader system as the exosystem and the follower system as the controlled plant. Generalizing the feedforward control approach and the IM approach for a single plant to their distributed counterparts, respectively, leads to the so-called DO approach and the so-called DIM approach. This paper will mainly focus on the DO approach.

A DO is a dynamic compensator aimed to estimate certain information of the leader system over a communication network and transmit the estimated information to the follower systems. Cascading a purely decentralized control law with DO gives a distributed control law. The DO was first developed in [26] over static networks for the purpose of solving the CORP for linear heterogeneous MASs, and was later generalized to jointly connected switching networks in [27]. Since 2011, the DO has experienced three phases of development. To be specific, the original DO as given in [26−27] only estimates and transmits the leader's signal to followers by assuming every follower knows the dynamics of the leader [26−29]. In practice, the dynamics of the leader may not be known by every follower. Thus, in the second phase, the capability of the original DO was enhanced to be able to estimate and transmit both the leader's signal and the dynamics of the leader. This enhanced DO only assumes that the leader's children know the dynamics of the leader and is thus fully distributed. Since this enhanced DO is able to estimate the leader's system matrix, it is called an adaptive distributed observer (ADO) for a known leader system [30, 31]. In a more realistic setting, the leaders' dynamics may contain uncertain parameters. In this case, none of the followers know the leader's exact dynamics. Thus, the third phase of DO needs to be able to estimate the leader's unknown parameters. Such a dynamic compensator is called ADO for an uncertain leader. The ADO for an uncertain leader was first studied in [32], which did not consider the convergence of the estimated unknown parameters of the leader to their real values. In [33], an ADO was further developed for an uncertain leader system and the sufficient condition was also provided on the convergence of the estimated unknown parameters to the actual parameters. The COPR subject to an uncertain leader system was further studied in [34]. More aspects of the CORP by the DO approach can be found in [35−40]. A comprehensive treatment on the CORP by the DO approach can be found in [41].

The rest of this paper is organized as: Section 2 formulates CORP of MASs. Section 3 overviews the three types of DOs. After reviewing the conventional feedforward control for solving linear ORP of one single control system in Section 4, we further summarize, in Section 5, the solvability conditions of the CORP by all three types of DOs. In Section 6, we present some variants and extensions of the DO as well as the applications of the DO to some other cooperative control problems such as the consensus problem of Euler-Lagrange systems, attitude synchronization problem of spacecraft systems, rendesvous/flocking problem, and formation problem. The CORP can also be handled by the DIM approach and the integrated approach that combines the DO approach and the DIM approach. In Section 7, we close this paper by giving a brief overview of the DIM approach and the integrated approach.

2. A Formulation of CORP

In this section, we give a general formation of the CORP for linear MASs. An MAS consists of a group of individual agents. A general representation for a continuous-time linear MAS having N agents can be described as:

$$\dot{x}_i = A_i(w)x_i + B_i(w)u_i + E_i(w)v_0 \tag{1a}$$

$$y_{mi} = C_{mi}(w)x_i + D_{mi}(w)u_i + F_{mi}(w)v_0 \tag{1b}$$

$$y_i = C_{pi}(w)x_i + D_{pi}(w)u_i + F_{pi}(w)v_0,\ i = 1,\ldots,N \tag{1c}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_{mi} \in \mathbb{R}^{p_{mi}}$, and $y_i \in \mathbb{R}^p$ are the state, control input, measurement output, and performance output of the i^{th} agent of (1), respectively, v_0 and w are the exogenous signal and the uncertain parametric vector, respectively. All nine matrices in (1) are smooth in w.

The exogenous signal v_0 is typically produced by the following system:

$$\dot{v}_0 = S_0(w_0)v_0 \tag{2a}$$

$$y_{m0} = W_0(w_0)v_0 \tag{2b}$$

$$y_0 = F_0 v_0 \tag{2c}$$

where $v_0 \in \mathbb{R}^{q_0}$, $y_{m0} \in \mathbb{R}^{p_{m0}}$, $y_0 \in \mathbb{R}^p$, and $w_0 \in \mathbb{R}^{n_{w0}}$ are the state, measurement output, reference output, and unknown constant vector, respectively, $S_0(w_0)$ and $W_0(w_0)$ are smooth in w_0, and F_0 is known and constant.

Systems (1) and (2) together are treated as an MAS of $N+1$ agents. Since (2) generates reference signals for the performance outputs of (1) to track, we call (2) and (1) as the leader system and the follower system, respectively. If the dynamics of (1) does not contain v_0, then (1) is called a leaderless MAS of N agents.

If (1) contains no uncertain parametric vectors, we can simplify it to:

$$\dot{x}_i = A_i x_i + B_i u_i + E_i v_0 \tag{3a}$$

$$y_{mi} = C_{mi} x_i + D_{mi} u_i + F_{mi} v_0 \tag{3b}$$

$$y_i = C_{pi}x_i + D_{pi}u_i + F_{pi}v_0,\ i = 1,\dots,N \tag{3c}$$

where all nine matrices are known. If (2) is known exactly, we can also simplify it to:

$$\dot{v}_0 = S_0 v_0 \tag{4a}$$

$$y_{m0} = W_0 v_0 \tag{4b}$$

$$y_0 = F_0 v_0 \tag{4c}$$

where S_0, W_0, and F_0 are all known.

The information exchange among the agents of a leader-follower MAS defined by (2) and (1) is described by a switching graph $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)})$[1] (See [41, Section 1.2] for a detailed description of a switching graph.), where $\bar{\mathcal{V}} = \{0,1,\dots,N\}$ is the node set, and $\bar{\mathcal{E}}_{\sigma(t)} \subset \bar{\mathcal{V}}\times\bar{\mathcal{V}}$ is the edge set. The node 0 is associated with the leader, and, for $i = 1,\dots,N$, the node i is associated with the i^{th} follower. For $i = 1,\dots,N$, $j = 0,1,2,\dots,N$, $i \neq j$, the edge $(j,i) \in \bar{\mathcal{E}}_{\sigma(t)}$ if and only if, at the time instant t, the control u_i can make use of the measurement output y_{mj}.

For $i, j = 0,1,2,\dots,N$, let $a_{ij}(t)$ be the entries of the adjacent matrix of $\bar{\mathcal{G}}_{\sigma(t)}$. Then, we consider the following control law:

$$u_i = k_i(\chi_i, y_{mi}, a_{ij}(t)\chi_j, a_{ij}(t)y_{mj}, j \in \bar{\mathcal{V}}/\{i\}) \tag{5a}$$

$$\dot{\chi}_i = g_i(\chi_i, y_{mi}, a_{ij}(t)\chi_j, a_{ij}(t)y_{mj}, j \in \bar{\mathcal{V}}/\{i\}) \tag{5b}$$

where k_i and g_i are smooth, χ_0 consists of the signals and /or the parameters of the leader system. Since the control law (5) satisfies the communication constraints, we call (5) a distributed control law.

The error output (or regulated output) $e_i \in \mathbb{R}^p$ of the i^{th} follower is defined as:

$$e_i = y_i - y_0 = C_i(w)x_i + D_i(w)u_i + F_i(w)v_0,\ i = 1,\dots,N \tag{6}$$

where $C_i(w) = C_{pi}(w)$, $D_i(w) = D_{pi}(w)$, and $F_i(w) = F_{pi}(w) - F_0$. If $C_i(w)$, $D_i(w)$, and $F_i(w)$ are all known, (6) takes the following simplified form:

$$e_i = C_i x_i + D_i u_i + F_i v_0,\ i = 1,\dots,N. \tag{7}$$

For simplicity, this paper focuses on the known follower system (3) without uncertain parametric vectors, while the leader system can be either known as in (4) or uncertain as in (2). The CORP is formulated as follows:

Problem 1. Given the systems (3), (4) (or (2)), and the graph $\bar{\mathcal{G}}_{\sigma(t)}$ (or a static communication graph $\bar{\mathcal{G}}$), find a distributed control law of the form (5) such that the closed-loop system satisfies the following properties:

• *Property 1:* The origin of the closed-loop system with v_0 set to zero is asymptotically stable.

• *Property 2:* For any initial condition, the solution of the closed-loop system satisfies $\lim_{t\to\infty} e_i(t) = 0,\ i = 1,\dots,N$.

3. The Purely Decentralized Feedforward Control

Let us first assume that the state v_0 can be used by the control input u_i of each follower. Then, Problem 1 can be handled by the classical feedforward control method under some standard assumptions as follows:

Assumption 1: The matrix pairs (A_i, B_i), $i = 1,\dots,N$, are stabilizable.

Assumption 2: The matrix pairs (C_{mi}, A_i), $i = 1,\dots,N$, are detectable.

Assumption 3: There exist matrix pairs (X_i, U_i), $i = 1,\dots,N$, that satisfy the following matrix equations:

$$X_i S_0 = A_i X_i + B_i U_i + E_i \tag{8a}$$

$$0 = C_i X_i + D_i U_i + F_i,\ i = 1,\dots,N \tag{8b}$$

Remark 1: Assumption 1 guarantees that Property 1 can be achieved by the state feedback control. Assumption 2 together with Assumption 1 guarantees that Property 1 can be achieved by the measurement output feedback control. Equations (8) are called the regulator equations, and the solvability of equations (8) is the necessary condition of the solvability of Problem 1 [11].

Two classes of control laws are considered.

1. *Purely Decentralized Full Information Static State Feedback*:

$$u_i = K_{1i}x_i + K_{2i}v_0,\ i = 1,\dots,N \tag{9}$$

where (K_1, K_2) are constant with appropriate dimensions.

2. *Purely Decentralized Measurement Output Feedback plus Feedforward*:

$$u_i = K_{1i}z_i + K_{2i}v_0, \tag{10a}$$

$$\dot{z}_i = \mathcal{G}_{zi}z_i + \mathcal{G}_{yi}y_{mi} + \mathcal{G}_{vi}v_0,\ i = 1,\ldots,N \tag{10b}$$

where $(K_1, K_2, \mathcal{G}_{zi}, \mathcal{G}_{yi}, \mathcal{G}_{vi})$ are constant with appropriate dimensions.

When N=1, the full information control law (9) was originally given in [11], where it is shown that, under Assumptions 1 and 3, the classical ORP can be solved by (9) if K_{1i} are chosen such that $A_i + B_iK_{1i}$ are Hurwitz, and $K_{2i} = U_i - K_{1i}X_i$ with U_i given in (8). The solvability of the classical ORP via the measurement output feedback plus feedforward control law (10) was considered in [29], where (K_{1i}, K_{2i}) can be chosen the same as those in (9), and under additional Assumption 2,

$$G_{zi} = A_i + B_iK_{1i} - \ell_iC_{mi},\ G_{yi} = \ell_i$$
$$G_{vi} = B_iK_{2i} + E_i - \ell_i(D_{mi}K_{2i} + F_{mi})$$

with $\ell_i \in \mathbb{R}^{n_i\times p_{mi}}$ being such that $A_i - \ell_iC_{mi}$ are Hurwitz. Moreover, when v_0 is unmeasurable for any follower, and

$$\text{the pairs } \left([C_{mi}\ \ F_{mi}], \begin{bmatrix} A_i & E_i \\ 0 & S_0 \end{bmatrix}\right) \text{ are detectable} \tag{11}$$

one can also find a measurement output feedback control law by estimating v_0 as well. Such a case can be found in [3, Section 1.2].

4. Three Types of the Distributed Observers

This section overviews three types of DOs, which can be viewed as a concentrate of [41, Chapter 4].

Given any switching communication graph $\bar{\mathcal{G}}_{\sigma(t)}$ with $N+1$ nodes, define a compensator of the following form: for $i = 1,\ldots,N$,

$$\dot{v}_i = \phi_i(W_iv_i, a_{ij}(t)W_jv_j, j \in \bar{\mathcal{V}}/\{i\}) \tag{12}$$

where ϕ_i is some globally defined smooth function. If, for any initial condition, the solution of systems (4) and (12) satisfies, for $i = 1,\ldots,N$, $\lim_{t\to\infty}(v_i(t) - v_0(t)) = 0$, then (12) is called a distributed observer (DO) for (4).

To construct a DO for (4), we list the following assumptions.

Assumption 4: There exists a positive number $T > 0$ such that, for all $i = 0,1,\cdots,$ and $t \geqslant 0$, the union graph $\bigcup_{t\leqslant t_j<t+T}\mathcal{G}_{\sigma(t_j)}$ contains a spanning tree with node 0 as the root.

To introduce the next assumption, let $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ be a subgraph of $\bar{\mathcal{G}}_{\sigma(t)}$ where $\mathcal{V} = \{1,\ldots,N\}$ and $\mathcal{E}_{\sigma(t)}$ is obtained from $\bar{\mathcal{E}}_{\sigma(t)}$ by removing all edges incident to node 0.

Assumption 5: $\mathcal{G}_{\sigma(t)}$ is undirected for any $t \geqslant 0$.

When the communication graph is static, we simplify the notation $\bar{\mathcal{G}}_{\sigma(t)}$ and $\mathcal{G}_{\sigma(t)}$ by $\bar{\mathcal{G}}$ and $\mathcal{G}$, respectively. In this case, Assumption 4 is simplified to the following one.

Assumption 6: $\bar{\mathcal{G}}$ contains a spanning tree with node 0 as the root.

Various assumptions for the leader system are as follows:

Assumption 7: (W_0, S_0) is detectable.

Assumption 8: The real parts of all the eigenvalues of the matrix S_0 are non-positive.

Assumption 9: The matrix S_0 is marginally stable.

4.1. Distributed Observer for a Known Leader System

In this subsection, let us allow the matrices S_0 and W_0 of the leader system (4) to be known by every follower for all $t \geqslant 0$.

Consider the following DO candidate:

$$\dot{v}_i = S_0v_i + L_0\sum_{j=0}^{N} a_{ij}(t)W_0(v_j - v_i),\ i = 1,\ldots,N \tag{13}$$

where $v_i \in \mathbb{R}^q$, and $L_0 \in \mathbb{R}^{q\times p_{m0}}$ is to be designed.

Let $\tilde{v}_i = v_i - v_0$, $i = 1,\ldots,N$, and $\tilde{v} = \text{col}(\tilde{v}_1,\ldots,\tilde{v}_N)$.[2](For n column vectors $x_1,\ldots,x_n$, $\text{col}(x_1,\ldots,x_n) = [x_1^\top,\ldots,x_n^\top]^\top$.) Then,

$$\dot{\tilde{v}} = (I_N \otimes S_0 - H_{\sigma(t)} \otimes (L_0W_0))\tilde{v}. \tag{14}$$

System (14) is called the error system of (13). Clearly, (13) is a DO for (4) if and only if the origin of (14) is asymptotically/exponentially stable.

If the leader's state is available, i.e., $y_{m0} = v_0$. Then, letting $L_0 = \mu_vI_q$ where μ_v is a positive real number, (13) is simplified to the following DO:

$$\dot{v}_i = S_0 v_i + \mu_v \sum_{j=0}^{N} a_{ij}(t)(v_j - v_i),\ i = 1,\ldots,N \tag{15}$$

and the error system (14) becomes:

$$\dot{\tilde{v}} = (I_N \otimes S_0 - \mu_v (H_{\sigma(t)} \otimes I_q)\tilde{v}. \tag{16}$$

In what follows, we call (13) and (15) the state-based DO and output-based DO, respectively.

The state-based DO was first given in [26] over static networks where it was shown that, under Assumptions 6, for sufficiently large μ_v, the origin of (16) is exponentially stable. This result was later extended to jointly-connected switching networks in [27] where it was shown that, under Assumptions 4 and 8, for any $\mu_v > 0$, the origin of (14) is exponentially stable.

Under Assumption 7, let $P_0 > 0$ be the unique solution of

$$P_0 S_0^{\mathrm{T}} + S_0 P_0 - P_0 W_0^{\mathrm{T}} W_0 P_0 + I_q = 0. \tag{17}$$

Then, under Assumptions 6 and 7, with $L_0 = \mu_v P_0 W_0^{\mathrm{T}}$ and $\mu_v \geqslant \frac{1}{2}\underline{\lambda}_H^{-1}$,[3] (Given any matrix $A \in \mathbb{R}^{n\times n}$, $\underline{\lambda}_A$ and $\overline{\lambda}_A$ denote the minimum and maximum real parts of eigenvalues of A, respectively.) the error system (14) with $H_{\sigma(t)} = H$ is exponentially stable [41, Theorem 3.1], which directly leads to the output-based DO over static networks. Under Assumptions 4, 5, 7, and 9, it was shown in [42] using the generalized Barbalat's lemma that (16) is asymptotically stable, which further leads to the output-based DO over jointly-connected switching networks as can be found in [29].

In some applications, the asymptotic stability of system (14) may not be enough. One needs system (14) to be exponentially stable. By using the generalized Krasovskii-LaSalle theorem together with the scaling invariant property [43], one can further show that, under Assumptions 4, 5, 7, and 9, the origin of (14) is exponentially stable, where $L_0 = \mu_v P_0 W_0^{\mathrm{T}}$ for any $\mu_v > 0$ with P_0 satisfying (9). By adopting the generalized Krasovskii–LaSalle theorem for switched time-varying systems [44], one can further show that the exponential stability is uniform w.r.t. a group of switching signals.

Remark 2: There are some other variants of this type of DOs. For example, [45] considered the case where each agent only has access to part of the output of the leader system. [38, 46] studied the local observer together with global distributed observer. [47] considered the finite-time convergent DO for a special leader system in chained integrator form. [48] treated leader systems with bounded inputs. DOs with self-tuning gain assignment can be found in [49–50], and the gain assignment for nonovershooting performance was proposed in [51].

4.2. Adaptive Distributed Observer for a Known Leader System

The DO (13) is not fully distributed in the sense that the system matrices S_0 and W_0 are used by every follower. To obtain a fully distributed DO, in this subsection, we further introduce the so-called adaptive distributed observer (ADO) for leader system (4), which not only estimates the state of the leader, but also the three matrices S_0, W_0, and F_0. Two scenarios are considered corresponding to the two scenarios in the previous subsection.

The output-based ADO candidate is designed as follows:

$$\dot{S}_i = \mu_S \sum_{j=0}^{N} a_{ij}(t)(S_j - S_i) \tag{18a}$$

$$\dot{F}_i = \mu_F \sum_{j=0}^{N} a_{ij}(t)(F_j - F_i) \tag{18b}$$

$$\dot{W}_i = \mu_W \sum_{j=0}^{N} a_{ij}(t)(W_j - W_i) \tag{18c}$$

$$\dot{L}_i = \mu_L \sum_{j=0}^{N} a_{ij}(t)(L_j - L_i) \tag{18d}$$

$$\dot{v}_i = S_i v_i + \mu_v L_i \sum_{j=0}^{N} a_{ij}(t)(W_j v_j - W_i v_i) \tag{18e}$$

$$\hat{y}_i = F_i v_i,\ i = 1,\ldots,N. \tag{18f}$$

The compensator (18) is further called an output-based ADO, if, for $i = 1,\ldots,N$,

$\lim_{t\to\infty}(S_i(t)-S_0)=0$, $\lim_{t\to\infty}(F_i(t)-F_0)=0$, $\lim_{t\to\infty}(W_i(t)-W_0)=0$, $\lim_{t\to\infty}(L_i(t)-L_0)=0$, $\lim_{t\to\infty}(v_i(t)-v_0(t))=0$, and $\lim_{t\to\infty}(\hat{y}_i(t)-y_0(t))=0$ all exponentially.

When $y_{m0}=v_0$, i.e., $W_0=I$, there is no need to estimate L. Then, we obtain the state-based ADO candidate in the following form:

$$\dot{S}_i=\mu_S\sum_{j=0}^{N}a_{ij}(t)(S_j-S_i) \tag{19a}$$

$$\dot{F}_i=\mu_F\sum_{j=0}^{N}a_{ij}(t)(F_j-F_i) \tag{19b}$$

$$\dot{v}_i=S_iv_i+\mu_v\sum_{j=0}^{N}a_{ij}(t)(v_j-v_i) \tag{19c}$$

$$\hat{y}_i=F_iv_i \tag{19d}$$

The compensator (19) is further called a state-based ADO if, for $i=1,\ldots,N$, $\lim_{t\to\infty}(S_i(t)-S_0)=0$, $\lim_{t\to\infty}(F_i(t)-F_0)=0$, $\lim_{t\to\infty}(v_i(t)-v_0(t))=0$, and $\lim_{t\to\infty}(\hat{y}_i(t)-y_0(t))=0$, all exponentially.

Remark 3: In many cases, the role of the matrices F_0 and W_0 is to select the elements of v_0. In these cases, we can assume F_0 and W_0 are known by every follower, and hence, there is no need to estimate F_0 and W_0. Consequently, the output-based ADO (18) can be simplified to:

$$\dot{S}_i=\mu_S\sum_{j=0}^{N}a_{ij}(t)(S_j-S_i) \tag{20a}$$

$$\dot{L}_i=\mu_L\sum_{j=0}^{N}a_{ij}(t)(L_j-L_i) \tag{20b}$$

$$\dot{v}_i=S_iv_i+\mu_vL_i\sum_{j=0}^{N}a_{ij}(t)(W_0v_j-W_0v_i) \tag{21c}$$

$$\hat{y}_i=F_0v_i \tag{20d}$$

and the state-based ADO (19) can be simplified to:

$$\dot{S}_i=\mu_S\sum_{j=0}^{N}a_{ij}(t)(S_j-S_i) \tag{21a}$$

$$\dot{v}_i=S_iv_i+\mu_v\sum_{j=0}^{N}a_{ij}(t)(v_j-v_i) \tag{21b}$$

$$\hat{y}_i=F_0v_i \tag{21c}$$

The state-based ADO (21) was first proposed in [30] over switching networks under Assumptions 4, 5, and the assumption that the system matrix S_0 is neutrally stable. Assumption 5 was removed in [52], and the neutral stability assumption on S_0 was relaxed to Assumption 8 in [53]. The case for static networks was established in [31] under Assumption 6. The study of the general form (19) over both switching and static networks can be found in [41, Section 4.4]. Technically, the error dynamics for this class of DOs is a perturbed system with the nominal system in the form (16). If the communication graph $\bar{\mathcal{G}}_{\sigma(t)}$ is switching, the state-based ADO (19) candidate is indeed an ADO if Assumptions 4 and 8 are satisfied and $\mu_S,\mu_F,\mu_v>0$. If the graph $\bar{\mathcal{G}}$ is static and Assumption 6 holds, then the state-based ADO (19) candidate is indeed an ADO with $\mu_S,\mu_F,\mu_v>\bar{\lambda}_{S_0}\underline{\lambda}_H^{-1}$.

A special type of the output-based ADO was given in [54], and a more general form of (18) can be found in [41, Section 4.4]. Technically, the error dynamics for this class of DOs is a perturbed system with the nominal system in the form (14). If the communication graph $\bar{\mathcal{G}}_{\sigma(t)}$ is switching, the output-based ADO (18) candidate is indeed an output-based ADO if Assumptions 4, 5, and 9 are satisfied and $\mu_S,\mu_F,\mu_W,\mu_L,\mu_v>0$. If the graph $\bar{\mathcal{G}}$ is static and Assumption 6 holds, then the output-based ADO (19) candidate is indeed an output-based ADO with $\mu_S,\mu_F,\mu_W>\bar{\lambda}_{S_0}\underline{\lambda}_H^{-1},\mu_L>0,\mu_v>\frac{1}{2}\underline{\lambda}_H^{-1}$.

Remark 4: Other ADO variants, for example, an ADO with self-turning gains was proposed in [55].

4.3. Adaptive Distributed Observer for an Uncertain Leader System

Both DO and ADO assume that the leader's dynamics are known by at least some followers. In this subsection, it is assumed that the leader's dynamics contains some uncertain parameters, and hence none of followers knows the exact dynamics of the leader system. For simplicity, let us consider a special case of uncertain leader system (2) as follows:

$$\dot{v}_0 = S_0(w_0)v_0 \tag{22a}$$

$$y_{m0} = v_0 \tag{22b}$$

where $v_0 \in \mathbb{R}^q$, $S_0(w_0) \in \mathbb{R}^{q\times q}$ satisfies the following assumption:

Assumption 10: For all $w_0 = \mathrm{col}(\omega_{01},\dots,\omega_{0l}) \in \mathbb{R}^l$, the matrix $S_0(w_0)$ is neutrally stable and nonsingular.

In fact, a neutrally stable matrix allows a simple zero eigenvalue. As the zero eigenvalue is known and hence can be dealt with by the DO, we exclude this case by requiring that the matrix $S_0(w_0)$ be nonsingular in Assumption 10. Thus, without loss of generality, we can assume that

$$S_0(w_0) = \mathrm{diag}(\omega_{01},\dots,\omega_{0l})\otimes a,\ a = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, q = 2l.$$

Let the mapping $\phi : \mathbb{R}^{2l} \mapsto \mathbb{R}^{l\times 2l}$ be such that, for any $x = \mathrm{col}(x_1,\dots,x_{2l}) \in \mathbb{R}^{2l}$,

$$\phi(x) = \begin{bmatrix} -x_2 & x_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -x_4 & x_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -x_{2l} & x_{2l-1} \end{bmatrix}. \tag{23}$$

Then, for $i = 1,\dots,N$, an ADO candidate, which is independent of the unknown vector w_0, is proposed as follows:

$$\dot{v}_i = S_0(w_i)v_i + \mu_v \sum_{j=0}^{N} a_{ij}(v_j - v_i) \tag{24a}$$

$$\dot{w}_i = \mu_w \phi\left(\sum_{j=0}^{N} a_{ij}(v_j - v_i)\right) v_i \tag{24b}$$

where $v_i \in \mathbb{R}^{2l}$, $w_i \in \mathbb{R}^l$, $\mu_v,\mu_w > 0$. The dynamic compensator (24) is further called ADO for the leader (22) if $\lim_{t\to\infty}(v_i(t) - v_0(t)) = 0, i = 1,\dots,N$.

The ADO for an unknown leader system was first studied in [32]. However, [32] did not consider the convergence of the estimated unknown parameters of the system matrix of the leader to their actual values. The same problem was further pursued in [34] with an ADO (24). It is shown in [34] that, under Assumptions 6 and 10, (24) is valid for any $\mu_v,\mu_w > 0$ if the communication graph $\bar{\mathcal{G}}$ is static and the subgraph $\mathcal{G}$ is undirected. It is also shown that, as long as the leader signal is persistently exciting, the estimated unknown parameters of the system matrix of the leader will asymptotically tend to their actual values, that is, $\lim_{t\to\infty}(w_i(t) - w_0) = 0, i = 1,\dots,N$.

Remark 5: Extensions of the ADO for an uncertain leader over directed acyclic static graphs and directed acyclic switching graphs were given in [56] and [57], respectively. Other results dealing with the uncertain leader can be found in [58–59].

5. The Cooperative Output Regulation via the Distributed Observer Approach

We are now ready to consider the solvability of the CORP by the DO approach. This section can be viewed as a concentrate of [41, Chapter 9].

Notice that the purely decentralized control laws (9) and (10) can be unified into the following form:

$$u_i = K_{zi}z_i + K_{yi}y_{mi} + K_{vi}v_0,\ i = 1,\dots,N \tag{25a}$$

$$\dot{z}_i = \mathcal{G}_{1i}z_i + \mathcal{G}_{2i}y_{mi} + \mathcal{G}_{3i}v_0 \tag{25b}$$

where $z_i \in \mathbb{R}^{n_{z_i}}$, K_{zi}, K_{yi}, K_{vi}, $\mathcal{G}_{1i}$, $\mathcal{G}_{2i}$, $\mathcal{G}_{3i}$ are some constant matrices.

Let the following system

$$\dot{\xi}_i = g_i(\xi_i, y_{mi}, a_{ij}(t)\xi_j, a_{ij}(t)y_{mj}, j \in \bar{\mathcal{V}}/\{i\}) \tag{26}$$

with a smooth function g_i in its argument, denote some DO of the leader presented in Section 4. Now, replacing v_0 in (25) by ξ_i gives the following cascade-connected control law:

$$u_i = K_{zi}z_i + K_{yi}y_{mi} + K_{vi}\xi_i,\ i = 1,\dots,N \tag{27a}$$

$$\dot{z}_i = \mathcal{G}_{1i} z_i + \mathcal{G}_{2i} y_{mi} + \mathcal{G}_{3i} \xi_i \tag{27b}$$

$$\dot{\xi}_i = g_i(\xi_i, y_{mi}, a_{ij}(t)\xi_j, a_{ij}(t) y_{mj}, j \in \bar{\mathcal{V}}/\{i\}) \tag{27c}$$

which is indeed a distributed control law since, at each time $t \geqslant 0$, for any $i = 1, \ldots, N$, u_i makes use of y_{m0} if and only if the leader is a neighbor of the ith follower. Moreover, it is shown in [29] that the distributed control law (27) solves the CORP as long as the purely decentralized control law (25) solves the ORP.

5.1. The Distributed Observer Based Approach for a Known Leader System

Let (26) be given by the DO (13), and let (25) be given by control laws (9) and (10), respectively. Then, we obtain two distributed control laws for solving the CORP (Problem 1) as follows.

1. Distributed full information control law:

$$u_i = K_{1i} x_i + K_{2i} v_i, \ i = 1, \ldots, N \tag{28a}$$

$$\dot{v}_i = S_0 v_i + \mu_v L_0 \left(\sum_{j=0}^{N} a_{ij}(t) W_0 (v_j - v_i) \right) \tag{28b}$$

2. Distributed measurement output feedback plus feedforward control law:

$$u_i = K_{1i} z_i + K_{2i} v_i, \ i = 1, \ldots, N \tag{29a}$$

$$\begin{aligned} \dot{z}_i = & A_i z_i + B_i u_i + E_i v_i \\ & + \ell_i (y_{mi} - C_{mi} z_i - D_{mi} u_i - F_{mi} v_i) \end{aligned} \tag{29b}$$

$$\dot{v}_i = S_0 v_i + \mu_v L_0 \left(\sum_{j=0}^{N} a_{ij}(t) W_0 (v_j - v_i) \right) \tag{29c}$$

where K_{1i}, K_{2i}, ℓ_i are the same as those in (9) and (10), and L_0 is given in (13).

Problem 1 was first formally formulated in [26] for the leader system with $W_0 = I_q$, and solved over static networks using distributed control law (28), and then solved over jointly connected switching networks in [27] using distributed control laws (28) and (29), respectively. The solution to the CORP with general output matrix W_0 was given in [29] by using the distributed control laws (28) and (29), respectively. It is noted that the solvability conditions are the combinations of the solvability conditions of the ORP (e.g. Assumptions 1 - 3) and those for establishing the DO (14) or its special case (15) (e.g. Assumptions 4 - 9). The same problem was also studied in [28] via the dynamic measurement output feedback design by combining the DO and the Luenberger observer.

5.2. The Adaptive Distributed Observer Based Approach for a Known Leader System

Employing ADO (18) enables us to solve the CORP without requiring the control of every follower knows the the leader's system matrix S_0. Since this design will result in a closed-loop system whose origin is not an equilibrium point when v_0 is set to zero, we modify Problem 1 to the following one.

Problem 2: Given systems (3), (4) (or (2)), and the graph $\bar{\mathcal{G}}_{\sigma(t)}$, find a distributed control law of the form (5) such that the closed-loop system satisfies the following properties.

• *Property 3:* The solution of the closed-loop system exists and is uniformly bounded over $[0, \infty)$ for uniformly bounded $v_0(t)$.

• *Property 4:* The solution of the closed-loop system satisfies $\lim_{t\to\infty} e_i(t) = 0, \ i = 1, \ldots, N$.

Clearly, if the closed-loop system satisfies Property 1, it also satisfies Property 3.

Since the solution of the regulator equations (8) associated with every follower system relies on S_0, the solution of (8) cannot be obtained without knowing S_0. To overcome this difficulty, an iterative approach for obtaining the solution of (8) was reported in [31]. To summarize the approach in [31], let $S_i(t)$, $i = 1, \ldots, N$, be a sequence such that $\lim_{t\to\infty}(S_i(t) - S_0) = 0$ exponentially, and let $Q_i(t)$ be defined as follows:

$$Q_i(t) = S_i^{\mathsf{T}}(t) \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}. \tag{30}$$

Then, under Assumption 3, for any initial condition, the time-varying compensator

$$\dot{\zeta}_i = -\mu_\zeta Q_i^{\mathsf{T}}(t)(Q_i(t)\zeta_i - b_i), \ i = 1, \ldots, N \tag{31}$$

where $b_i = \text{vec}\left(\begin{bmatrix} E_i \\ F_i \end{bmatrix} \right)$ and $\mu_\zeta > 0$, generates a uniformly bounded solution $\zeta_i(t)$ such that

$$\lim_{t\to\infty}\left(\Xi_i(t)-\begin{bmatrix} X_i \\ U_i \end{bmatrix}\right)=0, \text{ exponentially}$$

where $\Xi_i(t)=M^q_{(n_i+m_i)}(\zeta_i(t))^4$ (For any column vector $X\in\mathbb{R}^{nq}$ for some positive integers n and q, $M^q_n(X)=[X_1,\ldots,X_q]$, where, for $i=1,\ldots,q$, $X_i\in\mathbb{R}^n$). Moreover, if the exponential convergent rate of $S_i(t)$ is at least α, then there exists $\mu^*_\zeta>0$ such that, for any $\mu_\zeta>\mu^*_\zeta$, the exponential convergent rate of $\Xi_i(t)$ is at least α.

Consequently, one can find the approximation of K_{2i} in (9) and (10) by $K_{2i}(t)=[-K_{1i}\;\; I_{m_i}]\Xi_i(t)$. To save the notation, let us adopt the simpler ADO where $y_{m0}=v_0$ and assume F_0 is known by every follower. In this case, with the ADO (21), two types of distributed control laws corresponding to (9) and (10) for solving the CORP (Problem 2) can be synthesized as follows.

3. Distributed full information control law:

$$u_i=K_{1i}x_i+K_{2i}(t)v_i,\; i=1,\ldots,N \tag{32a}$$

$$\dot{S}_i=\mu_S\sum_{j=0}^{N}a_{ij}(t)(S_j-S_i) \tag{32b}$$

$$\dot{\zeta}_i=-\mu_\zeta Q_i^{\mathsf{T}}(t)(Q_i(t)\zeta_i-b_i) \tag{32c}$$

$$\dot{v}_i=S_iv_i+\mu_v\left(\sum_{j=0}^{N}a_{ij}(t)(v_j-v_i)\right) \tag{32d}$$

4. Distributed measurement output feedback plus feedforward control law:

$$u_i=K_{1i}z_i+K_{2i}(t)v_i,\; i=1,\ldots,N \tag{33a}$$

$$\dot{z}_i=A_iz_i+B_iu_i+E_iv_i+\ell_i(y_{mi}-C_{mi}z_i-D_{mi}u_i-F_{mi}v_i) \tag{33b}$$

$$\dot{S}_i=\mu_S\sum_{j=0}^{N}a_{ij}(t)(S_j-S_i) \tag{33c}$$

$$\dot{\zeta}_i=-\mu_\zeta Q_i^{\mathsf{T}}(t)(Q_i(t)\zeta_i-b_i) \tag{33d}$$

$$\dot{v}_i=S_iv_i+\mu_v\left(\sum_{j=0}^{N}a_{ij}(t)(v_j-v_i)\right) \tag{33e}$$

where K_{1i},ℓ_i are the same as those in (9) and (10), μ_S,μ_v are given in (21), $Q_i(t),\mu_\zeta$ are defined in (30).

Problem 2 over static networks was first formally formulated in [31] for the leader system with $W_0=I_q$ and solved using the distributed control law (32). The detailed study over both switching and static networks can be found in [41, Section 8.3].

5.3. The Adaptive Distributed Observer Based Approach for an uncertain Leader System

We now consider the uncertain leader system described in (22), which entails the ADO (24).

The regulator equations (8) in this case can be re-written as

$$\begin{aligned} X_{iw}S_0(w_0)&=A_iX_{iw}+B_iU_{iw}+E_i \\ 0&=C_iX_{iw}+D_iU_{iw}+F_i,\; i=1,\ldots,N \end{aligned} \tag{34}$$

where the solution may depend also on the uncertain parametric vector w_0, which is denoted by (X_{iw},U_{iw}). Like the previous subsection, let the sequence $S_0(w_i(t))$, $i=1,\ldots,N$, be generated by the ADO (24), and let $Q_0(w_i(t))$ be defined as follows:

$$Q_0(w_i)=S_0^{\mathsf{T}}(w_i)\otimes\begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix}-I_q\otimes\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}. \tag{35}$$

Whenever $\lim_{t\to\infty}(w_i(t)-w_0)=0$ exponentially, under Assumption 3, for any initial condition, the time-varying compensator

$$\dot{\zeta}_i=-\mu_\zeta Q_0^{\mathsf{T}}(w_i)(Q_0(w_i)\zeta_i-b_i),\; i=1,\ldots,N \tag{36}$$

where b_i and μ_ζ are the same as those in (31), generates a uniformly bounded solution $\zeta_i(t)$ such that

$$\lim_{t\to\infty}\left(\Xi_i(t)-\left[\begin{array}{c}X_{jw}\\U_{iw}\end{array}\right]\right)=0,\ \text{exponentially}$$

where $\Xi_i(t)=M^q_{(n_i+m_i)}(\zeta_i(t))$. Consequently, K_{2i} in (9) and (10) can be approximated by $K_{2i}(t)=[-K_{1i}\ \ I_{m_i}]\Xi_i(t)$. Therefore, as reported in [34], with the ADO (24), two types of the distributed control laws corresponding to (9) and (10) for solving the CORP (Problem 2) can be synthesized as follows.

5. Distributed full information control law:

$$u_i=K_{1i}x_i+K_{2i}(t)v_i,\ i=1,\dots,N \tag{37a}$$

$$\dot{v}_i=S_0(\omega_i)v_i+\mu_v\sum_{j=0}^N a_{ij}(v_j-v_i) \tag{37b}$$

$$\dot{\omega}_i=\mu_\omega\phi\left(\sum_{j=0}^N a_{ij}(v_j-v_i)\right)v_i \tag{37c}$$

$$\dot{\zeta}_i=-\mu_\zeta Q_0^{\mathrm{T}}(w_i)(Q_0(w_i)\zeta_i-b_i) \tag{37f}$$

6. Distributed measurement output feedback plus feedforward control law:

$$u_i=K_{1i}z_i+K_{2i}(t)v_i,\ i=1,\dots,N \tag{38a}$$

$$\dot{z}_i=A_iz_i+B_iu_i+E_iv_i+\ell_i(y_{mi}-C_{mi}z_i-D_{mi}u_i-F_{mi}v_i) \tag{38b}$$

$$\dot{v}_i=S_0(\omega_i)v_i+\mu_v\sum_{j=0}^N a_{ij}(v_j-v_i) \tag{38c}$$

$$\dot{\omega}_i=\mu_\omega\phi\left(\sum_{j=0}^N a_{ij}(v_j-v_i)\right)v_i \tag{38d}$$

$$\dot{\zeta}_i=-\mu_\zeta Q_0^{\mathrm{T}}(\omega_i)(Q_0(\omega_i)\zeta_i-b_i) \tag{38e}$$

where K_{1i},ℓ_i are the same as those in (9) and (10), μ_v,μ_w are given in (24), $Q_i(t),\mu_\zeta$ are defined in (36).

6. Other Variants and Extensions

This section briefly overviews some other variants and extensions of the CORP by the DO approach.

6.1. Other Types of Systems

The CORP of discrete-time linear MASs can be found in [60–62] based on various discrete-time distributed observers [62–65].

For the CORP of some other types of MASs, please refer to [66–70] for time-delay MASs, [71–73] for singular MASs, and [74–75] for switched MAS, and [76–77] for multiple parabolic PDE systems.

6.2. Cooperative Containment Control Problem

The cooperative containment control problem arises when there are multiple leaders. This problem can also be dealt with by the DO approach, where the states of the DO are steered to some weighted average of the leaders' states. Some representative publications can be found in, for example, [78–82].

6.3. Cooperative Synchronization Problem

While the CORP can be viewed as the extension of the leader-following consensus problem, the cooperative synchronization problem can be viewed as an extension of the leaderless consensus problem. This line of research can be found in a number of papers, for example, in [83–85].

6.4. Performance of the Distributed Observer

Extensive investigations on the performance of the DO were conducted including the input saturation design [86–87], the quantized feedback design [88], the fault tolerant control design [89], the indirect adaptive approach [90], and the H_∞ performance design [91].

6.5. Other Applications of the Distributed Observer

In addition to the CORP for linear MASs, the DO is also applicable to some other problems including some nonlinear systems. For example, the leader-following consensus problem of multiple Euler-Lagrange systems over jointly connected switching communication networks by the state-based DO was first studied in [92]. The same problem was further considered in [30] by a state-based ADO for the case where the leader system is neutrally stable. The neutral stability assumption in [30] was relaxed in [53] to allow the matrix S_0 to have eigenvalues with non-positive real parts. The uncertain leader system was considered in [33, 93]. Investigation on time delay issues for information exchange over the communication network can be found in [94] and [95]. References [53, 96] and [97] studied the rejection of external disturbances for Euler-Lagrange systems by the DO approach. The DO approach has also been applied to solve the leader-following attitude consensus problem for rigid body systems. For static communication networks, state feedback, output feedback, and adaptive control schemes were proposed in [98–99], and [100], respectively. These results were further extended in [101–104] to switching communication networks. Formation control of multiple rigid body systems was investigated in [105]. Rendezvous and flocking problems were reported in [106–107].

6.6. Other Issues

To implement the DO in digital platforms, various sampled-data feedback control and event-triggered feedback control approaches have been studied. The sampled-data distributed observer based approach was investigated in [108, 109], and various event-triggered DO-based approaches and DO-based integrated control approaches can be found in [110–121].

When the system model is unknown but the feedback data can be freely used, the reinforcement learning has been shown to be a useful tool for obtaining proper approximations for the solution of the regulator equations as well as the gain matrices in the DO and the distributed controller [122–129].

Security is one of the central issues for cyber-physical systems. Various DO-based distributed resilient control (robust against DOS attacks or Byzantine attacks) can be found in [130–137]. Private protect DO based controllers can be found in [138].

7. Conclusions

This paper has given an overview on the CORP of MASs by the DO approach. Since the DO approach needs to make use of the solution of the regulator equations to synthesize the control law, it cannot handle the model uncertainties by itself. On the other hand, like the classical IM approach, the DIM approach is also able to deal with the model uncertainties.

The distributed p-copy IM design was first studied in [139] over static and acyclic communication networks. This static and acyclic assumption was removed in [140] when agents have the same nominal dynamics. Some other attempts on this topic can be found in [141–144]. The distributed canonical IM design for SISO linear minimum phase systems with identical relative degree was treated in [143], and was extended to other uncertain linear MASs [145–146]. An advantage of the DIM approach is that it is able to deal with the global CORP of various uncertain nonlinear systems such as the output feedback systems with unity relative degree in [147], with higher relative degree in [148] and the strict feedback systems in [149]. The case where the exosystem is linear and contains uncertain parameters was studied for multiple nonlinear systems in output feedback form with unity relative degree in [150], and high relative degree in [151]. Other studies involving DIM control can be found in, for instance, [152–154].

Nevertheless, the DIM approach can only handle every time connected graphs. By integrating the DO approach and the DIM approach, one obtains a so-called integrated control approach which is capable of handling the nonlinearity and uncertainty of various systems over jointly connected switching networks.

Studies on integrating the DO approach with the p-copy IM can be found in [49, 155–159], which can deal with linear MASs with sufficiently small uncertain parametric vectors. Studies on integrating the DO approach with the canonical IM can be found in [160–162], which can deal with various linear/nonlinear MASs with arbitrary large uncertain parametric vectors.

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