

Receding Horizon Team Optimal Decentralized Filter for Discrete Multi-agent Systems with Delay

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Abstract: We investigate the receding horizon team-optimal decentralized filter for discrete-time multi-agent systems subject to delayed information sharing. By minimizing the team estimation error covariance over a finite horizon under unbiasedness constraints, both batch and recursive implementations of the RHTODF are derived. It is shown that the resulting estimator admits a decomposed structure consisting of a common-information component and local innovation-based correction terms, which enables distributed implementation under delayed communication constraints. Numerical simulations demonstrate that the proposed RHTODF achieves improved estimation accuracy and smaller estimation error than the local Kalman filter, thereby illustrating the effectiveness of the proposed finite-horizon team-optimal filtering framework.

Keywords: receding horizon team optimal decentralized filter; batch form; recursive form; matrix extremum principle

1. Introduction

Over the past few decades, state estimation has become a central topic in control theory and signal processing [1–8]. Research on dynamic state estimation can be traced back to the pioneering work of Kalman in the 1960s [9]. Since then, the Kalman filter has been extensively studied for both linear and nonlinear systems [10]. However, the standard Kalman filter may suffer from divergence in the presence of uncertainties in noise statistics [11]. Against this background, the receding horizon filter (RHF), also known as the moving horizon filter, has been proposed [12].

Since its introduction in the 1960s [12], the receding horizon filter has attracted increasing attention and has become a successful technique in control engineering, signal processing, and fault detection [13–16]. In particular, Ref. [12] derived an estimator based on the maximum likelihood criterion. A recursive solution for general systems was developed in [17] using scattering theory. The RHF based on the minimum variance criterion for time-varying systems was proposed in [18]. In [19], an unbiased RHF was derived via a reorganization technique. Furthermore, Ref. [20] introduced a new optimization criterion and established the convergence and unbiasedness of the estimator. Despite these advantages, classical RHF schemes are typically designed under centralized information structures, which limits their applicability to large-scale networked systems where global information is unavailable or communication constraints exist.

On the other hand, with the emergence of large-scale systems and the rapid development of network technologies, decentralized estimation has received significant attention in recent decades. In [21,22], decentralized estimation for large-scale systems was investigated by decomposing the overall system into interconnected subsystems under various interconnection structures. Under networked environments, agents can exchange information over a given communication graph, enabling each agent to estimate the system state based on its local observations and information from neighboring agents [23,24]. For instance, several consensus-based estimation algorithms were proposed in [25,26], while diffusion-based strategies were developed in [27]. However, consensus and diffusion-based methods often rely on asymptotic information fusion and may suffer from suboptimality under finite-horizon performance criteria or time-varying communication constraints. Additional results on networked decentralized estimation can be found in [28–33].

More recently, team-optimal decentralized estimation has attracted increasing attention [34–36]. In [34,35], a team-optimal estimation framework was proposed, where all agents cooperate to minimize a common team cost function. Due to communication delays among agents, each agent has access to different information when performing estimation. Although this framework provides optimality in a team sense, the resulting estimators are typically derived under static or full-horizon optimization settings, which may not be suitable for real-time implementation in large-scale dynamic networks.



Although the receding horizon filter and decentralized estimation have both been extensively studied, there are still fundamental gaps between them. Existing RHF methods are mainly centralized and computationally demanding in large-scale networks, while existing decentralized estimation schemes are often based on asymptotic consensus or static optimization and lack a receding-horizon structure that explicitly accounts for finite-time optimality and online adaptation. Moreover, only a limited number of works have attempted to integrate the receding horizon principle into decentralized estimation frameworks [37–39] and even fewer consider team-optimality in such settings.

Motivated by these observations, this paper investigates a receding horizon formulation for team-based decentralized estimation, referred to as the receding horizon team-optimal decentralized filter (RHTODF). Compared with existing receding horizon filtering methods, the proposed framework enables a distributed implementation under communication constraints while preserving the finite-horizon optimization nature. In contrast to existing decentralized estimation schemes, the RHTODF explicitly embeds a receding horizon optimization mechanism into a team decision-making structure, thereby improving adaptability and tracking performance in time-varying environments. The main contributions of this paper are summarized as follows:

- (i) A receding-horizon team-optimal estimation framework is established for multi-agent systems with delayed information sharing. Unlike existing team-optimal estimators based on static or full-horizon optimization, the proposed framework incorporates finite-horizon optimization and online updating mechanisms, thereby improving adaptability in dynamic environments.
- (ii) By minimizing the finite-horizon team estimation error covariance subject to the unbiasedness constraint, both batch and recursive forms of the RHTODF are derived. The recursive implementation significantly reduces computational complexity and is suitable for real-time applications.
- (iii) A structural decomposition of the optimal estimator is obtained, showing that each local estimate consists of a common-information estimate and a correction term driven by local innovations. This decomposition reveals the information structure underlying the team-optimal solution and enables distributed implementation while maintaining team optimality.

The organizational structure of this article is as follows. In Section 2, we present the problem formulation and related preparations of the study. The batch form of RHTODF is presented in Section 3. Section 4 further gives the recursive form of RHTODF. In Section 5, numerical examples are implemented. Finally, the conclusion of this paper is reached in Section 6.

2. Problem Statement

Consider the following linear discrete-time state-space model:

$$x(k+1) = Ax(k) + G\omega(k), \quad (1)$$

$$y_i(k) = C_i x(k) + v_i(k), \quad i = 1, \dots, n, \quad (2)$$

where $x(k) \in \mathbb{R}^{n_x}$ denotes the system state, and $y_i(k) \in \mathbb{R}^q$ represents the measurement of the i -th agent at time k . The matrices A , G , and C_i are of appropriate dimensions.

The process noise $\{\omega(k) \in \mathbb{R}^p\}$ and the measurement noise $\{v_i(k) \in \mathbb{R}^q\}$ are assumed to be mutually independent Gaussian white noises with zero mean and covariance matrices Q and R_i , respectively. The initial state $x(k_0)$ is a random vector with mean $\bar{x}(k_0)$ and covariance P_{k_0} . Moreover, the initial state, process noise, and measurement noise are mutually independent.

For convenience, the measurement equation in (2) can be written in compact vector form as

$$y(k) = Cx(k) + v(k), \quad (3)$$

where

$$C = [C_1^T \quad C_2^T \quad \dots \quad C_n^T]^T.$$

and

$$y(k) = \text{col}(y_1(k), \dots, y_n(k)), \quad v(k) = \text{col}(v_1(k), \dots, v_n(k)).$$

The agents considered in this paper are interconnected through a communication graph $\mathcal{G}$, which is assumed to be a strongly connected weighted directed graph. Due to the uncertainty of the network environment, communication delays are inevitable during information exchange among agents. In this paper, we assume that the communication delay between any two agents is a constant d -step delay.

For convenience of subsequent analysis, the system in (1) and (2) is rewritten in batch form over the latest finite time interval $[k - N, k]$, where N denotes the horizon length. The interval $[k - N, k]$ is referred to as a finite-time window, and the initial time instant $k - N$ is denoted for simplicity as k_N in the sequel.

Due to communication delays among agents, the information available to the i -th agent over the interval $[k_N, k]$ admits a decentralized structure of the form

$$Y_i(k - 1) = Y^{com}(k - d) \cup Y_i^{loc}(k - 1),$$

where $Y^{com}(k - d)$ denotes the commonly shared information available to all agents, and $Y_i^{loc}(k - 1)$ represents the local information accessible only to the i -th agent. To ensure that the common information $Y^{com}(k - d)$ lies within the finite-time window $[k_N, k]$, we assume $d \leq N$. Specifically, these information sets are given by

$$\begin{aligned} Y^{com}(k - d) &= [y^T(k_N) \quad y^T(k_N + 1) \quad \cdots \quad y^T(k - d)]^T, \\ Y_i^{loc}(k - 1) &= [y_i^T(k - d + 1) \quad \cdots \quad y_i^T(k - 1)]^T. \end{aligned}$$

On the finite time interval $[k_N, k]$, the estimate of the weighted state $L_i x(k)$ for the i -th agent, denoted by $\hat{z}_i(k)$, is designed as a linear function of the common information $Y^{com}(k - d)$ and the innovation of the local information $\tilde{Y}_i^{loc}(k - 1)$, i.e.,

$$\hat{z}_i(k) = L_i H Y^{com}(k - d) + M_i \tilde{Y}_i^{loc}(k - 1),$$

where

$$\tilde{Y}_i^{loc}(k - 1) = Y_i^{loc}(k - 1) - \mathbb{E}\{Y_i^{loc}(k - 1) \mid Y^{com}(k - d)\}.$$

Here, $L_i \in \mathbb{R}^{n_x \times n_x}$ is a prescribed weighting matrix associated with the i -th agent, while H and M_i are gain matrices to be determined.

The objective of this paper is to minimize a finite-horizon weighted estimation error covariance at each time instant k over the receding window $[k_N, k]$, which is formulated as

$$J_{\min} = \mathbb{E}\left\{(Lx(k) - \hat{z}(k))^T S (Lx(k) - \hat{z}(k))\right\}, \quad (4)$$

where $\hat{z}(k) = \text{col}(\hat{z}_1(k), \hat{z}_2(k), \dots, \hat{z}_n(k))$ denotes the stacked estimate of all agents. The matrix L is defined as $L = \text{col}(L_1, L_2, \dots, L_n)$, and $S = [s_{ij}] \in \mathbb{R}^{nn_x \times nn_x}$ is a given positive definite weighting matrix.

Remark 1. The objective in (4) adopts a team-optimal criterion, where the weighting matrix S introduces coupling among different agents. This is in contrast to most distributed estimation schemes, which typically consider local or decoupled performance indices. Different from centralized estimation, the proposed formulation preserves the distributed information structure while optimizing a globally coupled performance index. Hence, the estimator can be viewed as a distributed realization of a team-optimal estimation problem.

Remark 2. Although the output equation can be rewritten in a compact vectorized state-space form, it is important to emphasize that the resulting model is not equivalent to a standard centralized state-space estimation problem. In particular, the vectorization only provides a unified mathematical representation, whereas the underlying information structure remains fundamentally distributed. Each agent has access only to local measurements and delayed shared information, rather than the full stacked observation vector. Therefore, the estimation process is performed under a team decision-making framework with information asymmetry and communication delays.

Moreover, the coupling introduced by the receding horizon optimization further distinguishes the proposed problem from standard Kalman filtering or classical moving-horizon estimation. The optimization is carried out over a finite horizon with unbiasedness constraints and distributed information sets, which leads to a nontrivial decomposition of the optimal estimator into common-information and local innovation components. Hence, the main difficulty of the problem does not lie in the algebraic form of the model, but in the interaction between finite-horizon optimization, delayed information sharing, and team-optimal distributed decision-making.

Finally, we impose the following assumptions:

Assumption 1. *The weighting matrix S is positive definite, and the system matrix A is nonsingular.*

3. Batch Form of the RHTODF

In this section, the batch-form expression of the proposed RHTODF is developed. To exploit the receding horizon structure over the finite interval $[k_N, k]$, the common and local measurement sequences are first represented in terms of the current state $x(k)$. Such a representation converts the estimation problem into a finite-dimensional optimization problem and facilitates the derivation of the team-optimal filter.

According to (1) and (2), the common information vector $Y^{com}(k-d)$ and the local information vector $Y_i^{loc}(k-1)$ over the horizon $[k_N, k]$ can be expressed in terms of the current state $x(k)$ as

$$Y^{com}(k-d) = \bar{C}_N x(k) + \bar{G}_N W(k-1) + V(k-d), \quad (5)$$

$$Y_i^{loc}(k-1) = \bar{C}_i x(k) - \bar{G}_i W^{loc}(k-1) + V_i(k-1), \quad (6)$$

where the stacked process noise vector and measurement noise vectors are defined by

$$\begin{aligned} W(k-1) &\triangleq [\omega^T(k_N) \quad \cdots \quad \omega^T(k-1)]^T, \\ W^{loc}(k-1) &\triangleq [\omega^T(k-d+1) \quad \cdots \quad \omega^T(k-1)]^T, \\ V(k-d) &\triangleq [v^T(k_N) \quad \cdots \quad v^T(k-d)]^T, \\ V_i(k-1) &\triangleq [v_i^T(k-d+1) \quad \cdots \quad v_i^T(k-1)]^T, \end{aligned}$$

and

$$\bar{C}_N \triangleq \begin{bmatrix} CA^{-N} \\ CA^{-N+1} \\ \vdots \\ CA^{-d} \\ C \end{bmatrix},$$

$$\bar{G}_N = - \begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-(N-1)}G & CA^{-N}G \\ 0 & CA^{-1}G & \cdots & CA^{-(N-2)}G & CA^{-(N-1)}G \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & CA^{-1} & CA^{-2}G \\ 0 & \cdots & 0 & 0 & CA^{-1} \end{bmatrix}.$$

The matrices $\bar{C}_i$ and $\bar{G}_i$, $i = 1, \dots, n$, are given by

$$\bar{C}_i \triangleq \begin{bmatrix} C_i A^{-(d-1)} \\ C_i A^{-(d-2)} \\ \vdots \\ C_i A^{-1} \end{bmatrix},$$

$$\bar{G}_i \triangleq \begin{bmatrix} C_i A^{-1}G & C_i A^{-2}G & \cdots & C_i A^{-(d-1)}G \\ 0 & C_i A^{-1}G & \cdots & C_i A^{-(d-2)}G \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_i A^{-1}G \end{bmatrix}.$$

For convenience of computing the covariance matrix, we define

$$\bar{C}_{i,1} = C_i A^{-1}, \quad \bar{G}_{i,1} = -C_i A^{-1}G,$$

$$\bar{C}_{i,j} = \begin{bmatrix} C_i A^{-j} \\ C_i A^{-j+1} \\ \vdots \\ C_i A^{-1} \end{bmatrix} = \begin{bmatrix} \bar{C}_{i,j-1} \\ C_i \end{bmatrix} A^{-1},$$

and

$$\begin{aligned} \bar{G}_{i,j} &= - \begin{bmatrix} C_i A^{-1} G & C_i A^{-2} G & \cdots & C_i A^{-j} G \\ 0 & C_i A^{-1} G & \cdots & C_i A^{-j+1} G \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_i A^{-1} G \end{bmatrix} \\ &= \begin{bmatrix} \bar{G}_{i,j-1} & -\bar{C}_{i,j-1} A^{-1} G \\ 0 & -C_i A^{-1} G \end{bmatrix}, \quad 2 \leq j \leq N. \end{aligned}$$

Since the process noise and measurement noise are mutually independent zero-mean Gaussian white noises, the composite noise term

$$-\bar{G}_{i,j} W(k-1) + V_i(k-1)$$

associated with the i -th agent is also zero-mean. Its covariance matrix, denoted by Ξ_{ij} , can be recursively computed as

$$\begin{aligned} \Xi_{ij} &= \bar{G}_{ij} \left[\text{diag}(\underbrace{Q, \dots, Q}_j) \right] \bar{G}_{ij}^T + \left[\text{diag}(\underbrace{R_i, \dots, R_i}_j) \right] \\ &= \begin{bmatrix} \Xi_{i,j-1} & 0 \\ 0 & R_i \end{bmatrix} + \begin{bmatrix} \bar{C}_{i,j-1} \\ C_i \end{bmatrix} A^{-1} G Q G^T A^{-T} \begin{bmatrix} \bar{C}_{i,j-1} \\ C_i \end{bmatrix}^T, \end{aligned} \quad (7)$$

with the initial value $\Xi_{i1} = R_i + C_i A^{-1} G Q G^T A^{-T} C_i^T$.

For notational convenience, define the covariance matrix associated with the common information as

$$\Xi_N \triangleq \mathbb{E}[(\bar{G}_N W(k-1) + V(k-d))(\bar{G}_N W(k-1) + V(k-d))^T].$$

Before deriving the team-optimal receding horizon filter, the following property regarding the information structure will be needed.

Lemma 1. *Define*

$$\tilde{Y}_i^{\text{loc}}(k-1) = Y_i^{\text{loc}}(k-1) - \mathbb{E}[Y_i^{\text{loc}}(k-1) | Y^{\text{com}}(k-d)].$$

Then

$$\mathbb{E}[\tilde{Y}_i^{\text{loc}}(k-1)(Y^{\text{com}}(k-d))^T] = 0, \quad i = 1, \dots, n.$$

Proof. Since $\tilde{Y}_i^{\text{loc}}(k-1)$ is defined as the innovation, it is the orthogonal projection residual. Hence, by the orthogonality principle of conditional expectation,

$$\mathbb{E}[\tilde{Y}_i^{\text{loc}}(k-1)(Y^{\text{com}}(k-d))^T] = 0.$$

□

Now, we are in the position to give the main theorem.

Theorem 1. *Suppose that the pair $\{A, C\}$ is observable. Over the finite horizon $[k_N, k]$, the batch-form RHTODF for the i -th agent is given by*

$$\hat{z}_i(k) = L_i H Y^{\text{com}}(k-d) + M_i \tilde{Y}_i^{\text{loc}}(k-1). \quad (8)$$

The optimal gain matrices are given by

$$H = (\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1} \bar{C}_N^T \Xi_N^{-1}, \quad (9)$$

$$M = \Gamma^{-1} \mu, \quad (10)$$

where

$$\begin{aligned} M &= \text{vec}[M_1, \dots, M_n], \\ \mu &= \text{vec}[S_1 L H \Xi_{1N}^T, \dots, S_n L H \Xi_{nN}^T], \\ \Gamma &= [\Xi_{ij} \otimes S_{ij}]_{i,j=1}^n. \end{aligned}$$

Proof. According to the unbiased requirement of RHTODF, the condition $E[Lx(k)] = E[\hat{z}(k)]$ should be satisfied. Expand both sides of the above equation, thus we get the following from Lemma 1:

$$\begin{aligned} \mathbb{E}[Lx(k)] &= \mathbb{E} \left[\begin{pmatrix} L_1 \\ \vdots \\ L_n \end{pmatrix} x(k) \right] = \mathbb{E} \begin{bmatrix} \hat{z}_1(k) \\ \vdots \\ \hat{z}_n(k) \end{bmatrix} \\ &= \mathbb{E} \begin{bmatrix} L_1 H Y^{com}(k-d) + M_1 \tilde{Y}_1^{loc}(k-1) \\ \vdots \\ L_n H Y^{com}(k-d) + M_n \tilde{Y}_n^{loc}(k-1) \end{bmatrix} \\ &= \mathbb{E} \begin{bmatrix} L_1 H \bar{C}_N x(k) \\ \vdots \\ L_n H \bar{C}_N x(k) \end{bmatrix}. \end{aligned} \quad (11)$$

Then, we obtain from (11) that

$$L H \bar{C}_N = L. \quad (12)$$

Next, the objective is to obtain the optimal gain matrices H and M_i , subject to the unbiased constraint (12). Then, (4) becomes as follows

$$\begin{aligned} &\mathbb{E}[(Lx(k) - \hat{z}(k))^T S (Lx(k) - \hat{z}(k))] \\ &= \mathbb{E} \left\{ \begin{bmatrix} L_1 x(k) - \hat{z}_1(k) \\ \vdots \\ L_n x(k) - \hat{z}_n(k) \end{bmatrix}^T \begin{bmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nn} \end{bmatrix} \right. \\ &\quad \left. \times \begin{bmatrix} L_1 x(k) - \hat{z}_1(k) \\ \vdots \\ L_n x(k) - \hat{z}_n(k) \end{bmatrix} \right\} \\ &= \mathbb{E} \left\{ \text{Tr} \sum_{i=1}^N \sum_{j=1}^N S_{ij} [L_j H (\bar{G}_N W(k-1) + V(k-d)) \right. \\ &\quad - M_j (\bar{G}_j W^{loc}(k-1) - V_j(k-1))] [L_i H (\bar{G}_N W(k-1) \\ &\quad + V(k-d) - M_j (\bar{G}_j W^{loc}(k-1) - V_j(k-1))]^T \} \\ &= \sum_{i=1}^N \sum_{j=1}^N \{ \text{Tr} [S_{ij} L_j H \Xi_N H^T L_i^T] - \text{Tr} [S_{ij} M_j \Xi_{jN} H^T L_i^T] \\ &\quad - \text{Tr} [S_{ij} L_j H \Xi_{iN}^T M_i^T] + \text{Tr} [S_{ij} M_j \Xi_{ij}^T M_i^T] \}. \end{aligned}$$

Now, we construct the Hamiltonian function as below:

$$\begin{aligned}\mathcal{H} = & \sum_{i=1}^N \sum_{j=1}^N \{Tr(S_{ij}L_jH\Xi_NH^TL_i^T) - Tr(S_{ij}M_j\Xi_{jN}H^TL_i^T) \\ & - Tr(S_{ij}L_jH\Xi_{iN}^TM_i^T) + Tr(S_{ij}M_j\Xi_{ij}^TM_i^T)\} \\ & + Tr[(I - H\bar{C}_N)P^T(k+1)],\end{aligned}$$

then according to the matrix extremum principle, the necessary conditions are available

$$\frac{\partial \mathcal{H}}{\partial H} = 0, \quad \frac{\partial \mathcal{H}}{\partial M_i} = 0, \quad (13)$$

and the costate matrix $P(k+1)$ must meet the following equation

$$\frac{\partial \mathcal{H}}{\partial P(k+1)} = I - H\bar{C}_N = 0. \quad (14)$$

The above Equation (13) is expanded in detail as follows

$$\frac{\partial \mathcal{H}}{\partial H} = \sum_{j=1}^N S_{ij}[L_i^TL_jH\Xi_N - L_i^TM_j\Xi_{jN}] - P(k+1)\bar{C}_N^T = 0, \quad (15)$$

$$\frac{\partial \mathcal{H}}{\partial M_i} = \sum_{j=1}^N S_{ij}[M_j\Xi_{ij}^T - L_jH\Xi_{iN}^T] = 0. \quad (16)$$

Next, we will find the optimal gain matrix on the basis of (14)–(16). Above all, by vectorizing both sides of (16) and using Kronecker product $vec(ABC) = (C^T \otimes A) \times vec(B)$, we are able to gain

$$\sum_{j \in U} (\Xi_{ij} \otimes S_{ij})vec(M_j) - vec(S_iLH\Xi_{iN}^T) = 0.$$

We substitute $\Gamma_{ij} = \Xi_{ij} \otimes S_{ij}$ and $\mu_i = vec(S_iLH\Xi_{iN}^T)$ into above formula. Since $S > 0$, $\Xi_{ij} \geq 0$ and $\Xi_{iN} > 0$, $i, j = 1, \dots, n$, we know that $\Gamma > 0$ and invertible.

Hence, (16) has one and only solution, in other words the optimal gain matrix M_i has a unique solution that be able to simply denote

$$M = \Gamma^{-1}\mu, \quad (17)$$

where

$$\begin{aligned}M = & vec(M_1, \dots, M_n), \mu = vec(S_1LH\Xi_{1N}^T, \dots, S_nLH\Xi_{nN}^T), \\ S_i = & [S_{i1}, \dots, S_{in}], \Gamma = [\Gamma_{ij}]_{i,j=1,\dots,n}, \Gamma_{ij} = \Xi_{ij} \otimes S_{ij}.\end{aligned}$$

Substitute $M_j = \Gamma_{ij}^{-1}\mu_j$ into (15) to solve $P(k+1)$. And combine $P(k+1)$ with (15) and (17), we get

$$H = (\bar{C}_N^T\Xi_N^{-1}\bar{C}_N)^{-1}\bar{C}_N^T\Xi_N^{-1}, \quad j = 1, \dots, n, \quad (18)$$

□

Remark 3. The proposed batch-form RHTODF provides a unified closed-form solution for distributed state fusion over a finite horizon. Different from conventional distributed Kalman filtering or consensus-based estimators, the proposed framework explicitly separates the estimation structure into a common-information-driven component and a local-innovation-driven component.

In particular, the gain matrix H admits a projection-type closed-form expression determined by the global observation matrix $\bar{C}_N$ and the covariance matrix Ξ_N associated with the common information, while the local gain M_i is obtained from a structured linear matrix equation governed by the Kronecker product $\Gamma_{ij} = \Xi_{ij} \otimes S_{ij}$. This structural decomposition allows the global fusion gain to be computed first, followed by the local correction

gains through a structured linear matrix equation, thereby facilitating the implementation of the proposed estimator.

Moreover, the resulting estimator preserves the unbiasedness over the finite horizon while fully exploiting cross-agent coupling information encoded in the weighting matrix S and the covariance structure Ξ_{ij} .

4. Recursive form of the RHTODF

In this section, we reformulate the batch-form RHTODF in (8) into an equivalent recursive representation to reduce computational complexity associated with high-dimensional matrix inversion.

To this end, we exploit the inherent recursive structure of the finite-horizon information matrix underlying the batch estimator.

Theorem 2. Suppose that the pair $\{A, C\}$ is observable. Over the finite horizon $[k_N, k]$, the batch-form RHTODF for the i -th agent can be equivalently expressed in the following recursive form:

$$\hat{z}_i(k|k) = \bar{\Omega}_N^{-1} \hat{\eta}_i(k|k), \quad (19)$$

where $\bar{\Omega}_N = \bar{C}_N^T \bar{\Xi}_N^{-1} \bar{C}_N L_i^{-1}$.

For $0 \leq j \leq N-1$, the auxiliary estimate $\hat{\eta}_i(k_N + j + 1|k)$ is given by the recursive equation

$$\begin{aligned} & \hat{\eta}_i(k_N + j + 1|k) \\ &= [I + A^{-T}(\Omega_j + C^T R^{-1} C) A^{-1} G Q G^T]^{-1} A^{-T} \\ & \quad \times [\hat{\eta}_i(k_N + j|k) - \bar{\Omega}_j M_i \tilde{Y}_i^{\text{loc}}(k_N + j - 1) \\ & \quad + C^T R^{-1} y(k_N + j + 1 - d) \\ & \quad + (\Omega_j + C^T R^{-1} C) A^{-1} L_i^{-1} M_i \tilde{Y}_i^{\text{loc}}(k_N + j)]. \end{aligned}$$

Proof. To convert the batch form (8) into an recursive form, we first define

$$\Omega_{j+1} = \bar{C}_{j+1}^T \bar{\Xi}_{j+1}^{-1} \bar{C}_{j+1}, \quad \bar{\Omega}_{j+1} = \Omega_{j+1} L_i^{-1}, \quad (20)$$

and then Ω_{j+1} satisfies the following discrete Riccati equation:

$$\begin{aligned} \Omega_{j+1} &= [I + A^{-T}(\Omega_j + C^T R^{-1} C) A^{-1} G Q G^T]^{-1} \\ & \quad \times A^{-T}(\Omega_j + C^T R^{-1} C) A^{-1}, \end{aligned} \quad (21)$$

with initial conditions $\Omega_0 = 0$ and $\bar{\Omega}_0 = 0$.

Based on Theorem 1, $\hat{z}_i(k)$ can be written as

$$\bar{\Omega}_N \hat{z}_i(k) = \bar{C}_N^T \bar{\Xi}_N^{-1} Y^{\text{com}}(k - d) + \bar{\Omega}_N M_i \tilde{Y}_i^{\text{loc}}(k - 1). \quad (22)$$

We introduce the auxiliary estimate $\hat{\eta}_i(k_N + j | k)$ defined by

$$\begin{aligned} & \hat{\eta}_i(k_N + j | k) \\ & \triangleq \bar{\Omega}_j \hat{z}_i(k_N + j | k) \\ &= \bar{C}_j^T \bar{\Xi}_j^{-1} Y^{\text{com}}(k_N + j - d) + \bar{\Omega}_j M_i \tilde{Y}_i^{\text{loc}}(k_N + j - 1), \end{aligned} \quad (23)$$

for $j = 1, \dots, N$, where $\hat{\eta}_i(k_N | k) = \bar{\Omega}_0 \hat{z}_i(k_N | k) = 0$.

For initialization, the local innovation terms at $k_N - 1$ and k_N are taken as zero, i.e., $\tilde{Y}_i^{\text{loc}}(k_N - 1) = \tilde{Y}_i^{\text{loc}}(k_N) = 0$. Next, we derive the recursion for $\hat{\eta}_i(k_N + j + 1 | k)$. For $0 \leq j \leq N - 1$, using the definition (23), we have

$$\begin{aligned} & \hat{\eta}_i(k_N + j + 1 | k) \\ &= \bar{C}_{j+1}^T \bar{\Xi}_{j+1}^{-1} [Y^{\text{com}}(k_N + j - d) + y(k_N + j + 1 - d)] + \bar{\Omega}_{j+1} M_i \tilde{Y}_i^{\text{loc}}(k_N + j). \end{aligned} \quad (24)$$

Now we simplify the matrix $\bar{C}_{j+1}^T \bar{\Xi}_{j+1}^{-1}$. Using the recursive structure of Ξ_{j+1} , we obtain

$$\begin{aligned}
& \bar{C}_{j+1}^T \Xi_{j+1}^{-1} \\
&= A^{-T} \begin{bmatrix} \bar{C}_j \\ C \end{bmatrix}^T \left(\Delta_j + \begin{bmatrix} \bar{C}_j \\ C \end{bmatrix} A^{-1} G Q G^T A^{-T} \begin{bmatrix} \bar{C}_j \\ C \end{bmatrix}^T \right)^{-1} \\
&= [I + A^{-T} (\Omega_j + C^T R^{-1} C) A^{-1} G Q G^T]^{-1} A^{-T} \begin{bmatrix} \bar{C}_j \\ C \end{bmatrix}^T \Delta_j^{-1},
\end{aligned} \tag{25}$$

where $\Delta_j = \begin{bmatrix} \Xi_j & 0 \\ 0 & R \end{bmatrix}$.

Substituting (25) into (24) and employing (20), (21) together with the definition of the auxiliary estimate, after algebraic manipulation we obtain the recursion

$$\begin{aligned}
& \hat{\eta}_i(k_N + j + 1 | k) \\
&= [I + A^{-T} (\Omega_j + C^T R^{-1} C) A^{-1} G Q G^T]^{-1} A^{-T} \\
&\quad \times \left[\hat{\eta}_i(k_N + j | k) - \bar{\Omega}_j M_i \tilde{Y}_i^{\text{loc}}(k_N + j - 1) \right. \\
&\quad + C^T R^{-1} y(k_N + j + 1 - d) \\
&\quad \left. + (\Omega_j + C^T R^{-1} C) A^{-1} L_i^{-1} M_i \tilde{Y}_i^{\text{loc}}(k_N + j) \right].
\end{aligned} \tag{26}$$

Equation (26) provides the recursive relation for the auxiliary estimate. Finally, since $\hat{z}_i(k | k) = \bar{\Omega}_N^{-1} \hat{\eta}_i(k | k)$, the recursive form stated in Theorem 2 follows. This completes the proof. $\square$

Remark 4. Theorem 2 reveals that the proposed batch-form RHTODF admits an equivalent recursive implementation governed by a Riccati-type information matrix recursion. This transformation is nontrivial, since it bridges the high-dimensional batch optimization problem and a computationally efficient time-update structure. Different from conventional batch estimators that require repeated inversion of large-scale block matrices, the proposed recursive form propagates the information matrix Ω_j sequentially, avoiding direct inversion of the full batch matrix Ξ_N . Moreover, the estimator separates the evolution of the global information state from the local innovation-driven correction term, which enables a scalable implementation for large-scale distributed systems. This structural decomposition highlights the main advantage of the proposed RHTODF framework: it preserves the optimality of the batch solution while significantly reducing computational complexity through recursive information updating.

5. Simulation Results

In this section, numerical simulations are conducted to validate the effectiveness of the proposed RHTODF and to compare it with the conventional local Kalman filter (Local KF).

We consider a scenario with three agents ($n = 3$) that cooperatively estimate the state of a common two-dimensional linear dynamic system ($n_x = 2$). We consider the dynamic system and measurement model as described in (1) and (3), and adopt the following parameters for the simulation. The state transition matrix is chosen as $A = \begin{bmatrix} 0.85 & -0.2 \\ 0.2 & 0.85 \end{bmatrix}$, the input matrix is $G = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$, and the process noise $\omega(k)$ is assumed to be zero-mean Gaussian white noise with covariance $Q = 0.15^2$. The measurement matrices for the three agents are $C_1 = [1, 0]$, $C_2 = [0.6, 1]$, and $C_3 = [0.4, 0.9]$, respectively, and the measurement noises of the i -th agent is denoted by $v_i(k) \sim \mathcal{N}(0, R_i)$ with $R_1 = 0.1^2$, $R_2 = 0.12^2$, and $R_3 = 0.1^2$. The initial state is set to $x_0 = [1, -1]^T$, and the total simulation horizon is $T = 200$ time steps. All noise sequences are assumed to be mutually independent, zero-mean Gaussian white noise sequences, with their respective covariance matrices specified above.

For the RHTODF algorithm, the key parameters are chosen as follows: the horizon length $N = 5$ and the communication delay $d = 2$. The common gain H and the local correction matrices M_i are computed according to the method described in Theorem 1. As a benchmark, the local Kalman filter is implemented independently for each agent using only its own measurements.

All algorithms are tested under the same realisation of the true state trajectory and the corresponding measurement sequences. The simulation first generates the true state sequence and the three measurement sequences, then runs the local KF and the RHTODF to obtain the state estimates at each time step. The performance metric is the Euclidean norm of the estimation error, $\|\hat{x}_k^{(i)} - x_k\|_2$, and we plot both the state component trajectories and the error norm curves.

Figures 1–3 show the state trajectories of all agents, together with the true states. The blue solid line denotes the true state, the green dash-dotted line denotes the local KF estimate, and the red solid line denotes the RHTODF estimate. It can be observed that the proposed RHTODF closely tracks the true state for all agents and outperforms the local KF, especially during rapid variations by exploiting fused finite-horizon global information.

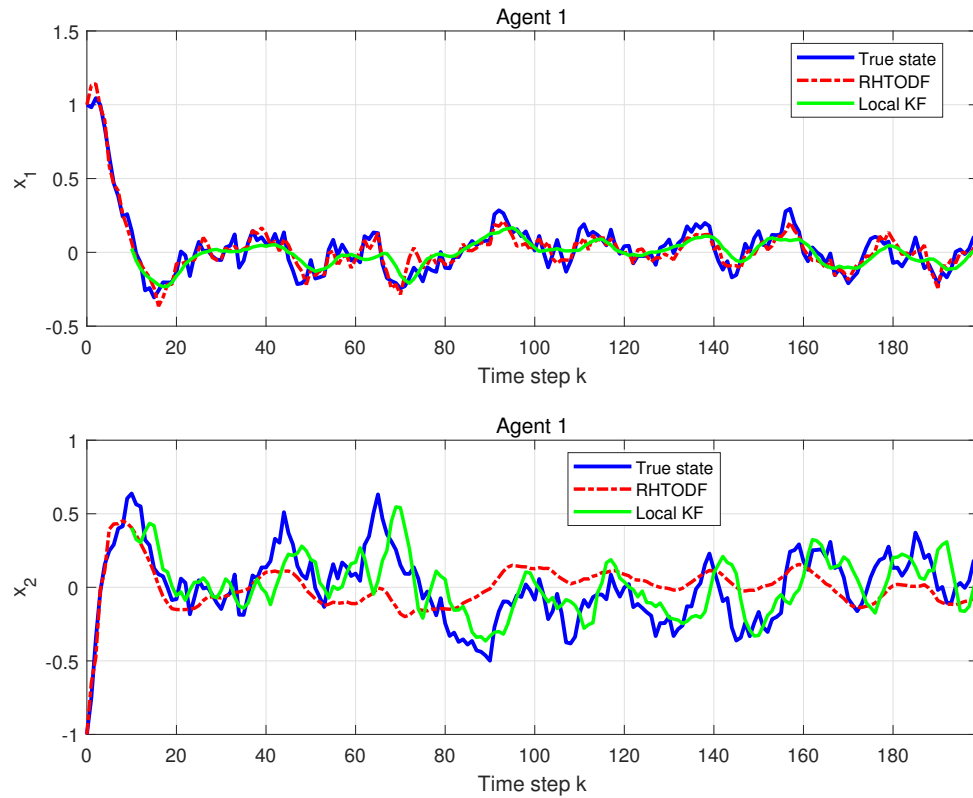


Figure 1. State estimation comparison for Agent 1.

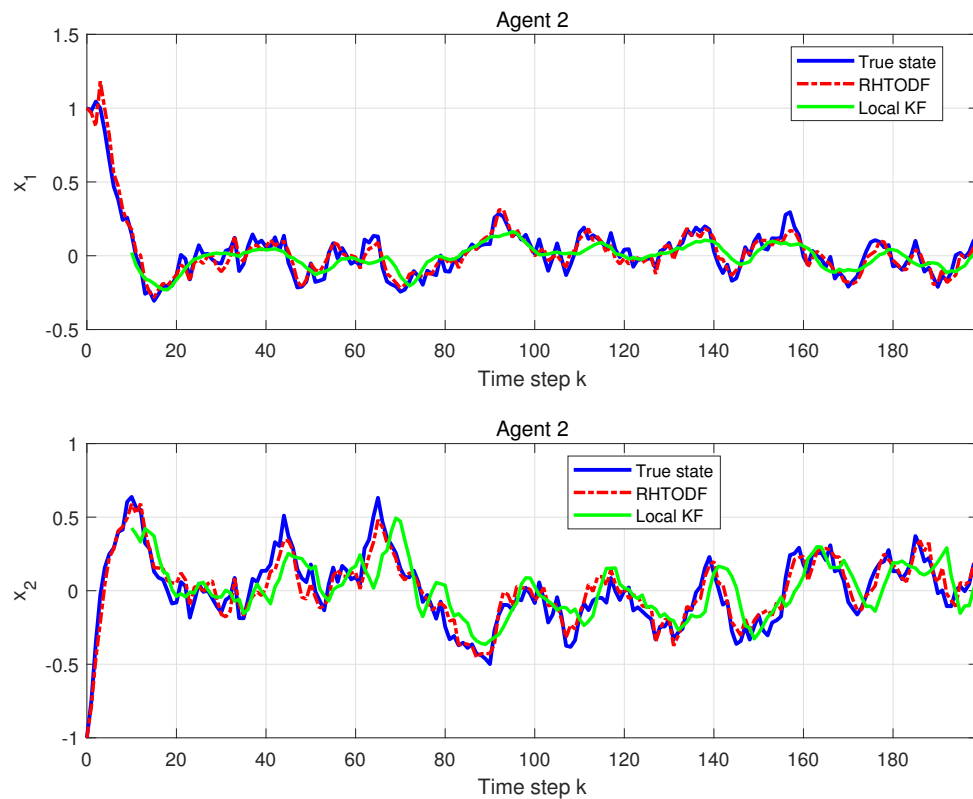


Figure 2. State estimation comparison for Agent 2.

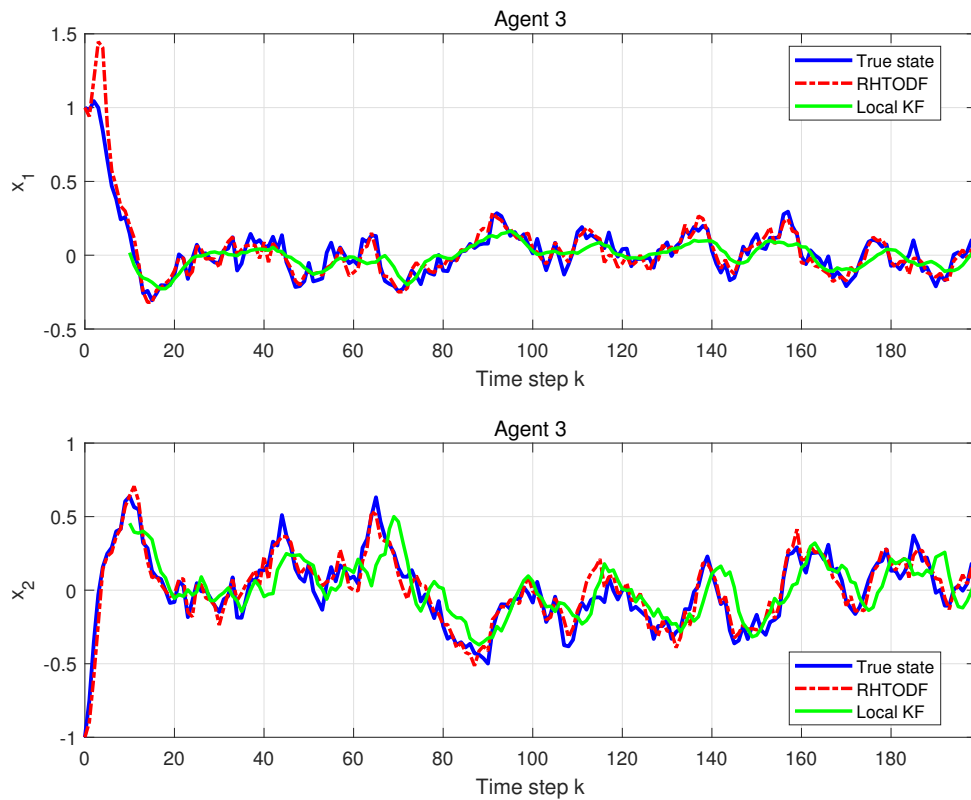


Figure 3. State estimation comparison for Agent 3.

Figures 4–6 further present the time evolution of the Euclidean error norm for each agent. The green dash-dotted lines correspond to the local KF and the red solid lines to RHTODF. The results show that the RHTODF error norm is consistently lower than or equal to that of the local KF at all time instants, and its fluctuation amplitude is smaller. Specifically, the average error norms of RHTODF are reduced by approximately 10.5%, 21.7%, and 22.1%, respectively for the three agents compared with the local KF (see Table 1). This demonstrates that RHTODF can effectively exploit historical multi-step measurements to achieve superior estimation performance over independent filtering in a distributed framework, and it exhibits stronger robustness against irregular dynamic variations.

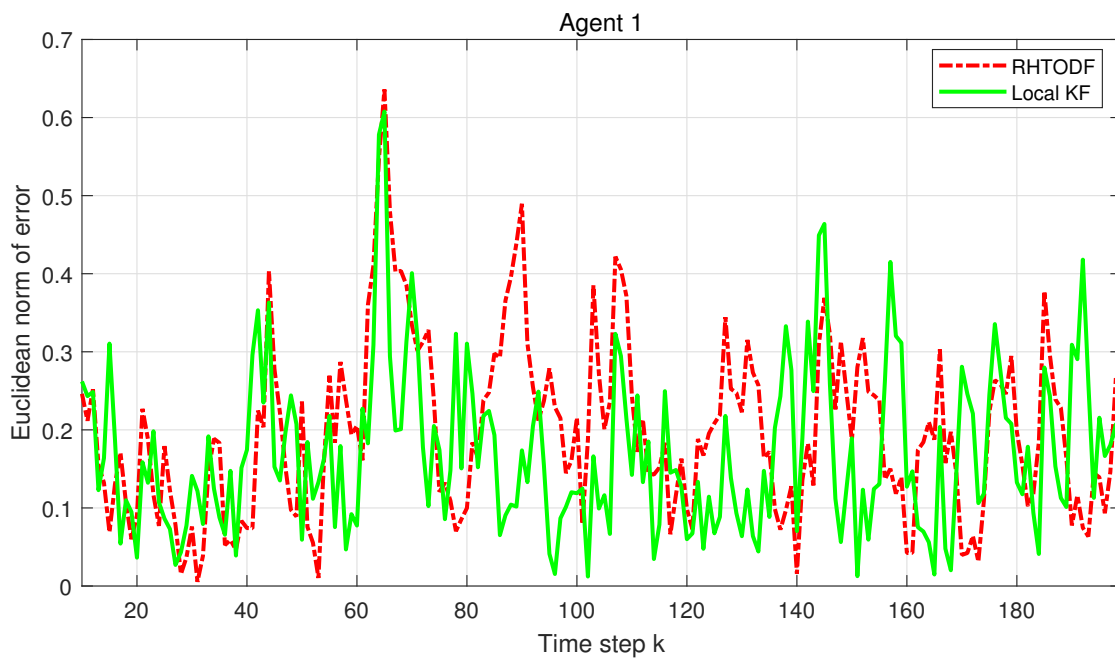
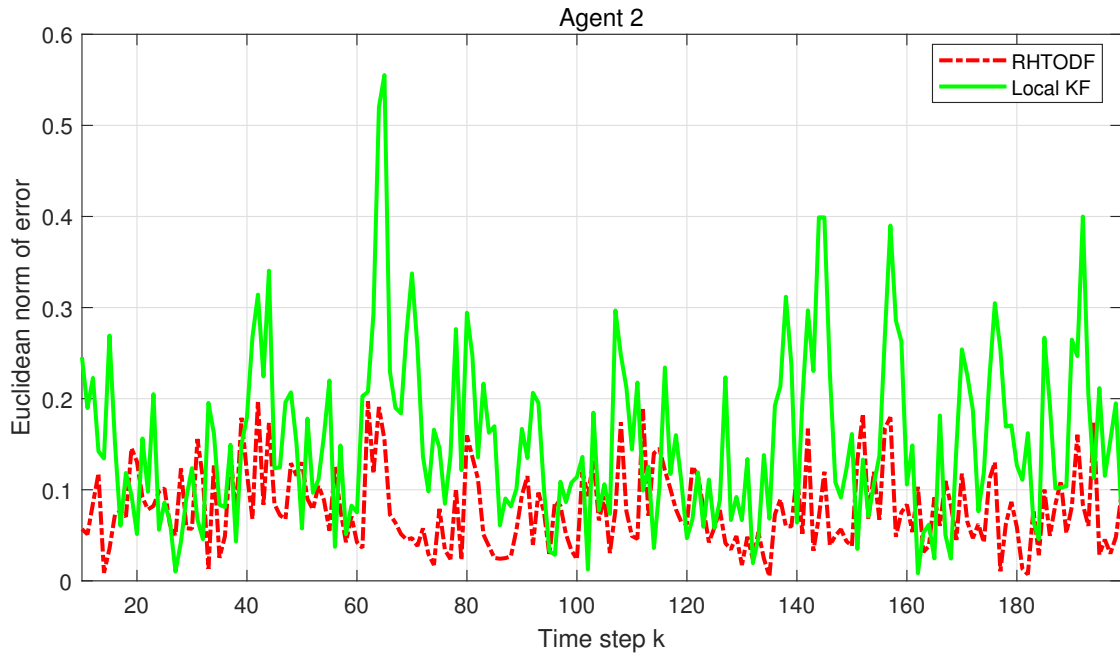
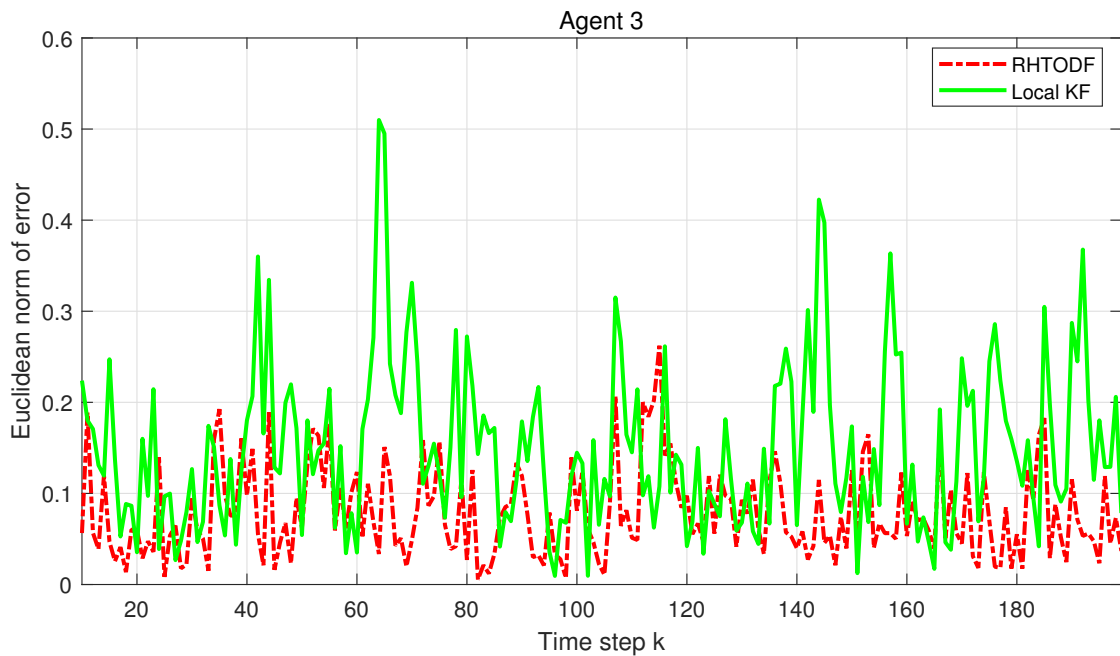


Figure 4. Estimation error norm comparison for Agent 1.

Table 1. Average error norms for each agent.

Agent	Local KF	RHTODF	Reduction
1	0.124	0.111	10.5%
2	0.138	0.108	21.7%
3	0.131	0.102	22.1%

**Figure 5.** Estimation error norm comparison for Agent 2.**Figure 6.** Estimation error norm comparison for Agent 3.

In summary, the simulation results confirm that the proposed RHTODF algorithm significantly enhances the estimation accuracy and stability of multi-agent systems with only modest communication and computational overhead, thereby validating the theoretical analysis.

6. Conclusions

This paper develops a receding horizon team-optimal decentralized filtering framework for discrete-time multi-agent systems with delayed information sharing. By incorporating the receding horizon principle into a team decision-making setting, a unified optimization framework is established that bridges finite-horizon estimation and decentralized information structures. A key contribution is the reformulation of the problem as a finite-horizon minimization of the estimation error covariance under unbiasedness constraints, from which both batch and recursive forms of the RHTODF are derived. Moreover, it is shown that the optimal estimator admits a structured decomposition into a common-information term and local innovation-based corrections, enabling a distributed implementation under delayed communication constraints.

The proposed framework is limited to linear Gaussian systems with fixed communication delays, and its computational complexity increases with the horizon length and network size. Future work will consider nonlinear and non-Gaussian extensions, time-varying or event-triggered communication protocols, and scalable implementations for large-scale networks. Overall, this work provides a structural perspective on integrating receding horizon optimization with team-optimal decentralized estimation under communication constraints.

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