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Practical Consensus of the Multi-Agent Systems on the Two-Layer Networks with Time-Varying Coupling Weights

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Abstract: In practical multi-agent systems, achieving strict consensus is often challenging due to complex communication environments and time-varying coupling relationships. Unlike conventional consensus, which requires the system state errors to converge exactly to the origin, practical consensus only requires the errors to converge to a bounded neighborhood of the origin, thereby better satisfying practical engineering requirements. Motivated by the above analysis, this paper investigates the practical consensus problem for two-layer leader-following multi-agent systems, where uncertain communication environments induce bounded time-varying deviations in coupling weights. Taking such bounded deviations of actual coupling weights into consideration, an error dynamics model is established for the time-varying two-layer network. By designing a distributed pinning control protocol and combining tools from graph theory, matrix theory, and strong practical stability theory, a sufficient condition is derived to guarantee practical consensus of the considered system. Numerical simulations are finally carried out to validate the correctness and effectiveness of the theoretical conclusions.

Keywords: multi-agent system; two-layer network; practical consensus; pinning control; coupling weight

1. Introduction

Owing to the rapid growth of drone swarms and multi-agent systems, consensus control has emerged as a core research focus in cooperative control [1–4]. A variety of consensus problems have been explored for different scenarios, such as partial component consensus [5], lag consensus [6], cluster consensus [7,8] and finite-time consensus [9,10]. Most existing results aim to drive agent state errors to zero using appropriately designed protocols. However, in practical operating environments, non-ideal communication conditions often make it difficult for state errors to converge exactly to zero. Meanwhile, from an engineering perspective, there is also no need to pursue absolute error convergence. The practical consensus relaxes this strict requirement, only requiring errors to converge to a small bounded region near the origin, which makes it better suited to real systems with noise and disturbances [11]. To relax this rigid requirement, Dong et al. [12] introduce the notion of practical consensus, under which state errors converge to a predefined bounded region. This notion better supports robust cooperative control in non-ideal situations and opens new directions for related research. Practical consensus has been extensively investigated in recent years. Ding et al. [13] propose a distributed practical consensus protocol for heterogeneous nonlinear systems, which broadens applications to complex dynamics. Zhang et al. [14] investigate the practical consensus of nonlinear leader-follower multi-agent systems with unknown coupling weights. By relaxing the ideal assumptions of constant coupling weights and asymptotic exact consensus, this

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work establishes an analysis for uncertain networks and provides fundamental theoretical support for the robust cooperative control of multi-agent systems with time-varying coupling weights in complex non-ideal environments. Wang et al. [15] devise dynamic event-triggered resilient control strategies to achieve prescribed-time practical consensus for nonlinear multi-agent systems in the presence of hybrid network attacks. To model communication link outages induced by denial-of-service attacks, they employ Markovian switching topologies and further derived consensus criteria in the form of linear matrix inequalities, which effectively alleviated communication resource burdens under hostile network conditions. Ni et al. [16] further study the practical fixed-time consensus of discontinuous multi-agent systems via adaptive intermittent control with time-varying gain approaches. By constructing novel stability lemmas for indefinite functions, they address bounded external disturbances and established settling-time bounds independent of initial states. Nevertheless, this work only considers single-layer network structures, and the two-layer coupling characteristic is not involved. In practical engineering, information interaction and physical interaction often rely on different network topologies. Simplifying such complex coupling relationships into a single-layer model will inevitably lead to inaccurate system characterization, which cannot reflect the multi-layer coupling structure widely seen in real systems.

Most real-world cooperative systems involve two-layer or multi-layer architectures, including a physical layer and an information layer, with intra-layer and inter-layer information flows coexisting. Single-layer models are insufficient to characterize such complicated interactions. For this reason, consensus problems in two-layer multi-agent systems have received increasing attention. Zhong et al. [17] investigate identical consensus for nonlinear cyber-physical systems under switched two-layer topologies, which supports later studies on time-varying cases. To handle uncertainties and disturbances, Xu et al. [18] present a distributed fixed-time consensus scheme using an adaptive two-layer disturbance observer, which enhances system robustness. Yang et al. [19] extend partial-component consensus to two-layer leader-follower multi-agent systems to break the limitation of single-layer network settings. Using pinning control, they derive sufficient conditions for heterogeneous intra-layer topologies. Wen et al. [20] investigate node-to-node practical consensus for leader-follower two-layer multi-agent systems. Breaking the restriction of identical topologies for both layers, this work addresses the state-tracking issue under heterogeneous layer topologies. Under Lipschitz nonlinearity, sufficient criteria are obtained by configuring coupling strengths to guarantee the bounded convergence of follower tracking errors. For the multi-agent systems with the two-layer subnetworks, the above results primarily focus on fixed topologies and constant coupling weight conditions, while time-varying coupling weights are rarely considered.

In practical communication scenarios, uncertain communication environments easily induce bounded time-varying deviations of coupling weights. Accordingly, this paper investigates the practical consensus problem of two-layer multi-agent systems with time-varying coupling weights under fixed network topology. The main contributions of this paper are summarized as follows:

(1) Most existing studies for two-layer multi-agent systems rely on the ideal constant-coupling assumption, which may fail to hold under uncertain communication environments where coupling coefficients fluctuate. In contrast, this paper focuses on the scenario of coupling weights with bounded time-varying deviations from nominal values, rather than external additive disturbances. This fills the research gap for two-layer networks under such time-varying coupling conditions and renders theoretical analysis more applicable to practical non-ideal communication environments.

(2) For two-layer multi-agent systems with time-varying coupling weights, exact asymptotic consensus is difficult to guarantee. For this reason, this paper addresses the practical consensus problem, where tracking errors are only required to converge to a bounded neighborhood of the origin. A distributed pinning control protocol is further constructed to achieve such practical consensus. Compared with the strict asymptotic consensus framework, the derived conditions relax convergence requirements and are better adapted to complex two-layer networks under coupling-weight fluctuations.

In this paper, a two-layer multi-agent network is investigated, where each subnetwork contains multiple follower agents, and a common leader is introduced to realize cooperative pinning control. Considering the inherent time-varying feature of coupling weights as well as their bounded time-varying deviations from ideal nominal values, an error dynamic model is established for the two-layer network with fixed topology. On this basis, the corresponding pinning control protocol is designed, and a sufficient condition on guaranteeing the practical consensus of the system is derived. Finally, numerical simulations are provided to verify the effectiveness of the theoretical conclusions.

2. Preliminaries

First, some fundamental graph-theoretic preliminaries are presented. For the considered graph, each node corresponds to an agent within the two-layer multi-agent system, while each edge characterizes information interactions between paired agents. Let $G=\{Y,\zeta\}$ denote an undirected graph, where $Y=\{1,...,N\}$ is the node index set, and $\zeta\subseteq Y\times Y$ is the edge set, representing the information exchange between agents. The adjacency matrix of *G* is denoted by $A=(a_{ij})\in\mathbb{R}^{N\times N}$, where a_{ij} represents the weight of information exchange between node i and node j in the multi-agent system. If $(i,j)\in\zeta$, then $a_{ij}>0$; otherwise $a_{ij}=0$. Let $D=\mathrm{diag}(d_1,d_2,\ldots,d_N)$, where $d_i=\sum_{j\neq i,j=1}^{N}a_{ij}$ represents the *i*-th diagonal entry of *D*, and *D* denotes the degree matrix of graph *G*. Define $L=D-A$ as the Laplacian matrix of *G*.

Next, some necessary theoretical preliminaries regarding stability are introduced. Consider the following *n*-dimensional non-autonomous differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=h(t,x), \tag{1}$$

Conditions for the existence and uniqueness of solutions to the equation are given, where $h(t,x)\in C[\mathbb{R}^+\times\mathbb{R}^n,\mathbb{R}^n], x=(x_1,...,x_n)^{\mathrm{T}}\in\mathbb{R}^n$, and $\mathbb{R}^+$ denotes the interval $[0,+\infty), t\in\mathbb{R}^+$, $h(t,x)$ further satisfies the condition:

$$h(t,0)\equiv 0. \tag{2}$$

This condition ensures that $x(t)=0$ is a solution to Equation (1), which is called the trivial solution.

Definition 1 ([21]). *A function $\varphi\in C[\mathbb{R}^+,\ \mathbb{R}^+]$ is said to belong to class K (denoted as $\varphi\in K$) if it is continuous, strictly increasing, and satisfies $\varphi(0)=0$.*

Definition 2 ([14]). *Let Q_0 and Q be bounded closed sets containing the origin $x=0$, with $Q_0\subset Q$. The zero solution of system (1) is said to be practically stable with respect to Q_0 and Q if, for any initial condition $x_0\in Q_0$, the solution $x(t,t_0,x_0)$ of system (1) satisfies $x(t,t_0,x_0)\in Q$ for $t\geq t_0$.*

Definition 3 ([14]). *The zero solution of system (1) is said to be strongly practically stable with respect to Q_0 and Q if it is practically stable with respect to Q_0 and Q, and for any $x_0\in\mathbb{R}^n$, there exists a $T(t_0,x_0)>0$ depending on t_0 and x_0, such that the solution $x(t,t_0,x_0)$ of system (1) belongs to Q whenever $t>t_0+T(t_0,x_0)$.*

Lemma 1 ([14]). *Suppose there exists a function $V(t,x)\in C[I\times\mathbb{R}^n,\mathbb{R}]$ that satisfies the following conditions*:

H1. *For $t\geq t_0$, and for any $x\in Q_0^c\cup\partial Q_0$, $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(1.1)}\leq-\delta<0$;*

H2. *For any $x_1\in Q_0, x_2\in Q^c$, and any $t_2\geq t_1\geq t_0, V(t_1,x_1)<V(t_2,x_2)$.*

Then, the zero solution of system (1) is strongly practically stable with respect to Q_0 and Q, where ∂Q_0 denotes the boundary of the set Q_0.

Lemma 2 ([22]). *Let $P=(p_{ij})^{n\times n}$ be an $n\times n$ matrix, and define the i-th row sum as $r_i(p)=\sum_{j=1,j\neq i}^{n}|p_{ij}|$. Then all eigenvalues of P lie in the Gershgorin disks, i.e.,*

$$G(P)=\bigcup_{i=1}^{n}\{z\,||\,z-p_{ii}|\leq r_i(p)\}.$$

3. Main Results

This paper investigates a multi-agent system with two-layer coupled subnetworks, where each subnetwork contains *N* follower agents, and both subnetworks are subject to pinning control by the same leader agent. To address the effects of uncertain communication environments that cause fluctuations in coupling weights, a

sufficient condition is derived to ensure λ-practical consensus of the considered time-varying two-layer multi-agent system, where the deviations between the actual and nominal coupling weights are assumed to be bounded.

Let $x_i(t)=(x_{i1},x_{i2},...,x_{in})^{\mathrm{T}}\in\mathbb{R}^n, y_i(t)=(y_{i1},y_{i2},...,y_{in})^{\mathrm{T}}\in\mathbb{R}^n$ denote the state variables of the *i*-th follower agent in the first layer and the *i*-th follower agent in the second layer, respectively, and let $x_0(t)$ denote the state variable of the leader agent. The ideal system is formulated as follows:

$$\begin{cases}\dot{x}_0(t)=f(x_0(t)),\\ \dot{x}_i(t)=f(x_i(t))-c\sum_{j=1}^{N}l_{ij}\Gamma x_j(t)-\beta_i\Gamma(x_i(t)-y_i(t))+u_i(t),\\ \dot{y}_i(t)=f(y_i(t))-c\sum_{j=1}^{N}w_{ij}\Gamma y_j(t)-\beta_i\Gamma(y_i(t)-x_i(t))+v_i(t),\end{cases}\tag{3}$$

where $f(x_i)=(f_1(x_i),f_2(x_i),f_3(x_i),\ldots,f_n(x_i))^{\mathrm{T}}$ is a nonlinear continuous function; $c>0$ represents the intra-layer coupling strength; $u_i(t)$ and $v_i(t)$ denote the pinning controllers for the first layer and the second layer, respectively; $B=\mathrm{diag}(\beta_1,\beta_2,...,\beta_N)\in\mathbb{R}^{N\times N}$, where $\beta_i\geq 0, i=1,2,\ldots,N$, represents the inter-layer coupling strength corresponding to the *i*-th agent in the first and second layers of the two-layer network; $\Gamma=\mathrm{diag}(\gamma_1,\gamma_2,...,\gamma_n)\in\mathbb{R}^{n\times n}$, where $\gamma_k\geq 0, k=1,2,...,n$ and is an *n*-dimensional matrix representing the coupling cases of each component in the two node state variables. Let $\tilde{L}(t)=(\tilde{l}_{ij}(t)), \tilde{W}(t)=(\tilde{w}_{ij}(t))$ denote the Laplacian matrices of the first and second layers, respectively. Then, the dynamics of the actual system can be described as:

$$\begin{cases}\dot{x}_0(t)=f(x_0(t)),\\ \dot{\tilde{x}}_i(t)=f(\tilde{x}_i(t))-c\sum_{j=1}^{N}\tilde{l}_{ij}(t)\Gamma\tilde{x}_j(t)-\beta_i\Gamma(\tilde{x}_i(t)-\tilde{y}_i(t))+u_i(t),\\ \dot{\tilde{y}}_i(t)=f(\tilde{y}_i(t))-c\sum_{j=1}^{N}\tilde{w}_{ij}(t)\Gamma\tilde{y}_j(t)-\beta_i\Gamma(\tilde{y}_i(t)-\tilde{x}_i(t))+v_i(t).\end{cases}\tag{4}$$

Next, the λ-practical consensus problem of the multi-agent system is considered under the condition that the errors between the actual coupling weights and the ideal ones satisfy $|\tilde{l}_{ij}(t)-l_{ij}|<\mu_1$, $|\tilde{w}_{ij}(t)-w_{ij}|<\mu_2$ $,(i\neq j)$.

Define the associated errors for the first and second layers as follows:

$$\begin{cases}e_i(t)=\tilde{x}_i(t)-x_0(t),\\ \delta_i(t)=\tilde{y}_i(t)-x_0(t).\end{cases}\tag{5}$$

The leader–follower pinning matrices for the first- and second-layer subnetworks are defined as $D_1=\mathrm{diag}(d_{11},d_{12},...,d_{1N}), D_2=\mathrm{diag}(d_{21},d_{22},...,d_{2N})\in\mathbb{R}^{N\times N}$, where $d_{ik}\geq 0, k=1,2.$ Design the pinning controllers for the actual system:

$$\begin{aligned}u_i(t)&=\begin{cases}-cd_{i1}\Gamma e_i(t), & \text{if } \|e_i(t)\|\geq\lambda,\\ -cd_{i1}\Gamma e_i(t)\exp(k\|e_i(t)\|-k\lambda), & \text{if } \|e_i(t)\|<\lambda,\end{cases}\\ v_i(t)&=\begin{cases}-cd_{i2}\Gamma\delta_i(t), & \text{if } \|\delta_i(t)\|\geq\lambda,\\ -cd_{i2}\Gamma\delta_i(t)\exp(k\|\delta_i(t)\|-k\lambda), & \text{if } \|\delta_i(t)\|<\lambda,\end{cases}\end{aligned}\tag{6}$$

where *c* and Γ are defined the same as in (3).

Afterwards, define the function $\eta:\mathbb{R}_{\geq 0}\times\mathbb{R}^+\times\mathbb{R}^+\to(0,1]$ as:

$$\eta(z,\lambda,k)=\begin{cases}1, & \text{if } z\geq\lambda,\\ \exp(kz-k\lambda), & \text{if } z<\lambda,\end{cases}$$

where $z\geq 0$. The function is continuous and at $z=\lambda$, $\exp(0)=1$, indicating no jump discontinuity. Thus, (5) can be rewritten as:

$$\begin{cases}\dot{e}_i(t)=f(\tilde{x}_i(t))-f(x_0(t))-c\sum_{j=1}^{N}\tilde{l}_{ij}(t)\Gamma e_j(t)-\beta_i\Gamma(e_i(t)-\delta_i(t))-cd_{i1}\Gamma e_i(t)\eta(\|e_i(t)\|,\lambda,k),\\ \dot{\delta}_i(t)=f(\tilde{y}_i(t))-f(x_0(t))-c\sum_{j=1}^{N}\tilde{w}_{ij}(t)\Gamma\delta_j(t)-\beta_i\Gamma(\delta_i(t)-e_i(t))-cd_{i2}\Gamma\delta_i(t)\eta(\|\delta_i(t)\|,\lambda,k).\end{cases}\tag{7}$$

The corresponding vector form of (7) is given by:

$$\begin{cases}\dot{e}(t)=F(e(t))-c(\tilde{L}(t)\otimes\Gamma)e(t)-(B\otimes\Gamma)e(t)+(B\otimes\Gamma)\delta(t)-c(D_1\otimes\Gamma)\Lambda_e e(t),\\ \dot{\delta}(t)=F(\delta(t))-c(\tilde{W}(t)\otimes\Gamma)\delta(t)-(B\otimes\Gamma)\delta(t)+(B\otimes\Gamma)e(t)-c(D_2\otimes\Gamma)\Lambda_\delta\delta(t),\end{cases}\tag{8}$$

where

$$F(e(t))=((f(x_1(t))-f(x_0(t)))^{\mathrm{T}},...,(f(x_N(t))-f(x_0(t))))^{\mathrm{T}},$$
$$F(\delta(t))=((f(y_1(t))-f(x_0(t)))^{\mathrm{T}},...,(f(y_N(t))-f(x_0(t))))^{\mathrm{T}},$$

the matrices Λ_e and Λ_δ are given by $\Lambda_e=\mathrm{diag}(\eta(\|e_1\|,\lambda,k),\eta(\|e_2\|,\lambda,k),\ldots,\eta(\|e_N\|,\lambda,k))$, $\Lambda_\delta=\mathrm{diag}(\eta(\|\delta_1\|,\lambda,k),\eta(\|\delta_2\|,\lambda,k)\ldots,\eta(\|\delta_N\|,\lambda,k))$.

Next, the overall error is defined as $E(t)=(e^{\mathrm{T}}(t),\delta^{\mathrm{T}}(t))^{\mathrm{T}}$, and the definition of practical consensus for two-layer networks is presented.

Definition 4. *If there exists a constant $\lambda>0$, and a finite time $T(t_0,E(t_0))>0$ depending on t_0 and $E(t_0)$, such that for all $t>t_0+T(t_0,E(t_0))$ the solutions of system (4) satisfy $\|E_i(t,t_0,E(t_0))\|\leq\lambda$, then system (4) is said to achieve λ-practical consensus, where λ denotes the upper bound of the allowable steady-state error.*

Assumption 1 (QUAD Condition). *There exists a constant $l>0$, such that for $x(t)\in\mathbb{R}^n, y(t)\in\mathbb{R}^n$, the nonlinear function $f(\bullet):\mathbb{R}^n\to\mathbb{R}^n$ satisfies*

$$(x(t)-y(t))^{\mathrm{T}}\Phi(f(x(t))-f(y(t)))\leq l(x(t)-y(t))^{\mathrm{T}}\Phi(x(t)-y(t)),$$

where $\Phi=\mathrm{diag}(\phi_1,...,\phi_n)>0$. In this case, *f* is said to satisfy the QUAD condition.

To simplify the subsequent derivations, the following notation is adopted:

$$P(t)=l(\Phi\otimes I_N)-c((\Phi\tilde{L}_D{}^s(t))\otimes\Gamma)-M+\varepsilon I_N,$$
$$U(t)=l(\Phi\otimes I_N)-c((\Phi\tilde{W}_D{}^s(t))\otimes\Gamma)-M+\frac{1}{\varepsilon}M^TM,$$

where

$$\tilde{L}_D(t)=\tilde{L}(t)+D_1,L_D=L+D_1,\tilde{W}_D(t)=\tilde{W}(t)+D_2,W_D=W+D_2,\tilde{L}_D^s(t)=\frac{1}{2}(\tilde{L}_D{}^{\mathrm{T}}(t)+$$

$$\tilde{L}_D(t)),L_D^s=\frac{1}{2}(L_D{}^{\mathrm{T}}+L_D),\tilde{W}_D^s(t)=\frac{1}{2}(\tilde{W}_D{}^{\mathrm{T}}(t)+\tilde{W}_D(t)),W_D^s=\frac{1}{2}(W_D{}^{\mathrm{T}}+W_D).$$

Theorem 1. *When $|\tilde{l}_{ij}(t)-l_{ij}|<\mu_1,|\tilde{W}_{ij}(t)-W_{ij}|<\mu_2$, if the matrices $l(\Phi\otimes I_N)-c((\Phi L_D^s)\otimes\Gamma)-M+(\varepsilon+2(N-1)\mu_1)I_N$ and $l(\Phi\otimes I_N)-c((\Phi W_D^s)\otimes\Gamma)-M+\frac{1}{\varepsilon}M^{\mathrm{T}}M+2(N-1)\mu_2 I_N$ are negative definite, then the matrices P(t) and U(t) are also negative definite.*

Proof. Let

$$P=(p_{ij})=l(\Phi\otimes I_N)-c((\Phi L_D^s)\otimes\Gamma)-M+\left(\varepsilon+2(N-1)\mu_1\right)I_N,$$

$$\tilde{P}(t)=(\tilde{p}_{ij}(t))=l(\Phi\otimes I_N)-c((\Phi\tilde{L}_D^s(t))\otimes\Gamma)-M+\varepsilon I_N.$$

Then

$$p_{ii}=l\phi_i+\varepsilon+2(N-1)\mu_1-c\phi_i\gamma_i l_{ii}-m_i,\ \ \tilde{p}_{ii}(t)=l\phi_i+\varepsilon-c\phi_i\gamma_i\tilde{l}_{ii}(t)-m_i,$$

$$\tilde{p}_{ij}(t)\ =-c\phi_i\gamma_i\tilde{l}_{ij}(t),\quad p_{ij}=-c\phi_i\gamma_i l_{ij},\ \ i\neq j.$$

By the Gershgorin circle theorem, the *i*-th Gershgorin disk of matrix *P* is centered at $(p_{ii},0)$ with radius $\sum_{j\neq i}^{N}|\ p_{ij}\ |$. The matrix *P* is symmetric with negative diagonal entries and strictly row diagonally dominant, i.e., $p_{ii}<0$ and $p_{ii}+\sum_{j\neq i}^{N}|\ p_{ij}|<0$, Accordingly, all its Gershgorin disks lie in the open left half-plane, which implies that P is negative definite. Consequently, for $i\neq j$, where $M=(\Phi B)\otimes\Gamma,$ the following inequalities hold:

$$\tilde{p}_{ii}(t)-p_{ii}\leq-c\phi_i\gamma_i\sum_{j\neq i}^{N}\left|l_{ij}-\tilde{l}_{ij}(t)\right|-2(N-1)\mu_1\leq-c(N-1)\mu_1\phi_i\gamma_i-2(N-1)\mu_1,$$

$$\tilde{p}_{ii}(t)+\sum_{j\neq i}^{N}\left|\tilde{p}_{ij}(t)\right|\leq p_{ii}+\sum_{j\neq i}^{N}\left|p_{ij}\right|-c(N-1)\mu_1\phi_i\gamma_i-2(N-1)\mu_1+c(N-1)\mu_1\phi_i\gamma_i,$$

then

$$\tilde{p}_{ii}(t)+\sum_{j\neq i}^{N}\left|\tilde{p}_{ij}(t)\right|<p_{ii}+\sum_{j\neq i}^{N}\left|p_{ij}\right|<0.$$

Therefore $\tilde{P}(t)$ is negative definite. Similarly, let $U=l(\Phi\otimes I_N)-c((\Phi W_D{}^s)\otimes\Gamma)-M$ $\frac{1}{\varepsilon}M^{\mathrm{T}}M+2(N-1)\mu_2 I_N$. If *U* is a negative definite matrix, then $\tilde{U}(t)$ is also negative definite. This completes the proof.

□

Theorem 2. *If systems (3) and (4) satisfy Assumption 1,* $\tilde{l}_{ij}(t)$ *and* $\tilde{w}_{ij}(t)$ *are continuously varying with respect to t, and matrices P and U are negative definite, then system (4) can achieve* λ*-practical consensus under the controller (6). Here, P and U are defined the same as in Theorem 1.*

Proof. Choose the Lyapunov function

$$V(t,E(t))=\frac{1}{2}E^{\mathrm{T}}(t)RE(t),$$

where $R=\begin{bmatrix}\Phi\otimes I_N & O\\ O & \Phi\otimes I_N\end{bmatrix}$, define the set

$$Q_0=Q=\left\{E(t)\in\mathbb{R}^{2Nn}\left|\left\|\left(\Phi\otimes I_{2N}\right)E(t)\right\|<\lambda\right.\right\},$$

for $\forall E^{\mathrm{T}}(t)\in Q_0^c\bigcup\partial Q=\left\{E(t)\in R^{2Nn}\left|\left\|(\Phi\otimes I_{2N})E(t)\right\|\geq\lambda\right.\right\}$, take the time derivative of $V(t)$ along the trajectory of system (7):

$$\begin{aligned}\dot{V}(t,E(t))=&\,e^{\mathrm{T}}(t)(\Phi\otimes I_N)F(e(t))+\delta^{\mathrm{T}}(t)(\Phi\otimes I_N)F(\delta(t))-ce^{\mathrm{T}}(t)((\Phi\tilde{L}_D^s(t))\otimes\Gamma)e(t)\\&-c\delta^{\mathrm{T}}(t)((\Phi\tilde{W}_D^s(t))\otimes\Gamma)\delta(t)-e^{\mathrm{T}}(t)((\Phi B)\otimes\Gamma)e(t)-\delta^{\mathrm{T}}(t)((\Phi B)\otimes\Gamma)\delta(t)\\&+e^{\mathrm{T}}(t)((\Phi B)\otimes\Gamma)\delta(t)+\delta^{\mathrm{T}}(t)((\Phi B)\otimes\Gamma)e(t).\end{aligned}$$

Here, let $M=(\Phi B)\otimes\Gamma$, by Lemma 1, the following inequalities hold

$$\begin{aligned}\dot{V}(t)&\le le^{\mathrm{T}}(t)(\Phi\otimes I_N)e(t)-ce^{\mathrm{T}}(t)((\Phi\tilde{L}_D{}^s(t))\otimes\Gamma)e(t)-e^{\mathrm{T}}(t)Me(t)+l\delta^{\mathrm{T}}(t)(\Phi\otimes I_N)\delta(t)\\&\quad-c\delta^{\mathrm{T}}(t)((\Phi\tilde{W}_D{}^s(t))\otimes\Gamma)\delta(t)-\delta^{\mathrm{T}}(t)M\delta(t)+2\delta^{\mathrm{T}}(t)Me(t)\\&\le le^{\mathrm{T}}(t)(\Phi\otimes I_N)e(t)-ce^{\mathrm{T}}(t)((\Phi\tilde{L}_D{}^s(t))\otimes\Gamma)e(t)-e^{\mathrm{T}}(t)Me(t)+\varepsilon e^{\mathrm{T}}(t)e(t)-\delta^{\mathrm{T}}(t)M\delta(t)\\&\quad-c\delta^{\mathrm{T}}(t)((\Phi\tilde{W}_D{}^s(t))\otimes\Gamma)\delta(t)+\frac{1}{\varepsilon}\delta^{\mathrm{T}}(t)M^{\mathrm{T}}M\delta(t)+l\delta^{\mathrm{T}}(t)(\Phi\otimes I_N)\delta(t)\\&\le e^{\mathrm{T}}(t)\tilde{P}(t)e(t)+\delta^{\mathrm{T}}(t)\tilde{U}(t)\delta(t)\\&<\lambda_{\max}(\tilde{P}(t))\|e(t)\|^2+\lambda_{\max}(\tilde{U}(t))\|\delta(t)\|^2\\&<\lambda\|E(t)\|^2.\end{aligned}$$

From Theorem 1, $\tilde{P}(t)$ and $\tilde{U}(t)$ are negative definite matrices. Let $m=-2\alpha$, where $\alpha=\dfrac{\lambda}{\lambda_{\max}(R)}$. Then for any $E(t)\in Q_0^c\bigcup\partial Q$, $\dot{V}\le mV<0$; On the other hand, for any $E_1(t)\in Q_0, E_2(t)\in Q^c$, it holds that $\|(\Phi\otimes \mathrm{I}_{2N})E_2(t)\|>\|(\Phi\otimes \mathrm{I}_{2N})E_1(t)\|$, which clearly implies $V_1(t)<V_2(t)$ for any $t_2\ge t_1\ge t_0$.

In summary, the trivial solution of system (7) is strongly practically stable with respect to Q_0, Q. Therefore, there exists a finite time $T(t_0,E(t_0))>0$ such that for all $t>t_0+T(t_0,E(t_0))$, it holds that $\|E(t,t_0,E(t_0))\|\le\lambda$. This means that the two-layer network (4) achieves λ-practical consensus. The proof is complete.

□

4. Numerical Simulation

To verify the correctness of the theoretical results on λ-practical consensus for the multi-agent systems with two-layer networks, numerical simulations are carried out in this section. A two-layer network model is established to compare the differences in error evolution between the ideal and actual systems. On this basis, the influence of parameters μ and λ on the practical consensus of the actual system is further explored.

Specifically, the considered system consists of two mutually coupled network layers. Each layer contains one leader agent and four follower agents, i.e., $N=4$. The dynamics of each agent are governed by Chua's circuit, with the state dimension $n=3$. The communication topology is depicted in the Figure 1.

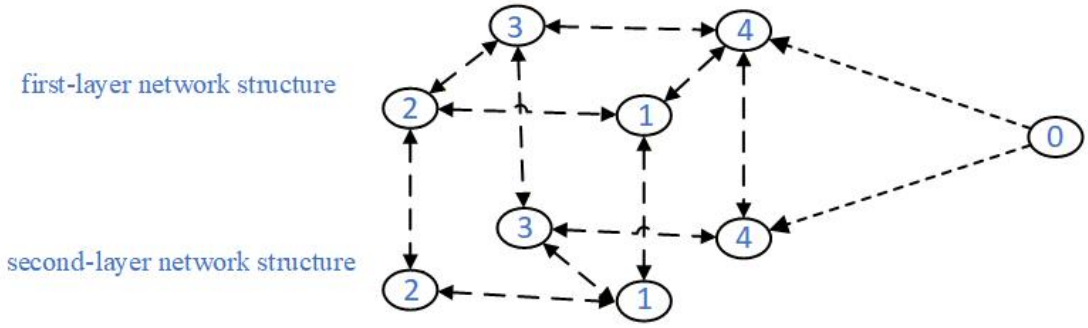


Figure 1. Communication topology of the two-layer leader-follower multi-agent system (4).

Let the dynamic equations of individual agents satisfy Chua's circuit law. The Chua's circuit can be expressed as

$$\begin{cases}\dot{x}_{i1}=\alpha\left(x_{i2}-x_{i1}-h(x_{i1})\right),\\ \dot{x}_{i2}=x_{i1}-x_{i2}+x_{i3},\\ \dot{x}_{i3}=-\beta x_{i2},\end{cases}$$

where $h(s)=m_1 s+\dfrac{1}{2}(m_0-m_1)\left(|s+1|-|s-1|\right)$, the parameter values are set as $\alpha=10$, $\beta=16$, $m_0=-1.27$, and $m_1=-0.75$. Since the Chua's circuit is globally Lipschitz continuous, numerical calculations show that when $l=10$, the *f* function satisfies the QUAD condition.

Based on the above dynamical equations, two cases are studied: the ideal two-layer network system and the actual system with time-varying coupling weights. The corresponding models are described as follows:

$$\begin{cases} \dot{x}_0 = f(x_0), \\ \dot{x}_i = f(x_i) - c\sum_{j=1}^{4} l_{ij}\Gamma x_j - \beta_i\Gamma(x_i - y_i) + u_i, \\ \dot{y}_i = f(y_i) - c\sum_{j=1}^{4} w_{ij}\Gamma y_j - \beta_i\Gamma(y_i - x_i) + v_i, \end{cases}$$

$$\begin{cases} \dot{x}_0 = f(x_0), \\ \dot{\tilde{x}}_i = f(x_i) - c\sum_{j=1}^{4} \tilde{l}_{ij}(t)\Gamma\tilde{x}_j - \beta_i\Gamma(\tilde{x}_i - \tilde{y}_i) + u_i, \\ \dot{\tilde{y}}_i = f(y_i) - c\sum_{j=1}^{4} \tilde{w}_{ij}(t)\Gamma\tilde{y}_j - \beta_i\Gamma(\tilde{y}_i - \tilde{x}_i) + v_i. \end{cases}$$

The Laplacian matrices of the first-layer and second-layer networks are $L = (l_{ij})$ and $W = (w_{ij})$, respectively, given by

$$L = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix}, \quad W = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$$

Accordingly, the corresponding time-varying Laplacian matrices are defined as $\tilde{L}(t) = (\tilde{l}_{ij}(t)) = l_{ij} + \mu_1 \sin(t)$, $\tilde{W}(t) = (\tilde{w}_{ij}(t)) = w_{ij} + \mu_2 \sin(t)$, where μ_1 and μ_2 represent the upper bounds of coupling perturbations.

Design the practical consensus pinning controller as follows,

$$u_i = \begin{cases} -cd_{i1}\Gamma e_i, & \text{if } \|e_i\| \geq \lambda, \\ -cd_{i1}\Gamma e_i \exp(k\|e_i\| - k\lambda), & \text{if } \|e_i\| < \lambda, \end{cases}$$

$$v_i = \begin{cases} -cd_{i2}\Gamma \delta_i, & \text{if } \|\delta_i\| \geq \lambda, \\ -cd_{i2}\Gamma \delta_i \exp(k\|\delta_i\| - k\lambda), & \text{if } \|\delta_i\| < \lambda. \end{cases}$$

The control gain is set as c = 18, the exponential decay coefficient k = 2, and the coupling matrix $\Gamma = \mathrm{diag}(1,1,1)$ .The pinning gain matrices for the two layers are set to $D_1 = \mathrm{diag}(3,0,3,3)$ and $D_2 = \mathrm{diag}(3,3,0,3)$, where the zero entries meet both the practical requirements of pinning control and the constraints given in Theorem 1. Under these parameters, both the ideal system and the perturbed system achieve λ-practical consensus with the leader agent.

With the above parameters, the negative-definiteness condition required in Theorem 1 is further verified. The corresponding matrices P and U are constructed as

$$P = l(\Phi \otimes I_N) - c((\Phi L_D^s) \otimes \Gamma) - M + (\varepsilon + 2(N-1)\mu_1) I_N,$$

$$U = l(\Phi \otimes I_N) - c((\Phi W_D^s) \otimes \Gamma) - M + \frac{1}{\varepsilon} M^{\mathrm{T}} M + 2(N-1)\mu_2 I_N.$$

For $\mu_1 = 0.01, \mu_2 = 0.01,$ numerical calculation yields the maximum eigenvalues $\lambda_{\max}(P) \approx -6.15$ and $\lambda_{\max}(U) \approx -6.14.$ For the larger perturbation bounds $\mu_1 = 0.5,$ $\mu_2 = 0.5,$ numerical calculation yields the maximum eigenvalues $\lambda_{\max}(P) \approx -3.21$ and $\lambda_{\max}(U) \approx -3.20$. In both cases, the maximum eigenvalues of P and U are negative. Therefore, the negative-definiteness condition of Theorem 1 holds, which guarantees the theoretical validity of the following simulation results.

Let the initial condition of the leader be $x_0 = (10,15,20)^{\mathrm{T}}$. For the first layer, the initial conditions are $x_i = (a_i, b_i, z_i)^{\mathrm{T}}$ for $i = 1,...,4$, where $x_1 = (10,0,0)^{\mathrm{T}}, x_2 = (0,10,0)^{\mathrm{T}}, x_3 = (0,0,10)^{\mathrm{T}}$, $x_4 = (10,10,0)^{\mathrm{T}}$. For the second layer, the initial conditions are $y_i = (a_i, b_i, z_i)^{\mathrm{T}}$ for $i = 1,...,4$, where $y_1 = (5,0,0)^{\mathrm{T}}, y_2 = (0,5,0)^{\mathrm{T}}, y_3 = (0,0,5)^{\mathrm{T}}, y_4 = (10,0,10)^{\mathrm{T}}$.

Figure 2 depicts the error trajectories of the ideal fixed-topology system and the actual system with time-varying coupling weights. The left column corresponds to the ideal scenario, while the right column shows system responses under bounded sinusoidal coupling perturbations. The red dashed lines represent the predefined error threshold $\lambda = 3$.

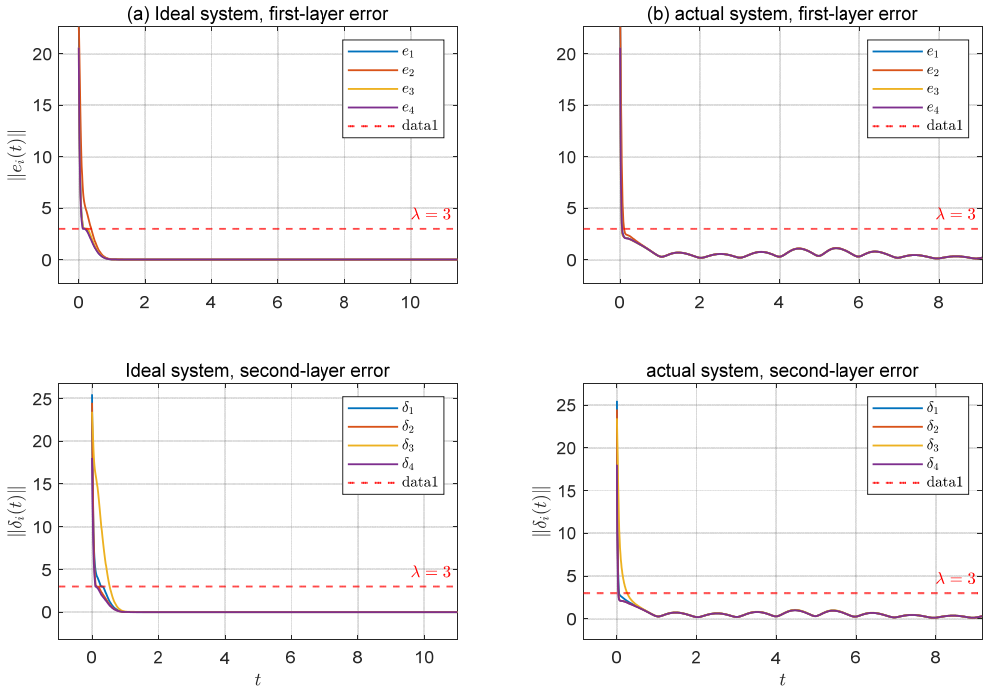


Figure 2. Error evolution between followers and the leader. (**a**) Ideal system (3): first-layer and second-layer consensus errors; (**b**) Actual system (4): first-layer and second-layer consensus errors.

It can be observed that all consensus errors stay below this threshold for both cases. Owing to time-varying couplings, the actual system exhibits mild steady-state oscillations. These results verify that the proposed pinning control protocol achieves λ-practical consensus, ensuring tracking errors are bounded within the prescribed bound as time evolves.

Subsequently, the effects of μ and λ on the λ-practical consensus of the actual system are investigated. To further examine the effect of coupling perturbation bounds on the consensus performance, two additional groups of μ_1 and μ_2 are tested while keeping λ unchanged.

It can be observed from Figure 3 that all error trajectories converge and stay within the predefined threshold $\lambda = 3$. As the perturbation bounds increase from $\mu_1 = \mu_2 = 0.05$ to $\mu_1 = \mu_2 = 0.1$, the amplitude of steady-state oscillations of consensus error norms increases accordingly. This phenomenon demonstrates that larger time-varying coupling perturbations amplify the fluctuations of consensus errors. Even for coupling perturbations with different magnitudes, the designed controller achieves λ-practical consensus, which validates the robustness of the proposed control protocol against time-varying coupling weights.

In addition, to investigate the impact of the threshold parameter λ, comparative simulations are performed with two distinct values, while maintaining the coupling perturbation bounds as $\mu_1 = \mu_2 = 0.05$.

It can be observed from Figure 4 that the system achieves λ-practical consensus in both cases. When λ decreases from 4 to 2, the allowable steady-state error bound becomes tighter, and the amplitude of steady-state oscillations is reduced. This result demonstrates that a smaller threshold yields higher control accuracy. Although a λ imposes stricter requirements on the control protocol, the proposed controller can still guarantee satisfactory consensus performance.

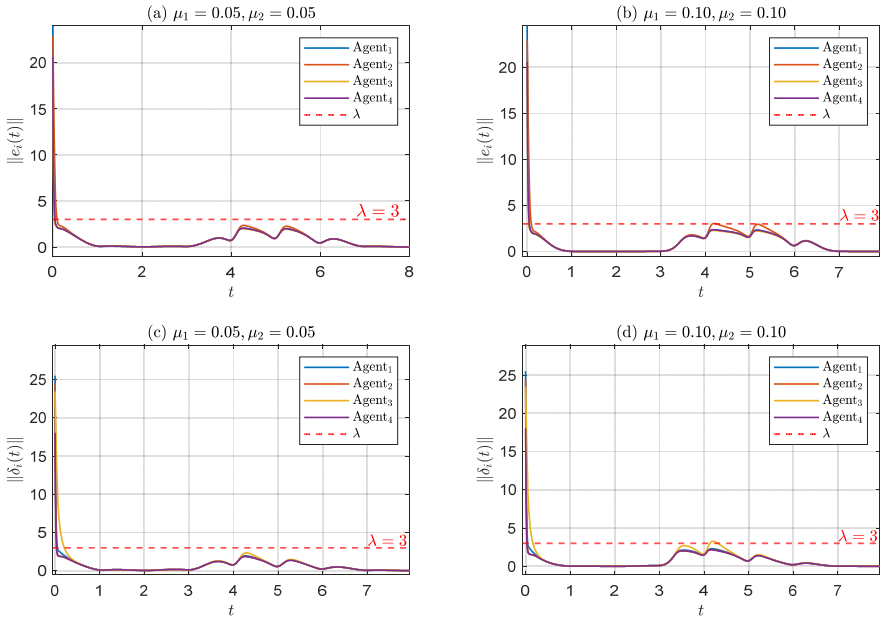


Figure 3. Error norm trajectories of the two-layer actual system with fixed threshold $\lambda = 3$ under different μ_1, μ_2. (**a**) $\mu_1 = 0.05, \mu_2 = 0.05$; (**b**) $\mu_1 = 0.10, \mu_2 = 0.10$; (**c**) $\mu_1 = 0.05, \mu_2 = 0.05$; (**d**) $\mu_1 = 0.10, \mu_2 = 0.10$. $\|e_i(t)\|$ denotes the error norm of agents in the first-layer subnet, and $\|\delta_i(t)\|$ represents the error norm of agents in the second-layer subnet.

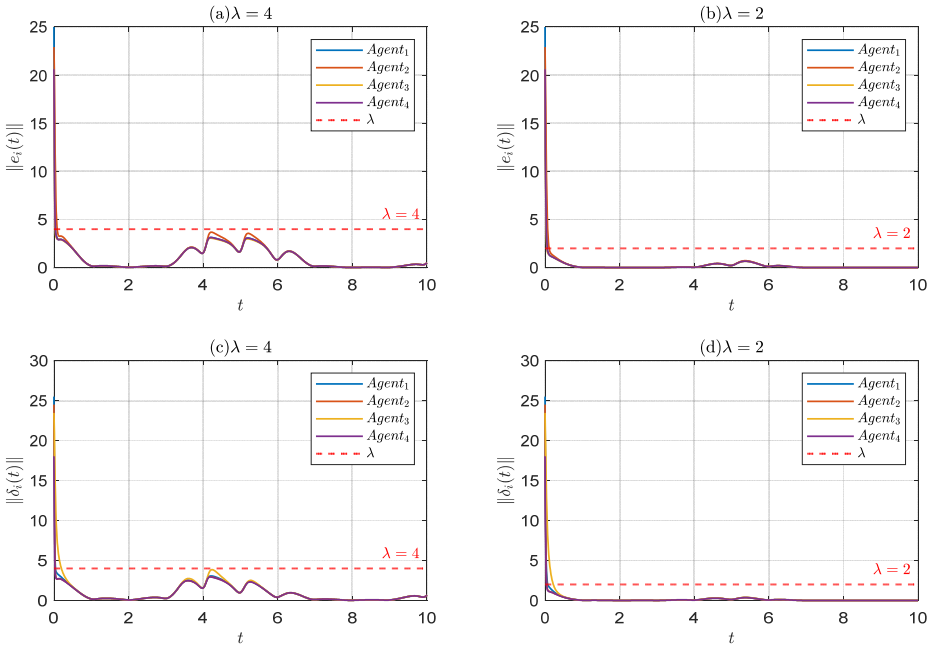


Figure 4. Error norm trajectories of the two-layer multi-agent system with fixed coupling perturbation bounds $\mu_1 = \mu_2 = 0.05$. (**a**) $\lambda = 4$, error norm $\|e_i(t)\|$ of the first-layer subnet; (**b**) $\lambda = 2$, error norm $\|e_i(t)\|$ of the first-layer subnet; (**c**) $\lambda = 4$, error norm $\|\delta_i(t)\|$ of the second-layer subnet; (**d**) $\lambda = 2$, error norm $\|\delta_i(t)\|$ of the second-layer subnet.

5. Conclusions

This paper investigates the practical consensus problem for two-layer leader-following multi-agent systems, where the time-varying coupling weights arise from bounded deviations caused by uncertain communication environments. By relaxing the ideal assumption of constant coupling weights, an error dynamics model is established for two-layer networks with fixed topology, where the deviations between actual time-varying coupling

weights and nominal values is bounded. A distributed pinning control protocol is designed, and a sufficient condition for realizing λ-practical consensus is derived by combining graph theory, matrix theory and strong practical stability theory. Numerical simulations based on Chua's circuit are conducted to verify the validity of the theoretical results. Future research will extend the proposed results to switching topologies, directed communication graphs and event-triggered strategies, to reduce control costs and further enhance system scalability.

Author Contributions

Y.J.: conceptualization, methodology, formal analysis, validation, writing—original draft; W.Z.: supervision, writing—review and editing; Z.M.: funding acquisition, investigation. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The simulation codes supporting the results of this work are available from the corresponding author upon reasonable request. Conflicts of Interest

Conflicts of Interest

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results. Use of AI and AI-Assisted Technologies

Use of AI and AI-Assisted Technologies

During the preparation of this work, the authors used AI tools to polish the English language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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