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Exploring Low-Carbon Strategy Adoption in Manufacturing: An Evolutionary Game Theory Perspective on Government Oversight and Rent-Seeking by Third-Party Testing Organizations

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Abstract: Low-carbon transition in manufacturing has been widely studied using two-party game models. However, the strategic interactions among governments, enterprises, and third-party testing organizations (TTOs), particularly under rent-seeking risks, remain underexplored. To address this gap, this study developed a tripartite evolutionary game model that incorporates bounded rationality and strategic interdependence. In detail, the study analyzed the dynamic decision-making processes of these three actors under low-carbon objectives, focusing on rent-seeking behavior and its systemic implications. By using replicator dynamics and Jacobian matrix-based stability analysis, the study identified the asymptotic stability conditions for equilibrium strategies and validated them through numerical simulations. After correcting the payoff matrix and replicator dynamics, the re-derived Jacobian analysis shows that no pure-strategy equilibrium satisfies all negative eigenvalues; hence, the system exhibits no pure evolutionarily stable strategy (ESS). Instead, numerical simulations reveal that the system converges to different stable states depending on initial conditions and parameter combinations. Key findings of the study reveal that: (1) enterprises' adoption of low-carbon production is influenced not only by government oversight but also by the cost differential between low- and non-low-carbon options; (2) governments' active regulation is driven by both the likelihood of corporate compliance and the penalties imposed on non-compliant TTOs; and (3) TTOs' refusal of rent-seeking is negatively associated with rent-seeking costs but positively correlated with forgery expenses and penalties. Moreover, the initial strategy probabilities of TTOs significantly shape the evolutionary trajectory of the entire system. The theoretical contribution of this study centers on the integration of rent-seeking dynamics into a tripartite evolutionary game framework. This demonstrates how collusion risks fundamentally reshape strategic interactions among governments, enterprises, and third-party verifiers. In effect, this will provide a theoretical basis for designing robust regulatory and anti-collusion mechanisms for carbon emission verification.

Keywords: low carbon goal; manufacturing enterprise; third-party testing organizations; evolution game; simulation

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1. Introduction

The global commitment to low-carbon transition represents one of the most significant societal and economic shifts in the 21st century, driven by the urgent need to mitigate climate change. Within this broad landscape, the manufacturing sector occupies a dual and often contradictory position: it is a primary engine of economic growth and social wealth creation, yet it remains a major contributor to global carbon emissions [1,2]. This tension has placed manufacturing enterprises at the forefront of policy and academic debates on environmental governance. Governments worldwide have responded with regulatory measures, including carbon emission caps, trading schemes, and fiscal incentives, aiming to steer corporate behavior towards sustainable production [3,4]. However, the pathway to effective decarbonization in manufacturing is saddled with challenges, characterized by complex strategic interactions among multiple stakeholders with often divergent interests.

Evolutionary game theory (EGT) has emerged as a prominent analytical framework for studying these complex interactions in environmental governance [5]. Its strength lies in accommodating the bounded rationality of actors, who learn and adapt their strategies over time through imitation and experimentation, rather than assuming perfect information and instantaneous optimal decision-making [6]. The extant literature applying EGT to low-carbon transitions has largely converged on two predominant model structures. First, studies on bilateral games between governments and enterprises. Seminal works in this vein, such as Jiao et al. [7], have examined how carbon quotas, trading prices, and supervision costs influence the evolutionary stability of corporate strategies. Similarly, Cao et al. [8] analyzed the impact of government regulation on the nascent and new energy vehicle industry, while Chen et al. [9] explored the strategic interplay from an energy security perspective. Second, studies on multi-level governance expand this dyad to incorporate multi-level governance, typically involving central and local governments alongside enterprises. For instance, Lian and He [10] constructed a tripartite model to simulate how parameter changes affect strategic choices of central government, local government, and firms. Shen et al. [11] further extended this to a four-party model, including the public, by analyzing stable strategy combinations in environmental governance.

Third, while studies incorporating third-party actors or collusion risks have undoubtedly enriched our understanding of low-carbon governance dynamics, they exhibit critical theoretical limitations that we seek to address. The most pronounced gap is the inadequate treatment of Third-Party Testing Organizations (TTOs). TTOs are independent, accredited entities responsible for verifying and certifying corporate carbon emission data, ensuring the accuracy and credibility of environmental information submitted to regulators. In reality, due to resource constraints, limited professional expertise, impracticality of real-time, and comprehensive monitoring by government bodies, TTOs have become indispensable actors in the carbon verification ecosystem. Their independence and professionalism are intended to ensure the objectivity and accuracy of carbon emission data, thereby underpinning the credibility of the entire regulatory system. However, existing EGT models have either completely overlooked TTOs or, at best, relegated them to a passive, exogenous factor that merely influences the payoffs of primary players (government and enterprises), without granting them agency as strategic players in their own right. This omission constitutes a significant simplification of the real-world governance structure.

Furthermore, the existing literature pays insufficient attention to the pervasive risk of collusion and rent-seeking, a vulnerability that is inherent in the delegation of monitoring authority [12,13]. The strategic choice for a manufacturing enterprise engaged in non-low-carbon production is not only to comply or not, but often involves the decision on whether to seek illicit cooperation from a TTO to fabricate compliance reports [14,15]. Conversely, the TTO faces a strategic choice between upholding professional integrity and succumbing to the temptation of illicit gains offered by enterprises. This potential for collusion creates a "second-order" moral hazard that can severely undermine the effectiveness of top-down regulatory policies [16,17]. The failure of existing game-theoretic models to incorporate this critical strategic dimension— the rent-seeking link between enterprises and TTOs—amounts to an incomplete and potentially optimistic portrayal of the low-carbon governance system [18]. The claimed novelty of simply adding a third player (i.e., TTO) is, therefore, insufficient. Specifically, although Guo et al. [19] constructed a tripartite model involving government, enterprises, and inspectors, their study focused on general supervision of industrial upgrading and did not explicitly model the rent-seeking channel as a strategic choice of third-party verifiers for carbon data fabrication. Similarly, Weng et al. [12] examined quality supervision under reward-punishment mechanisms but treated the verifier's role as passive, without endogenizing the TTO's decision to accept or reject rent-seeking. Our work differs fundamentally by making the TTO an active strategic player whose collusion decision is endogenous, thereby uncovering the collusion dynamics that are central to the credibility of carbon verification. The true theoretical innovation lies in modeling the strategic interdependence, including the rent-seeking channel [1].

This study aims to bridge these gaps by developing a refined tripartite evolutionary game model that explicitly incorporates governments, manufacturing enterprises, and TTOs as boundedly rational and strategic actors. The model's core theoretical contribution is its formalization of the rent-seeking relationship as an integral part of the strategic landscape, allowing for a systemic analysis of how collusion risks influence the stability and efficacy of low-carbon governance. Specifically, this study is guided by the following research questions: (1) How do the strategic interactions among these three actors evolve over time, particularly under the shadow of potential rent-seeking? (2) What are the asymptotic stability conditions for different strategic combinations, and which parameters are most critical in driving the system towards a desirable equilibrium (i.e., enterprise low-carbon production, government active regulation, TTO refusal of rent-seeking)? (3) How do the initial strategy probabilities of each party shape the evolutionary trajectory of the entire system?

To address these questions, we constructed a replicator dynamic system and employed Jacobian matrix stability analysis to identify evolutionarily stable strategies (ESS). Numerical simulations were then conducted to validate the theoretical findings and explored the dynamic sensitivity of the system to key parameters, such as the cost differential between production modes, rent-seeking costs, forgery expenses, and the magnitude of governmental fines and rewards.

This study offers several significant contributions to the literature. Theoretically, it advances EGT literature on low-carbon governance by providing a more holistic and realistic framework that captures critical, yet previously underexplored, strategic vulnerability -collusion. It moves beyond the simplistic two-party or multi-level government models to a more nuanced tripartite structure with an endogenous rent-seeking channel. Practically, the findings offer empirically grounded insights for policymakers seeking to design robust, anti-collusion regulatory mechanisms. By identifying the key leverage points and threshold effects that encourage compliance and deter fraud, this study can inform the design of more effective carbon verification systems, which in effect enhances the integrity and success of the global low-carbon transition in the manufacturing sector.

The remainder of the study is organized as follows. Section 2 details the methodology, including problem description, model assumptions, and payoff matrix construction. Section 3 derives the expected returns and replication dynamic equations for each player. Section 4 involves a comprehensive stability analysis of the strategic choices for all parties and equilibrium points of the tripartite system. Section 5 presents and discusses the results of numerical simulations. Section 6 synthesizes the findings and proposes targeted policy recommendations. Finally, Section 7 concludes the study by summarizing the key insights, acknowledging limitations, and suggesting avenues for future research.

2. Methodology

2.1. Problem Description

Low-carbon development is a critical component of manufacturing enterprises' reforms. Moreover, pathways to achieve high-quality development and mechanisms to maximize interests among all parties have become key concerns for manufacturing enterprises [20]. In the process of environmental governance with low-carbon emission reduction, the participants are mainly the government, manufacturing enterprises, and the public [21]. In view of this, all the parties are expected to meet the assumption of limited rationality in the evolutionary game [22,23]. Therefore, this study intends to construct a three-party evolutionary game model that includes the government, manufacturing enterprises, and TTO, with the relationships between each subject as shown in Figure 1. The strategy set of the government is {active regulation, passive regulation}. On the other hand, the strategy set of manufacturing enterprises is {low-carbon production, non-low-carbon production}, and the strategy combination of third-party testing organizations is {accepting rent-seeking, not accepting rent-seeking}.

2.2. Assumptions and Revenue Expectation Function Construction

Based on the limited rationality of government, enterprises, and third-party carbon emission testing institutions, coupled with the research needs and the actual situation, this study makes the following assumptions.

Assumption 1. *The subjects of the game are manufacturing enterprises, the government, and TTO, and all parties in the game have limited rationality [24].*

Assumption 2. *The probability of manufacturing enterprises choosing low-carbon production is x ($0 \le x \le 1$), and the probability of selecting non-low-carbon production is $1 - x$; the probability of government choosing active supervision is y ($0 \le y \le 1$) and the probability of choosing passive supervision is $1 - y$; the probability of TTO refusing rent-seeking is z ($0 \le z \le 1$) and the probability of accepting rent-seeking is $1 - z$ [25].*

Assumption 3. *The manufacturing enterprises can get subsidy m only when they genuinely adopt low-carbon production and their emission report is verified as compliant, and the cost of choosing low-carbon production and non-low-carbon production is c_1 and c_2 respectively. When manufacturers conduct low-carbon production activities, their carbon emissions meet the standard level; when manufacturers produce non-low-carbon production, they will seek rent from TTO to maximize their own interests. The rent-seeking cost is c_3, and $c_3 < (c_1 - c_2)$. The opportunity cost for manufacturing enterprises to miss the low-carbon transition is c_4 [24].*

Assumption 4. *The testing income of the TTO is r, which is earned regardless of the enterprise's production mode, as long as the TTO performs the verification service. When the enterprise chooses non-low-carbon production, if the TTO refuses rent-seeking, the carbon emission detection will not reach the standard level. If the TTO accepts rent-seeking, it will help the enterprise forge the report of carbon emission compliance. The cost of forgery is c_5, and $c_5 < c_3$ [25].*

Assumption 5. *When the government takes the initiative to regulate, manufacturers will be fined f for choosing non-low-carbon production. Given this scenario, the TTO will be fined f_1. Moreover, if the manufacturer chooses the low-carbon production option, it will receive a reward j. And if the TTO does not practice favoritism, it will be rewarded j_1. The cost of active regulation by the government is c_6. When the government passively regulates, it will only maintain the minimum supervision intensity, which will cost c_7, and $c_6 > c_7$ [26].*

Assumption 6. *When the manufacturing enterprise conducts a low-carbon production initiative, the government's social and economic benefits are r_1. However, if the manufacturing enterprises choose non-low-carbon production and the TTO accepts rent-seeking, the government needs to incur cost c_8 to promote low-carbon policies, regulate the manufacturing enterprises and the TTO. If the manufacturing enterprise carries out non-low-carbon production and the testing organization accepts rent-seeking, the government will subject it to administrative punishment from the higher authorities f_2, $f_2 > c_6$ [25,26]. The specific symbol description is shown in Table 1.*

In this study, a combination matrix of manufacturing enterprises, governments, and TTO is constructed, as shown in Table 2. From the parameters in the definition settings in Tables 1 and 2, the payoff function of the three participants under different policy selections can be obtained. The results are shown in Table 3.

Table 1. Main parameters and their interpretation.

Variables	Description
m	Subsidies that manufacturers can get when they genuinely meet carbon emissions standards and the report is verified.
c_1	The cost of manufacturing enterprises choosing low-carbon production.
c_2	The cost of manufacturing enterprises choosing non-low-carbon production.
c_3	Rent-seeking costs.
c_4	Due to rent-seeking behavior, manufacturing companies miss the opportunity cost of low-carbon transformation.
r	The testing revenue of third-party testing organizations.
c_5	The cost of forging carbon emission compliance reports by third-party testing organizations.
f	Fines imposed on manufacturing companies for choosing non-low-carbon production when the government actively regulates.
f_1	When the government actively regulates, third-party testing organizations that accept rent-seeking will be fined.
j	When the government actively regulates, manufacturing companies that choose low-carbon production will receive rewards.
j_1	Third-party testing organizations are rewarded for not practicing favoritism.
c_6	The cost of proactive government regulation.
c_7	When the government passively regulates, the cost required for the government to maintain the minimum level of regulation.
r_1	The social and economic benefits of the government when manufacturing enterprises engage in low-carbon production.
c_8	The cost required for the government to promote low-carbon policies, rectify manufacturing enterprises, and third-party testing organizations.
f_2	The administrative penalties faced by the government when adopting passive supervision.

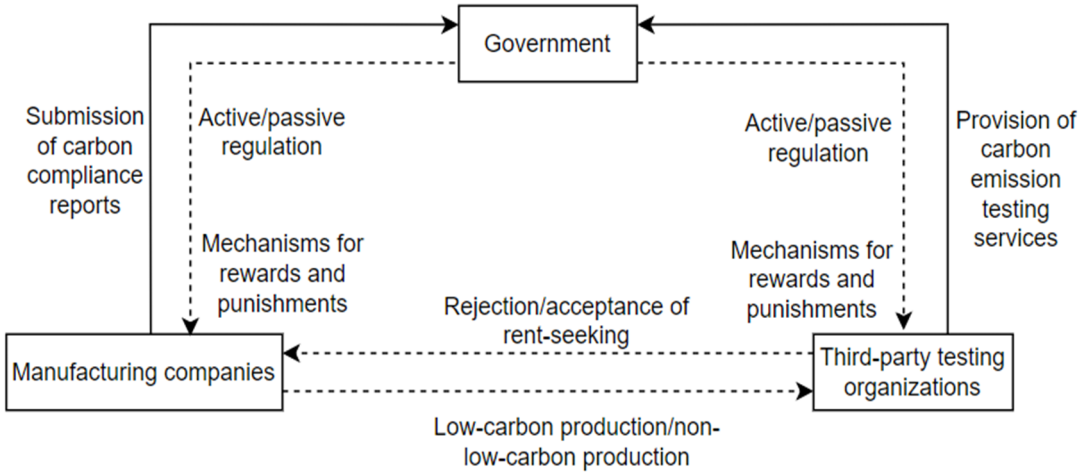


Figure 1. The three-party evolutionary game model.

Table 2. Combination matrix of the tripartite game strategies.

Manufacturing Enterprise	Government Initiative Supervision		Government Passive Regulation	
	TTO Do Not Accept Rent-Seeking	TTO Accept Rent-Seeking	TTO Do Not Accept Rent-Seeking	TTO Accept Rent-Seeking
Low carbon production	(a_1,b_1,c_1)	(a_2,b_2,c_2)	(a_3,b_3,c_3)	(a_4,b_4,c_4)
Non-low carbon production	(a_5,b_5,c_5)	(a_6,b_6,c_6)	(a_7,b_7,c_7)	(a_8,b_8,c_8)

Table 3. The return value of different game strategies.

Strategy Combination	Enterprise Income	Government Revenue	Revenue of TTO
(a_1,b_1,c_1)	$m - c_1 + j$	$r_1 - c_6 - j - j_1$	$r + j_1$
(a_2,b_2,c_2)	$m - c_1 + j$	$r_1 - c_6 - j + f_1$	$r - c_5 - f_1$
(a_3,b_3,c_3)	$m - c_1$	$r_1 - c_7$	r
(a_4,b_4,c_4)	$m - c_1$	$r_1 - c_7$	$r - c_5$
(a_5,b_5,c_5)	$-c_2 - c_4 - f$	$-c_6 - j_1 + f$	$r + j_1$
(a_6,b_6,c_6)	$-c_2 - c_3 - c_4 - f$	$-c_6 - c_8 + f + f1$	$r - c_5 - f_1 + c_3$
(a_7,b_7,c_7)	$-c_2 - c_4$	$-c_7$	r
(a_8,b_8,c_8)	$-c_2 - c_3 - c_4$	$-c_7 - c_8 - f_2$	$r - c_5 + c_3$

3. Expected Returns and Replicated Dynamic Equation

3.1. Enterprise Strategy

Based on the above propositions, the tripartite game revenue can be obtained. The expected returns when the manufacturing enterprise adopts low-carbon behavior and non-low-carbon behavior strategies are E_x and E_{1-x}, respectively. We can also get the average earnings E_1 for manufacturing companies as:

$$E_x = m - c_1 + jy \tag{1}$$

$$E_{1-x} = -c_2 - c_4 - fy - (1 - z)c_3 \tag{2}$$

$$E_1 = xE_x + (1 - x)E_{1-x} \tag{3}$$

According to evolutionary game theory, the replication dynamic equation of the enterprise behavior strategy is obtained from formulas (1)–(3):

$$F(x) = x(E_x - E_1) = x(1 - x)[m + c_2 - c_1 + c_4 + y(j + f) + c_3(1 - z)]) \tag{4}$$

3.2. Government Strategy

In this study, E_y represents the expected return when the government chooses active regulation. On the other hand, E_{1-y} represents the expected return when the government chooses passive regulation. From the foregoing, the average return on E_2 is calculated.

$$E_y = xz(r_1 - c_6 - j - j_1) + x(1-z)(r_1 - c_6 - j + f_1) + (1-x)z(-c_6 - j_1 + f) + (1-x)(1-z)(-c_6 - c_8 + f + f_1) \quad (5)$$

$$E_{1-y} = xz(r_1 - c_7) + x(1-z)(r_1 - c_7) + (1-x)z(-c_7) + (1-x)(1-z)(-c_8 - f_2 - c_7)) \quad (6)$$

$$E_2 = yE_y + (1-y)E_{1-y} \quad (7)$$

According to evolutionary game theory, the replicated dynamic equation can be obtained from formulas (5)–(7):

$$F(y) = y(1-y)[f + f_1 + f_2 - c_6 + c_7 - (j + f + f_2)x - (j_1 + f_1 + f_2)z + f_2 \cdot xz] \quad (8)$$

3.3. TTO Strategy

The present study uses E_z to express the expected income that the government chooses to refuse rent-seeking. On the other hand, E_{1-z} expresses the expected income that the government chooses to accept rent-seeking. Thus, we can calculate the average return of E_3 as:

$$E_z = xy(r + j_1) + x(1-y)(r) + (1-x)y(r + j_1) + (1-x)(1-y)r \quad (9)$$

$$E_{1-z} = r - c_5 + c_3(1-x) - yf_1 \quad (10)$$

$$E_3 = zE_z + (1-z)E_{1-z} \quad (11)$$

According to evolutionary game theory, the replicated dynamic equation can be obtained from formulas (9)–(11):

$$F(z) = z(1-z)[c_5 - c_3 + c_3x + y(f_1 + j_1)] \quad (12)$$

4. Strategy Combination Stability Test

4.1. Stability Analysis of the Enterprise Strategy

According to the stability theorem of differential equations, the probability of a firm's strategic choice can reach a stable state when the following conditions are met: $F(x) = 0$ and $\frac{dF(x)}{dx} < 0$. $x = 1$ and $x = 0$ represent the stable strategies of choosing low-carbon and non-low-carbon production, respectively.

Next, we analyzed the different scenarios.

$$y_0 = \frac{c_1 - c_2 - c_4 - m - c_3(1-z)}{f + j}$$

when $y > y_0$, $G = c_2 - c_1 + c_4 + m + y(j + f) + c_3(1-z) > 0$, so $\frac{dF(x)}{dx}|_{x=1} < 0$. At this point, $x = 1$ is in an evolutionary stable state.

When $y < y_0$, $G < 0$, so $\frac{dF(x)}{dx}|x = 0 < 0$. At this point, $x = 0$ is in an evolutionarily stable state. Enterprises are more willing to choose non-low-carbon production strategies.

The evolution of the enterprise behavioral selection strategy is shown in Figure 2. When the government chooses the probability $y > y_0$ of active supervision, the state of enterprise strategy selection is in space V_1 (Let V_1 represents the probability of enterprises engaging in low-carbon production). At this point, $x = 1$, is the equilibrium point, indicating that the enterprise is willing to engage in low-carbon production. When the government chooses the probability $y < y_0$ of active supervision, the state of enterprise strategy selection is in space V_2 (where V_2 represents the probability of enterprises engaging in non-low-carbon production). At this point, $x = 0$ is the equilibrium point. This indicates that companies are more willing to stick to the old ways and are unwilling to engage in low-carbon production. Overall, it can be seen that the government's attitude influences the behavioral strategy choices of enterprises.

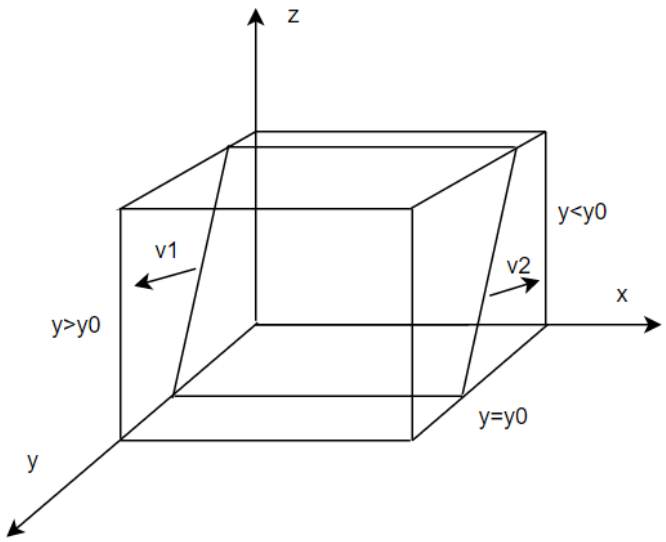


Figure 2. Evolution process of enterprise behavioral selection strategies.

Next, we performed the analysis of the parameters.

$$y_0 = \frac{c_1 - c_2 - c_4 - m - c_3(1 - z)}{j + f}$$

From Figure 2, the following phenomenon occurs when the other parameters remain constant.

When c_1 increases, y_0 increases. The cross-section moves forward. That is, the volume of V_1 decreases with the increase of c_1. This indicates that the higher the cost of low-carbon production for manufacturing enterprises, the lower the probability that they will choose it. At this point, the probability of enterprises choosing non-low-carbon production strategies is also higher. As c_2, c_3, c_4, m, f, and j increase, y_0 gradually decreases, and the cross-section shifts backwards. At this point, the volume of V_1 gradually increases with an increase in c_2, c_3, c_4, m, f and j. The above indicates that as c_2, c_3, c_4, m, f, and j increase, the probability of enterprises adopting low-carbon production strategies will increase. Consequently, their probability of choosing a non-low-carbon production strategy becomes smaller.

4.2. Stability Analysis of Government Strategy

According to the stability theorem of differential equations, if the probability of government strategy selection is in a stable state, it must satisfy the condition $F(y) = 0$ and $dF(y)/dy < 0$. From Equation (8), we have $F(y) = y(1 - y)\cdot H$, where $H = f + f_1 + f_2 - c_6 + c_7 - (j + f + f_2)x - (j_1 + f_1 + f_2)z + (c_6 - c_7 + f_2)xz$. Since H does not depend on y, the derivative is $dF(y)/dy = (1 - 2y)\cdot H$.

If $H = 0$, then $F(y) \equiv 0$, meaning any y is stable. If $H > 0$, then $dF(y)/dy \rightarrow \_\{y = 1\} = -H < 0$, so $y = 1$ (active regulation) is stable. If $H < 0$, then $dF(y)/dy \rightarrow \_\{y = 0\} = H < 0$, so $y = 0$ (passive regulation) is stable.

Inference 2:

The government chooses active regulation ($y = 1$) if $H > 0$, i.e.,: $f + f_1 + f_2 - c_6 + c_7 - (j + f + f_2)x - (j_1 + f_1 + f_2)z + (c_6 - c_7 + f_2)xz > 0$. Conversely, the government chooses passive regulation ($y = 0$) if $H < 0$.

The evolution process of government behavior selection strategies is shown in Figure 3. From Figure 3, it can be observed that when the probability of enterprises choosing low-carbon production (x) is low, H tends to be positive, leading the government to choose active regulation. When x is high, H tends to be negative, leading the government to choose passive regulation.

Parameter analysis:

From the expression of H, we can analyze the impact of key parameters on the government's regulatory choice. When f_1 (the fine on TTOs for accepting rent-seeking) increases, H becomes larger, making active regulation more likely. When c_7 (the cost of maintaining minimum regulation under passive supervision) increases, H also becomes larger, encouraging active regulation. When c_6 (the cost of active regulation) increases, H becomes smaller, making passive regulation more likely. When j (reward to enterprises for low-carbon

production) or j_1 (reward to TTOs for refusing rent-seeking) increases, H decreases, discouraging active regulation. These results are consistent with economic intuition.

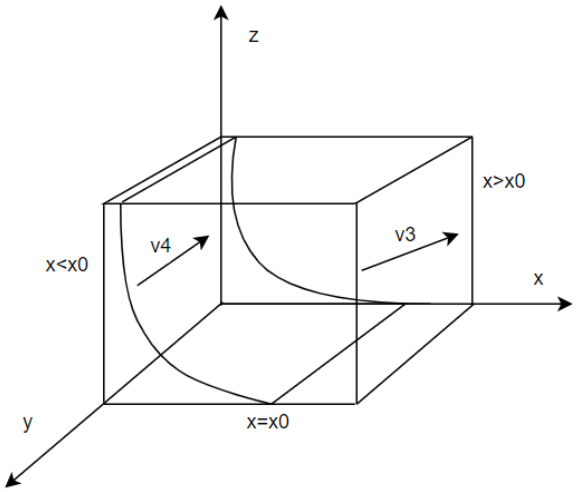


Figure 3. Evolution process of government behavioral selection strategies.

4.3. Stability Analysis of the Third-Party Testing Organization Strategy

According to the stability theorem of differential equations, if the probability of the strategy selection of the TTO is in a stable state, it must satisfy $F(z) = 0$ and $dF(z)/dz < 0$.

From Equation (12), we have $F(z) = z(1 - z)\cdot I$, where $I = c_5 - c_3 + c_3x + y(f_1 + j_1)$.

Since I does not depend on z, the derivative is $dF(z)/dz = (1\text{-}2z)\cdot I$.

If $I = 0$, then $F(z) \equiv 0$, meaning any z is stable. If $I > 0$, then $dF(z)/dz \rightarrow \_\{z = 1\} = -I < 0$, so $z = 1$ (refuse rent-seeking) is stable. If $I < 0$, then $dF(z)/dz \rightarrow \_\{z = 0\} = I < 0$, so $z = 0$ (accept rent-seeking) is stable.

Inference 3:

The TTO refuses rent-seeking ($z = 1$) if $I > 0$, i.e.,: $c_5 - c_3 + c_3x + y(f_1 + j_1) > 0$, or equivalently, $x > 1 - [c_5 + y(f_1 + j_1)]/c_3$ (assuming $c_3 > 0$). Conversely, the TTO accepts rent-seeking ($z = 0$) if $I < 0$.

The evolution process of the behavior selection strategy of the TTOs is shown in Figure 4. From Figure 4, it can be concluded that when the probability of enterprises choosing low-carbon production (x) is high, I tends to be positive, leading TTOs to refuse rent-seeking. When x is low, I tends to be negative, leading TTOs to accept rent-seeking.

From $I = c_5 - c_3 + c_3x + y(f_1 + j_1)$, we observe the following: when c_3 (rent-seeking cost paid to the TTO) increases, I decreases, making TTOs more likely to accept rent-seeking. When c_5 (forgery cost) increases, I increases, making TTOs more likely to refuse rent-seeking. When f_1 (fine for accepting rent-seeking under active government regulation) or j_1 (reward for refusing rent-seeking) increases, I also increases, making TTOs more likely to refuse rent-seeking. These findings are consistent with economic intuition.

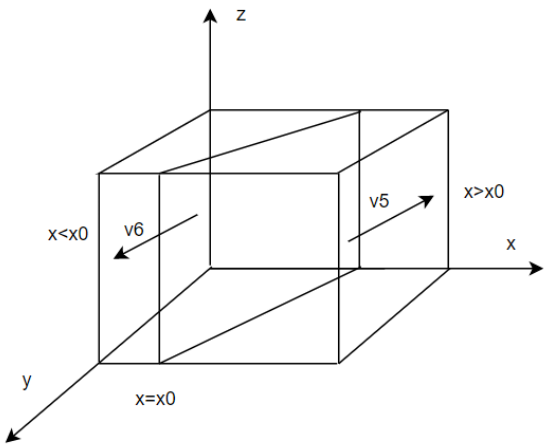


Figure 4. Evolution process of TTO behavioral strategies.

4.4. Stability Analysis of Equilibrium Points in the Tripartite Evolutionary Game System

The primary goal of enterprises and third-party testing institutions is to maximize their own benefits, while the government aims to maximize public welfare. In pursuit of their respective objectives, the three parties continuously adjust their strategies through mutual observation and learning until an optimal evolutionary stable strategy (ESS) is achieved [27]. Before identifying the ESS, it is essential to determine the equilibrium points of the evolutionary game. Thus, the replication dynamic differential equations for the strategic choices of the enterprises, third-party testing institutions, and the government are combined to form the following three-dimensional dynamical system:

$$F(x) = x(1-x)[c_2 - c_1 + c_4 + m + y(j+f) + c_3(1-z)]$$

$$F(y) = y(1-y)[f + f_1 + f_2 - c_6 + c_7 - (j + f + f_2)x - (j_1 + f_1 + f_2)z + (c_6 - c_7 + f_2)xz]$$

$$F(z) = z(1-z)[c_5 - c_3 + c_3x + y(f_1 + j_1)]$$

By setting $F(x) = 0$, $F(y) = 0$, and $F(z) = 0$, the strategic equilibrium points of the dynamical system can be obtained. Solving these equations yields eight pure-strategy equilibria: $E_1(0,0,0)$, $E_2(1,0,0)$, $E_3(0,1,0)$, $E_4(0,0,1)$, $E_5(1,1,0)$, $E_6(1,0,1)$, $E_7(0,1,1)$, and $E_8(1,1,1)$. If any one of the variables x, y, or z is set to 0 or 1, several mixed-strategy equilibrium points can be derived from the replication dynamic equations. However, due to the complex form of the eigenvalues of the corresponding Jacobian matrix, and given that the eigenvalue expressions for mixed-strategy equilibria are extremely complex and analytically intractable, this study follows the common practice in evolutionary game literature [24,25] by focusing on the pure-strategy equilibrium points. Mixed-strategy equilibria are not analyzed in detail, as they are less relevant for the clear policy implications derived from pure-strategy outcomes.

Next, the partial derivatives with respect to x, y, and z are calculated to obtain the Jacobian matrix J.

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \\ J_7 & J_8 & J_9 \end{bmatrix}$$

$$J_{11} = \partial F(x)/\partial x = (1-2x)[m + c_2 - c_1 + c_4 + y(j+f) + c_3(1-z)]$$

$$J_{12} = \partial F(x)/\partial y = x(1-x)(j+f)$$

$$J_{13} = \partial F(x)/\partial z = -c_3x(1-x)$$

$$J_{21} = \partial F(y)/\partial x = y(1-y)[-(j+f+f_2) + (c_6 - c_7 + f_2)z]$$

$$J_{22} = \partial F(y)/\partial y = (1-2y)[f + f_1 + f_2 - c_6 + c_7 - (j + f + f_2)x - (j_1 + f_1 + f_2)z + (c_6 - c_7 + f_2)xz]$$

$$J_{23} = \partial F(y)/\partial z = y(1-y)[-(j_1 + f_1 + f_2) + (c_6 - c_7 + f_2)x]$$

$$J_{31} = \partial F(z)/\partial x = z(1-z)c_3$$

$$J_{32} = \partial F(z)/\partial y = z(1-z)(f_1 + j_1)$$

$$J_{33} = \partial F(z)/\partial z = (1-2z)[c_5 - c_3 + c_3x + y(f_1 + j_1)]$$

Finally, the stability of the equilibrium points in the evolutionary game system is determined using Lyapunov's first method [25]. The specific procedure is as follows: each equilibrium point is substituted into the Jacobian matrix, and the eigenvalues of the matrix are computed. If the real parts of the eigenvalues of the Jacobian matrix are negative at a given equilibrium point, then that equilibrium is an evolutionarily stable state (ESS) [28]. Using MATLAB, the eigenvalues of the Jacobian matrix corresponding to each equilibrium point are calculated, and their stability is analyzed.

After substituting each equilibrium point into the corrected Jacobian matrix and computing the eigenvalues using MATLAB, we find that none of the eight pure-strategy equilibria has all eigenvalues with negative real parts

simultaneously under any feasible parameter set (with positive costs and penalties). In many cases, at least one eigenvalue is positive or zero. Therefore, the system does not possess a pure-strategy ESS. This result indicates that the low-carbon governance system with endogenous rent-seeking is path-dependent; its final state depends on initial conditions and parameter combinations rather than converging to a single fixed equilibrium. This finding is further validated by numerical simulations in Section 5.

5. Simulation Analysis

5.1. Simulation and Analysis of the Enterprise Strategy

$F(x) = x(E_x - E_1) = x(1-x)[c_2 - c_1 + m + c_4 + y(j+f) + c_3(1-z)]$ is the replicated dynamic differential equation of the firm. Our evolutionary game simulation analysis of the firms' strategy choices leads to Figures 5–7. The following conclusions can be drawn from Figures 5–7. To ensure the plausibility of simulation parameters, we anchor them to realistic ranges based on prior empirical studies and policy documents. For instance, penalties f and f1 are typically set at 2–3 times the cost savings from non-compliance; subsidies m and rewards j are generally lower than the cost difference $c_1 - c_2$. The baseline values adopted in this study (e.g., $f = 20$, $m = 10$, $j = 3$, $c_1 = 30$, $c_2 = 20$) satisfy the core assumptions $c_3 < c_1 - c_2$, $c_5 < c_3$, and $f > c_6$, ensuring that the simulation scenario reflects realistic incentive structures. These parameter settings are for illustrative purposes to demonstrate the model's behavioral predictions, and policymakers can adjust them based on specific regulatory contexts.

When $y0 <$ y, $y = 0.75$, $y = 0.8$, $y = 0.85$, $y = 0.9$, and $y = 0.95$, are all chosen for simulation analysis. Let $m = 10$, $c_1 = 30$, $c_2 = 20$, $c_3 = 4$, $r = 1$, $f = 20$, $f_1 = 5$, $j = 3$, $j_1 = 1$, $c_4 = 4$, $c_5 = 1$, $r_1 = 15$, $c_6 = 10$, $f_2 = 12$, $c_7 = 1$, $c_8 = 15$; when $y0 > y$, $y = 0.3$, $y = 0.4$, $y = 0.5$, $y = 0.6$, $y = 0.7$ are chosen for the evolutionary analysis. Furthermore, let $m = 10$, $c_1 = 40$, $c_2 = 12$, $c_3 = 4$, $r = 1$, $f = 10$, $f_1 = 5$, $j = 3$, $j_1 = 1$, $c_4 = 4$, $c_5 = 1$, $r_1 = 15$, $c_6 = 10$, $f_2 = 12$, $c_7 = 1$, $c_8 = 15$. Here, we assigned different values to y to carry out evolutionary simulation analysis on the enterprise's choice of low-carbon production strategy, as shown in Figure 5. According to Figure 5, when the probability of the government choosing an active regulation strategy is less than a certain fixed value, the probability of the enterprise carrying out a low-carbon production strategy tends to 0. Moreover, when the probability of the government choosing an active regulation strategy is greater than a certain fixed value, the probability of the enterprise choosing a low-carbon production strategy tends to 1.

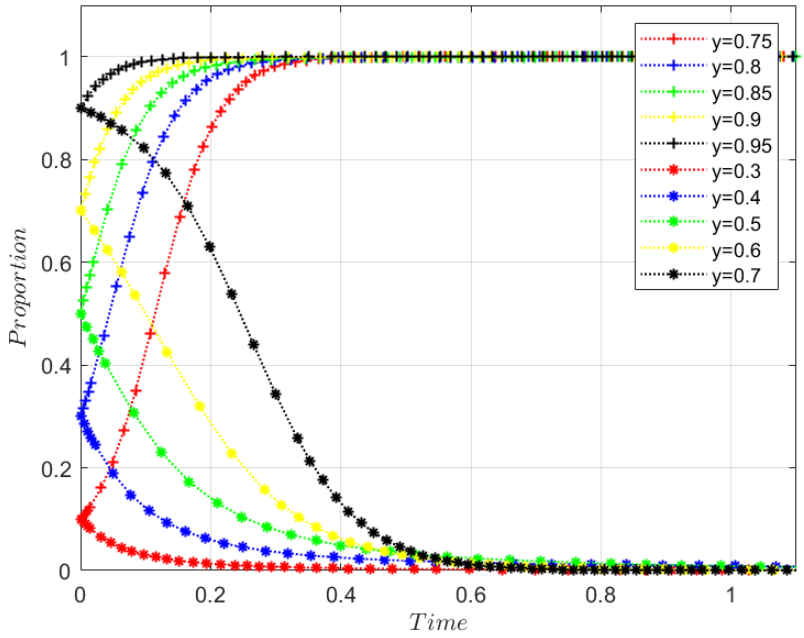


Figure 5. Evolutionary simulation of firms' strategic choices.

We still choose $m = 10$, $c_1 = 30$, $c_2 = 20$, $c_3 = 4$, $r = 1$, $f = 20$, $f_1 = 5$, $j = 3$, $j_1 = 1$, $c_4 = 4$, $c_5 = 1$, $r_1 = 15$, $c_6 = 10$, $f_2 = 12$, $c_7 = 1$, $c_8 = 15$, as the initial comparison state. The simulation of the factors affecting the enterprise strategy choice is shown in Figure 6. From Figure 6, it can be seen that when c_1 increases to 35, the probability of a firm's low-carbon production decreases. At this time, the probability of enterprise low-carbon production has a negative correlation with the cost of enterprise low-carbon production.

The probability of low-carbon production of enterprises increases when c_2 increases to 25, c_3 increases to 9, c_4 increases to 9, m increases to 15, f increases to 25, and j increases to 6. This indicates a positive correlation between the probability of low-carbon production and factors such as "the cost of non-low-carbon production", "rent-seeking cost", "the opportunity cost of manufacturing enterprises missing low-carbon transformation due to rent-seeking behavior", "subsidies to manufacturing enterprises after carbon emission standards", and "fines for manufacturing enterprises choosing non-low-carbon production when the government actively regulates" [29].

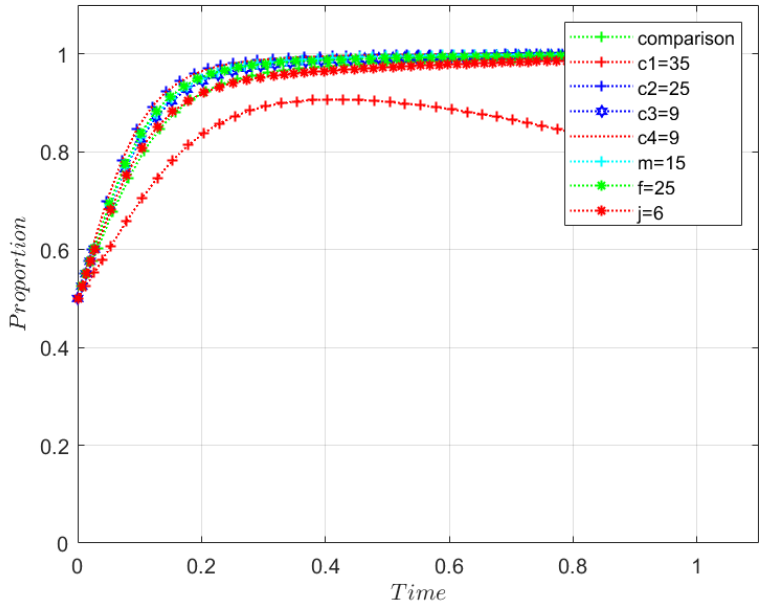
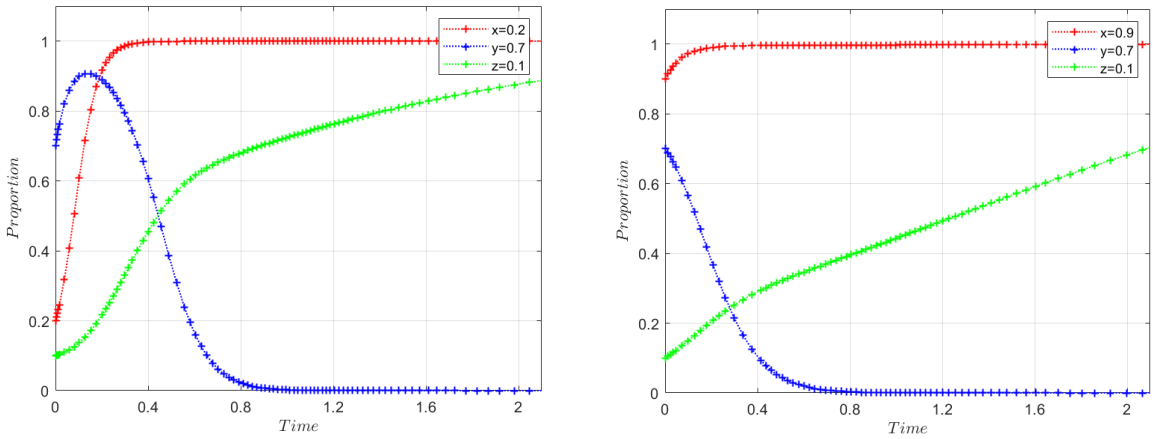


Figure 6. Simulation of factors influencing firms' strategy choices.

The simulation of enterprise strategies under different initial states is shown in Figure 7. When $t = 0, x = 0.2$ (the probability of enterprises choosing low-carbon production is small), the probability of enterprises choosing low-carbon production gradually increases as the probability of government-initiated regulation increases. However, as the probability of government-initiated supervision gradually decreases, the probability of enterprises choosing low-carbon production gradually decreases. In this case, the TTOs tend to reject rent-seeking proposals. When $t = 0, x = 0.9$ (the probability of enterprises choosing low-carbon production is larger). This implies that the enterprise's initial willingness to carry out low-carbon production is stronger. At this point, the government's low-carbon pressure is smaller for economic cost considerations, and thus, it gradually turned to a passive regulation mode. Enterprises will gradually shift to non-low-carbon production strategies and away from profit maximization considerations when they perceive a decline in government regulation.



(**a**) x = 0.2 (**b**) x = 0.9

Figure 7. Simulation of enterprise strategy selection under different initial states.

5.2. Simulation and Analysis of the Government Strategy

According to the government's replicated dynamic differential equation, the evolutionary game simulation analysis of the firm's strategic choices can be obtained in Figures 8 and 9. From Figures 8 and 9, the following conclusions can be obtained.

When the probability of enterprises choosing low-carbon production is relatively low (i.e., $H > 0$, which leads the government to actively regulate), we chose $x = 0.2$, $x = 0.25$, $x = 0.3$, $x = 0.35$, $x = 0.4$ for simulation analysis. Moreover, we made $m = 10, c_1 = 80, c_2 = 20, c_3 = 20, r = 10, f = 4, f_1 = 2, j = 20, j_1 = 15, c_4 = 10, c_5 = 1, r_1 = 1, c_6 = 10, f_2 = 12, c_7 = 1, c_8 = 15$, for simulation analysis. When the probability of enterprises choosing low-carbon production is relatively high (i.e., $H < 0$, which leads the government to passively regulate), we chose $x = 0.55$, $x = 0.65$, $x = 0.75$, $x = 0.85$, $x = 0.95$ for simulation analysis, and let $m = 30, c_1 = 42, c_2 = 20, c_3 = 11, r = 15, f = 30, f_1 = 40, j = 10, j_1 = 15, c_4 = 25, c_5 = 1, r_1 = 15, c_6 = 10, f_2 = 12, c_7 = 1, c_8 = 15$. A simulation analysis of the probability of the government choosing active regulation is shown in Figure 8. According to Figure 8, when the probability of enterprises choosing a low-carbon production strategy is less than a certain fixed value, the probability of the government choosing an active supervision strategy tends to 1. On the other hand, when the probability of enterprises choosing a low-carbon production strategy is greater than a certain fixed value, the probability of the government choosing an active supervision strategy tends to 0.

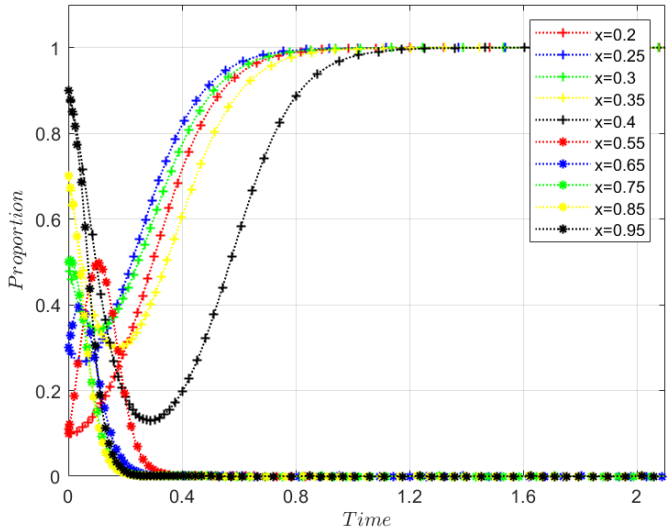


Figure 8. Evolutionary simulation of governmental strategy choices.

The evolutionary simulation of the government's strategic choice is shown in Figure 9. It can be observed from Figure 9 that when c_7 increases to 6 and f_1 increases to 17, the probability of government active regulation increases. Therefore, the probability of active government regulation has a positive correlation with "the fine that will be imposed if the TTOs accept rent-seeking when the government actively regulates", and "the cost of maintaining minimum regulation by the government when the government passively regulates". As c_6 increases to 11, j increases to 11, and j_1 increases to 17, the probability of government active regulation y decreases.

The simulation of government strategies under different initial states is shown in Figure 10. In particular, $when\ t = 0,\ y = 0.2$ (the probability that the government chooses to actively regulate is small), the government gradually shifts to actively regulate low-carbon pressure, and accordingly, the environmental pressure on the government increases. This also forces the probability of enterprises choosing low-carbon production and the TTOs choosing to reject rent-seeking to gradually increase. When the government's low-carbon pressure is small as a result of economic cost considerations, the government gradually turns to passive regulation. The enterprises will gradually shift to non-low-carbon production to maximize their profits when they find out that government regulation is declining. However, when the probability of enterprises choosing non-low-carbon production is less than a certain value, the probability of the government choosing active supervision will gradually increase. In this case, the TTOs tend to reject rent-seeking initiatives. Furthermore, when $t = 0$, $y = 0.7$, and at this point, the probability that the government chooses active regulation is larger. The evolutionary trend is roughly the same as when the government chooses the probability of active regulation ($y = 0.2$), but the evolution is faster. In addition, we found significant differences in the equilibrium results.

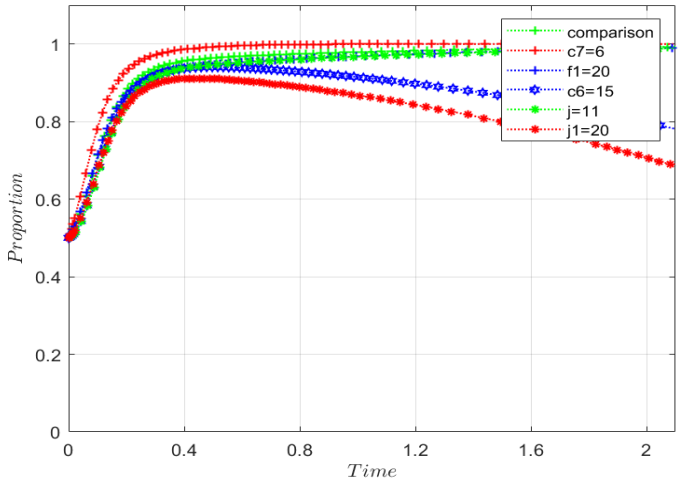
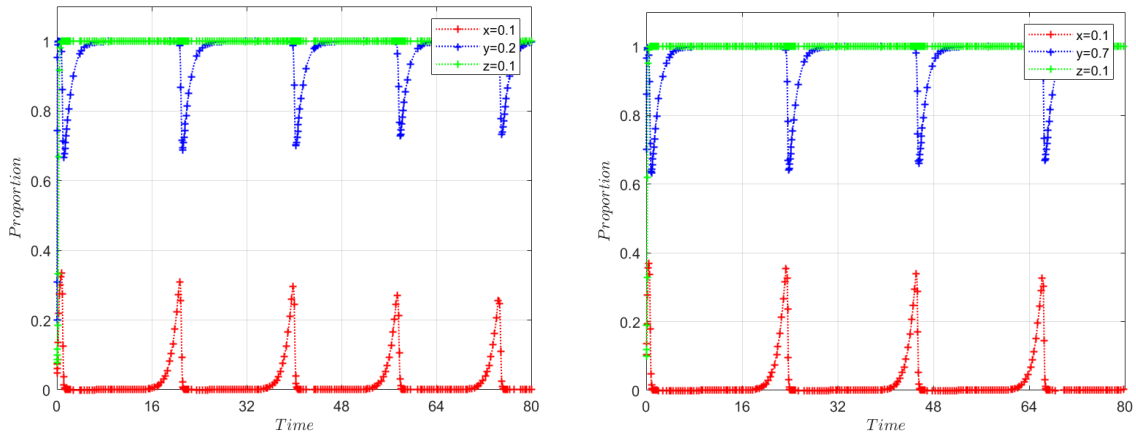


Figure 9. Simulation of Factors Influencing the Choice of Government Strategies.



(**a**) y = 0.2 (**b**) y = 0.7

Figure 10. Simulation of government strategy selection under different initial states.

5.3. Simulation and Analysis of the TTOs Strategy

According to the replicated dynamic differential equation of the TTO, we conducted the evolutionary game simulation analysis on the strategy selection of enterprises as follows:

When the probability of enterprises choosing low-carbon production is relatively high (i.e., $I > 0$, which leads TTOs to refuse rent-seeking), choose $x = 0.7$, $x = 0.75$, $x = 0.8$, $x = 0.85$, $x = 0.9$, and let $m = 20$, $c_1 = 40$, $c_2 = 20$, $c_3 = 5$, $r = 5$, $f = 30$, $f_1 = 20$, $j = 18$, $j_1 = 12$, $c_4 = 10$, $c_5 = 5$, $r_1 = 58$, $c_6 = 35$, $f_2 = 40$, $c_7 = 30$, $c_8 = 100$. We performed the simulation as follows. When the probability of enterprises choosing low-carbon production is relatively low (i.e., $I < 0$, which leads TTOs to accept rent-seeking), choose $x = 0.0$, $x = 0.2$, $x = 0.35$, $x = 0.55$, $x = 0.65$, and let $m = 20$, $c_1 = 80$, $c_2 = 10$, $c_3 = 55$, $r = 2$, $f = 30$, $f_1 = 10$, $j = 18$, $j_1 = 10$, $c_4 = 10$, $c_5 = 3$, $r_1 = 58$, $c_6 = 35$, $f_2 = 40$, $c_7 = 30$, $c_8 = 100$. The simulation analysis of the probability of TTOs choosing to reject rent-seeking strategies is shown in Figure 11. According to Figure 11, when x is less than a certain constant value, the probability of TTO choosing to refuse rent-seeking tends to zero. However, when the probability of a company choosing a low-carbon production strategy is greater than a specific value, the probability of TTO choosing to refuse tends to be 1.

The initial comparison state is $m = 20$, $c_1 = 40$, $c_2 = 20$, $c_3 = 15$, $r = 5$, $f = 30$, $f_1 = 20$, $j = 18$, $j_1 = 12$, $c_4 = 10$, $c_5 = 5$, $r_1 = 58$, $c_6 = 35$, $f_2 = 40$, $c_7 = 30$, $c_8 = 100$. The evolutionary simulation of strategic selection for TTO is shown in Figure 12. Specifically, it can be observed that when c3 increases to 18, the probability z of TTO refusing rent-seeking decreases.. This indicates a positive correlation between the probability of TTO accepting rent-seeking and the cost of rent-seeking. On the other hand, when c_5 increases to 8, f_1 increases to 25, and j_1 increases to 15, the probability of the TTOs refusing rent-seeking increases. This indicates that the probability of the TTOs refusing rent-seeking is positively correlated with these variables.

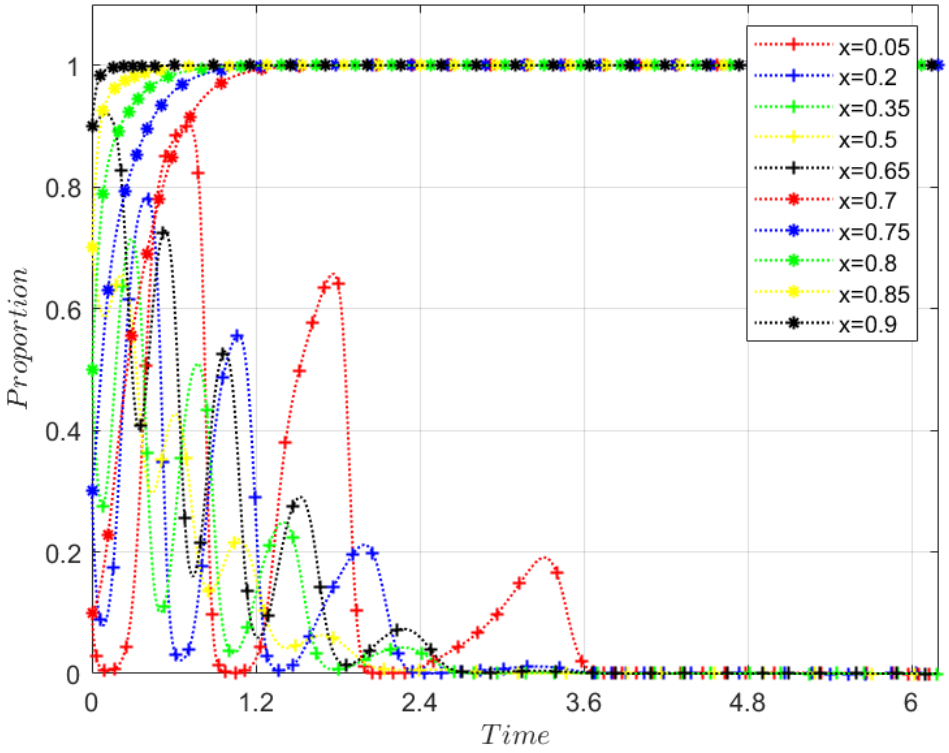


Figure 11. Evolutionary simulation of strategy selection of TTO.

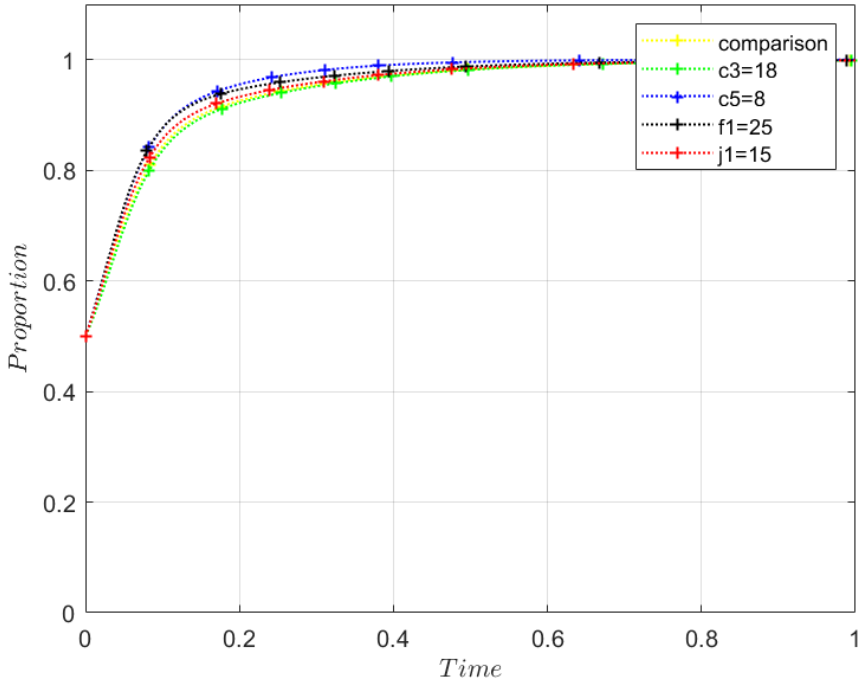
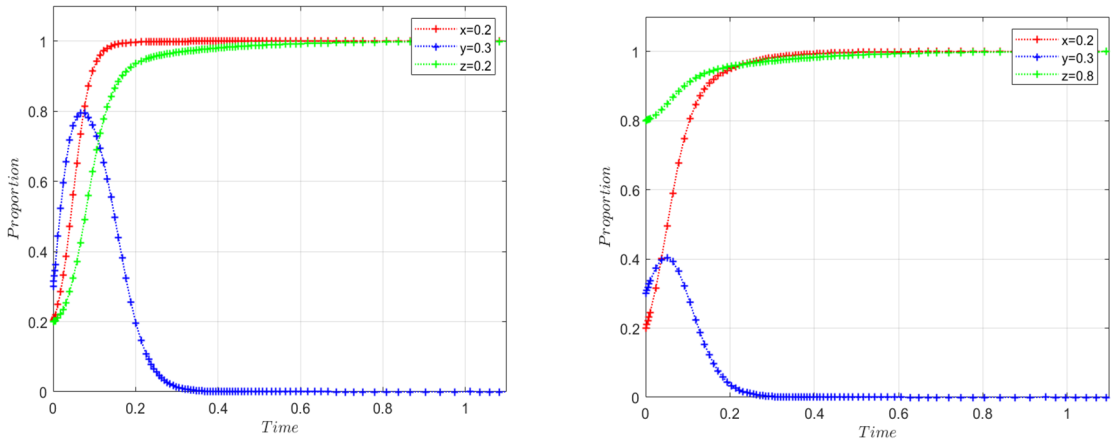


Figure 12. Simulation of factors influencing strategy selection of TTO.

The strategy selection simulation of TTO under different initial states is shown in Figure 13 ($m = 20$, $c_1 = 40$, $c_2 = 20$, $c_3 = 15$, $r = 5$, $f = 30$, $f_1 = 20$, $j = 18$, $j_1 = 12$, $c_4 = 10$, $c_5 = 5$, $r_1 = 58$, $c_6 = 35$, $f_2 = 40$, $c_7 = 30$, $c_8 = 100$). When $t = 0$ and $z = 0.2$, the probability of TTO refusing rent-seeking is small, and the probability of successful rent-seeking by enterprises is large. To avoid widespread rent-seeking, the government quickly chose to actively regulate this situation. However, when the probability of enterprises choosing low-carbon production and the TTOs refusing rent-seeking approaches 1, the government gradually shifts to passive regulation measures to maximize profits. Moreover, when $t = 0$ and $z = 0.8$, the probability of the

TTOs refusing rent-seeking is higher, and the willingness to refuse rent-seeking is strong. Given this condition, the probability of successful rent-seeking by enterprises is relatively lower. To avoid punishment from the government, the companies will reduce rent-seeking behavior and gradually shift towards low-carbon production options. But when the probability of enterprises choosing low-carbon production and the TTO refusing rent-seeking approaches 1, the government gradually shifts to passive regulation to maximize profit.



(**a**) z = 0.2 (**b**) z = 0.8

Figure 13. Simulation of strategy selection of TTO under different initial states.

6. Discussion and Implications

6.1. Discussion

As shown in Figure 5, when the probability of the government adopting active regulatory strategies is low, the probability of enterprises choosing low-carbon production approaches 0. This is because, when the probability of the government choosing to adopt an active regulatory strategy is lower, despite the initial strong willingness of enterprises to engage in low-carbon production, their initiative decreases as the government remains inactive [30]. At this point, choosing a low-carbon production strategy would increase the costs for enterprises. By considering profit maximization, enterprises gradually tend towards non-low-carbon production. However, when non-low-carbon production by enterprises leads to significant pollution, causing substantial losses to the public and society, the government, under pressure from upper-level authorities and environmental concerns, will strengthen its regulation on enterprises. To avoid government penalties, enterprises then begin to shift towards low-carbon production [31]. We also found that when the probability of the government choosing active regulatory strategies exceeds a certain threshold, the probability of enterprises opting for low-carbon production approaches 1. This is because, with increased governmental regulatory efforts, enterprises choose low-carbon production to avoid penalties. This is evidence that the choice of enterprise behavioral strategies is influenced by the likelihood of government supervision. Therefore, in the process of promoting low-carbon development among manufacturing enterprises, active government regulation is particularly crucial. This conclusion supports prior research conducted by Jiang et al. [28]. As observed in Figure 6, with an increase in c_1, the probability x of enterprises engaging in low-carbon production decreases. This is because, as the cost of low-carbon production for enterprises increases, their profits decline. Consequently, their willingness to engage in low-carbon production diminishes. When variables such as c_2, c_3, c_4, m, f, and j all increase, the probability of enterprises choosing low-carbon production x rises. This indicates a positive correlation between the probability of enterprises adopting low-carbon production and these variables. The reason is that as related factors (cost of non-low-carbon production for manufacturing enterprises, rent-seeking costs, opportunity costs missed by manufacturing enterprises due to rent-seeking behavior during low-carbon transition, subsidies available to manufacturing enterprises after reaching carbon emission standards, fines imposed on manufacturing enterprises for choosing non-low-carbon production under active government regulation, and rewards given to manufacturing enterprises for opting for low-carbon production under active government regulation) increase, the costs of non-low-carbon production for enterprises become higher while the benefits of low-carbon production increase [32,33]. Therefore, by considering profit maximization options, enterprises will choose a low-carbon production strategy. By comparing the simulation situations of enterprise strategies under different initial states (Figure 7), it can be observed that the initial state of enterprises

affects the evolutionary process, and the probability of enterprises engaging in low-carbon production plays a significant role in the choice of government strategies.

According to Figure 8, when the probability of an enterprise choosing a low-carbon production strategy is less than a certain value, the probability of the government choosing the active regulatory strategy will gradually change to 1. This is because when the probability of enterprises choosing to carry out low-carbon production initiatives is low, enterprises are not only unable to effectively use green resources, but will also cause severe pollution, bringing heavy losses to the public and society as found by Zhu et al. [31]. To avoid this situation, the government will strengthen the regulatory schemes of corporate behavior. When the probability of an enterprise choosing a low-carbon production strategy is greater than a certain threshold, the probability of the government choosing an active regulatory strategy tends to be 0. This is because when the probability of enterprises choosing a low-carbon production strategy is higher, enterprise production has less impact on the environment [34]. At this point, the interests of the public and society will not be negatively affected, and the government's low-carbon pressure will be less. More specifically, the government tends to regulate passively to maximize profits. It can be seen that the strategic choice of government behavior is affected by the probability of enterprises choosing low-carbon production. As observed in Figure 9, "the probability of the government's active supervision" is positively correlated with what "the TTO is fined when the government is actively supervising" and "the cost of the government maintaining the minimum supervision when the government is passively supervising". The reason is that when the government actively regulates, the TTOs accepting rent-seeking will be fined more. When the government passively regulates, the increase in the cost of maintaining the minimum level of regulation by the government leads to an increase in the benefits of active regulation compared to passive regulation. In addition, the study found a negative correlation between "the probability of government active supervision" and "the cost of active supervision", "when the government actively supervises, the manufacturing enterprises will be rewarded for choosing low-carbon production", and "if the TTOs do not accept rent-seeking". This is because an increase in government's active regulatory cost and the reward of the TTOs leads to an increase in economic burden of the government. In order to relieve the economic pressure, the government will relax the regulation of enterprises and increase the probability of choosing the passive regulatory strategy [35]. From Figure 10, it can be found that the equilibrium results are different when the initial states of the participants are different. Through the analysis of the two situations above, it can be found that when the initial state of the TTOs and enterprises remains unchanged and the probability of the government choosing the active regulatory changes, the probability of the government choosing the active regulatory strategy affects the probability and evolution process of enterprises choosing the low-carbon production strategy.

According to Figure 11, as the probability of enterprises choosing low-carbon production strategies decreases, they strive to obtain government subsidies and engage in rent-seeking activities. With more and more rent-seeking behaviors, the TTOs will gradually turn to accept rent-seeking in order to maximize their own interests. On the other hand, when the probability of an enterprise choosing a low-carbon production strategy is greater than a specific value, the probability of a TTO choosing to reject tends to 1. The study found that when the probability of enterprises choosing a low-carbon production strategy is higher, the rent-seeking behavior. At this point, the TTOs' acceptance of rent-seeking income is low, and they still have to face the government's supervision and punishment. Therefore, due to economic, reputation, and other factors, the TTOs will refuse rent-seeking behavior. This shows that the behavioral choice of the TTOs is influenced by the choice of enterprise strategies. It can be seen in Figure 12 that the probability of accepting rent-seeking by a TTO is positively correlated with the rent-seeking cost. This conclusion is consistent with the findings of Guo et al. [27]. This may be because even though there is a high cost of rent-seeking, the higher the income that can be obtained, the higher the probability of accepting rent-seeking [36]. When c_5, f_1, and j_1 increase, the probability of the TTOs refusing rent-seeking z increases. This indicates that there is a positive correlation between "the probability of the TTOs refusing rent-seeking" and "the forgery cost of the TTO forging the carbon emission standard report", "the TTOs accept rent-seeking when the government actively supervises", and "the TTOs will be rewarded if they do not practice for personal gain". This is because when the cost of forgery increases, along with the fines and rewards, not only maximize the self-interest of TTOs when rejecting rent-seeking, but also enhance their willingness to do so. By comparing Figure 13, the authors found that, under the condition that the initial states of the government and enterprises remain unchanged, the probability of the TTOs refusing rent-seeking is affected by the probability of low-carbon production by enterprises and the probability of active government regulation. From the above two scenarios, the authors observed that the initial state of the TTO affects the evolutionary process. In addition, the probability of TTOs refusing rent-seeking plays an important role in guiding the behavioral strategies of enterprises and the government.

6.2. Managerial and Policy Implications

This study models the strategic interactions among manufacturers, governments, and third-party testing organizations (TTOs) during the low-carbon transition. Beyond theoretical insights, the findings provide concrete recommendations for policymakers and managers seeking to foster credible and collusion-resistant carbon governance, as shown below.

(1) Strategic and Adaptive Regulation for Policymakers

Government intervention is essential and indispensable. However, it must be strategically calibrated. A key finding of the study is that the effectiveness of oversight depends less on its constant intensity and more on its intelligent design and adaptability. First, regulatory mechanisms should be dynamically aligned with economic realities. The model shows that a persistent high-cost gap between low- and high-carbon production can weaken compliance, even under strict supervision. In view of this, governments should implement flexible incentive schemes where subsidies and tax benefits for green technology are periodically reviewed and adjusted in response to evolving production costs and market behaviors. This will ensure that financial support effectively bridges the competitiveness gap for early adopters. Second, a robust deterrent system against verification fraud needs urgent consideration. The stability of systems where the TTOs act with integrity hinges on imposing meaningful penalties for collusion that outweigh its potential gains. In light of this, policymakers must establish significant and consistent fines for TTOs involved in falsifying reports, paired with tangible rewards for auditors demonstrating sustained integrity. This will create a clear economic framework which favors honest verification. Finally, regulatory frameworks should be adaptive. Specifically, the evolutionary paths indicate that once a high compliance norm is established, regulatory intensity can be cautiously scaled back without causing reversal to reduce administrative costs. This will promote risk-based monitoring frameworks where inspection frequency is dynamically tailor-made to the past compliance performance of firms and the audit history of their assigned TTOs. In effect, this will optimize the allocation of regulatory resources.

(2) Proactive Transition and Credible Reporting of Manufacturing Firms

For enterprises, the transition is seen as a strategic investment rather than mere compliance. In particular, the model underscores that long-term advantage resides in internalizing decarbonization into core business strategic frameworks.

In light of this, firms should proactively invest heavily in low-carbon technologies and processes. By anticipating that policies will increasingly internalize the ecological costs of carbon-intensive production via mechanisms such as carbon taxes or stricter emissions caps, forward-looking companies can gain a first-mover advantage. Arguably, proactive investment mitigates future cost shocks and avoids potential reputational damage from non-compliance. It also reduces exposure to the risks and expenses associated with seeking unapproved avenues for production.

Furthermore, companies, especially those within high transition-cost sectors, should be encouraged to invest in a transparent manner to enhance effective monitoring of their emissions. By making accurate, real-time data accessible, firms will constantly promote rent-seeking initiatives, cement their claim for sustainability, enhance their market trust, and investor confidence.

(3) Safeguarding Institutional Value of TTOs

TTOs are not considered passive intermediaries. In fact, they are guardians of systemic credibility. Arguably, their strategic choices overwhelmingly stimulate the entire ecosystem's trajectory toward integrity or collusion.

The primary recommendation for TTOs is to meticulously invest in and showcase their unique institutional integrity. This involves adopting technically innovative, transparent, and standardized verification protocols that are complex to compromise. By doing so, the TTOs inherently raise the forgery cost (c5), and thus make unapproved alterations economically unattractive. Moreover, the TTOs are encouraged to actively participate in industry-wide schemes that appreciate and reward long-term ethical performance, including public integrity rankings or premium accreditation status. These measures invariably transform professional honesty into an indispensable market differentiator.

Furthermore, the TTOs should actively embrace their roles as systemic actors. They should engage more in tripartite dialogues with regulators and industry associations. By providing guidance on compliance challenges and emerging fraud tactics, the TTOs can assist in shaping effective and pragmatic oversight mechanisms, and reinforce their position as indispensable partners in the low-carbon transition ecosystem.

(4) Integrated Governance Ecosystem

The model's sensitivity to initial conditions suggests that disjointed interventions are suboptimal. In particular, the most effective and potential pathway demands coordinated action across all three parties.

In view of this, we recommend fostering a collaborative governance platform involving standing forum representatives from governmental bodies, industry leaders, and accredited TTOs. This body will not set policy unilaterally. Per their terms of reference, they will be required to share data, identify early warning signs of market failure or collusion, and provide cogent feedback on the practical effects of regulatory parameters (like subsidy m or fine f1). Such continuous and structured dialogue can promote mutual understanding and calibrate policy instruments more responsively. It will also provide the necessary platform required to build the collective vigilance to sustain a transition that is both efficient and ethically sound.

7. Conclusions

The core theoretical contribution of this study is the application of rent-seeking dynamics into a tripartite evolutionary game framework. This demonstrates how collusion risks essentially reshape the strategic interplay among governments, enterprises, and third-party verifiers -a perspective principally absent in extant literature on two-party or multi-level governance models. According to the three-party evolutionary game model constructed in the present study, the analyses of the stability of each participant and associated simulations provided the following conclusion.

(1) At the enterprise level: (1) The selection of low-carbon production strategy by an enterprise is not only influenced by the probability of government active supervision, but also by the cost of low-carbon production; (2) The probability of low-carbon production among enterprises is negatively correlated with the cost of low-carbon production; (3) There is a positive correlation between "whether enterprises choose low-carbon production policies" and "the costs of non-low-carbon production", "rent-seeking costs", "the opportunity costs for manufacturing enterprises to miss low-carbon transformation due to rent-seeking behavior", "subsidies that manufacturing enterprises can obtain after meeting carbon emission standards", "fines imposed on manufacturing enterprises for choosing non low-carbon production when the government actively regulates", and "rewards received by manufacturing enterprises for choosing low-carbon production when the government actively regulates"; (4) The initial state of manufacturing enterprises affects the evolutionary process, and the probability of government active regulation plays an important role in the choice of corporate behavior strategies.

(2) At the government level: (1)The government decision to actively regulate is not only influenced by the probability of enterprises choosing low-carbon production, but also by the fines imposed on TTOs for rent-seeking when the government actively regulates; (2) The probability of active government regulation is positively correlated with the fines imposed on TTO for rent-seeking during active government regulation; (3) There is a negative correlation between "whether the government chooses active regulation" and "fines imposed on third-party testing organizations for rent-seeking", "costs of active regulation", and "rewards received by manufacturing enterprises for choosing low-carbon production when the government actively regulates".

(3) At the level of TTO: (1) The decision by the TTOs to choose to reject rent-seeking strategies is not only influenced by the probability of low-carbon production of enterprises, but also by the cost of rent-seeking; (2) The probability of TTO refusing rent-seeking is negatively correlated with the cost of rent-seeking; (3) "The probability of third-party testing organizations rejecting rent-seeking" is positively correlated with "the forgery cost of third-party testing organizations forgery of carbon emission compliance reports", "the fine that third-party testing organizations will be imposed if they accept rent-seeking when the government actively supervises them", and "third-party testing organizations will be rewarded if they do not engage in malpractice for personal gain"; (4) The initial state of the TTOs affects the evolutionary process, and the probability of the TTOs choosing to refuse rent-seeking plays an important role in the strategic choices of enterprises and governments. Furthermore, due to the absence of a pure-strategy ESS, the system's final outcome is path-dependent; coordinated policy mixes that simultaneously adjust subsidies, fines, and rewards are essential to steer the system towards a desirable low-carbon equilibrium. From a practical standpoint, the model serves as a decision-support tool for policymakers. By adjusting key parameters (e.g., fines, subsidies, rewards), regulators can simulate long-term behavioral responses of firms and TTOs under different policy mixes, thereby pre-testing the effectiveness and robustness of regulatory frameworks before implementation.

Author Contributions

J.Y.G.: conceptualization, methodology, formal analysis, supervision, project administration, funding acquisition, writing—original draft; C.W.: software, validation, formal analysis, data curation, writing—original draft; E.M.A.: investigation, resources, writing—review & editing; N.Z.: data curation, visualization, investigation;

X.L.: Writing—review & editing, Visualization, Software. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement

Not applicable. This study does not involve humans or animals. All data used in this study are derived from publicly available databases of listed companies.

Informed Consent Statement

Not applicable. This study does not involve human participants. All data analyzed in this study were obtained from publicly available databases of listed companies.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

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