

Distributed Two-Stage Fixed-Time Optimization Algorithm for Formation Control of Multi-Agent System Based on Local Voting Event-Triggered Mechanism

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Abstract: This article proposes a two-stage fixed-time distributed optimization algorithm based on a local voting event-triggered mechanism for continuous-time multi-agent systems, which simultaneously guarantees fast convergence, high 3D formation precision, and communication efficiency. This algorithm consists of two stages: the gradient-driven stage ensures each agent quickly converges from the initial position to the local optimal solution, and the event-triggered consensus stage realizes coordinated formation and cuts down communication frequency. In this case, this algorithm avoids the insufficient convergence speed existing in single-stage counterparts. First, an exponential trajectory compensation term is introduced into the second stage, aiming to eliminate precision loss induced by spatial dimensional conversion in formation control. Second, a local voting-based event-triggered mechanism that adopts exponential decay thresholds and introduces neighbor state feedback is developed, under which an agent can only trigger communication when its local state error exceeds the threshold and at least one neighbor satisfies the same triggering condition. The designed cooperative triggering strategy eliminates isolated triggering and stale information utilization, which reduces controller updates and communication consumption. Furthermore, a timeout fallback module is embedded to avoid triggering deadlock caused by strict neighbor-consensus constraints, ensuring stable and reliable triggering performance. Third, theoretical analysis is demonstrated that the proposed algorithm achieves the fixed time convergence, while achieving optimal formation control for multi-agent systems. It is also strictly proven that the proposed event-triggered mechanism avoids Zeno behavior. Finally, numerical simulations verified the precise convergence of the proposed algorithm to the optimal value in 3D formation tasks.

Keywords: fixed-time distributed optimization; formation control; event-triggered control; multi-agent system

1. Introduction

In recent years, formation control of multi-agent systems has become a pivotal research direction in coordinated optimization, with widespread applications in unmanned system formation [1], underwater vehicle collaboration [2], intelligent logistics [3], and complex infrastructure planning [4]. The core objective is to enable a group of agents to achieve and maintain a predefined spatial configuration through local information interaction and distributed decision-making, which provides essential support for complex engineering tasks. To achieve this objective, various distributed optimization algorithms have been developed to solve the formation control problem. Among them, continuous-time distributed algorithms have been widely investigated to ensure smooth state synchronization and formation execution. Specifically, to handle time-varying optimization under formation constraints, Wu et al. [5]

designed a continuous-time algorithm that enables agents to track dynamic optimal solutions while maintaining affine formation configurations. Shi et al. [6] proposed an extended zero-gradient-sum continuous-time approach for constrained distributed optimization, and Sun et al. [7] developed continuous-time algorithms for time-varying constrained convex optimization. To address the optimal formation with safety requirements, Huang et al. [8] developed a continuous-time distributed optimal formation control scheme for second-order multi-agent systems with obstacle avoidance. Furthermore, to realize complex formation maneuvers, Su et al. [2] proposed distributed time-varying optimization-based continuous-time protocols for affine formation maneuvering, achieving translation, rotation, and scaling of the formation. These continuous-time algorithms realize formation synchronization by continuously updating control inputs and exchanging information with neighbors. However, the continuous communication manner leads to excessive communication consumption and heavy computational burden. Furthermore, such continuous update mechanisms typically guarantee only asymptotic convergence, failing to meet the stringent convergence speed requirements of time-critical formation tasks.

To address the slow convergence issue of continuous-time algorithms, a large number of studies focus on asymptotic, finite-time, or fixed-time formation control algorithms, but such methods still face inherent algorithmic drawbacks that restrict their performance in time-sensitive scenarios. Specifically, for asymptotic formation algorithms, Chen et al. [9] proposed a dual-layer leader-follower strategy combined with backstepping techniques to achieve affine formation maneuvering, but this hierarchical algorithmic structure inherently relies on continuous neighbor communication and only guarantees asymptotic convergence, leading to slow convergence speed and high energy consumption. To accelerate convergence, finite-time distributed optimization algorithms have been developed. Liu et al. [10] proposed a continuous-time algorithm based on finite-time consensus for constrained convex optimization, and Shi et al. [11] designed a finite-time distributed average tracking algorithm for multi-agent optimization with bounded inputs, and Gao et al. [12] investigated finite-time event-triggered output consensus of heterogeneous fractional-order multi-agent systems. Although these finite-time algorithms improve convergence speed, their convergence time is inherently dependent on initial states; that is, under large initial deviations, the convergence time may become unpredictably long, failing to meet the requirements of high-reliability real-time tasks. This motivates the investigation of fixed-time distributed optimization algorithms. As theoretically proved by Yue et al. [13], fixed-time control ensures that the system converges to the desired configuration within a bounded time independent of initial conditions, making it suitable for time-critical tasks. Significant efforts have been devoted to fixed-time formation and optimization. Chen et al. [14] designed a fixed-time flocking algorithm for second-order multi-agent systems to realize coordinated motion. Ma et al. [15] proposed fixed-time distributed optimization via edge-based adaptive algorithms. Lei et al. [16] studied fixed-time distributed optimization for high-order nonlinear systems under switching topologies using a two-layer framework. Garg et al. [17] proposed fixed-time convergent algorithms for non-convex optimization problems. Wang et al. [18] applied fixed-time formation control to uncertain nonlinear systems with actuator failures, and Ma et al. [19] applied fixed-time formation tracking control for multi-aircraft. Furthermore, fixed-time consensus strategies have been developed to counteract uncertain disturbances under complex topologies [20]. Although predefined-time optimization algorithms have been proposed to achieve time-varying convergence and avoid singularity [21,22], they still rely on single-stage frameworks and strict parameter tuning. Most existing fixed-time methods adopt a single-stage algorithmic framework, which cannot simultaneously guarantee fast transient convergence from arbitrary initial positions and high-precision formation maintenance. From a theoretical perspective, this explicit decoupling design has two distinct advantages over single-stage counterparts such as [14,23]. First, in terms of Lyapunov function construction, single-stage fixed-time methods have to incorporate both gradient optimization terms and consensus terms into a unified control law. The corresponding Lyapunov function needs to bound both optimization error and formation consistency error simultaneously, which introduces coupled cross terms in the derivative derivation. To ensure the negative definiteness of the Lyapunov derivative, more conservative inequality scaling has to be adopted. In contrast, our two-stage framework constructs independent Lyapunov functions for the local optimization stage and the global consensus stage respectively, with no coupled cross terms. This simplifies the stability derivation and avoids redundant inequality scaling. Second, regarding the tightness of convergence bounds, the settling time bound of single-stage methods is jointly determined by coupled optimization gains and consensus gains, making it difficult to tune parameters for a tight bound. In our design, the fixed-time bounds of the two stages are derived independently, and the total settling time is the sum of two independent bounds. Parameters of each stage can be tuned separately, which helps reduce the overall conservatism of the convergence bound. This motivates us to develop a two-stage fixed-time distributed optimization algorithm that explicitly decouples local optimization and global formation tasks. Furthermore, with the development of practical applications, 3D formation control has attracted extensive attention. However, the interaction between optimization dynamics and formation accuracy is rarely considered, especially in 3D scenarios where dimensional transformation causes unavoidable precision degradation [24,25].

Moreover, the above algorithms incur excessive communication consumption and heavy computational burden. Event-triggered mechanisms have been proven to be effective strategies to solve these problems, but existing designs still face critical limitations. Most event-triggered mechanisms determine triggering behaviors merely according to individual local state errors, without involving local voting mechanisms. For instance, Liu et al. [26] proposed an event-triggered communication scheme for nonlinear multi-agent optimization based solely on local state error thresholds. Gao et al. [27] designed an event-triggered approach for gradient tracking where each agent independently evaluates its triggering condition. Shi et al. [28] proposed an event-triggered distributed optimization algorithm over jointly connected balanced digraphs, but their triggering condition still solely depends on local state deviations. Such self-centered triggering policies may induce isolated triggers and outdated information utilization, resulting in degraded consensus performance. Furthermore, Song et al. [23] developed a fixed-time distributed optimization algorithm based on an event-triggered strategy, yet its independent triggering mechanism fails to balance communication efficiency and formation precision in complex scenarios. To improve triggering flexibility, Chen et al. [29] proposed a dynamic event-triggered control for predefined-time optimization, and Malik et al. [30] studied consensus-based event-triggered distributed optimization for cubic objective functions, but these schemes still rely on individual judgment without neighbor-supported validation. Guo et al. [31] studied distributed time-varying constrained convex optimization with finite-time and fixed-time convergence, but still adopted individual triggering policies. Event-triggered related discontinuous-communication schemes can also be applied to other cooperative control scenarios. For example, sampled-data mechanisms have been adopted for connectivity-preserving consensus of heterogeneous Euler-Lagrange systems [32], and event-triggered frameworks have also been utilized for connectivity-preserving formation control of heterogeneous multiple unmanned surface vehicles [33]. To overcome the isolated triggering issue, it is highly necessary to introduce a local voting mechanism. This mechanism ensures that an agent triggers only when both its own error constraint and the local voting requirement are met, specifically, at least one neighbor also satisfies the identical trigger criterion, which effectively avoids isolated and redundant triggers. However, solely relying on the local voting mechanism may lead to triggering deadlock, where an agent indefinitely waits for its neighbors to meet the local voting requirement. This motivates us to introduce a timeout fallback mechanism to guarantee reliable triggering when the voting condition is persistently unsatisfied, ultimately motivating us to develop a distributed event-triggered mechanism that integrates local error conditions, local voting, and timeout fallback.

The proposed local-voting triggering strategy abandons the single-agent independent triggering mode and introduces neighbor collaborative verification for trigger activation. Communication updates are executed only when neighboring agents meet the preset triggering conditions collectively. The voting-driven trigger judgment avoids unnecessary triggering behaviors caused by transient local state fluctuations of individual agents, which effectively reduces redundant communication overhead during the system convergence process. This multi-agent joint judgment mode integrates network neighbor correlation information into the triggering criterion, rather than relying only on isolated individual information. To further highlight the distinctions between our scheme and existing representative triggering strategies, key feature comparisons are summarized in Table 1.

Table 1. Comparison of typical event-triggered schemes.

Scheme	Information Dependence	Isolated Trigger	Deadlock Avoid	Compute Cost
Dynamic [29]	Self + local var	Yes	No	Low
Consensus-based [30]	Self + neighbor samples	Yes	No	Medium
This work	Self + buffered neighbors	No	Yes	Low

Based on the above analysis, this paper addresses three key drawbacks of existing approaches: the difficulty of balancing convergence speed and accuracy in single-stage frameworks, precision loss caused by dimensional transformation in 3D scenarios, and isolated triggering as well as potential deadlock under individual event-triggered rules. A two-stage fixed-time distributed optimization algorithm with a local voting-based event-triggered mechanism is proposed to solve the formation control problem of continuous-time multi-agent systems. The algorithm consists of a gradient-driven local optimization stage, an event-triggered consensus stage with exponential compensation, and a local voting-based event-triggered mechanism. The specific contributions are encapsulated as follows:

- (1) A two-stage fixed-time distributed optimization algorithm is proposed. This algorithm is divided into a gradient-driven local optimization stage and an event-triggered global consensus stage, which explicitly decouples local optimization tasks from global formation tasks. It overcomes the slow convergence inherent in conventional single-stage schemes and enables all agents to reach the desired formation within a fixed time. Compared with the single-stage methods in [14,23], which fail to achieve a good balance between convergence speed and formation accuracy, the proposed design achieves better overall performance.

- (2) A synergistic consensus design is constructed to simultaneously address spatial precision degradation and communication inefficiency in the second stage. On one hand, a state-dependent exponential decay compensation term is embedded into the control law to rectify accuracy loss induced by spatial dimensional conversion, which greatly enhances spatial adaptability compared with existing three-dimensional distributed formation strategies in [34]. On the other hand, a local voting-based event-triggered mechanism integrated with a timeout fallback strategy is developed to govern communication behaviors. Distinct from conventional individual policies in [29, 30] that easily lead to isolated triggers and stale information, this cooperative mechanism imposes dual constraints—an exponential decay threshold and a neighbor-supported voting requirement—on triggering actions, while the timeout fallback module prevents potential triggering deadlock. This integrated design maintains reliable performance in complex three-dimensional scenarios and balances communication efficiency with formation precision effectively.
- (3) We perform strict theoretical analyses for the proposed two-stage algorithm and the supporting event-triggered mechanism. The analytical results verify that the algorithm realizes fixed-time convergence and completes optimal formation control for multi-agent systems. Furthermore, strict mathematical proofs confirm that the local voting-based event-triggered mechanism successfully excludes Zeno behavior [35].

The remainder of this paper is organized as follows. Section 2 formulates the optimization problem and introduces preliminaries. Section 3 presents the two-stage fixed-time distributed optimization algorithm and the local voting-based event-triggered mechanism. Section 4 provides simulation results to verify the effectiveness of the proposed method. Section 5 concludes this paper.

Notation: Let $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{N}^+$ represent the sets of real numbers, respectively. The vector $\mathbf{1}_n = [1, \dots, 1]^T \in \mathbb{R}^n$ and I_n denotes an $n \times n$ identity matrix. $\|\cdot\|$ denotes the Euclidean norm and $(\cdot)^T$ is the transpose of $(\cdot)$. Let $x_i = [x_{i1}, \dots, x_{in}]^T \in \mathbb{R}^n$, the gradient and Hessian matrix of function $f(x)$ are represented individually by $\nabla f(x)$ and $\nabla^2 f(x)$, $|x_i| = [|x_{i1}|, \dots, |x_{in}|]^T$, $x_i^\alpha = [x_{i1}^\alpha, x_{i2}^\alpha, \dots, x_{in}^\alpha]^T$, $\text{sig}^\alpha(x) = [\text{sig}^\alpha(x_{i1}), \text{sig}^\alpha(x_{i2}), \dots, \text{sig}^\alpha(x_{in})]^T$. $\text{sig}^\alpha(z) = \text{sign}(z)|z|^\alpha$ if $z \in \mathbb{R}$, where $\text{sign}(\cdot)$ is a sign function. $\lambda_2(A)$ is the algebraic connectivity of symmetric matrix A .

2. Problem Formulation and Preliminaries

2.1. Convex Property

A function $f: \mathbb{R}^m \rightarrow \mathbb{R}$ is convex if $f(x_1) - f(x_2) \geq \nabla f^T(x_2)(x_1 - x_2)$ holds for $\forall x_1, x_2 \in \mathbb{R}^m$. ∇f is θ -Lipschitz continuous if $\|\nabla f(x_1) - \nabla f(x_2)\| \leq \theta\|x_1 - x_2\|$, where θ is a positive constant.

2.2. Graph Theory

Let $G = (\mathcal{V}, \mathcal{E})$ denote a graph, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. The corresponding weighted adjacency matrix is $A(G) = \text{diag}(a_{ij}) \in \mathbb{R}^{N \times N}$ with $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Since there is no self-loop, $a_{ii} = 0 \forall i \in \mathcal{V}$. The neighbor set of agent i is defined as $N_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. An undirected graph is a particular case in which $A = A^T$. It is considered connected if there exists at least one path between any two agents. The degree of vertex i is defined as $d_i = \sum_{j \in N_i} a_{ij}$ and the degree matrix is defined as $D(G) = \text{diag}([d_1, \dots, d_N])$. If the graph is undirected and connected, the eigenvalues of the Laplacian matrix are $0 < \lambda_2(L) \leq \dots \leq \lambda_N(L)$, where $\lambda_2(L)$ is the algebraic connectivity of the graph.

2.3. Problem Formulation

The objective is to design the control input u_i for each agent which achieves leadless formation with fixed offsets $h_i \in \mathbb{R}^m$, $i \in \mathcal{V}$ around the global minimizer of an objective function consisting of a sum of local θ -strongly convex functions. In addition, it is required that the formation is achieved within a fixed-time $T_1 > 0$ using only information from the neighbors. It can be formulated as the following unconstrained optimization problem:

$$F(p) = \min_{x \in \mathbb{R}^m} \sum_{i=1}^N f_i(x) \quad \text{subject to} \quad \begin{cases} \lim_{t \rightarrow T_1} \sum_{i=1}^N \|p_i(t) - x^*\| = 0 \\ \sum_{i=1}^N \|p_i(t) - x^*\| = 0 \quad \forall t \geq T_1 \end{cases} \quad (1)$$

where $f_i: \mathbb{R}^n \rightarrow \mathbb{R}$ is the local objective function of agents, $h_i \in \mathbb{R}^n$ is a constant and defined as the formation configuration of agents, $p_i(t) = x_i(t) - h_i$ is the distance offset of the agent i , and $x^* \in \mathbb{R}^n$ is an unknown optimal formation center.

Consider continuous-time multi-agent systems with N agents as follows:

$$\dot{x}_i(t) = u_{i1}(t) + u_{i2}(t) \quad i \in \mathcal{V} \quad (2)$$

where $x_i(t) = [x_{i1}(t), \dots, x_{i2}(t)]^T \in \mathbb{R}^n$ is state of agent i , $u_i(t) = [u_{i1}(t), \dots, u_{i2}(t)]^T \in \mathbb{R}^n$ is the input of agent i .

2.4. Assumptions and Lemmas

The subsequent theoretical analysis relies on the following fundamental assumptions and supporting lemmas.

Assumption 1. Local objective function $f_i(x_i(t))$, $i \in \mathcal{V}$ is m_i -strongly convex and quadratically continuous differentiable. Additionally, there exist two constants m and M , where $m_i \geq m$, $M_i \leq M$, $\forall i \in \mathcal{V}$, and there is a local Lipschitz Hessian $\nabla^2 f_i(p_i(t))$.

Assumption 2. The communication topology G is fixed, connected and undirected graph that can be accessed by every intelligent agent.

Assumption 3. During the entire convergence process, the state of each agent remains uniformly bounded, i.e., there exists a positive constant R such that $\|p_i(t)\| \leq R$ holds for all $i \in \mathcal{V}$ and $t \geq 0$.

Lemma 1 ([2]). Let $k_1, k_2, \dots, k_N \geq 0$, then $\sum_{i=1}^N k_i^a \geq \left(\sum_{i=1}^N k_i\right)^a$ for $0 < a \leq 1$, and $\sum_{i=1}^N k_i^a \geq N^{1-a} \left(\sum_{i=1}^N k_i\right)^a$ for $a > 1$.

Lemma 2 ([3]). Consider a dynamic system, if there is a radially unbounded and continuous positive definite function $V(x(t)) : \mathbb{R}^n \rightarrow \mathbb{R}$ meeting $\dot{V}(x(t)) \leq -k_1 V^a(x(t)) - k_2 V^b(x(t))$ for any solution $x(t)$, where $k_1 > 0$, $k_2 > 0$, $0 < a < 1$ and $b > 1$, then the origin is globally fixed-time stable with setting time $T \leq \frac{1}{k_1(1-a)} + \frac{1}{k_2(b-1)}$.

Lemma 3 ([5]). For the Laplacian matrix of an undirected connected graph, where the minimum non-zero eigenvalue $\lambda_2(L)$ satisfies $\sum_{i=1}^N \sum_{j \in N_i} a_{ij} (p_i - p_j)^T \text{sig}^\alpha(p_i - p_j) \geq \lambda_2(L) \sum_{i=1}^N \|p_i\|^{\alpha+1}$.

3. Main Results

In order to seek the optimal position x^* in fixed time, a two-stage distributed optimization algorithm is proposed in this section. The first stage is to find out the locally optimal state of each agent in fixed time, and the second stage is to achieve the global optimum in fixed time.

For system (2), the control input law u_i is designed as:

Stage 1:

$$u_{i1}(t) = -\varepsilon_1 \text{sig}^{\mu_0}(\nabla f_i(p_i(t))) - \varepsilon_2 \text{sig}^{\nu_0}(\nabla f_i(p_i(t))) \quad (3)$$

Stage 2:

$$\begin{aligned} u_{i2}(t) = & -k_1 \left(\nabla^2 f_i(p_i(t_k^i))\right)^{-1} \sum_{j \in N_i} \text{sig}^{\mu_1} \left(p_i(t_k^i) - p_j(t_k^j)\right) \\ & - k_2 \left(\nabla^2 f_i(p_i(t_k^i))\right)^{-1} \sum_{j \in N_i} \text{sig}^{\nu_1} \left(p_i(t_k^i) - p_j(t_k^j)\right) \\ & - k \exp \left(\|p_i(t_k^i)\|^2\right) p_i(t_k^i) \end{aligned} \quad (4)$$

where $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, N_i is the neighboring set of agent i , $p_i = x_i - h_i$, $i \in \mathcal{V}$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $0 < \mu_0 < 1$, $\nu_0 > 1$; $k_1 > 0$, $k_2 > 0$, $0 < \mu_1 < 1$, $\nu_1 > 1$; $\nabla f_i(p_i(t))$ and $\nabla^2 f_i(p_i(t_k^i))$ are first-order and second-order derivatives of the objective function of the agent i ; t_k^i and t_k^j denote the triggered moments of the i th and j th agents.

To implement the local voting mechanism, the trigger condition for each neighbor of agent i is first defined.

For any neighbor $j \in N_i$, where N_i is the neighbor set of agent i , its state x_j , deviation relative to the global target $p_j = x_j - h_j$, and trigger error $e_j = p_j - \hat{p}_j(t_k^j)$ are computed. Here, h_j represents the global target of agent j , and $\hat{p}_j(t_k^j)$ denotes the deviation of agent j at its latest triggering instant.

The trigger condition for neighbor j is consistent with agent i , given by:

$$\|e_j\|^2 > \Theta_j(t) \quad (5)$$

where $\Theta_j(t)$ denotes the fixed threshold function of agent j . The term $\sum_{k \in N_j} (\hat{p}_j - \hat{p}_k)$ stands for the sum of deviation differences between agent j and its neighbors based on their latest triggering states. An indicator variable I_j is set to 1 if neighbor j satisfies the trigger condition, and 0 otherwise.

The triggered error is defined as:

$$e_i(t) = p_i - \hat{p}_i \quad (6)$$

The triggering condition for each agent i is defined by the local voting rule with a timeout fallback guarantee:

$$\left(\|e_i(t)\|^2 > \Theta_i(t) \wedge \sum_{j \in N_i} I_j \geq \gamma_i \right) \vee (t - t_k^i \geq T_{max}) \quad (7)$$

where $\Theta_i(t)$ is the threshold function of agent i . $\sum_{j \in N_i} I_j$ is the number of neighbors that satisfy the trigger condition, and γ_i is the local voting requirement. The voting requirement is set as $\gamma_i = 1$ for agents 1–14, and $\gamma_{15} = 0$ as a seed node to break potential system-level deadlock.

Remark 1. When one neighbor broadcasts its latest sampled state $p_j(t_k^j)$ together with the triggering timestamp, each agent locally stores the most recently received information of all its neighbors. During the voting-condition evaluation, agent i computes the triggering error $e_j(t)$ of its neighbors locally by virtue of the stored sampled data and known system dynamics. No additional real-time communication is required for voting judgment, and hence no extra communication burden is introduced.

The threshold function for agent i is defined as:

$$\Theta_i(t) = \eta \left\| \sum_{j \in N_i} (p_i(t_k^i) - p_j(t_k^j)) \right\|^2 + \phi e^{-\tau t} \quad (8)$$

This threshold is consistent with the neighbor trigger judgment threshold (5), ensuring uniform triggering standards for all agents in the local voting process.

Benefiting from the decoupled architecture, the Lyapunov functions and convergence bounds of the two stages are derived independently, which is consistent with the theoretical analysis in the introduction. Next, we conduct theoretical analysis. Theorem 1 proves that all agents converge to their local optimal solutions within the fixed time T_2 under the gradient-driven control law. Theorem 2 proves that all agents achieve global convergence to the optimal formation within the fixed time T_2 . Theorem 3 proves that the event-triggered interval has a strictly positive lower bound, thus excluding Zeno behavior for the entire system.

Theorem 1. If Assumption 1 and Assumption 2 hold, then all agents of system (2) can converge to the local optimal solution within a fixed time $T_1 \leq \frac{1}{2\varepsilon_1 m^{\mu_0} (1 - \frac{\mu_0+1}{2})} + \frac{1}{2\varepsilon_2 m^{\nu_0} N^{\frac{1-\nu_0}{2}} (\frac{\nu_0+1}{2} - 1)}$ under control law (3).

Proof. When $t \leq T_1$, construct a Lyapunov function:

$$V_1(t) = \sum_{i=1}^N \|p_i(t) - x_i^*\|^2 \quad (9)$$

where x_i^* is the local optimal solution, then there is $\nabla f_i(x_i^*) = 0$.

Let V_1 take the derivative of time t along x_i , we can obtain:

$$\begin{aligned} \dot{V}_1(t) &= 2 \sum_{i=1}^N (p_i(t) - x_i^*)^T \dot{p}_i(t) \\ &= -2\varepsilon_1 \sum_{i=1}^N (p_i(t) - x_i^*)^T \text{sig}^{\mu_0} (\nabla f_i(p_i) - \nabla f_i(x_i^*)) \\ &\quad - 2\varepsilon_2 \sum_{i=1}^N (p_i(t) - x_i^*)^T \text{sig}^{\nu_0} (\nabla f_i(p_i) - \nabla f_i(x_i^*)) \end{aligned} \quad (10)$$

Since (9) satisfies Assumption 1, utilizing the m_i -strong convexity of the local objective function, there is:

$$\begin{aligned} \dot{V}_1(t) &\leq -2\varepsilon_1 \sum_{i=1}^N \left| (p_i - x_i^*)^T (m_i (p_i(t) - x_i^*))^{\mu_0} \right| - 2\varepsilon_2 \sum_{i=1}^N \left| (p_i - x_i^*)^T (m_i (p_i(t) - x_i^*))^{\nu_0} \right| \\ &\leq -2\varepsilon_1 m^{\mu_0} \sum_{i=1}^N \left\| (p_i - x_i^*)^{\mu_0+1} \right\| - 2\varepsilon_2 m^{\nu_0} \sum_{i=1}^N \left\| (p_i - x_i^*)^{\nu_0+1} \right\| \end{aligned} \quad (11)$$

Derive from Lemma 1:

$$\begin{aligned}\dot{V}_1(t) &\leq -2\varepsilon_1 m^{\mu_0} \left(\sum_{i=1}^N \|(p_i - x_i^*)^2\| \right)^{\frac{\mu_0+1}{2}} - 2\varepsilon_2 m^{\nu_0} N^{\frac{1-\nu_0}{2}} \left(\sum_{i=1}^N \|(p_i - x_i^*)^2\| \right)^{\frac{\nu_0+1}{2}} \\ &= -2\varepsilon_1 m^{\mu_0} (V_1(t))^{\frac{\mu_0+1}{2}} - 2\varepsilon_2 m^{\nu_0} N^{\frac{1-\nu_0}{2}} (V_1(t))^{\frac{\nu_0+1}{2}}\end{aligned}\quad (12)$$

where $k_1 = 2\varepsilon_1 m^{\mu_0}$, $k_2 = 2\varepsilon_2 m^{\nu_0} N^{\frac{1-\nu_0}{2}}$, $a = \frac{\mu_0+1}{2}$, $b = \frac{\nu_0+1}{2}$

According to Lemma 2, we can obtain the upper bound of time T_1 :

$$T_1 \leq \frac{1}{2\varepsilon_1 m^{\mu_0} (1 - \frac{\mu_0+1}{2})} + \frac{1}{2\varepsilon_2 m^{\nu_0} N^{\frac{1-\nu_0}{2}} (\frac{\nu_0+1}{2} - 1)} \quad (13)$$

It should be noted that the above fixed-time bound is a conservative upper bound. The conservatism mainly comes from multiple inequality scalings in the derivation, including the strong convexity scaling of the local cost function, the norm inequality scaling, and the power mean inequality scaling introduced by Lemma 1. The conservatism can be adjusted by parameter selection: Increasing the control gains ε_1 and ε_2 will reduce the upper bound of settling time and lower the conservatism, but will increase the amplitude of control inputs. When the power parameters μ_0 and ν_0 are closer to 1, the inequality scaling becomes slighter and the convergence bound is tighter. A larger strong convexity parameter m leads to a tighter bound, while a larger number of agents N will increase the conservatism of the bound.

The proof of Theorem 1 is completed. $\square$

Theorem 2. Assuming that Assumptions 1 and 2 are satisfied, all agents of system (2) under control law (4) will converge to the global optimal formation configuration within a fixed time T_2 . Specifically, the offset state $p_i(t)$ converges to the global optimal solution x^* , and the actual state $x_i(t) = p_i(t) + h_i$ converges to $x^* + h_i$ to form the desired spatial formation.

Proof. Inspired by [28], we construct the Lyapunov function based on the state error with respect to the global optimal solution x^* :

$$V(t) = \frac{1}{2} \sum_{i=1}^N \|p_i(t) - x^*\|^2 \quad (14)$$

Since x^* is a constant optimal point, its time derivative is zero. Thus the derivative of the error satisfies $\frac{d}{dt}(p_i - x^*) = \dot{p}_i = u_{i2}(t)$.

Calculate the derivative of Lyapunov function:

$$\dot{V}(t) = \sum_{i=1}^N (p_i(t) - x^*)^T \cdot \dot{p}_i(t) = \sum_{i=1}^N (p_i(t) - x^*)^T \cdot u_{i2}(t) \quad (15)$$

Note that the control law $u_{i2}(t)$ is constructed based on the sampled states at the latest triggering instant $p_i(t_k^i)$ and $p_j(t_k^j)$, rather than the continuous states. Define the sampling error of agent i as:

$$e_i(t) = p_i(t) - p_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i) \quad (16)$$

Substituting the sampled states into the Lyapunov derivative yields:

$$\dot{V}(t) = \sum_{i=1}^N (p_i(t) - x^*)^T u_{i2}(p_i(t_k^i), p_j(t_k^j)) = \sum_{i=1}^N (p_i(t) - x^*)^T u_{i2}(p_i(t) - e_i(t), p_j(t) - e_j(t)) \quad (17)$$

Accordingly, $\dot{V}(t)$ is decomposed into two parts:

$$\dot{V}(t) = \dot{V}_{\text{nom}}(t) + \Delta V_e(t) \quad (18)$$

where $\dot{V}_{\text{nom}}(t)$ corresponds to the error-free continuous-time Lyapunov derivative, and $\Delta V_e(t)$ denotes the deviation term arising from event-triggered sampling errors.

Based on the event-triggered condition (7) and threshold function (8), the sampling error satisfies $\|e_i(t)\|^2 \leq \Theta_i(t)$. The threshold function $\Theta_i(t)$ is uniformly bounded during the finite-time convergence process, which guarantees the uniform boundedness of $e_i(t)$. Since the control law is Lipschitz continuous with respect to system states, the sampling-error-induced deviation term is bounded by:

$$|\Delta V_e(t)| \leq \bar{M} \sum_{i=1}^N |p_i(t) - x^*| \quad (19)$$

where $\bar{M} > 0$ is a constant.

Recall that $V(t) = \frac{1}{2} \sum_{i=1}^N \|p_i(t) - x^*\|^2$. By the Cauchy–Schwarz inequality:

$$\sum_{i=1}^N |p_i(t) - x^*| \leq \sqrt{2NV(t)} \quad (20)$$

Substituting the above bound yields:

$$|\Delta V_e(t)| \leq cV^{\frac{1}{2}}(t) \quad (21)$$

where $c = \bar{M}\sqrt{2N} > 0$. Combining with the nominal stability result, we obtain:

$$\dot{V}(t) \leq -\alpha_0 V^{a_0}(t) - \beta_0 V^{b_0}(t) + cV^{\frac{1}{2}}(t) \quad (22)$$

with $a_0 = \frac{\mu_1+1}{2}$ and $b_0 = \frac{\nu_1+1}{2}$, $0 < a_0, b_0 < 1$.

As $V(t) \rightarrow 0$, the negative-definite nominal convergence terms $-\alpha_0 V^{a_0}(t) - \beta_0 V^{b_0}(t)$ decay faster than the perturbation term $cV^{\frac{1}{2}}(t)$, and thus dominate the dynamics near the origin. Although the above differential inequality contains a state-dependent perturbation term induced by event-triggered sampling error, the perturbation term vanishes as $V(t)$ approaches zero. Consequently, practical fixed-time convergence can still be guaranteed. Therefore, a uniformly bounded settling-time upper bound T_2 can still be obtained for the closed-loop system.

Let $u_{i2} = u_y + u_t$. The consistency and constraint terms of control law (4) are:

$$\begin{aligned} u_y = & -k_1 \left(\nabla^2 f_i(p_i(t_k^i)) \right)^{-1} \sum_{j \in N_i} \text{sig}^{\mu_1} \left(p_i(t_k^i) - p_j(t_k^j) \right) \\ & - k_2 \left(\nabla^2 f_i(p_i(t_k^i)) \right)^{-1} \sum_{j \in N_i} \text{sig}^{\nu_1} \left(p_i(t_k^i) - p_j(t_k^j) \right) \end{aligned} \quad (23)$$

$$u_t = -k \cdot e^{\|p_i(t_k^i)\|^2} \cdot p_i(t_k^i) \quad (24)$$

According to Lemma 3: Substitute the consistent term in the algorithm into the properties of the Laplacian matrix, extract the minimum eigenvalue of the inverse of the Hessian matrix, and obtain:

$$\begin{aligned} \sum_{i=1}^N p_i^T \cdot u_y \leq & -k_1 \cdot \lambda_{\min} \left((\nabla^2 f_i)^{-1} \right) \cdot \lambda_2(L) \sum_{i=1}^N \|p_i\|^{\mu_1+1} \\ & - k_2 \cdot \lambda_{\min} \left((\nabla^2 f_i)^{-1} \right) \cdot \lambda_2(L) \sum_{i=1}^N \|p_i\|^{\nu_1+1} \end{aligned} \quad (25)$$

By substituting the constraint term, we obtain:

$$\sum_{i=1}^N p_i^T \cdot u_t = -k \cdot e^{\|p_i(t_k^i)\|^2} \cdot p_i(t_k^i) \leq 0 \quad (26)$$

After merging the consistency term and constraint term, we obtain:

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^N p_i^T \cdot (u_y + u_t) \leq -\alpha \sum_{i=1}^N \|p_i(t_k^i)\|^{\mu_1+1} - \beta \sum_{i=1}^N \|p_i(t_k^i)\|^{\nu_1+1} \\ = & -\alpha V^{\frac{\mu_1+1}{2}}(p_i(t_k^i)) - \beta V^{\frac{\nu_1+1}{2}}(p_i(t_k^i)) \end{aligned} \quad (27)$$

both α and β are positive numbers, $0 < \mu_1 < 1$, $0 < \nu_1 < 1$.

According to Lemma 1, we obtain:

$$\begin{aligned} T_2 \leq & \frac{1}{2^{\frac{\mu_1+1}{2}} \cdot k_1 \cdot \lambda_{\min}((\nabla^2 f_i)^{-1}) \cdot \lambda_2(L) \cdot \left(1 - \frac{\mu_1+1}{2}\right)} \\ & + \frac{1}{2^{\frac{\nu_1+1}{2}} \cdot k_2 \cdot \lambda_{\min}((\nabla^2 f_i)^{-1}) \cdot \lambda_2(L) \cdot \left(\frac{\nu_1+1}{2} - 1\right)} \end{aligned} \quad (28)$$

where $\alpha = 2^{\frac{\mu_1+1}{2}} \cdot k_1 \cdot \lambda_{\min}((\nabla^2 f_i)^{-1}) \cdot \lambda_2(L)$, $\beta = 2^{\frac{\nu_1+1}{2}} \cdot k_2 \cdot \lambda_{\min}((\nabla^2 f_i)^{-1}) \cdot \lambda_2(L)$, $p = \frac{\mu_1+1}{2}$, $q = \frac{\nu_1+1}{2}$.

Remark 2. Similar to Theorem 1, the fixed-time settling time upper bound derived in Theorem 2 is conservative. Such conservatism stems from the lower bound estimation of the Laplacian algebraic connectivity, the minimum eigenvalue approximation of the inverse Hessian matrix, and the relaxation of power mean inequalities during derivation. Larger control gains k_1 and k_2 , parameters μ_1, ν_1 approaching 1, as well as larger algebraic connectivity $\lambda_2(L)$ and minimum eigenvalue of $(\nabla^2 f_i)^{-1}$ can effectively reduce the conservatism and tighten the convergence bound, at the cost of increased control input magnitude.

Proof of fixed time convergence. The proof of Theorem 2 is completed. $\square$

Theorem 3. The event-triggered interval has a strictly positive lower bound, and Zeno behavior is excluded for the overall system. Consider system (2) with control law (4). Suppose that Assumptions 1–3 hold. The triggering rule is given by (7), which incorporates a local voting condition and a timeout fallback mechanism. The triggering interval is lower bounded by a positive constant $\Delta t_k^i \geq \frac{\sqrt{\Theta_{\min}}}{M}$ ($\Theta_{\min}, M, \tau > 0$) such that Zeno behavior is avoided.

Proof. Define the Lyapunov function as:

$$V_i(t) = \frac{1}{2} \|e_i(t)\|^2 = \frac{1}{2} \|p_i(t_k^i) - p_i(t)\|^2 \quad (29)$$

where $p_i(t_k^i)$ is a fixed value during the interval $[t_k^i, t_{k+1}^i)$, so $\dot{e}_i(t) = -\dot{p}_i(t) = -u_{i2}(t)$.

First, we prove the boundedness of the control input $u_{i2}(t)$. Based on Assumption 3, the state of each agent remains uniformly bounded. Under Assumption 1, $\nabla^2 f_i(p_i(t_k^i))$ is continuous and positive-definite on the bounded set of agent states, such that its inverse is also uniformly bounded. Combining the boundedness of neighboring agents' states and the property of the sigmoid function, every term contained in the designed controller is bounded. Therefore, there exists a positive constant M such that: $\|u_{i2}(t)\| \leq M, \forall t \geq 0, \forall i \in \mathcal{V}$.

By taking the derivative of the Lyapunov function, we obtain:

$$\dot{V}_i(t) = e_i(t)^T \dot{e}_i(t) = -e_i(t)^T u_{i2}(t) \quad (30)$$

$$\|p_i(t)\| = \|x_i(t) - h_i\| \leq \|x_i(t)\| + \|h_i\| \leq (R + \epsilon) + R = 2R + \epsilon \quad (31)$$

where ϵ denotes a bounded residual term. It follows from (26) that $p_i(t)$ is bounded.

Since $(\nabla^2 f_i(p_i(t_k^i)))^{-1}$ in (4) and $f_i(p_i)$ is a convex function, $\nabla^2 f_i$ is continuous and positive definite, so $\nabla^2 f_i(p_i(t_k^i))$ is bounded and $(\nabla^2 f_i(p_i(t_k^i)))^{-1}$ exists.

Therefore, there exists a constant $K_1 > 0$ that makes (32) hold true:

$$\|(\nabla^2 f_i(p_i(t_k^i)))^{-1}\| \leq K_1 \quad (32)$$

Because both $p_i(t_k^i)$ and $p_j(t_k^j)$ have boundaries, from (31), we obtain $\|p_i(t_k^i)\| \leq 2R + \epsilon, \|p_j(t_k^j)\| \leq 2R + \epsilon$. Thus $\|p_i(t_k^i) - p_j(t_k^j)\| \leq 4R + 2\epsilon$, for $\|\text{sig}^\alpha(z)\| = |z|^\alpha \leq (4R + 2\epsilon)^\alpha$ ($\alpha > 0$). For the fixed bounded-degree communication graph, every agent satisfies $N_i < d_{\max}$, where constant $d_{\max}$ is the maximum graph degree.

We can obtain:

$$\left\| \sum_{j \in N_i} \text{sig}^\alpha(p_i(t_k^i) - p_j(t_k^j)) \right\| \leq d_{\max} (4R + 2\epsilon)^\alpha \quad (33)$$

From (31), we have $-k \exp(\|p_i(t_k^i)\|^2) \leq \exp((2R + \epsilon)^2)$. Take the norm of the whole term it yields:

$$\left\| -k \exp(\|p_i(t_k^i)\|^2) p_i(t_k^i) \right\| \leq k \exp((2R + \epsilon)^2) (2R + \epsilon) \quad (34)$$

Merge and organize the previous (31)–(34) to obtain:

$$\|u_{i2}(t)\| \leq k_1 K_1 d_{\max} (4R + 2\epsilon)^{\mu_1} + k_2 K_2 d_{\max} (4R + 2\epsilon)^{\nu_1} + k \exp((2R + \epsilon)^2) (2R + \epsilon) = M \quad (35)$$

where $K_2 > 0$ is its norm bound, as for K_1 . Since $M > 0$, $\|u_{i2}(t)\| \leq M$ for $\forall t \geq 0, \forall i$, so $u_{i2}(t)$ is bounded.

The local voting mechanism requires that agent i triggers only when its own error exceeds the threshold and the number of supporting neighbors meets the voting requirement $\sum_{j \in N_i} I_j \geq \gamma_i$. Since each $I_j \in \{0, 1\}$ and $|N_i| \leq N_{\max}$, the voting condition introduces an additional constraint that prevents isolated, frequent triggers.

The timeout fallback mechanism further ensures that $t - t_k^i \geq T_{\max}$ even if the voting condition is temporarily not satisfied. Thus, the triggering interval of agent i satisfies $t_{k+1}^i - t_k^i \geq \min \{\Delta t_{\text{vote}}, T_{\max}\}$, where Δt_{vote} is the interval determined by the threshold and boundedness of $u_{i2}(t)$. This combined mechanism guarantees that the triggering interval cannot approach zero, eliminating Zeno behavior.

For the last agent with $\gamma_i = 0$, its triggering condition is relaxed to self-triggering, but it is still constrained by the fixed threshold $\Theta_i(t)$ and the timeout interval $T_{\max}$.

According to Cauchy-Schwarz inequality, we obtain:

$$\dot{V}_i(t) = -e_i(t)^T u_{i2}(t) \leq \|e_i(t)\| \cdot \|u_{i2}(t)\| \leq M \|e_i(t)\| \quad (36)$$

By substituting $\|e_i(t)\| = \sqrt{2V_i(t)}$, there is:

$$\dot{V}_i(t) \leq M \sqrt{2V_i(t)} \quad (37)$$

After integrating both sides from t_k^i to $t \in [t_k^i, t_{k+1}^i]$, and we note that $V_i(t_k^i) = 0$ at each triggering instant. so:

$$\sqrt{V_i(t)} \leq \frac{M}{\sqrt{2}} (t - t_k^i) \quad (38)$$

Combining the triggered condition (7) with the threshold function (8), the following is compiled:

$$2 \left(\frac{M}{\sqrt{2}} (t - t_k^i) \right)^2 > \Theta_i(t) \Rightarrow M^2 (t - t_k^i)^2 > \Theta_i(t) \quad (39)$$

In the threshold function (8), $\phi e^{-\tau t}$ is a decreasing function, and $\phi e^{-\tau t} \geq \phi e^{-\tau T}$, $\tau > 0$, and T is the total time. Since $\Theta_i(t) \geq \phi e^{-\tau t} \geq \phi e^{-\tau T}$ for all $t \in [0, T]$, we define the positive lower bound of threshold as:

$$\Theta_{\min} = \phi e^{-\tau T} \quad (40)$$

Substituting lower bound (40) into (39) yields:

$$t - t_k^i > \frac{\sqrt{\Theta_{\min}}}{M} \quad (41)$$

Consequently, the triggering interval satisfies $\Delta t_k^i = t_{k+1}^i - t_k^i \geq \min \left\{ \frac{\sqrt{\Theta_{\min}}}{M}, T_{\max} \right\} > 0$. This strictly positive lower bound holds for every agent over the whole execution process, which strictly demonstrates that Zeno behavior is excluded from the closed-loop system.

Remark 3. From the mechanism perspective, the local voting rule introduces an extra neighbor-based constraint to reduce frequent isolated triggers. Meanwhile, the timeout fallback condition $t - t_k^i \geq T_{\max}$ serves as a high-priority safeguard. If the voting condition remains unsatisfied over a long period, the agent will unconditionally trigger communication once the inter-event interval reaches $T_{\max}$, irrespective of the magnitude of the sampling error. This mechanism effectively avoids potential triggering deadlock.

The proof of Theorem 3 is completed. $\square$

4. Simulation

Numerical simulations are presented to verify the effectiveness of the proposed two-stage fixed-time distributed optimization algorithm and the local voting event-triggered mechanism. Two cases are considered: leaderless consensus control ($h_i = 0$) and distributed three-dimensional spherical formation control. All simulations are conducted in Python, and simulations are presented to verify the effectiveness of the proposed two-stage fixed-time distributed optimization algorithm and the local voting event-triggered mechanism. Two cases are considered: leaderless consensus control ($h_i = 0$) and distributed three-dimensional spherical formation control. All simulations are conducted in Python, with a maximum iteration 10,000 and a time step of 0.001. In both cases, a multi-agent system consisting of $N = 15$ agents with dynamics given in (1) is considered. The communication topology is a connected undirected ring graph, as shown in Figure 1, which satisfies Assumption 2. Each agent has a local strongly convex cost function, and Assumption 1 holds accordingly. The initial states of all agents are randomly selected within the interval $[-1, 1]$. Specifically, the first case validates the basic fixed-time optimization and consensus

performance without the local voting mechanism and formation offsets, while the second case demonstrates the complete formation control scheme with the local voting event-triggered mechanism in 3D space.

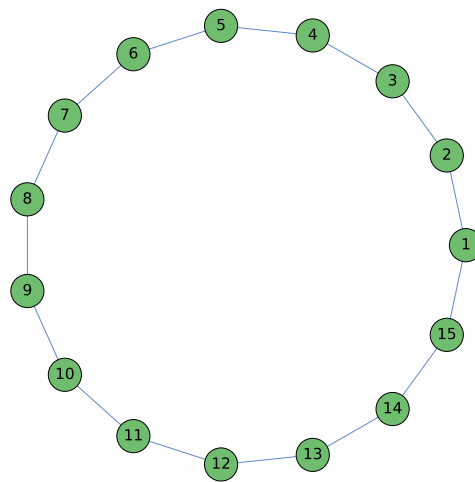


Figure 1. Communication topology for 15 agents formation control.

4.1. Consensus Control

In this subsection, the basic fixed-time optimization and consensus performance is validated. The formation offset is set to zero ($h_i = 0$), and a standard event-triggered mechanism without local voting is adopted to evaluate the core two-stage optimization framework. Figures 2–4 show the system trajectories, state norms, and control input norms, respectively. The results demonstrate that the proposed controller drives all agents to their local minima and achieves leaderless consensus within a fixed time, independent of initial conditions.

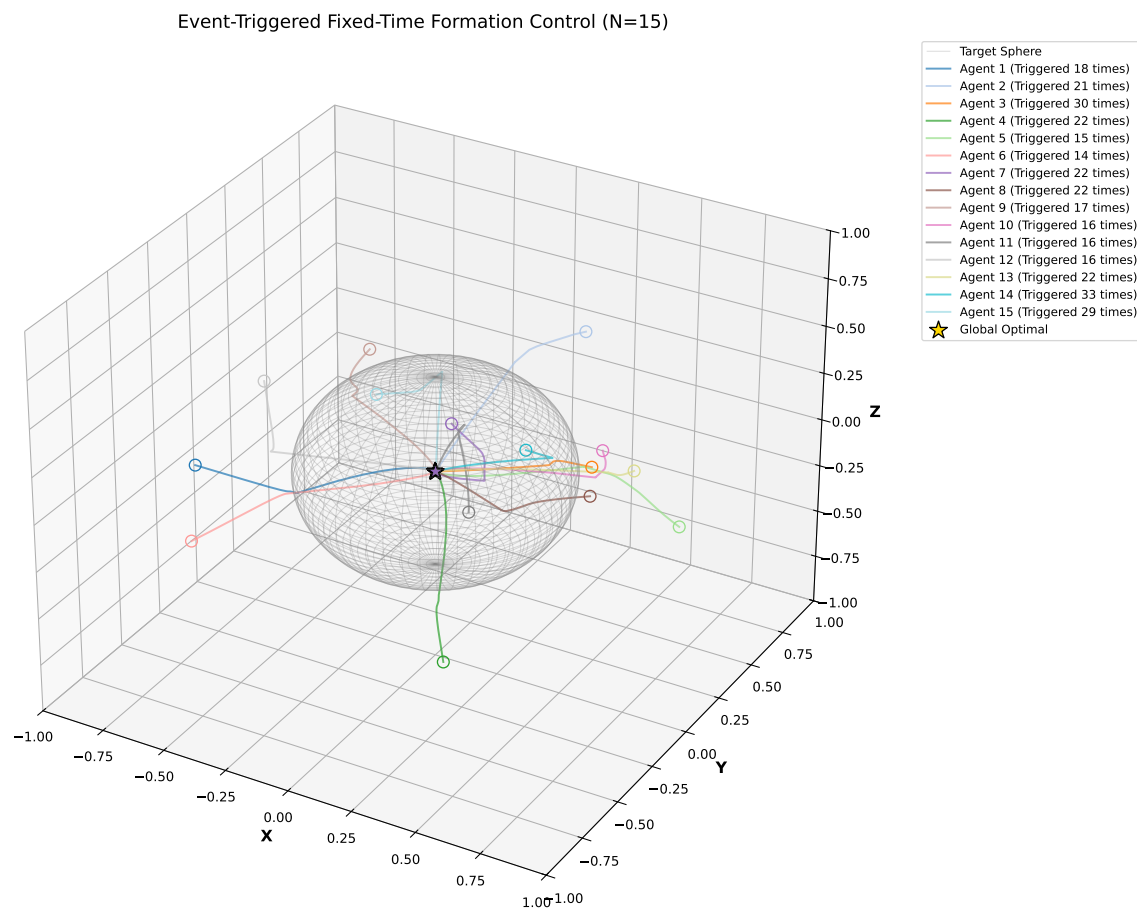


Figure 2. Three-dimensional trajectory of each agent.

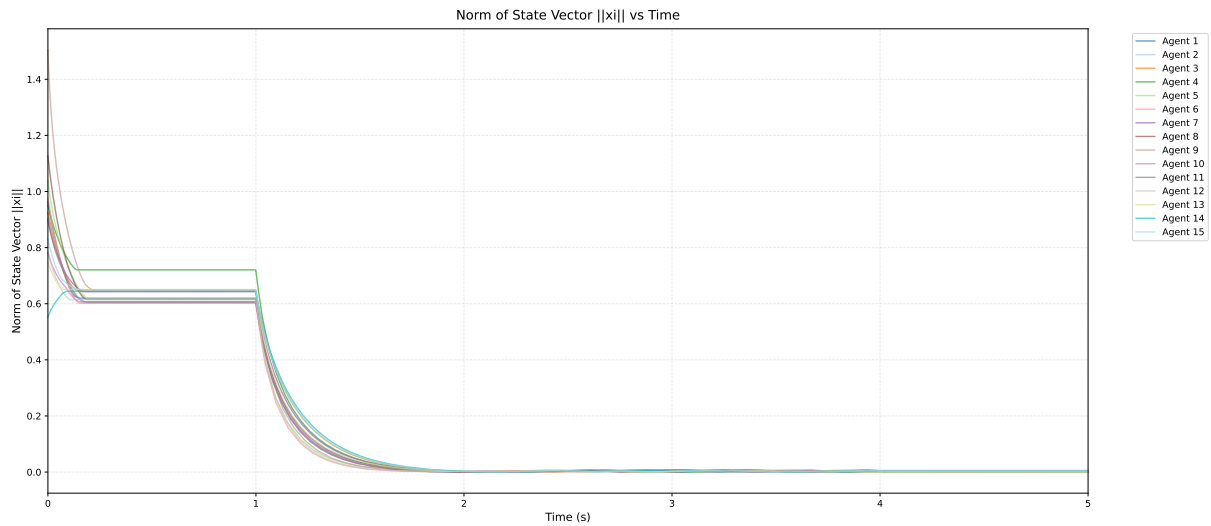


Figure 3. Evolution of the state norm $\|x_i\|$.

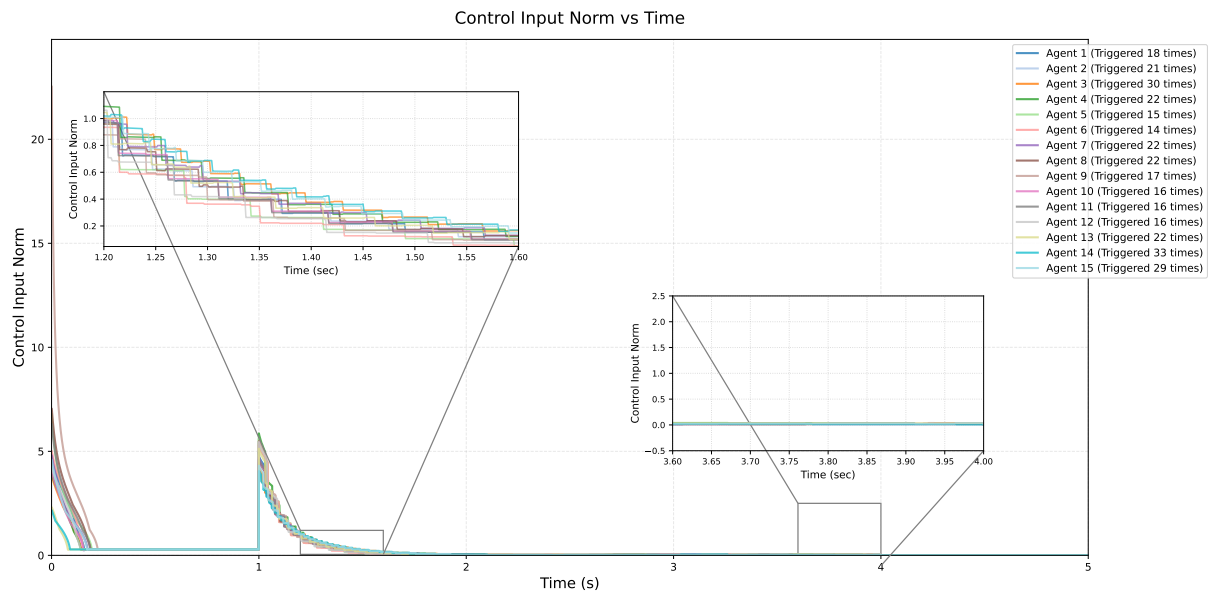


Figure 4. Evolution of the control input norm $\|u_i\|$.

4.2. Formation Control

This subsection verifies the performance of the proposed local voting event-triggered mechanism for three-dimensional fixed-time formation control. Simulations are conducted on a spherical formation with radius $R = 0.5$, where all agents are initially deployed outside the sphere, and Assumption 2 is satisfied. The overall control goal is two-fold: all agents shall converge to their local optimal states within a fixed time, and subsequently move to their individual target points on the spherical surface. To realize this goal, we design a local cost function for each agent, whose minimum value corresponds precisely to its target position on the sphere.

$$h_i = R \cdot \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} = R \cdot \begin{bmatrix} \cos(\Phi \cdot i) \cdot \sqrt{1 - \left(1 - 2 \cdot \frac{i}{N-1}\right)^2} \\ 1 - 2 \cdot \frac{i}{N-1} \\ \sin(\Phi \cdot i) \cdot \sqrt{1 - \left(1 - 2 \cdot \frac{i}{N-1}\right)^2} \end{bmatrix}$$

Golden ratio $\Phi = \pi(3 - \sqrt{5})$, and $N = 15$, $i = 1, 2, \dots, N$. $[x_{i1}^L, x_{i2}^L, x_{i3}^L]$ is the local optimal 3D position for agent i . The communication topology is described by the undirected graph depicted in Figure 1. One can easily see that Assumption 2 is fulfilled. The local cost function for each agent is defined as:

$$f_i = (x_{i1} - x_{i1}^L)^2 + (x_{i2} - x_{i2}^L)^2 + (x_{i3} - x_{i3}^L)^2$$

Let $Q_i^T = [x_{i1}^L, x_{i2}^L, x_{i3}^L]^T$, $i \in \{1, 2, \dots, N\}$, then $Q^T = [Q_1^T, Q_2^T, \dots, Q_N^T]$. The local optimal position of each agent are given as:

$$\begin{aligned} Q_1^T &= [0.03, 0.70, 0.09]^T, & Q_2^T &= [-0.28, 0.74, 0.02]^T, & Q_3^T &= [0.11, 0.37, -0.49]^T, \\ Q_4^T &= [0.35, 0.63, 0.41]^T, & Q_5^T &= [-0.87, 0.38, -0.27]^T, & Q_6^T &= [0.39, 0.29, -0.61]^T, \\ Q_7^T &= [-0.04, 0.11, 0.58]^T, & Q_8^T &= [-0.36, 0.02, -0.64]^T, & Q_9^T &= [0.52, 0.09, 0.50]^T, \\ Q_{10}^T &= [-0.65, -0.23, 0.36]^T, & Q_{11}^T &= [0.47, -0.12, -0.69]^T, & Q_{12}^T &= [0.15, -0.40, 0.61]^T, \\ Q_{13}^T &= [-0.44, -0.19, -0.18]^T, & Q_{14}^T &= [0.28, -0.54, -0.21]^T, & Q_{15}^T &= [-0.35, -0.43, -0.14]^T. \end{aligned}$$

The control parameters are chosen as $\mu_0 = \frac{1}{3}$, $\mu_1 = 0.5$, $\nu_0 = 3$, $\nu_1 = 1.5$, $k_1 = 0.1$, $k_2 = 1.5$, and $k = 4$. For the event-triggered mechanism, parameters are set as $\phi = 10^{-4}$, $\eta = 0.01$, $\tau = 0.01$, the maximum triggering interval is $T_{\max} = 0.8$, and the error threshold $\varepsilon = 0.005$. The voting requirement is set as $\gamma_i = 1$ for agents 1–14, and $\gamma_{15} = 0$ to avoid triggering deadlock. Besides, the uniform-bound constant in Assumption 3 is set as $R = 0.5$ in this simulation. The global optimal consensus target for all agents is chosen as $x^* = [0, 0, 0]^T$.

Figure 5 shows the three-dimensional trajectories of all agents. It can be observed that agents first converge to local optimal positions and then reach the predefined spherical formation precisely. Figure 6 shows the norm of agent states, which converges to the unit sphere scale within a fixed time. Figure 7 presents the evolution of control inputs, which decrease stepwise under the event-triggered mechanism and finally converge to zero.

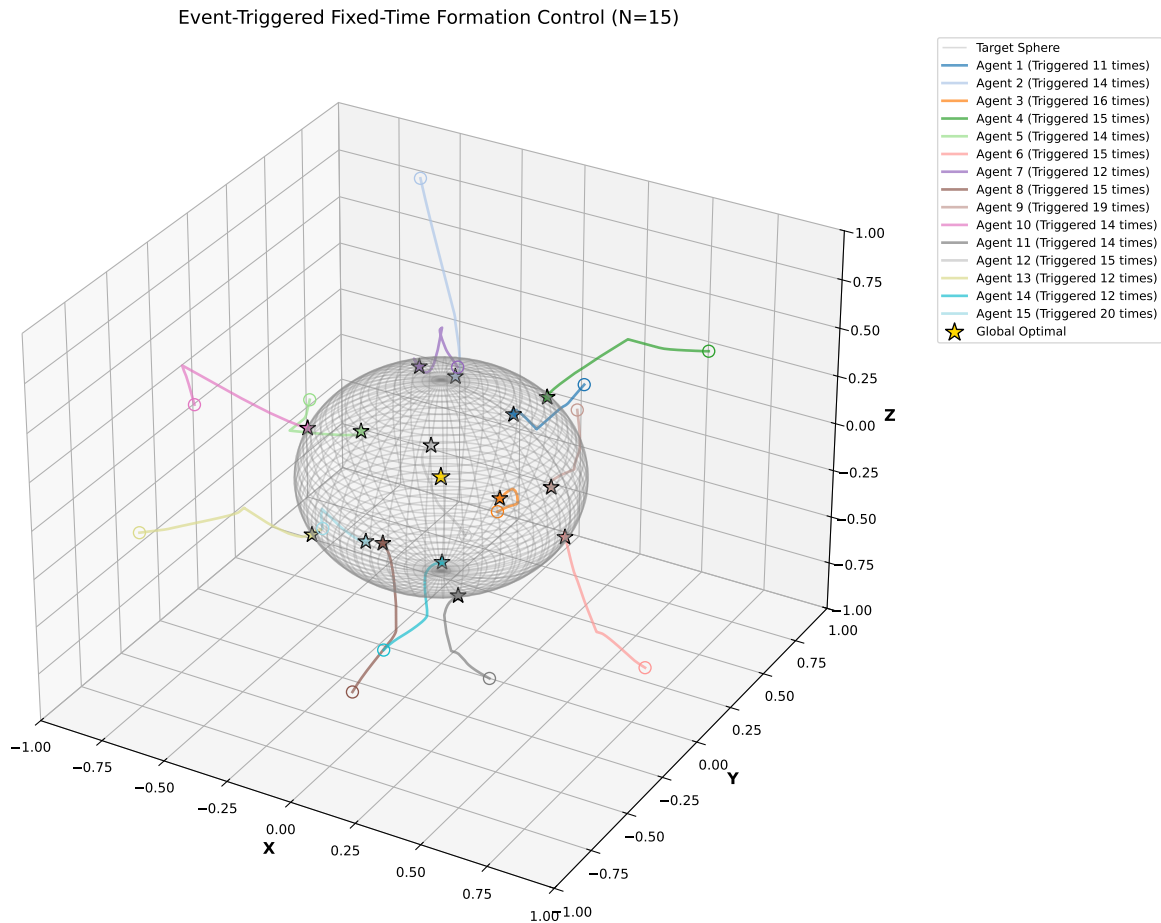


Figure 5. Three-dimensional trajectories of all agents.

Figure 8 shows the event-triggered time instants for all 15 agents. The local voting mechanism significantly reduces redundant triggering. The special voting rule for the last agent eliminates triggering deadlock. The timeout mechanism ensures reliable updates and balances communication efficiency with convergence performance.

Figure 9 shows the evolution of the global objective function, which converges to zero within a fixed time, verifying the global optimization performance.

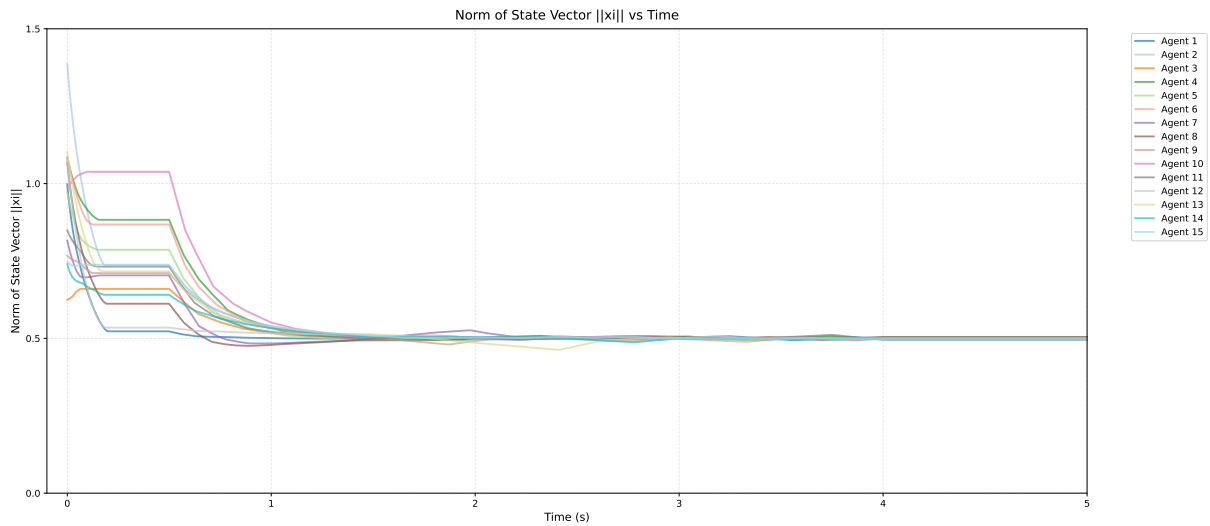


Figure 6. Evolution of the state norm $\|x_i\|$.

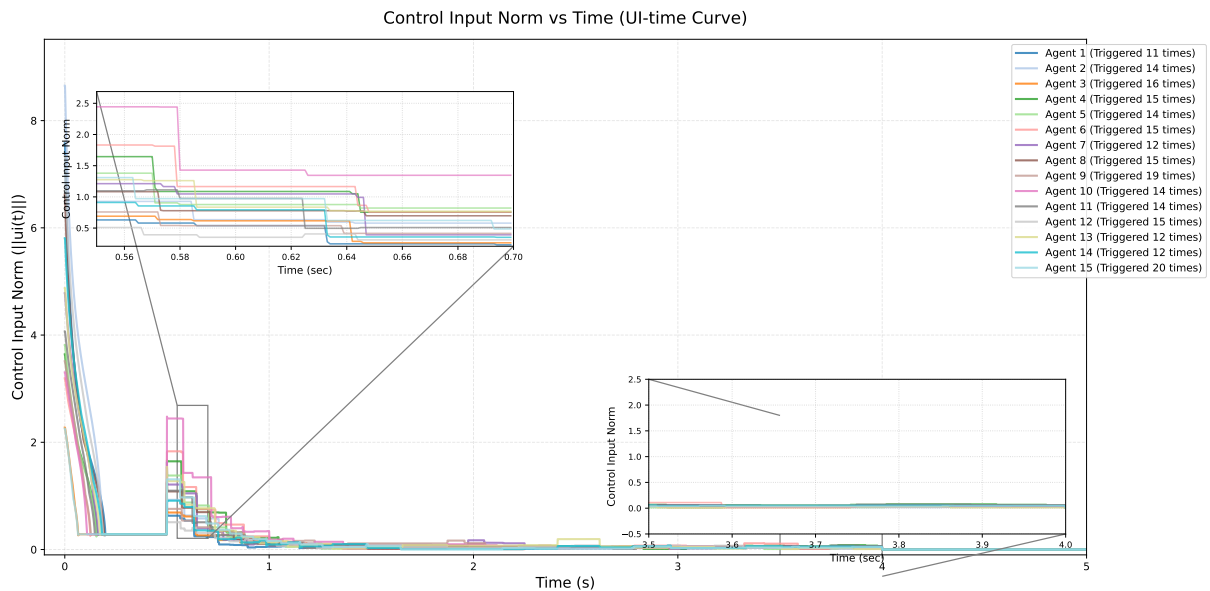


Figure 7. Evolution of control input $\|u_i\|$.

Simulation results confirm that the proposed algorithm achieves fixed-time consensus and high-precision spherical formation. The local voting event-triggered mechanism reduces communication costs while maintaining fixed-time convergence and effectively avoids Zeno behavior.

To evaluate the performance of the proposed local-voting event-triggered mechanism, two representative event-triggered schemes are selected as baselines for quantitative comparison: the fixed-threshold triggering scheme [26] and the periodic event-triggered scheme [18].

All comparison schemes are implemented under the same two-stage fixed-time formation control framework proposed in this paper, with only the event-triggering decision logic replaced. To ensure fair comparison under uniform parameter configurations, all schemes adopt fixed control parameters. For the periodic event-triggered baseline adapted from ref. [18], only its core periodic-sampling triggering mechanism is retained, while its original adaptive period-adjustment parameter module is not included, so that all baselines operate on the same fixed-parameter benchmark.

The quantitative comparison results are summarized in Table 2. The proposed local-voting triggering scheme significantly reduces the total number of triggering events and extends the average triggering interval, which effectively lowers communication overhead. Specifically, the total number of triggers is reduced by approximately 86.8% compared with the fixed-threshold baseline [26]. As a reasonable trade-off for reduced communication frequency, the steady-state formation error increases slightly but still remains at the 10^{-3} high-precision level. In addition, the convergence time of all three schemes stays at roughly the same level, which confirms that the fixed-time convergence property of the control framework is well preserved across different triggering mechanisms.

Overall, the proposed scheme achieves a more favorable balance between communication efficiency and formation control accuracy.

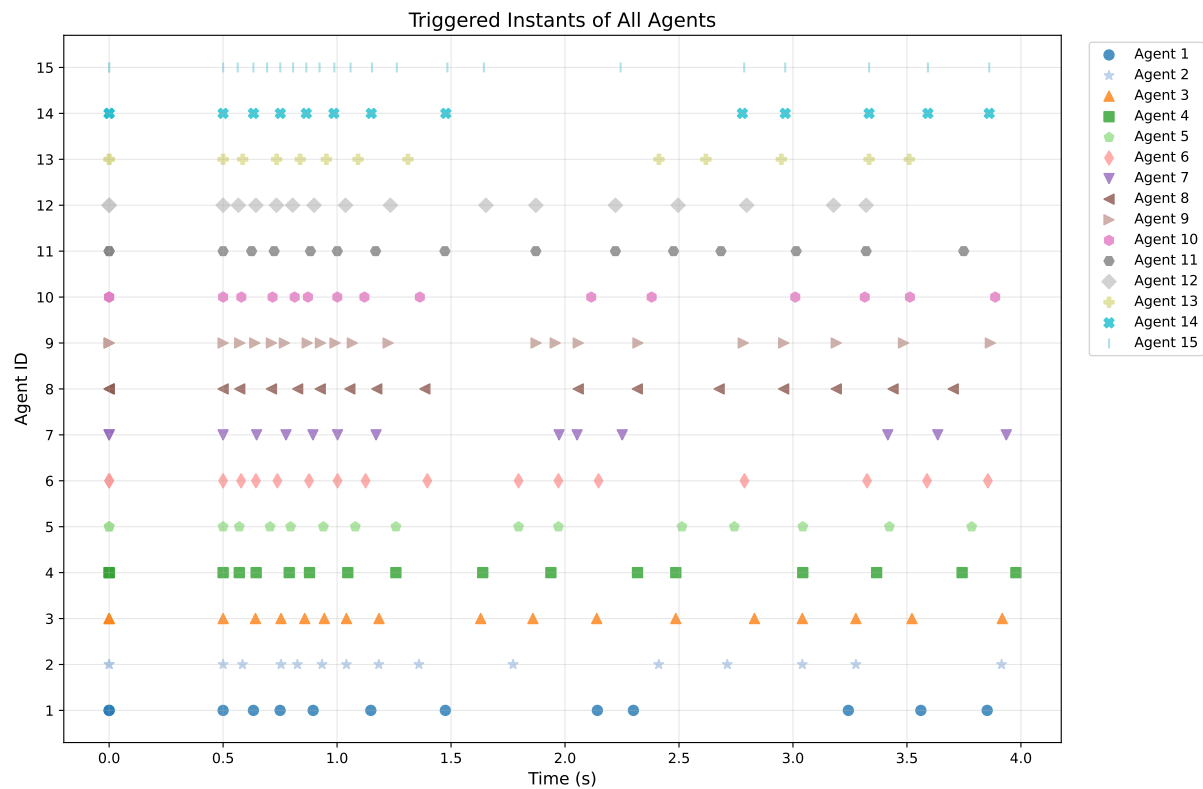


Figure 8. Event-triggered time instants for 15 agents.

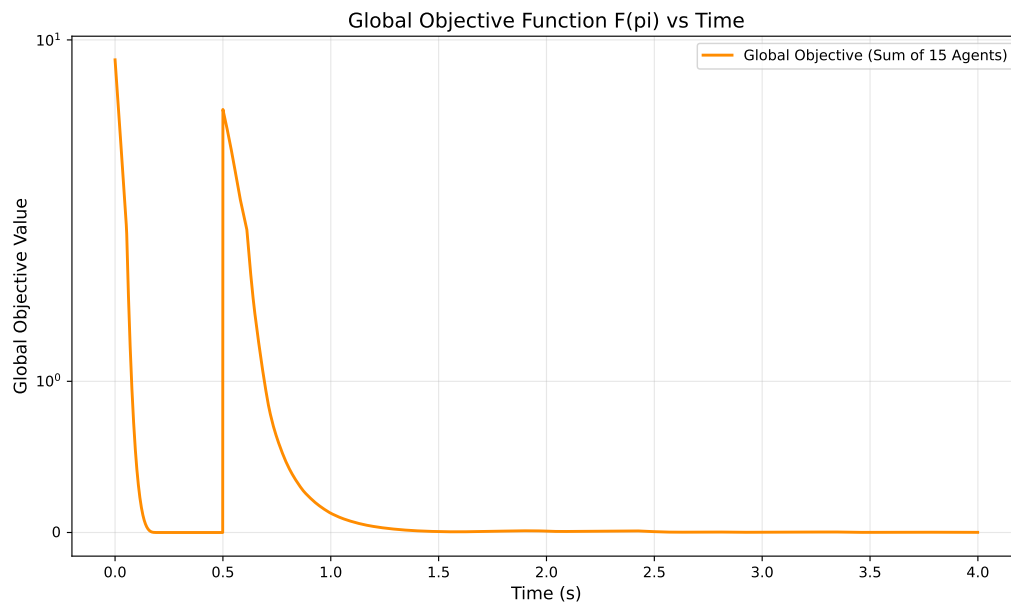


Figure 9. Evolution of the global objective function value.

Table 2. Quantitative comparison of different event-triggered schemes.

Scheme	Total Triggering Times	Average Interval (s)	Steady-State Formation Error
Fixed-threshold triggering [26]	1867	0.0021	0.000132
Periodic event-triggering [18]	1515	0.0026	0.000133
Local-voting triggering (Proposed)	247	0.0162	0.005478

5. Conclusions and Future Work

In this paper, a two-stage fixed-time distributed optimization framework has been developed for leaderless formation control of continuous-time multi-agent systems. By separating gradient-driven local convergence from event-triggered global consensus, the proposed algorithm ensures that each agent reaches its local optimum and the entire team achieves the desired formation within a uniformly bounded time independent of initial conditions. A state-dependent exponential compensation term has been integrated into the consensus stage to rectify precision losses arising from three-dimensional dimensional transformation. Furthermore, a cooperative local voting event-triggered mechanism has been established to eliminate isolated triggers and stale information utilization via neighbor-supported decisions, thereby markedly alleviating communication and computational loads while resolving potential triggering deadlock and preventing Zeno behavior through a timeout fallback and a specialized voting rule. Both theoretical analysis and numerical experiments on consensus and spherical formation tasks corroborate that the proposed algorithm attains fixed-time convergence, high formation accuracy, and superior communication efficiency.

Author Contributions

P.Z.: conceptualization, methodology, software, writing—original draft preparation; T.L.: writing—original draft preparation, data curation; Y.Y.: writing—review and editing; A.W.: writing—review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

During the preparation of this work, the authors used DeepSeek to polish the language of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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