

Robust Fault and Disturbance Reconstruction for Continuous-Time Markovian Jump Systems

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Abstract: This paper presents the robust actuator fault and disturbance reconstruction scheme for a kind of continuous-time Markovian jump systems (MJSs). A novel observer with the output derivative signal is proposed to realize the estimation for system states, actuator faults and disturbances simultaneously. By using the disturbance decoupled method, the influence of disturbances is completely eliminated from estimation error dynamics, which can facilitate the stability analysis and improve estimation accuracy. Moreover, the proposed method does not impose known boundedness assumptions on disturbances, thereby enlarging its applicability compared with the existing schemes. Based on the Lyapunov stability theory and the robust H_∞ method, sufficient conditions ensuring robust stochastic stability of the resulting error systems including system states, actuator faults and disturbances are established in the form of strict linear matrix inequalities (LMIs), by which observer gains are also solved. Finally, the simulation study on an electronic circuit model is presented to demonstrate the effectiveness of the proposed multivariate estimation approach.

Keywords: Markovian jump systems; fault and disturbance reconstruction; observer design; robust stochastic stability

1. Introduction

Markovian jump systems (MJSs), as a class of stochastic hybrid systems, consist of multiple subsystems which switch according to a Markovian process [1–3]. Owing to their capability to model complex systems subject to abrupt structural or parameter variations, MJSs have been widely applied in aerospace, power systems, telecommunications, and other industrial domains [4,5]. However, it is worth emphasizing that the existence of stochastic mode transitions makes the stability analysis for MJSs significantly more challenging than those for deterministic systems.

In modern industrial environment, actuator faults are always unavoidable due to component aging, unexpected failures, or harsh operating conditions [6–10]. Such faults may severely degrade system performances or even cause serious accidents such as economic losses and potential casualties. For MJSs, the presence of stochastic mode switching further complicates the estimation process. Therefore, developing effective fault reconstruction schemes that can accurately estimate actuator faults under stochastic switching is of great importance for ensuring system reliability and safety [11–14].

Apart from actuator faults, external disturbances such as load variations, environmental fluctuations, and unmodeled dynamics widely exist in practical systems, which may also bring negative impact on system performances [15–18]. Currently, various observer-based fault estimation approaches for continuous-time MJSs have been reported in [19–21]. Although effective fault reconstruction can be achieved in these studies, disturbances generally remain coupled in the error dynamics. In other words, disturbances in these studies are treated as bounded vectors to be attenuated rather than explicitly reconstructed. It should be noted that explicit disturbance information is often essential for

monitoring and performance evaluation. Thus, it is meaningful to develop estimation schemes that can simultaneously reconstruct actuator faults and disturbances. Furthermore, most of existing observer-based methods still require known boundedness assumptions on disturbances to guarantee robustness. However, in practical applications, disturbances may be stochastic, time-varying, or of unknown magnitude, making such assumptions restrictive or unrealistic. This limitation may restrict practical applicability. Hence, achieving simultaneous system states, actuator faults, and disturbances reconstruction without imposing the assumption on disturbances having a known bound is still a challenging problem, which requires in-depth exploration.

Inspired by the above discussions, this paper focuses on the simultaneous reconstruction of system states, actuator faults, and external disturbances for continuous-time MJSs under stochastic mode switching. The key issue is how to remove the disturbance from the estimation error dynamics without relying on its magnitude bound, while retaining sufficient information for explicit disturbance reconstruction. To address this issue, an output-derivative-based observer together with a structural disturbance-decoupling mechanism is developed. Furthermore, a mode-dependent Lyapunov function and the robust H_∞ technique are employed to derive strict LMI conditions for observer synthesis and robust stochastic stability.

The main contributions of the developed approach are summarized in the following:

- (1) A novel observer using the output derivative signal is designed for continuous-time MJSs, which realizes simultaneous reconstruction of the system states, disturbances, and actuator faults. Besides, unlike conventional augmented-system-based approaches, the proposed design enables direct estimation rather than augmenting the system states, disturbances, and actuator faults into a new state.
- (2) The proposed framework does not impose known boundedness assumptions on the disturbances, that is, the upper bound of disturbances can be unknown in our paper. Besides, by introducing a disturbance-decoupling condition, the disturbance is completely removed from the state/fault estimation error dynamics, and it is reconstructed by introducing the output derivative signal. Consequently, the proposed approach can provide explicit disturbance information and enlarge its applicability to disturbances whose bound exists but is unknown.

Notation: $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, I and 0 denote the identity and zero matrices with appropriate dimensions, respectively, and the superscripts T and $+$ represent the matrix transpose and Moore-Penrose pseudoinverse, respectively. $P > 0$ ($P < 0$) means that P is a positive (negative) definite matrix. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation, and $*$ denotes the symmetric counterpart in a symmetric matrix.

2. Problem Formulation

Let us consider the following continuous-time MJSs which are affected by actuator faults and disturbances

$$\begin{cases} \dot{x}(t) = A(m(t))x(t) + B(m(t))u(t) \\ \quad + F(m(t))f_a(t) + D(m(t))d(t) \\ y(t) = C(m(t))x(t), \end{cases} \quad (1)$$

in which $x(t) \in \mathbb{R}^n$ represents the system state, $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^{n_y}$ stand for the control input and the measurable output. $f_a(t) \in \mathbb{R}^{n_a}$ stands for the actuator fault. $d(t) \in \mathbb{R}^{n_d}$ stands for the disturbance. $A(m(t))$, $B(m(t))$, $C(m(t))$, $F(m(t))$, $D(m(t))$ are known real matrices determined by $m(t)$. We assume that $D(m(t))$ has full column rank. Consider a right-continuous Markovian process $\{m(t), t \geq 0\}$ on the probability space taking values in the finite set $S = \{1, 2, \dots, s\}$. The stochastic process is governed by the mode transition probabilities specified as

$$P(m(t + \Delta t) = j \mid m(t) = i) = \pi_{ij}\Delta t + o(\Delta t) + \mathcal{X}_{ij}, \quad (2)$$

in which $\lim_{\Delta t \rightarrow 0} \frac{o(\Delta t)}{\Delta t} = 0$, $\Delta t > 0$. $\mathcal{X}_{ij} = \begin{cases} 1, & j = i, \\ 0, & j \neq i, \end{cases}$ and π_{ij} stands for the transition rate from mode i to mode j . In the case of $i = j$, $\pi_{ij} < 0$, when $i \neq j$, $\pi_{ij} > 0$. Additionally, $\sum_{j \neq i} \pi_{ij} = -\pi_{ii}$. It can be noticed that, when $m(t) = i \in S$, the corresponding system parameter matrices are rewritten as A_i , B_i , C_i , F_i , D_i . Thus, for simplicity, when $m(t) = i \in S$, system (1) can be expressed as

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + F_i f_a(t) + D_i d(t) \\ y(t) = C_i x(t). \end{cases} \quad (3)$$

The following definition is necessary in the stability analysis.

Definition 1 ([22]). We consider the continuous-time MJSs given as follows

$$\begin{cases} \dot{x}(t) = \mathfrak{A}_i x(t) + \mathfrak{B}_i u(t) + \mathfrak{F}_i \eta(t) \\ y(t) = \mathfrak{C}_i x(t), \end{cases} \quad (4)$$

where $\eta(t)$ stands for the external disturbance. For a given constant $\alpha > 0$, when $u(t) \equiv 0$, system (4) is said to be robust stochastic stable with H_∞ performance if for any initial mode $m_0 \in S$, there exists a constant $X(x_0, m_0)$ with $X(0, m_0) = 0$ such that the L_2 norm of $x(t)$, defined as $\|x(t)\|_2 = [\mathbb{E}\{\int_0^\infty x^T(t)x(t)dt \mid x_0, m_0\}]^{1/2}$ satisfies the following inequality

$$\|x(t)\|_2^2 \leq \alpha^2 [\|\eta(t)\|_2^2 + X(x_0, m_0)]. \quad (5)$$

3. Observer-Based Actuator Fault and Disturbance Reconstruction Design

In this section, a novel observer with the derivative of output is proposed for system (1) to simultaneously estimate the system states, disturbances and actuator faults, which can be formulated as

$$\begin{cases} \dot{x}_o(t) = G_i x_o(t) + E_i u(t) + M_i y(t) + S_i \hat{f}_a(t) \\ \hat{x}(t) = x_o(t) + T_i y(t) \\ \hat{y}(t) = C_i \hat{x}(t) \\ \hat{f}_a(t) = -U_i(y(t) - \hat{y}(t)) \\ \hat{d}(t) = (C_i D_i)^+ \dot{y}(t) - J_i u(t) - K_i \hat{x}(t) \\ \quad - (C_i D_i)^+ C_i F_i \hat{f}_a(t), \end{cases} \quad (6)$$

in which $G_i, E_i, M_i, S_i, T_i, U_i, J_i, K_i$ are the observer parameters to be solved later. $x_o(t) \in \mathbb{R}^n$ denotes the observer state vector. $\hat{x}(t), \hat{f}_a(t)$ and $\hat{d}(t)$ represent the estimation of system states, actuator faults and disturbances, respectively. Here, $(C_i D_i)^+$ denotes the Moore-Penrose pseudoinverse of $C_i D_i$. The reconstruction behavior of observer (6) is explicitly reflected in three channels: $\hat{x}(t) = x_o(t) + T_i y(t)$ provides the state reconstruction, $\hat{f}_a(t) = -U_i(y(t) - \hat{y}(t))$ dynamically generates the actuator fault reconstruction, and the last equation in (6) algebraically reconstructs the disturbance by using $\dot{y}(t), u(t), \hat{x}(t)$, and $\hat{f}_a(t)$. Thus, the proposed observer simultaneously generates $\hat{x}(t), \hat{f}_a(t)$, and $\hat{d}(t)$.

Next, the definitions of errors $e_x(t)$ and $e_f(t)$ are given as

$$e_x(t) = x(t) - \hat{x}(t), \quad (7)$$

$$e_f(t) = f_a(t) - \hat{f}_a(t). \quad (8)$$

Based on (6) and (7), one has the state estimation error as

$$\begin{aligned} e_x(t) &= x(t) - x_o(t) - T_i y(t) \\ &= H_i x(t) - x_o(t), \end{aligned} \quad (9)$$

where $H_i = I - T_i C_i$. As a consequence, from (3) and (6), we can further deduce that

$$\begin{aligned} \dot{e}_x(t) &= H_i A_i x(t) + H_i B_i u(t) + H_i F_i f_a(t) + H_i D_i d(t) \\ &\quad - G_i x_o(t) - E_i u(t) - M_i y(t) - S_i \hat{f}_a(t) \\ &= (H_i A_i - M_i C_i) x(t) + (H_i B_i - E_i) u(t) \\ &\quad + (H_i F_i - S_i) f_a(t) + H_i D_i d(t) \\ &\quad + S_i e_f(t) - G_i (H_i x(t) - e_x(t)) \\ &= G_i e_x(t) + (H_i A_i - M_i C_i - G_i H_i) x(t) \\ &\quad + (H_i B_i - E_i) u(t) + (H_i F_i - S_i) f_a(t) \\ &\quad + S_i e_f(t) + H_i D_i d(t). \end{aligned} \quad (10)$$

In order to eliminate certain negative effects on error dynamics (10), we let

$$H_i A_i - M_i C_i - G_i H_i = 0, H_i D_i = 0, \quad (11)$$

$$H_i B_i = E_i, H_i F_i = S_i. \quad (12)$$

Note that since $H_i = I - T_i C_i$, we have $G_i = H_i A_i + G_i T_i C_i - M_i C_i$ from (11). Then, defining $R_i = G_i T_i - M_i$ gives $G_i = H_i A_i + R_i C_i$. Based on the above statements, error dynamics (10) can be re-expressed as

$$\dot{e}_x(t) = (H_i A_i + R_i C_i) e_x(t) + S_i e_f(t). \quad (13)$$

It is obviously that error dynamics (13) are not influenced by $d(t)$ at all, and $d(t)$ is totally decoupled. Here, we provide an essential assumption.

Assumption 1. It is assumed that $\text{rank}(C_i D_i) = \text{rank}(D_i)$.

Remark 1. The condition $\text{rank}(C_i D_i) = \text{rank}(D_i)$ is a structural matching condition for the designed disturbance-decoupling and reconstruction mechanism. Specifically, since $H_i = I - T_i C_i$, the disturbance-decoupling requirement $H_i D_i = 0$ is equivalent to $T_i C_i D_i = D_i$. Therefore, $\text{rank}(C_i D_i) = \text{rank}(D_i)$ is a necessary and sufficient condition for the solvability of the above matrix equation. When D_i has full column rank, this condition reduces to $\text{rank}(C_i D_i) = n_d$, which means that the disturbance channels remain distinguishable through the output-derivative channel. Hence, the condition ensures that the effect of the disturbance can be decoupled from the state estimation error dynamics while retaining sufficient information for disturbance reconstruction.

Next, we deal with $e_f(t)$. Based on (6) and (8), we get

$$\begin{aligned} \dot{e}_f(t) &= \dot{f}_a(t) - \dot{\hat{f}}_a(t) \\ &= U_i C_i (x(t) - \hat{x}(t)) + \dot{f}_a(t) \\ &= U_i C_i e_x(t) + \dot{f}_a(t). \end{aligned} \quad (14)$$

We now proceed to derive the error equation of disturbances. We define $e_d(t) = d(t) - \hat{d}(t)$. Based on (6), we can obtain

$$\begin{aligned} e_d(t) &= (C_i D_i)^+ C_i F_i \hat{f}_a(t) + d(t) - (C_i D_i)^+ \dot{y}(t) \\ &\quad + J_i u(t) + K_i \hat{x}(t) \\ &= d(t) - (C_i D_i)^+ C_i (A_i x(t) + B_i u(t) + F_i f_a(t) \\ &\quad + D_i d(t)) + J_i u(t) + K_i \hat{x}(t) \\ &\quad + (C_i D_i)^+ C_i F_i \hat{f}_a(t) \\ &= (K_i - (C_i D_i)^+ C_i A_i) x(t) \\ &\quad + (J_i - (C_i D_i)^+ C_i B_i) u(t) \\ &\quad - (C_i D_i)^+ C_i F_i e_f(t) - K_i e_x(t). \end{aligned} \quad (15)$$

If we set

$$K_i = (C_i D_i)^+ C_i A_i, J_i = (C_i D_i)^+ C_i B_i. \quad (16)$$

Then, Equation (15) can be rewritten as

$$e_d(t) = -(C_i D_i)^+ C_i F_i e_f(t) - K_i e_x(t). \quad (17)$$

The removal of the boundedness requirement on $d(t)$ results from the structural disturbance-decoupling mechanism. Specifically, the condition $H_i D_i = 0$ eliminates $d(t)$ exactly from the state estimation error dynamics rather than merely attenuating its influence. Moreover, according to (17), the disturbance reconstruction error is expressed only in terms of $e_x(t)$ and $e_f(t)$, without containing $d(t)$ itself. Therefore, the stability analysis does not require any a priori upper bound on the magnitude of $d(t)$. This feature fundamentally differs from observer schemes in which the disturbance remains as an external input in the estimation error dynamics. It can be noticed that the disturbance error (17) is purely algebraic. Consequently, driving $e_x(t)$ and $e_f(t)$ stability automatically forces $e_d(t)$ stability. Aiming at stabilizing both of these two errors, we pack them into a single vector, that is $\mu(t) = \begin{bmatrix} e_x^T(t) & e_f^T(t) \end{bmatrix}^T$. As a result, error dynamics (13) and (14) can be re-expressed as

$$\dot{\mu}(t) = (\mathcal{A}_i + \Theta_i \mathcal{C}_i) \mu(t) + \Upsilon_i \omega, \quad (18)$$

in which

$$\mathcal{A}_i = \begin{bmatrix} H_i A_i & H_i F_i \\ 0 & 0 \end{bmatrix}, \Theta_i = \begin{bmatrix} R_i \\ U_i \end{bmatrix}, \\ \mathcal{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \Upsilon_i = \begin{bmatrix} 0 \\ I \end{bmatrix}, \omega = \dot{f}_a(t).$$

4. Robust Stochastic Stability Analysis

Based on the above augmented error system, the remaining task is to determine the observer gains such that the coupled state and actuator fault estimation errors are robust stochastic stable in the presence of fault variation $\omega = \dot{f}_a(t)$. To this end, a mode-dependent Lyapunov function and the H_∞ performance criterion are introduced. The following theorem provides a tractable LMI-based sufficient condition for robust stochastic stability and observer gain synthesis.

Theorem 1. For a given H_∞ performance index $\tau > 0$, if there exist matrices X_i and $P_i = P_i^T > 0$ satisfying

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & P_i \Upsilon_i \\ * & -\tau^2 I \end{bmatrix} < 0, \quad (19)$$

where

$$\Omega_{11i} = P_i \mathcal{A}_i + \mathcal{A}_i^T P_i + X_i \mathcal{C}_i + \mathcal{C}_i^T X_i^T + I + \sum_{j=1}^s \pi_{ij} P_j.$$

Then, error dynamics (13) and (14) exhibit robust stochastic stability under the H_∞ performance index τ , and parameter Θ_i defined in (18) is solved as $\Theta_i = P_i^{-1} X_i$.

Proof. We now consider the Lyapunov candidate function defined as follows

$$V(\mu(t), m(t)) = \mu^T(t) P(m(t)) \mu(t). \quad (20)$$

Define ℓ to be the infinitesimal operator associated with the Lyapunov function, for $m(t) = i \in S$, it can be obtained that

$$\ell V(\mu(t), i) = 2\mu^T(t) P_i \dot{\mu}(t) + \mu^T(t) \sum_{j=1}^s \pi_{ij} P_j \mu(t). \quad (21)$$

Substituting (18) into (21) yields

$$\begin{aligned} \ell V(\mu(t), i) &= 2\mu^T(t) P_i (\Upsilon_i \omega + (\mathcal{A}_i + \Theta_i \mathcal{C}_i) \mu(t)) \\ &\quad + \mu^T(t) \sum_{j=1}^s \pi_{ij} P_j \mu(t) \\ &= \mu^T(t) (P_i \mathcal{A}_i + \mathcal{A}_i^T P_i + P_i \Theta_i \mathcal{C}_i + \mathcal{C}_i^T \Theta_i^T P_i \\ &\quad + \sum_{j=1}^s \pi_{ij} P_j) \mu(t) + 2\mu^T(t) P_i \Upsilon_i \omega. \end{aligned} \quad (22)$$

Let $P_i \Theta_i = X_i$, (22) can be rewritten as

$$\begin{aligned} \ell V(\mu(t), i) &= \mu^T(t) (P_i \mathcal{A}_i + \mathcal{A}_i^T P_i + X_i \mathcal{C}_i + \mathcal{C}_i^T X_i^T \\ &\quad + \sum_{j=1}^s \pi_{ij} P_j) \mu(t) + 2\mu^T(t) P_i \Upsilon_i \omega. \end{aligned} \quad (23)$$

Subsequently, for the purpose of analyzing the robustness of the estimation error system, let $\mathfrak{R}(t) = \mathbb{E}[\mu^T(t) \mu(t) - \tau^2 \omega^T \omega]$, $\mathfrak{J}(t) = \ell V(\mu(t), i) + \mathfrak{R}(t)$. Thus, from (23), we have

$$\begin{aligned}
\mathfrak{J}(t) &= \mu^T(t)(P_i \mathcal{A}_i + \mathcal{A}_i^T P_i + X_i \mathcal{C}_i + \mathcal{C}_i^T X_i^T \\
&\quad + \sum_{j=1}^s \pi_{ij} P_j + I) \mu(t) + 2\mu^T(t) P_i \mathfrak{T}_i \omega - \tau^2 \omega^T \omega \\
&= \begin{bmatrix} \mu^T(t) & \omega^T \end{bmatrix} \Upsilon_i \begin{bmatrix} \mu(t) \\ \omega \end{bmatrix},
\end{aligned} \tag{24}$$

in which $\Upsilon_i = \begin{bmatrix} \Upsilon_{11i} & \Upsilon_{12i} \\ * & \Upsilon_{22i} \end{bmatrix}$ with

$$\begin{aligned}
\Upsilon_{11i} &= P_i \mathcal{A}_i + \mathcal{A}_i^T P_i + X_i \mathcal{C}_i + \mathcal{C}_i^T X_i^T + \sum_{j=1}^s \pi_{ij} P_j + I, \\
\Upsilon_{12i} &= P_i \mathfrak{T}_i, \Upsilon_{22i} = -\tau^2 I.
\end{aligned}$$

We can find that $\mathfrak{J}(t) < 0$ is guaranteed by the feasibility of (19). It can be noticed that $\mathfrak{J}(t) < 0$ is equivalent to

$$\ell V(\mu(t), i) + \mathbb{E}[\mu^T(t) \mu(t) - \tau^2 \omega^T \omega] < 0. \tag{25}$$

By integrating both sides of (25), we have

$$\begin{aligned}
&\mathbb{E} \left[\int_0^T \mu^T(t) \mu(t) dt \right] - \tau^2 \mathbb{E} \left[\int_0^T \omega^T(t) \omega(t) dt \right] \\
&+ \mathbb{E}[V(\mu(T), m(T))] - V(\mu_0, m_0) < 0,
\end{aligned} \tag{26}$$

in which μ_0 and m_0 represent the initial value of $\mu(t)$ and $m(t)$. On the basis of $\mathbb{E}[V(\mu(T), m(T))] > 0$, we can deduce

$$\begin{aligned}
&\mathbb{E} \left[\int_0^T \mu^T(t) \mu(t) dt \right] - \tau^2 \mathbb{E} \left[\int_0^T \omega^T(t) \omega(t) dt \right] \\
&- V(\mu_0, m_0) < 0,
\end{aligned} \tag{27}$$

which is equivalent to $\|\mu(t)\|_2^2 \leq \tau^2 \|\omega\|_2^2 + V(\mu_0, m_0)$. On the basis of Definition 1, error dynamics (13) and (14) possess robust stochastic stability with H_∞ performance index under condition (19). Note that $P_i \Theta_i = X_i$ gives $\Theta_i = P_i^{-1} X_i$, which means R_i and U_i are solved. $\square$

Remark 2. With the aim of stating the observer synthesis conditions within a full LMIs framework, the unknown matrix H_i should be attained in advance. Substituting $H_i = I - T_i C_i$ into $H_i D_i = 0$ yields $D_i = T_i C_i D_i$. Under the assumption of $\text{rank}(C_i D_i) = \text{rank}(D_i)$, this matrix equation is solvable, and the general form of T_i can be expressed as

$$T_i = D_i (C_i D_i)^+ - \Xi_i (I - (C_i D_i)(C_i D_i)^+), \tag{28}$$

in which Ξ_i is an arbitrary matrix. Once T_i is determined, H_i is directly available.

Finally, with the purpose of achieving the simulation estimation of system states, actuator faults and disturbances, we present Algorithm 1 as follows.

Remark 3. The current stability condition is established under a completely known transition rate matrix because the term $\sum_{j=1}^s \pi_{ij} P_j$ is explicitly involved in the infinitesimal operator. For partially known transition rates, the known and unknown transition subsets may be treated separately, while appropriate bounding or slack-matrix techniques can be introduced to handle the unknown transition terms. Accordingly, the present reconstruction framework has the potential to be extended to MJSSs with partially known transition information. This issue will be investigated in our future work.

Remark 4. The main technical difficulties of this work can be summarized as follows. First, because of the coexistence of actuator faults, external disturbances, and stochastic mode transitions, the simultaneous reconstruction of multiple unknown quantities under stochastic mode transitions becomes challenging. Second, retaining the

disturbance in the estimation error dynamics generally makes the reconstruction performance dependent on its magnitude and may require a priori boundedness information. Third, the stability analysis should simultaneously accommodate the coupled state/fault estimation errors, fault variations, and mode-transition-dependent stochastic terms. To overcome these difficulties, the output derivative is exploited to construct the disturbance reconstruction channel, a structural decoupling condition is introduced to eliminate the disturbance from the state/fault error dynamics, and a mode-dependent Lyapunov function combined with the robust H_∞ method is employed to establish tractable LMI-based synthesis conditions.

Algorithm 1 The Estimation Process of System States, Actuator Faults and Disturbances

Step 1: Under Assumption 1, compute the matrix T_i based on (28). Then using the obtained T_i to calculate $H_i = I - T_i C_i$.

Step 2: Determine E_i and S_i via (12).

Step 3: Obtain P_i and X_i by solving LMIs (19). Then the observer parameters R_i and U_i can be computed as $R_i = [I_n \ 0] \Theta_i$ and $U_i = [0 \ I_a] \Theta_i$, where $\Theta_i = P_i^{-1} X_i$.

Step 4: According to $G_i = H_i A_i + R_i C_i$ and $M_i = G_i T_i - R_i$, compute G_i and M_i .

Step 5: On the basis of (16), determine K_i and J_i . Finally, synthetic observer (6) to produce $\hat{x}(t)$, $\hat{d}(t)$ and $\hat{f}_a(t)$.

5. Simulation Analysis

In this section, we consider an electronic circuit model to verify the feasibility of the designed approach in this paper. The structure of the circuit is shown in Figure 1. The circuit consists of resistors R_1, R_2, R_3 , inductors L_1, L_2 , capacitors C_1, C_2, C_3 . In addition, $V(t)$ is the power supply, the voltages across C_1, C_2 and C_3 are denoted by $V_1(t), V_2(t), V_3(t)$. The currents through R_1, L_1 and L_2 are represented by $i_s(t), i_1(t)$ and $i_2(t)$. The relevant parameters of the circuit model are listed in Table 1. It is assumed that $f_a(t)$ appears in the power supply $V(t)$. The disturbance $d(t)$ affects $V_3(t)$. Thus, the circuit can be presented by the following model

$$\begin{cases} \frac{di_1(t)}{dt} = \frac{V_1(t) - V_2(t)}{L_1} \\ \frac{di_2(t)}{dt} = \frac{V_2(t) - V_3(t)}{L_2} \\ \frac{dV_1(t)}{dt} = \frac{V(t) + f_a(t) - V_1(t)}{C_1 R_1} - \frac{1}{C_1} i_1(t) \\ \frac{dV_2(t)}{dt} = \frac{i_1(t) - i_2(t)}{C_2} - \frac{s(t)}{C_2 R_2} V_2(t) \\ \frac{dV_3(t)}{dt} = \frac{d(t) - V_3(t)}{C_3 R_3} + \frac{1}{C_3} i_2(t), \end{cases}$$

in which the variable $s(t)$ is supposed to be governed by a Markovian process. It can be written as

$$s(t) = \begin{cases} 0, & \text{switch turn off or } i = 1 \\ 1, & \text{switch turn on or } i = 2 \end{cases}$$

Then, let $x(t) = [i_1^T(t) \ i_2^T(t) \ V_1^T(t) \ V_2^T(t) \ V_3^T(t)]^T$, $u(t) = V(t)$. Thus, the MJSs can be recast as

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + F_i f_a(t) + D_i d(t), \\ y(t) = C_i x(t). \end{cases} \quad (29)$$

in which

$$D_i = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{C_3 R_3} \end{bmatrix}^T, i = 1, 2$$

$$C_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, B_i = F_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{C_1 R_1} \\ 0 \\ 0 \end{bmatrix},$$

$$A_i = \begin{bmatrix} 0 & 0 & \frac{1}{L_1} & -\frac{1}{L_1} & 0 \\ 0 & 0 & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ -\frac{1}{C_1} & 0 & -\frac{1}{C_1 R_1} & 0 & 0 \\ \frac{1}{C_2} & -\frac{1}{C_2} & 0 & -\frac{s(t)}{C_2 R_2} & 0 \\ 0 & \frac{1}{C_3} & 0 & 0 & -\frac{1}{C_3 R_3} \end{bmatrix}.$$

The transition rate matrix in this simulation is selected as $\Pi = \begin{bmatrix} -1 & 1 \\ 0.2 & -0.2 \end{bmatrix}$, the switching signal $m(t)$ is shown in Figure 2. The actuator fault $f_a(t)$ is chosen as $f_a(t) = 0$, $0s \leq t \leq 5s$ and $f_a(t) = 0.2\sin(2t + 6)$, $5s \leq t \leq 30s$. We set the disturbance signal $d(t)$ to be a uniformly distributed random signal within $[-1, 1]$. The simulation results are shown in Figures 3 and 4. We can clearly find that the system state $x(t)$, the disturbance $d(t)$ and the actuator fault $f_a(t)$ are all well estimated. Therefore, the proposed method of this paper is demonstrated to be effective.

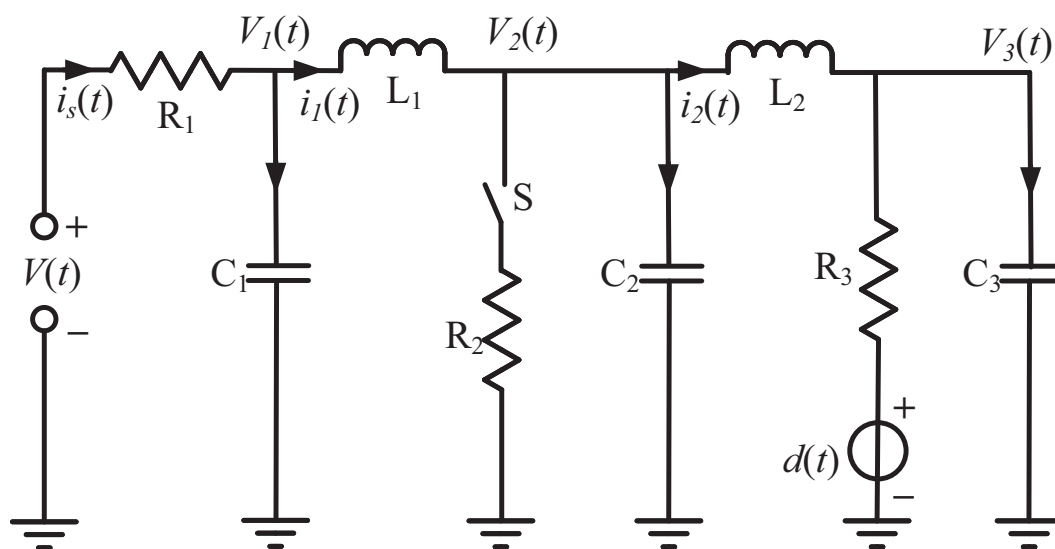


Figure 1. Electronic circuit.

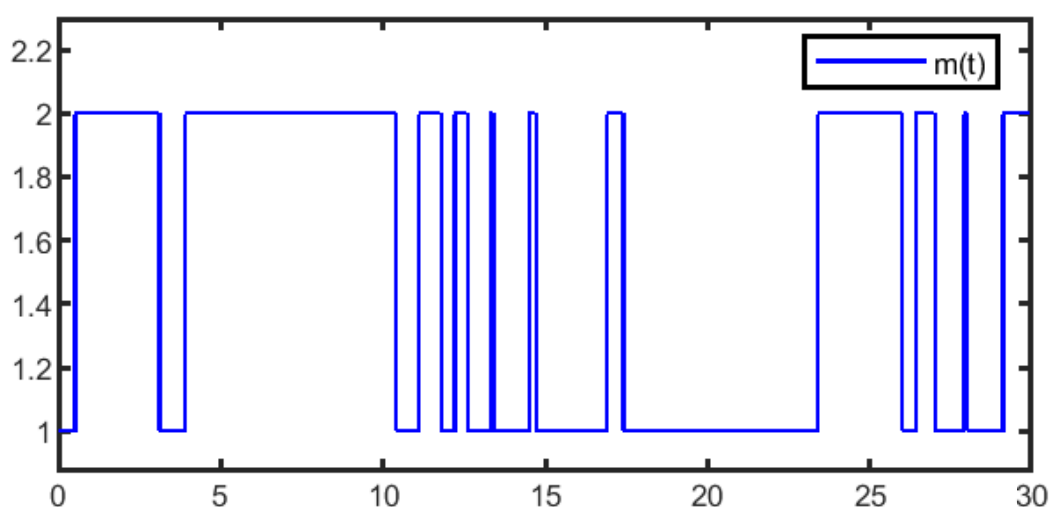


Figure 2. Switching signal.

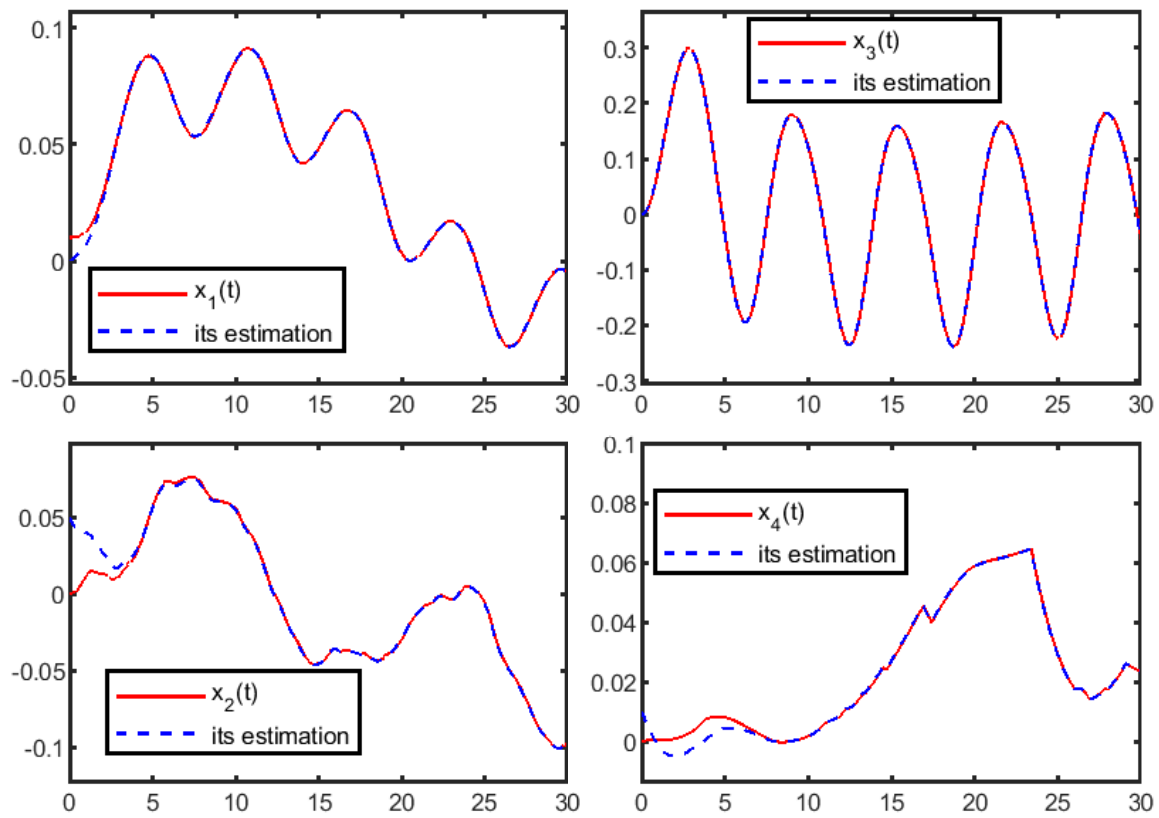


Figure 3. $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$ and their estimations.

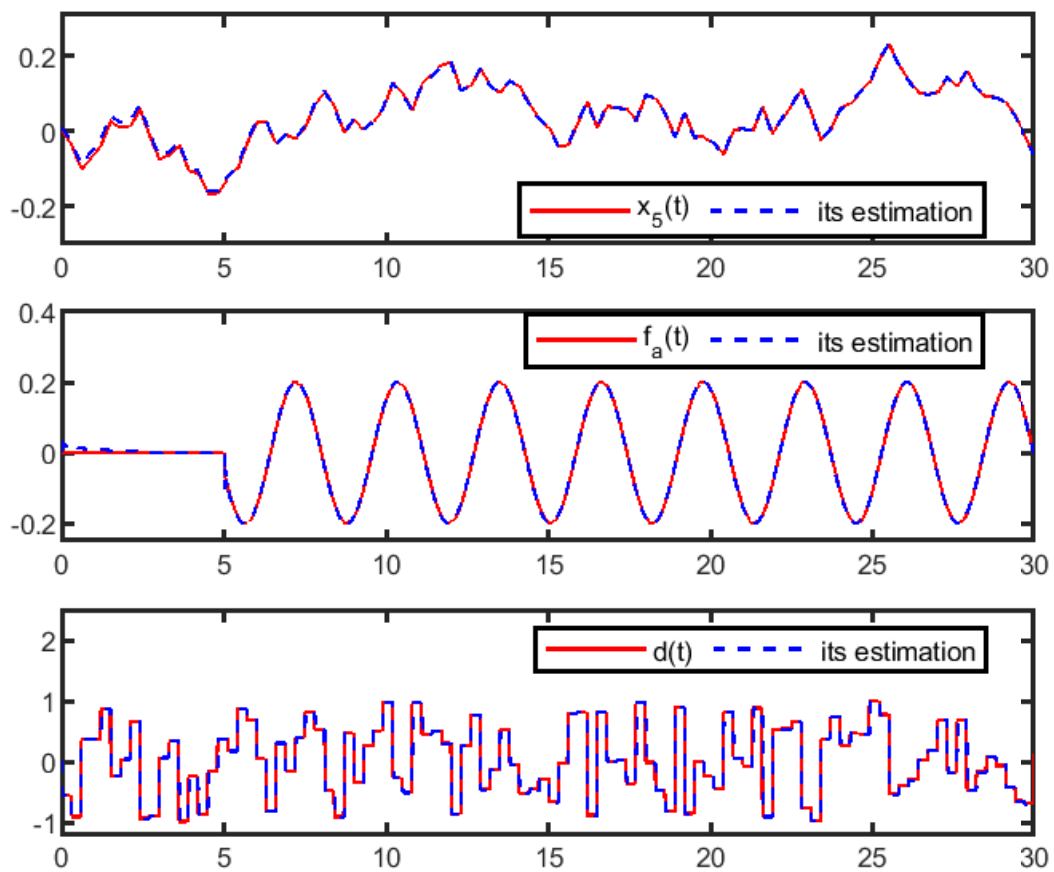


Figure 4. $x_5(t)$, $f_a(t)$, $d(t)$ and their estimations.

Table 1. Model parameters for the circuit.

Symbol	Parameter	Value
$V(t)$	Power resource	$\sin t$
R_1	Resistor	$2\ \Omega$
R_2	Resistor	$5\ \Omega$
R_3	Resistor	$5\ \Omega$
L_1	Inductor	$10\ \text{H}$
L_2	Inductor	$5\ \text{H}$
C_1	Capacitor	$10\ \text{F}$
C_2	Capacitor	$10\ \text{F}$
C_3	Capacitor	$20\ \text{F}$

6. Conclusions

This paper investigated the robust simultaneous reconstruction problem of system states, actuator faults, and external disturbances for continuous-time MJSs. An observer with the derivative of output was designed, in which a structural disturbance-decoupling mechanism completely removes the external disturbance from the state/fault error dynamics. Thus, no known upper bound on the disturbance is required in the stability analysis, while the disturbance itself can still be reconstructed. Based on the Lyapunov stability theory, sufficient conditions ensuring robust stochastic stability of error systems were derived in terms of LMIs. Finally, the feasibility of the designed approach was verified through simulation results on an electronic circuit model. The current framework assumes that the transition rate matrix is completely known. Future research will consider nonlinear MJSs with partially known or generally uncertain transition rates.

Author Contributions

J.L.: Simulation and writing; Y.M.: Simulation and writing; J.W.: Writing, editing; Y.L.: Writing, editing; D.Y.: Writing, editing. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

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