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Can an Einasto Dark-Matter Halo Support a Traversable Wormhole?

Alp Cesur

Vocational School of Health Services, Antalya Bilim University, 07190 Antalya, Türkiye; alp.cesur@antalya.edu.tr

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Abstract: We revisit and extend the proposal that an Einasto dark-matter halo can support a traversable wormhole [B. Sen, *Can. J. Phys.* **96**, 1242–1245 (2018)]. Although the corresponding Einstein equations can be integrated exactly, the density profile fixes only part of the geometry: the boundary condition distinguishes a regular halo from a spacetime in which a throat has been imposed. We give a dimensionally consistent reconstruction and identify algebraic, normalization and unit inconsistencies in the original treatment. For realistic halo parameters, the geometrical deformation remains far too weak to form a throat. Treating the density and rotation curve consistently also yields pressures too small to provide the exotic stress required by a traversable wormhole. A wormhole geometry requires an additional throat-supporting contribution whose influence on galactic motion depends on the associated redshift function. We illustrate these results using the SPARC rotation curve of NGC 3198. The regular halo remains safely in the weak-field regime and contains no halo-generated throat. Under a localized-support interpretation, the data restrict a possible throat to approximately astronomical-unit scales, subject to spherical symmetry and the adopted stellar model. Exact Einasto-dressed wormholes are therefore mathematically admissible, but ordinary Einasto dark matter does not itself generate the throat.

Keywords: traversable wormholes; Einasto profile; dark-matter halos; rotation curves; NGC 3198

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1. Introduction

The relation between the distribution of matter and the geometry of spacetime is one of the central themes of general relativity. Most astrophysical applications concern weak gravitational fields or compact objects with conventional topology, but the Einstein equations also admit geometries connecting otherwise separated spacetime regions. Traversable wormholes are among the best-known examples. They are valuable not only as hypothetical passages between distant regions, but also as theoretical laboratories for studying energy conditions, global topology and the observational distinction between geometry and matter.

A particularly useful framework is the static, spherically symmetric construction introduced by Morris and Thorne [1, 2]. In this description a wormhole contains a minimal two-sphere, called the throat, and remains traversable only if the redshift function is regular there. Within general relativity the required flare-out of the spatial geometry implies violation of the radial null energy condition (NEC) at or near the throat [3]. This requirement is the main physical difficulty of the model: ordinary matter does not naturally provide the necessary combination of positive density and sufficiently negative radial pressure. Considerable effort has therefore been devoted to identifying possible sources of the supporting stress, including scalar fields, semiclassical corrections, effective stresses arising in modified gravity, and anisotropic matter distributions.

Dark matter offers an interesting setting in which to examine this question. Galactic rotation curves require an extended gravitating component whose density can be described by several empirical profiles. The Navarro–Frenk



–White profile is widely used, while the Einasto profile provides a smooth, finite-density alternative with a continuously varying logarithmic slope [4–6]. These profiles are normally introduced to reproduce the mass distribution of a galactic halo, not to describe exotic matter. Nevertheless, because a specified density can be inserted directly into the spherical Einstein equations, it is natural to ask whether a dark-matter halo might also generate or support a wormhole geometry.

This possibility was first explored using Navarro–Frenk–White and universal-rotation-curve models [7–9]. Sen subsequently applied the Einasto profile to a Morris–Thorne geometry and considered both constant and polynomial representations of galactic rotation curves [10]. Later work examined related configurations with isotropic pressure, monopole and semiclassical contributions, alternative gravitational theories and nonconstant redshift functions [11–14]. More recently, an observational reconstruction based on NGC 3198 treated the small value of the shape-function ratio in the measured region as compatible with the exterior of an unresolved wormhole [15]. These studies demonstrate that dark-matter profiles can be incorporated into formal wormhole solutions, but they leave open the more restrictive question of whether a realistic halo actually produces a throat.

The distinction is important because the density determines only the radial derivative of the shape function. Its additive integration constant, fixed by a boundary condition, determines whether the solution has a regular centre or begins at an imposed throat. Consequently, finding that the shape function is smaller than the radial coordinate throughout an observed galactic region does not demonstrate the existence of a throat at an unresolved radius. It may instead describe an ordinary weak-field halo with no throat anywhere. Similarly, the derivative condition usually associated with flare-out has geometrical significance only after a radius satisfying the throat condition has been identified. The density profile, the rotation curve, the integration constant and the required pressures must therefore be treated within one consistent normalization and system of units.

In the present paper we carry out this consistency analysis for the Einasto profile. We first derive the exact shape function for an arbitrary Einasto index and compare the result with the numerical expressions used in the original construction. The regular-centre solution then leads to a simple global criterion showing that a realistic Einasto halo cannot develop a throat. We also relate the halo mass to its circular speed and reconstruct the associated pressures, which shows that the radial pressure is only a very small weak-field correction and does not supply the exotic stress required by a wormhole. We then introduce a throat through the integration constant and determine how this additional geometrical scale enters the pressure distribution and the exterior circular motion.

To test the astrophysical scale of these effects, we fit the Einasto model together with the observed baryonic components to the rotation curve of NGC 3198 from the Spitzer Photometry and Accurate Rotation Curves (SPARC) database [16]; an independent analysis of the dark-matter distribution in this galaxy is given in Ref. [17]. The resulting parameters provide a direct comparison between the weak geometry of a realistic halo and the order-unity deformation required at a wormhole throat. We then use the same data to obtain a conditional limit on a localized throat-supporting contribution and discuss the assumptions under which this limit is applicable. The analysis clarifies the difference between an exact wormhole surrounded by Einasto matter and a wormhole physically produced by that matter.

The paper is organized as follows. Section 2 reviews the spherical Einstein equations and the throat conditions. Section 3 gives the exact Einasto reconstruction and the no-throat result. Section 4 discusses consistency issues in the earlier treatment, while Section 5 examines the geometries obtained by imposing a throat. The NGC 3198 fit and observational constraint are presented in Section 6, and the physical interpretation and limitations are discussed in Section 7. We use $G = c = 1$ in the formal relativistic equations. In the observational analysis ρ denotes mass density, all factors of G and c are restored explicitly, and the corresponding geometrized variables are $\rho_{\text{geo}} = G\rho/c^2$ and $p_{\text{geo}} = Gp/c^4$. Here G is Newton’s gravitational constant, c is the speed of light, ρ_{geo} is the geometrized mass density and p_{geo} is the geometrized pressure.

2. Morris–Thorne Reconstruction

The geometry of a static, spherically symmetric wormhole can be represented in the general Morris–Thorne form

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 d\Omega^2, \quad (1)$$

In Equation (1), ds^2 is the spacetime line element, t is the time coordinate, r is the areal radial coordinate and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element on the unit two-sphere, with θ and φ the angular coordinates. The function $\Phi(r)$ is the redshift function, which controls gravitational time dilation and must remain finite in a traversable geometry, while $b(r)$ is the shape function governing the spatial geometry. The matter supporting this spacetime is described by the anisotropic stress tensor $T^\mu{}_\nu = \text{diag}(-\rho, p_r, p_t, p_t)$, where the indices μ and ν run over the four spacetime coordinates, and ρ , p_r and p_t denote the energy density, radial pressure and tangential

pressure, respectively. A prime denotes differentiation with respect to r . With these definitions, the independent Einstein equations take the form

$$8\pi\rho = \frac{b'}{r^2}, \quad (2)$$

$$8\pi p_r = -\frac{b}{r^3} + \frac{2}{r}\left(1 - \frac{b}{r}\right)\Phi', \quad (3)$$

$$8\pi p_t = \left(1 - \frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r(r-b)}\left(\Phi' + \frac{1}{r}\right)\right]. \quad (4)$$

The conservation equation,

$$p_r' = -(\rho + p_r)\Phi' + \frac{2}{r}(p_t - p_r), \quad (5)$$

is then an identity rather than an additional field equation.

The metric in Equation (1) represents a traversable wormhole only when the spatial geometry develops a regular minimum of the areal radius without producing an event horizon. Denoting the position of this minimum by r_{th} , the corresponding throat conditions are

$$b(r_{\text{th}}) = r_{\text{th}}, \quad b'(r_{\text{th}}) < 1, \quad |\Phi(r_{\text{th}})| < \infty, \quad (6)$$

and $b(r) < r$ must hold throughout each exterior. Equation (3) immediately gives

$$p_r(r_{\text{th}}) = -\frac{1}{8\pi r_{\text{th}}^2}, \quad (\rho + p_r)_{r_{\text{th}}} = \frac{b'(r_{\text{th}}) - 1}{8\pi r_{\text{th}}^2} < 0. \quad (7)$$

Thus a positive density at the throat does not remove the required radial tension.

The redshift function can also be related directly to galactic observations. For a test particle moving on an equatorial circular geodesic, let $v_c(r)$ denote the tangential circular speed measured by a static observer. It obeys

$$\frac{v_c^2(r)}{c^2} = r\Phi'(r). \quad (8)$$

The explicit c^{-2} is essential when observed velocities are supplied in km s^{-1} .

3. Exact Einasto Reconstruction

We now specialize the general reconstruction to an astrophysically motivated dark-matter distribution. The Einasto model is particularly convenient because it is nonsingular at the centre, decreases rapidly at large radii and retains a flexible logarithmic slope. We write it in the standard three-parameter form [5,6]

$$\rho_E(r) = \rho_{-2} \exp\left[-\frac{2}{\alpha}\left(\left(\frac{r}{r_{-2}}\right)^\alpha - 1\right)\right], \quad (9)$$

Here $\rho_E(r)$ is the Einasto mass density, ρ_{-2} is its value at $r = r_{-2}$, r_{-2} is the radius at which the logarithmic slope equals -2 , and the positive dimensionless parameter α controls the curvature of the density profile. Defining

$$x = \frac{r}{r_{-2}}, \quad z = \frac{2}{\alpha}x^\alpha, \quad (10)$$

where x is a dimensionless radius and z is the corresponding incomplete-gamma argument, the mass $M_E(r)$ enclosed within a sphere of radius r is

$$M_E(r) = \frac{4\pi\rho_{-2}r_{-2}^3}{\alpha} e^{2/\alpha} \left(\frac{\alpha}{2}\right)^{3/\alpha} \gamma\left(\frac{3}{\alpha}, z\right), \quad (11)$$

where $\gamma(a, z)$ is the lower incomplete gamma function. Restoring units, Equation (2) integrates to

$$b(r) = C + \frac{2G}{c^2}M_E(r) \equiv C + b_E(r). \quad (12)$$

In Equation (12), C is an integration constant with dimensions of length and $b_E(r) = 2GM_E(r)/c^2$ is the contribution of the Einasto halo to the shape function. The density fixes b' but does not fix C .

3.1. Regular Normalization and a No-Throat Theorem

The first possibility is to impose the usual boundary condition for an ordinary galactic halo, namely that the enclosed geometrical mass vanish at the centre. This regular-centre normalization sets $C = 0$ and provides the appropriate reference solution against which a wormhole geometry should be compared. The derivative of the resulting shape function is

$$b'_E(r) = \lambda_E x^2 \exp\left[-\frac{2}{\alpha}(x^\alpha - 1)\right], \quad \lambda_E = \frac{8\pi G}{c^2} \rho_{-2} r_{-2}^2. \quad (13)$$

Its logarithmic derivative is $2 - 2x^\alpha$, so the unique maximum occurs at $x = 1$ and

$$\max_{r \geq 0} b'_E(r) = \lambda_E. \quad (14)$$

The dimensionless quantity λ_E therefore measures the maximum geometrical strength of the Einasto halo, while b'_E denotes the derivative of its shape-function contribution.

No-Throat Result

If $C = 0$ and $\lambda_E < 1$, then $b_E(r) < r$ for every $r > 0$ and the spacetime has no wormhole throat.

Proof. Since $b_E(0) = 0$ and $b'_E(r) < 1$,

$$\frac{d}{dr} [r - b_E(r)] = 1 - b'_E(r) > 0. \quad (15)$$

Therefore $r - b_E(r) > 0$ for all $r > 0$. □

The same inequality often described as “satisfying flare-out everywhere” therefore proves the absence of a throat when combined with the regular normalization. A flare-out derivative is not a throat-existence criterion.

3.2. Rotation-Consistent Redshift and Effective Pressure

A density reconstruction becomes physically meaningful only when it is compatible with the gravitational potential inferred from orbital motion. Let $v_h(r)$ denote the circular speed generated by the Einasto halo alone. For a spherical halo whose enclosed mass is $M_E(r)$, the standard Newtonian mass–velocity relation is

$$v_h^2(r) = \frac{GM_E(r)}{r}, \quad (16)$$

Equations (8) and (12), with $C = 0$, give

$$\Phi'_E(r) = \frac{b_E(r)}{2r^2}, \quad \frac{b_E(r)}{r} = 2 \frac{v_h^2(r)}{c^2}. \quad (17)$$

We shall refer to the dimensionless ratio $b_E(r)/r$ as the compactness of the regularly normalized Einasto halo. The symbol $\Phi_E(r)$ will denote the redshift function generated by the Einasto contribution. Choosing its additive constant so that $\Phi_E \rightarrow 0$ at spatial infinity gives the closed form

$$\Phi_E(r) = -\frac{G}{c^2} \left[\frac{M_E(r)}{r} + \frac{4\pi\rho_{-2}r_{-2}^2}{\alpha} e^{2/\alpha} \left(\frac{\alpha}{2}\right)^{2/\alpha} \Gamma\left(\frac{2}{\alpha}, z\right) \right], \quad (18)$$

where $\Gamma(a, z)$ is the upper incomplete gamma function.

Substitution of Equation (17) into the full Einstein equations yields, in geometrized units,

$$8\pi p_r = -\frac{b_E^2}{r^4}, \quad (19)$$

$$8\pi p_t = -\frac{b_E}{4r^5} (3r^2 b'_E - 4r b_E + b_E^2), \quad (20)$$

$$8\pi(\rho + p_r) = \frac{r^2 b'_E - b_E^2}{r^4}. \quad (21)$$

Thus the radial pressure is second order in the weak-field compactness. In a galactic region with $b/r \sim 10^{-6}$

it is not of the same order as the density and cannot supply throat exoticity. An apparent first-order NEC violation obtained from the same mass and velocity profile indicates inconsistent units, incompatible normalizations, or independent inputs that do not describe one spherical source.

An infinitely extended Einasto tail eventually becomes too dilute for Equation (21) to remain positive, while Equation (18) is not exactly the exterior Schwarzschild redshift at second order. A physical halo should be truncated or matched to an exterior solution at a finite radius; this does not affect the observed weak-field result.

4. Consistency Checks of the Original Einasto Construction

Having obtained the general Einasto solution, we can compare it directly with the expressions used in the original Canadian Journal of Physics analysis. Ref. [10] established the useful idea of inserting an Einasto profile into the Morris–Thorne equations, but some of its printed formulae require further consistency checks. In the discussion below, $f(r)$ denotes the redshift function used in Ref. [10]; it plays the same role as $\Phi(r)$ in our notation. Several formulae in the accepted manuscript do not pass direct differentiation or dimensional checks. They may partly be typographical or transcription errors, but they materially affect the geometrical interpretation. Table 1 summarizes the main points.

For comparison, when $\alpha = 2$, Equation (9) becomes a Gaussian in r and its regular shape function is

$$b_E(r) = \frac{8\pi G \rho_{-2} e}{c^2} \left[\frac{\sqrt{\pi}}{4} r_{-2}^3 \operatorname{erf}\left(\frac{r}{r_{-2}}\right) - \frac{1}{2} r_{-2}^2 r e^{-r^2/r_{-2}^2} \right]. \quad (22)$$

Here erf denotes the error function. There is no term proportional to r . Equation (22) also makes it straightforward to check every numerical coefficient before plotting.

Table 1. Diagnostic checks on the formulae printed in Ref. [10].

Location	Printed or Implied Step	Diagnostic Consequence
Equations (8) and (15)	The shape function contains a term $+9r$.	The remaining terms decay or approach constants, so $b' \rightarrow 9$ and $b/r \rightarrow 9$. This contradicts both $b' = 8\pi\rho r^2$ for a decaying Einasto density and the plotted claim $b' < 1$. No linear term arises in the exact density integral.
Equation (13)	The displayed expression is labelled $f'(r)$.	Direct differentiation of Equation (12) shows that Equation (13) equals $f'(r)/r$, while Equation (14) is consistent with $f''(r)$. Using Equation (13) as f' changes both pressures.
Equation (11)	A polynomial rotation speed is inserted after setting $c = 1$.	The observational polynomial used in Ref. [9] carries a conversion of order 3×10^{-6} from km s^{-1} to units of c . Omitting it makes $r\Phi'$ too large by roughly eleven orders of magnitude.
Notation	r_0 denotes a profile scale and later an integration constant.	Neither role should be identified automatically with the throat. The throat must be found or imposed from $b(r_{\text{th}}) = r_{\text{th}}$.
Global geometry	$b' < 1$ and NEC plots are shown on a selected interval.	These tests do not demonstrate a root $b = r$, absence of forbidden regions, an asymptotically valid redshift, or matching at the halo boundary.

5. Einasto-Dressed Wormhole Geometries

The regular-centre solution does not contain a throat, but the same local Einasto density can be embedded in a wormhole spacetime by taking the radial domain to begin at $r = r_{\text{th}}$ and fixing the integration constant through $b(r_{\text{th}}) = r_{\text{th}}$. The corresponding shape function is

$$b(r) = r_{\text{th}} + \frac{2G}{c^2} [M_E(r) - M_E(r_{\text{th}})] = C + b_E(r), \quad C = r_{\text{th}} - b_E(r_{\text{th}}). \quad (23)$$

If $\lambda_E < 1$, then $b' < 1$ globally and $b(r) < r$ for every $r > r_{\text{th}}$. Because the Einasto mass is finite, $b/r \rightarrow 0$. Any finite, asymptotically vanishing and sufficiently smooth Φ then gives an exact traversable geometry with pressures determined by Equations (3) and (4).

For realistic halo parameters $b_E(r_{\text{th}}) \ll r_{\text{th}}$, and hence $C \simeq r_{\text{th}}$. The integration constant therefore carries the throat scale. Its observable effect depends on how the redshift potential is assigned.

5.1. Exterior Dynamics

The simplest choice is to let the full shape function determine the potential measured by circular motion,

$$\Phi'_{\text{vis}} = \frac{b}{2r^2}, \quad \Phi_{\text{vis}} = \Phi_E - \frac{C}{2r}. \quad (24)$$

Here Φ_{vis} denotes the redshift function obtained when the constant C contributes to the exterior potential. This choice is finite at $r = r_{\text{th}}$, asymptotically flat, and produces

$$\frac{v_c^2}{c^2} = \frac{b}{2r}, \quad 8\pi p_r = -\frac{b^2}{r^4}. \quad (25)$$

The constant acts in the rotation curve as a central mass

$$M_C = \frac{c^2 C}{2G} = 5.07 \times 10^7 M_\odot \left(\frac{C}{1 \text{ AU}} \right). \quad (26)$$

Here M_C is the central mass equivalent of the integration constant, $M_\odot$ denotes one solar mass and AU denotes one astronomical unit. The pressure is small far from the throat, but Equation (7) still enforces NEC violation near it. For an approximately constant central halo density, writing $b'_E \simeq \kappa r^2$ with $\kappa = 8\pi G \rho_E(0)/c^2$, Equation (21) generalizes to $r^2 b'_E - b^2$. The approximate outer edge of the inner NEC-violating region is therefore

$$r_{\text{NEC}} \simeq \left(\frac{C^2}{\kappa} \right)^{1/4}. \quad (27)$$

Even an astronomical-unit throat can consequently require exotic stress over parsec scales in a dilute galactic core.

One may instead retain the halo redshift gradient $\Phi' = b_E/(2r^2)$ while keeping C only in the spatial geometry. Equation (3) then gives

$$8\pi p_r = -\frac{C}{r^3} - \frac{b_E b}{r^4}. \quad (28)$$

The leading tension $-C/(8\pi r^3)$ removes the corresponding Keplerian term from Φ . Thus the same density profile admits different relations between the two metric potentials, but only at the cost of changing the pressure distribution. Rotation data alone cannot select between these geometrical completions.

5.2. Finite-Radius Matching

An astrophysical halo occupies a finite region and must ultimately be connected to an exterior spacetime. Let R denote the outer matching radius and let M denote the total Schwarzschild mass measured outside that radius. If the matter region is matched at R to a Schwarzschild exterior without a thin shell, continuity gives $b(R) = 2GM/c^2$ and $e^{2\Phi(R)} = 1 - b(R)/R$, while continuity of the extrinsic curvature is equivalent to $p_r(R) = 0$. Equation (3) then implies

$$u(R) \equiv \frac{v_c^2(R)}{c^2} = \frac{b(R)}{2[R - b(R)]}, \quad \frac{b(R)}{R} = \frac{2u(R)}{1 + 2u(R)}. \quad (29)$$

The quantity $u(R)$ introduced here is the squared circular speed at the matching surface in units of c^2 . Thus localized exotic support makes C observable through outer dynamics and lensing. Avoiding this conclusion requires a nonzero pressure or a junction layer at R , which must itself be included in the matter budget.

Table 2 illustrates the enormous mass scale geometrically associated with macroscopic values of $C \simeq r_{\text{th}}$. In the table, pc denotes a parsec and kpc denotes a kiloparsec, equal to 10^3 pc.

Table 2. Central mass equivalent of a throat-determining integration constant, $M_C = c^2 C/(2G)$.

$C \simeq r_{\text{th}}$	M_C
1 AU	$5.07 \times 10^7 M_\odot$
1 pc	$1.04 \times 10^{13} M_\odot$
1 kpc	$1.04 \times 10^{16} M_\odot$
5 kpc	$5.22 \times 10^{16} M_\odot$

6. Application to NGC 3198

We now apply the reconstruction to NGC 3198, whose extended rotation curve is accompanied by a decomposition of the visible matter. Distances below are expressed in parsecs (pc), kiloparsecs (kpc) or megaparsecs (Mpc), with $1 \text{ kpc} = 10^3 \text{ pc}$ and $1 \text{ Mpc} = 10^6 \text{ pc}$.

6.1. Data and Einasto Fit

NGC 3198 is a benchmark spiral galaxy whose approximately flat rotation curve extends far beyond the luminous disk [17]. We use its 43-point Spitzer Photometry and Accurate Rotation Curves (SPARC) Newtonian mass model [16], including the tabulated gas and stellar-disk velocity contributions. The nominal distance is 13.8 Mpc and the observed interval is 0.32–44.08 kpc. We fix the disk mass-to-light ratio to $\Upsilon_d = 0.5$ in solar units at wavelength $3.6 \mu\text{m}$, where μm denotes micrometres, and model

$$v_{\text{mod}}^2(r) = v_{\text{gas}}|v_{\text{gas}}| + \Upsilon_d v_{\text{disk}}^2 + \frac{GM_E(r)}{r}. \quad (30)$$

In Equation (30), $v_{\text{mod}}(r)$ is the predicted total circular speed, $v_{\text{gas}}(r)$ and $v_{\text{disk}}(r)$ are the tabulated gas and unit-mass-to-light stellar-disk contributions, and Υ_d rescales the stellar contribution to the adopted disk mass-to-light ratio.

The sign convention in the gas term follows the SPARC table. The three logarithmic Einasto parameters are fitted by minimizing

$$\chi^2 = \sum_i \frac{[v_{\text{mod}}(r_i) - v_{\text{obs}}(r_i)]^2}{\sigma_i^2}. \quad (31)$$

The index i labels the 43 measured radii, r_i is the i th radius, $v_{\text{obs}}(r_i)$ is the observed circular speed, σ_i is its quoted one-standard-deviation uncertainty, and χ^2 is the corresponding goodness-of-fit statistic. Formal covariance errors are quoted conditional on the adopted distance, inclination and Υ_d .

The fit in Figure 1 gives $\chi^2 = 39.99$ for 40 degrees of freedom. We denote the number of degrees of freedom by ν , so that the reduced statistic is χ^2/ν . Table 3 lists the parameters and the resulting geometry. The central density inferred from the fit is $\rho_E(0) = 5.85 \times 10^8 M_\odot \text{ kpc}^{-3}$, and $M_E(44.08 \text{ kpc}) = 1.71 \times 10^{11} M_\odot$.

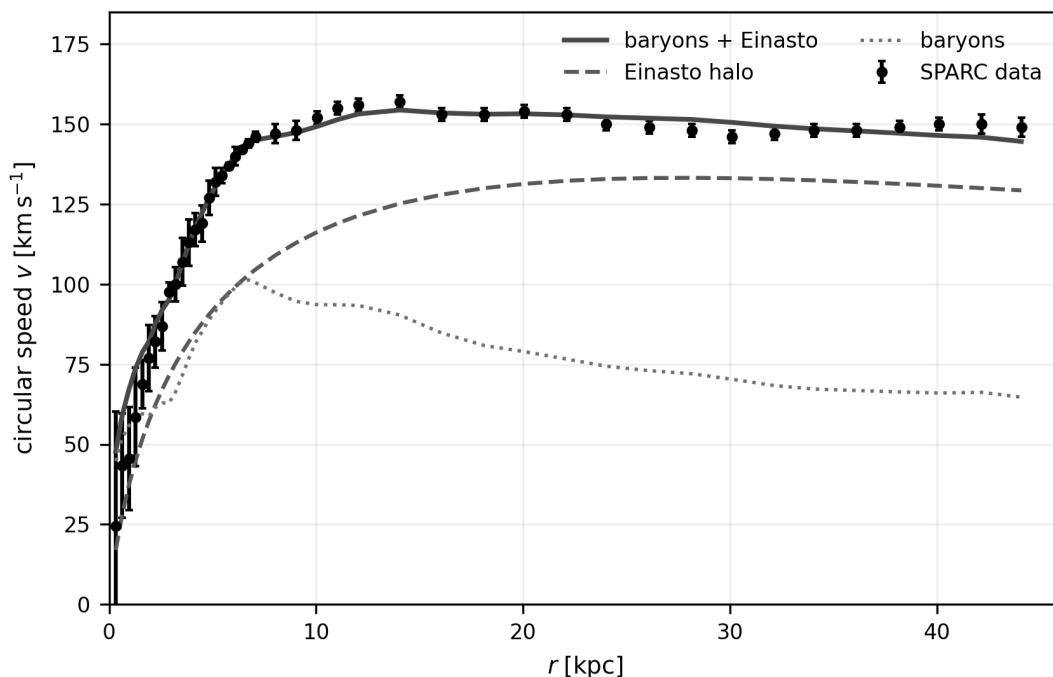


Figure 1. SPARC rotation curve of NGC 3198 and the best-fitting baryon-plus-Einasto model. Unlike a halo-only fit, the calculation includes the published gas and stellar-disk contributions.

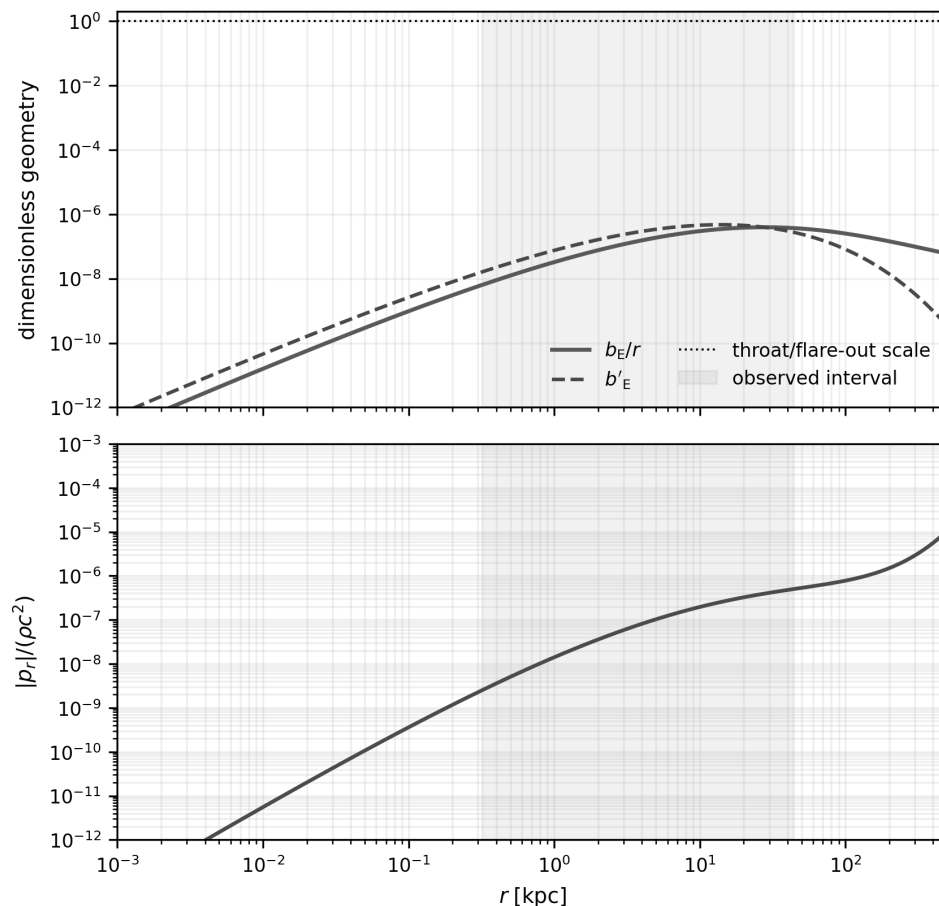
In Table 3, $M_E(\infty)$ is the asymptotic total Einasto mass, and the subscript “data” indicates that the maximum is taken over the 43 observed radii.

Table 3. Best-fitting NGC 3198 Einasto parameters and derived diagnostics. Errors are formal one-standard-deviation covariance estimates.

Quantity	Value
ρ_{-2}	$(2.04 \pm 0.17) \times 10^6 M_{\odot} \text{ kpc}^{-3}$
r_{-2}	$13.89 \pm 0.52 \text{ kpc}$
α	0.354 ± 0.029
χ^2/ν	$39.99/40 = 1.00$
$M_E(44.08 \text{ kpc})$	$1.71 \times 10^{11} M_{\odot}$
$M_E(\infty)$	$3.12 \times 10^{11} M_{\odot}$
$\max b'_E = \lambda_E$	4.74×10^{-7}
$\max_{\text{data}}(b_E/r)$	3.95×10^{-7}
$\max_{\text{data}} p_r /(\rho c^2)$	5.02×10^{-7}

6.2. Geometrical Implications

The fitted parameters determine the geometrical scale of the halo. Figure 2 compares b_E/r and b'_E with unity. Both are smaller by more than six orders of magnitude, and $\lambda_E < 1$ establishes that the regular Einasto solution has no positive-radius root $b_E = r$.

**Figure 2.** Geometry and pressure of the regularly normalized NGC 3198 Einasto halo. Top: the compactness b_E/r and derivative b'_E remain far below unity. Bottom: the rotation-consistent radial pressure is a sub-part-per-million correction to the positive density throughout the measured interval.

Equation (21) is positive at every observed point. Numerically,

$$\frac{\rho c^2 + p_r}{\rho c^2} = 1 - \frac{(b_E/r)^2}{b'_E} > 0.99999949 \quad (32)$$

over the measured range. The fitted halo therefore satisfies the radial NEC throughout the observed region and does not generate a throat. In particular, a small value of b/r at the measured radii does not by itself identify a throat at smaller radius [15]. The effect of changing the integration constant while keeping the Einasto density fixed is illustrated in Figure 3.

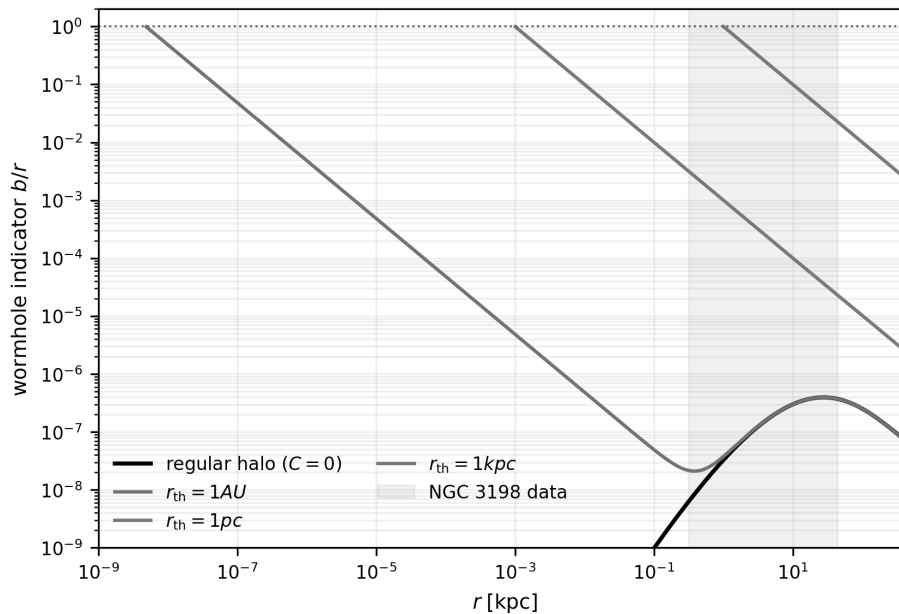


Figure 3. Examples obtained by changing the additive constant while keeping the same Einasto density derivative. The imposed-throat curves satisfy $b(r_{\text{th}})/r_{\text{th}} = 1$, whereas the regular-centre solution has no throat.

6.3. Constraint on a Central Contribution

To estimate the allowed scale of a central contribution to the exterior potential, we add GM_C/r to Equation (30) and re-optimize the Einasto parameters at each fixed nonnegative M_C . The likelihood peaks at $M_C = 0$. The corresponding profile likelihood is shown in Figure 4. Here $\Delta\chi^2$ is the increase in the optimized chi-square relative to its minimum. The one-sided 95% profile criterion $\Delta\chi^2 = 2.71$ gives

$$M_C < 8.78 \times 10^7 M_\odot, \quad C = \frac{2GM_C}{c^2} < 1.73 \text{ AU}. \quad (33)$$

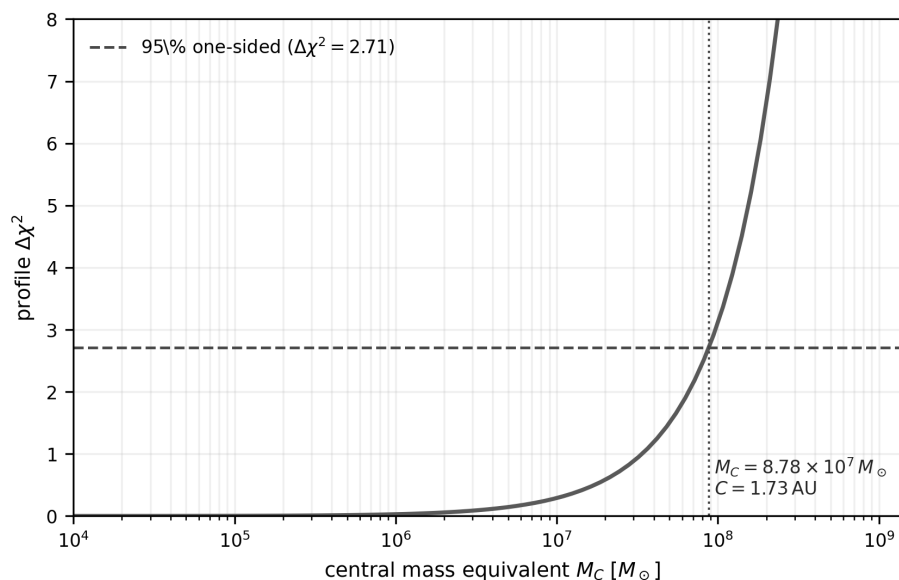


Figure 4. Profile likelihood for the central mass equivalent of a localized throat-supporting integration constant. The three Einasto parameters are re-optimized at each M_C ; $\Upsilon_d = 0.5$ is held fixed.

At this scale $b_E(r_{\text{th}})$ is negligible, so $r_{\text{th}} \simeq C$. This interpretation assumes that the central contribution enters the exterior redshift according to Equation (24); rotation data alone do not constrain pressure prescriptions that remove it from the temporal potential.

For the limiting value in Equation (33), direct evaluation of Equation (27) gives an inner NEC-violating region of approximately 3.4 pc. The supporting stress in this model is therefore distributed beyond the immediate neighbourhood of the throat.

7. Physical Interpretation

An Einasto profile specifies the density but not the complete matter dynamics. Once a redshift function is chosen, the Einstein equations determine the radial and tangential pressures, yet this reconstruction does not supply an equation of state or a microscopic Lagrangian. In particular, collisionless dark matter described by a nonnegative phase-space distribution satisfies the NEC and cannot provide the throat tension in Equation (7). The pressure associated with the constant C must therefore be attributed to an additional matter component, or to an effective stress tensor outside ordinary Einstein gravity.

The NGC 3198 calculation should be read within the spherical model used to derive it. Circular motion constrains Φ' , whereas lensing is also sensitive to the spatial potential and would provide an independent test of the integration constant and pressure distribution. Moreover, the observed baryonic components belong to a disk even though the halo reconstruction is spherical. These features do not affect the no-throat result for the regular Einasto solution, but they make the quoted central-mass limit conditional on the adopted distance, inclination and stellar mass-to-light ratio.

8. Conclusions

The Einstein equations admit exact Morris–Thorne geometries whose density component follows an Einasto profile. The exact integral involves an incomplete gamma function, or an error function for $\alpha = 2$. This mathematical statement is substantially weaker than saying that an astrophysical Einasto halo generates a wormhole.

For the regular boundary condition $b(0) = 0$, the global maximum of b' is the single dimensionless parameter $\lambda_E = 8\pi G\rho_{-2}r_{-2}^2/c^2$. When $\lambda_E < 1$, $r - b(r)$ increases monotonically and no throat exists. Realistic halos lie deep in this regime. If the same spherical mass profile generates the rotation curve, $b/r = 2v_h^2/c^2$ and the effective radial pressure is second order in the galactic compactness. The observed halo therefore satisfies the radial NEC with an enormous margin.

A throat can be imposed through $C = r_{\text{th}} - b_E(r_{\text{th}})$, but the resulting spacetime is an Einasto-dressed wormhole rather than a wormhole generated by the halo. For a central contribution that enters the exterior potential, the NGC 3198 data give the preliminary conditional limit $r_{\text{th}} \lesssim 1.73$ AU.

The present analysis is static. A dynamical stability calculation would require a time-dependent model for the supporting matter and its perturbations, as in earlier studies of other wormhole configurations [18–23]. We do not attempt such a calculation here.

A natural continuation is to specify an explicit two-component source, $T_{\mu\nu} = T_{\mu\nu}^{\text{Einasto}} + T_{\mu\nu}^{\text{exotic}}$, and then study its matching, perturbations and combined rotation–lensing signal. This would turn the present geometrical reconstruction into a definite physical model.

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Data Availability Statement

The numerical analysis uses the public SPARC Newtonian mass model for NGC 3198 [16]. The addition numerical data can be provided by the corresponding author upon reasonable request.

Conflicts of Interest

The author declares no conflict of interest.

Use of AI and AI-Assisted Technologies

During the preparation of this work, the author used ChatGPT (OpenAI) to assist with English-language editing, including refining the language and correcting grammatical errors. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

1. Morris, M.S.; Thorne, K.S. Wormholes in Spacetime and Their Use for Interstellar Travel: A Tool for Teaching General Relativity. *Am. J. Phys.* **1988**, *56*, 395–412. <https://doi.org/10.1119/1.15620>.
2. Visser, M. *Lorentzian Wormholes: From Einstein to Hawking*; AIP Press: Woodbury, NY, USA, 1995.
3. Hochberg, D.; Visser, M. Null Energy Condition in Dynamic Wormholes. *Phys. Rev. Lett.* **1998**, *81*, 746–749. <https://doi.org/10.1103/physrevlett.81.746>.
4. Navarro, J.F.; Frenk, C.S.; White, S.D.M. The Structure of Cold Dark Matter Halos. *Astrophys. J.* **1996**, *462*, 563–575. <https://doi.org/10.1086/177173>.
5. Merritt, D.; Graham, A.W.; Moore, B.; et al. Empirical Models for Dark Matter Halos. I. Nonparametric Construction of Density Profiles and Comparison with Parametric Models. *Astron. J.* **2006**, *132*, 2685–2700. <https://doi.org/10.1086/508988>.
6. Retana-Montenegro, E.; Van Hese, E.; Gentile, G.; et al. Analytical Properties of Einasto Dark Matter Haloes. *Astron. Astrophys.* **2012**, *540*, A70. <https://doi.org/10.1051/0004-6361/201118543>.
7. Kuhfittig, P.K.F. Gravitational Lensing of Wormholes in the Galactic Halo Region. *Eur. Phys. J. C* **2014**, *74*, 2818. <https://doi.org/10.1140/epjc/s10052-014-2818-2>.
8. Rahaman, F.; Salucci, P.; Kuhfittig, P.K.F.; et al. Possible Existence of Wormholes in the Central Regions of Halos. *Ann. Phys.* **2014**, *350*, 561–567. <https://doi.org/10.1016/j.aop.2014.08.003>.
9. Rahaman, F.; Shit, G.C.; Sen, B. Could Wormholes Form in Dark Matter Galactic Halos? *Astrophys. Space Sci.* **2016**, *361*, 37. <https://doi.org/10.1007/s10509-015-2626-7>.
10. Sen, B. Existence of Wormhole Supported by the Einasto Density Profile. *Can. J. Phys.* **2018**, *96*, 1242–1245. <https://doi.org/10.1139/cjp-2017-0524>.
11. Xu, Z.; Tang, M.; Cao, G.; et al. Possibility of Traversable Wormhole Formation in the Dark Matter Halo with Isotropic Pressure. *Eur. Phys. J. C* **2020**, *80*, 70. <https://doi.org/10.1140/epjc/s10052-020-7636-0>.
12. Rahaman, F.; Samanta, B.; Sarkar, N.; et al. Traversable Wormholes Supported by Dark Matter and Monopoles with Semiclassical Effects. *Eur. Phys. J. C* **2023**, *83*, 395. <https://doi.org/10.1140/epjc/s10052-023-11456-4>.
13. Yousaf, Z.; Bhatti, M.Z.; Rizwan, M.; et al. From Dark Halos to Exotic Bridges: Wormhole Structures with Einasto Density Profiles. *Nucl. Phys. B* **2025**, *1019*, 117128. <https://doi.org/10.1016/j.nuclphysb.2025.117128>.
14. Albalahi, A.M.; Yousaf, Z.; Rizwan, M.; et al. Constructing Traversable Wormholes in Dark Matter Halos with a Non-Trivial Redshift Function. *Eur. Phys. J. C* **2026**, *86*, 142. <https://doi.org/10.1140/epjc/s10052-026-15386-9>.
15. Ray, S.; Sanyal, A. Traversable Wormhole Geometry Reconstruction from the Rotation Curve of NGC 3198: A Comparative Study of Dark Matter Halo Profiles. *arXiv* **2026**, arXiv:2605.20221.
16. Lelli, F.; McGaugh, S.S.; Schombert, J.M. SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. *Astron. J.* **2016**, *152*, 157. <https://doi.org/10.3847/0004-6256/152/6/157>.
17. Karukes, E.V.; Salucci, P.; Gentile, G. The Dark Matter Distribution in the Spiral NGC 3198 out to $0.22R_{\text{vir}}$. *Astron. Astrophys.* **2015**, *578*, A13. <https://doi.org/10.1051/0004-6361/201425339>.
18. Konoplya, R.A.; Molina, C. Ringing Wormholes. *Phys. Rev. D* **2005**, *71*, 124009. <https://doi.org/10.1103/physrevd.71.124009>.
19. Bronnikov, K.A.; Konoplya, R.A.; Zhidenko, A. Instabilities of Wormholes and Regular Black Holes Supported by a Phantom Scalar Field. *Phys. Rev. D* **2012**, *86*, 024028. <https://doi.org/10.1103/physrevd.86.024028>.
20. Armendáriz-Picón, C. On a Class of Stable, Traversable Lorentzian Wormholes in Classical General Relativity. *Phys. Rev. D* **2002**, *65*, 104010. <https://doi.org/10.1103/physrevd.65.104010>.
21. Shinkai, H.A.; Hayward, S.A. Fate of the First Traversable Wormhole: Black-Hole Collapse or Inflationary Expansion. *Phys. Rev. D* **2002**, *66*, 044005. <https://doi.org/10.1103/physrevd.66.044005>.
22. González, J.A.; Guzmán, F.S.; Sarbach, O. Instability of Wormholes Supported by a Ghost Scalar Field: I. Linear Stability Analysis. *Class. Quantum Grav.* **2009**, *26*, 015010. <https://doi.org/10.1088/0264-9381/26/1/015010>.
23. Cremona, F.; Pirota, F.; Pizzocchero, L. On the Linear Instability of the Ellis–Bronnikov–Morris–Thorne Wormhole. *Gen. Relativ. Gravit.* **2019**, *51*, 19. <https://doi.org/10.1007/s10714-019-2501-x>.