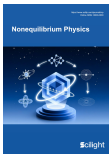
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Article

Neural Oscillations Model Using Basin-Hopping Optimization

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Abstract: Brain oscillations are a measurable characteristic of collective neuronal activity and reflect the dynamic organisation of the nervous system across multiple timescales. The main bands of the electroencephalogram (EEG), i.e., Delta (0.5–4 Hz), Theta (4–7 Hz), Alpha (8–12 Hz), Beta (13–30 Hz) and Gamma (30–100 Hz), are conventionally associated with functional and cognitive states, although their interpretation in terms of general dynamic principles remains an open question. This paper proposes a minimal physical-mathematical model for investigating selected spectral properties associated with neural oscillations. The model consists of a linear chain of identical elastically coupled masses. The normal modes of the system are obtained by solving the associated eigenvalue problem, using a global-local optimization process to align the model eigenfrequencies with representative frequencies of the five EEG bands. More specifically, the Basin-hopping and Powell algorithms are combined to explore the parameter space and minimise deviations from the prescribed target frequencies. The results achieved show quantitative alignment within numerical tolerance between simulated frequencies and target reference frequencies. This agreement is interpreted as the solution of an inverse spectral fitting problem rather than as independent validation of a mechanism generating neural oscillations. A 50-run multi-start analysis showed high spectral reproducibility despite substantial variability in the optimized parameters, while local sensitivity analysis revealed a structured, mode-dependent response to parameter perturbations. The parallelism between physical and biological systems does not lie in a direct correspondence between components, and the model is not intended to reproduce specific neuronal or synaptic mechanisms. The results thus indicate that a highly simplified coupled system can provide a mathematically transparent framework for investigating relationships between coupling parameters, eigenfrequencies and collective modes, opening up prospects for future applications in neural system modelling.

Keywords: EEG oscillations; inverse spectral fitting; basin-hopping optimization

1. Introduction

The human brain is a highly organised complex system, consisting of a huge number of interacting units distributed across multiple spatial and temporal scales. Its dynamic activity emerges from the collective interaction of neuronal populations, giving rise to global phenomena that cannot be directly deduced from the properties of individual neurons. In this context, neural oscillations represent one of the most evident and studied manifestations of collective brain dynamics, observable through electroencephalography (EEG) and local field potential recordings [1,2]. Brain oscillations are conventionally classified into Delta (approximately 0.5–4 Hz), Theta (4–7 Hz), Alpha (8–12 Hz), Beta (13–30 Hz) and Gamma (30–100 Hz) frequency bands (see Figure 1), which have been associated with different functional and cognitive states, including sleep, memory, attention, motor control and high-level cognitive

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processing [3–6]. Although this conventional classification is extremely useful from a descriptive and clinical perspective, the boundaries between frequency bands are not absolute and may vary across studies, subjects, behavioural states, and experimental conditions. The classification therefore remains primarily phenomenological and does not, by itself, explain the dynamical mechanisms underlying the observed organisation of neural timescales. From the perspective of complex systems biophysics, neural oscillations can be interpreted as emergent collective modes of a network of coupled dynamic units. In this view, rhythmic activity is not the result of isolated oscillators, but a global property that reflects the coupling structure and organisation of the neural network [7,8]. Numerous studies have shown that synchronisation between neurons and neuronal populations is crucial for the generation of measurable EEG signals, suggesting that the oscillations observed reflect coordinated collective states rather than independent local dynamics [9]. A particularly relevant element that has recently emerged in the literature [10] concerns the empirical observation of a hierarchical organisation of EEG frequencies, which seems to suggest the existence of an underlying structuring principle in the distribution of brain time scales. In this context, Magazù and Caccamo have highlighted how the average frequencies associated with the main EEG bands (Delta, Theta, Alpha, Beta and Gamma) show an orderly progression, with systematic relationships between adjacent bands, interpreting this regularity as an indication of a coherent dynamic organisation of neural activity across multiple scales. Beyond the specific mechanism proposed by the authors, this observation is of more general interest from the point of view of the biophysics of complex systems, as it suggests that the neural oscillations observed experimentally are not distributed arbitrarily, but reflect an intrinsic spectral structure of the brain system. This experimental evidence provides strong motivation for approaches based on a spectral description of neural dynamics. A broad range of theoretical frameworks has been developed to investigate the mechanisms underlying neural oscillations. These include neural-mass and neural-field models, nonlinear models of neuronal excitability and synaptic interactions, synchronization-based approaches, and, more recently, connectome-based and whole-brain models. Such frameworks have provided fundamental insights into the roles of excitation-inhibition balance, coupling, delays, nonlinearities, bifurcations, multistability, and network topology in shaping oscillatory neural activity [6,11]. These mechanistic approaches remain essential for understanding the biological generation and modulation of brain rhythms. The present work does not attempt to replace or reproduce these levels of description; rather, it addresses a deliberately narrower question concerning the spectral properties of a minimal coupled system. From a complementary physical perspective, one may ask a more restricted question: whether the hierarchy of characteristic frequencies can be represented as the spectrum of collective modes of a minimal coupled system, independently of the detailed biological mechanisms responsible for generating those rhythms. In physics, the analysis of the eigenvalues and eigenvectors of a dynamic system is a fundamental tool for identifying the intrinsic time scales (eigenfrequencies) and collective spatial configurations (eigenmodes) that govern the system's response. This formalism is widely used in classical mechanics, vibration physics and dynamic systems theory, and has found increasing applications in the study of complex networks and biological systems [12,13]. Neurophysical and dynamical-systems approaches have likewise contributed substantially to understanding how oscillatory activity can arise, interact, and respond to external perturbations in neural systems [11]. In the context of theoretical neuroscience, the use of spectral language has received increasing attention. In particular, it has been shown that the structure of neural connectivity matrices can be analysed in terms of eigenvalues and eigenmodes, which provide a natural basis for describing coherent patterns of large-scale activity and their dynamic evolution [14]. Within such a spectral description, experimentally observed oscillatory activity may be related to preferential contributions of particular collective modes, although this correspondence does not by itself specify the biological mechanisms responsible for their generation. Alongside detailed neural models based on nonlinear equations and microscopic descriptions of membrane dynamics and synapses [15–17], simplified models play a fundamental role in identifying global organisational principles. In particular, neuromechanical models, in which neuronal populations are represented as coupled oscillators, provide a mathematically transparent context for studying how the coupling structure influences the organisation of collective oscillations [18]. In such models, the objective is not to reproduce the detailed biological mechanisms responsible for neural oscillations, but to isolate selected mathematical properties such as coupling, eigenfrequencies, and collective modes in a transparent and tractable framework. Eigenfrequencies provide a compact description of the system's privileged time scales, while eigenmodes allow us to interpret the spatial patterns associated with different frequency bands. This approach is particularly suited to the biophysics of complex systems, where the main objective is to understand how global properties emerge from the interaction of many components. The present study builds on our previously proposed Neuromechanical Network Model [19], in which a simplified coupled mechanical system was introduced as a spectral analogue of neural oscillatory organisation. Here, rather than introducing a new biological model, we investigate whether a global-local optimization strategy can effectively address the associated inverse spectral problem and examine the robustness and parameter dependence of the resulting spectral reconstruction. In this

work, we investigate a minimal model based on a coupled discrete system and formulate the correspondence with representative EEG frequencies as an inverse spectral problem. Starting from a second-order linear dynamical system, we consider five dominant eigenmodes and optimize the coupling parameters so that their eigenfrequencies correspond to five representative frequencies within the conventional Delta, Theta, Alpha, Beta, and Gamma bands. Importantly, because these frequencies are explicitly used as optimization targets, their subsequent agreement with the model eigenfrequencies is not interpreted as independent validation of the model or as evidence of a biological mechanism generating EEG rhythms. Rather, the analysis addresses whether a minimal coupled system can realise the prescribed spectral hierarchy, how effectively this inverse problem can be solved through global local optimization, and how the resulting spectrum depends on the inferred coupling parameters. The model therefore constitutes a spectral analogue rather than a mechanistic model of neural oscillations and does not incorporate neuronal excitability, synaptic dynamics, delays, stochastic activity, nonlinear interactions, or spatial brain connectivity. Within these deliberately restricted assumptions, the aim is to explore the mathematical relationship between coupling structure, eigenmodes, and EEG-like spectral organisation.

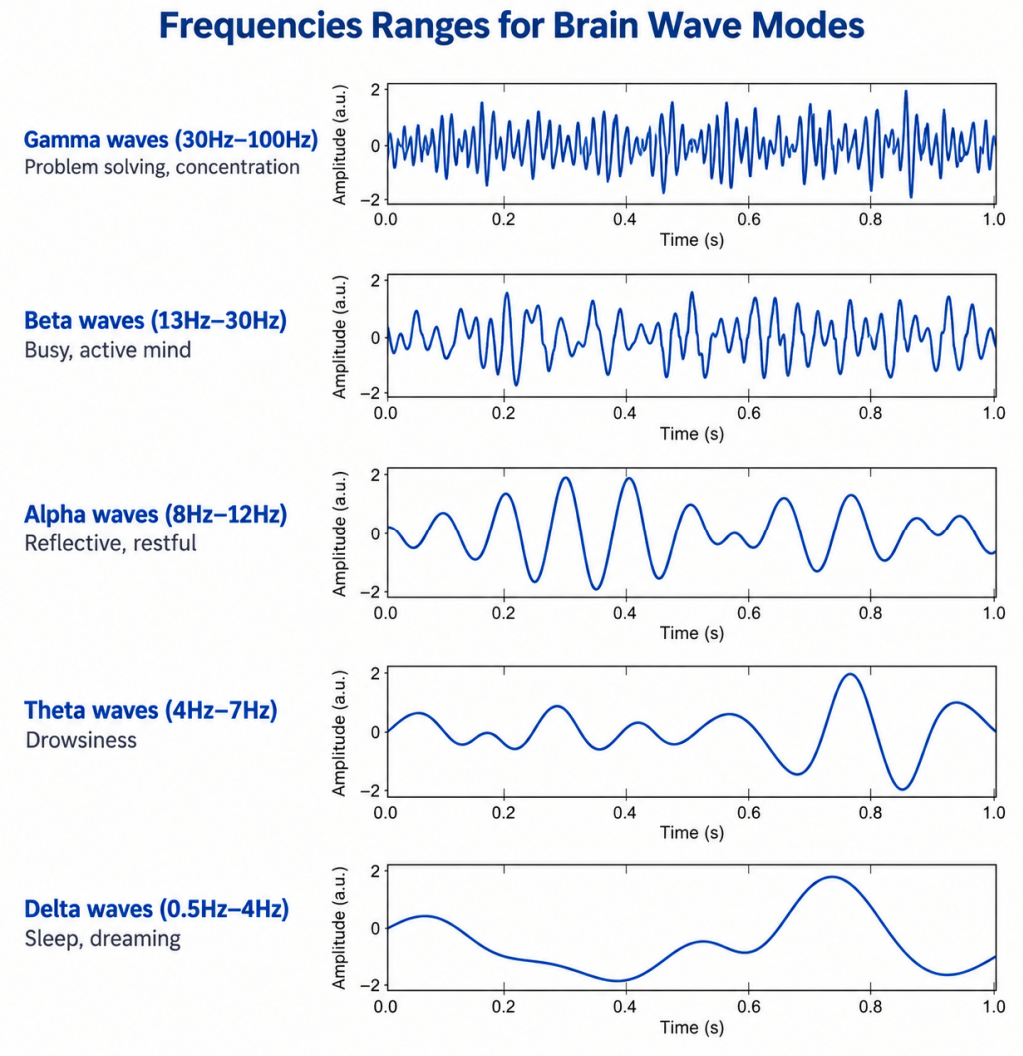


Figure 1. Characteristic frequency ranges of the main EEG bands. Brain oscillations are commonly classified into five main bands: Delta (0.5–4 Hz), Theta (4–7 Hz), Alpha (8–12 Hz), Beta (13–30 Hz) and Gamma (30–100 Hz). The figure shows qualitative examples of signals in the time domain associated with each band, highlighting the progressive variation in time scales from slow (Delta) to fast (Gamma) oscillations. These bands are commonly associated with different functional and cognitive states and represent privileged time scales of collective neural activity.

2. Physical-Mathematical Model

The system under study is modelled as a discrete network composed of N degrees of freedom, described by the vector of generalised displacements $x(t) \in \mathbb{R}^N$. This formulation provides an abstract description of the collective dynamics of interacting units, designed to study the global properties of the system rather than the

behaviour of individual elements. Accordingly, the generalized coordinates should not be interpreted as literal displacements of neurons or neuronal populations, but as abstract dynamical variables of the model. Within the limit of small oscillations around the equilibrium configuration, the dynamics of the system are governed by a set of second-order linear equations of the form:

$$M\ddot{x}(t) + Hx(t) = 0 \tag{1}$$

In this formulation, the symmetric and positive definite matrix M describes the inertial properties of the system, while the coupling matrix H encodes the architecture of the interactions between the dynamic units. Within the present formulation, these quantities define the mathematical structure of the model and are not intended to represent specific neuronal or synaptic mechanisms. The dynamic properties of the system are determined by the generalised eigenvalue problem associated with the pair (M, H). To characterise the oscillations of the system, harmonic solutions of the equations of motion are sought in the form:

$$x(t) = u\, sin(\omega t + \phi) \tag{2}$$

where u is a vector of constant modal amplitudes and ω is the associated angular frequency. Substituting this expression into the equation of motion, we obtain the generalised eigenvalue problem:

$$Hu = \lambda Mu, \qquad \lambda = \omega^2 \tag{3}$$

whose solution provides a discrete set of eigenvalues $\{\lambda_n\}$ and corresponding eigenvectors $\{u_n\}$. The quantities λ_n determine the eigenfrequencies of the system according to the relation:

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{\lambda_n}}{2\pi} \tag{4}$$

while the eigenvectors u_n define the eigenmodes, i.e., the collective spatial patterns associated with each time scale. Since the matrices M and H are symmetric, the eigenmodes can be chosen orthogonal to M, allowing a complete modal decomposition of the system dynamics. In this representation, the temporal evolution can be interpreted as a superposition of independent oscillations, each associated with a specific eigenfrequency and eigenmode, providing a natural description of the collective dynamics.

In the following, while maintaining the general formulation just introduced, the analysis is reduced to a system with five degrees of freedom, corresponding to five dominant collective modes. The choice of five degrees of freedom is a deliberate modelling reduction introduced to construct a minimal spectral analogue with the same dimensionality as the five conventional EEG bands considered here; it should not be interpreted as evidence that brain dynamics is intrinsically restricted to five oscillatory modes. This choice is motivated by the goal of comparing the spectral structure of the model with the five representative frequencies selected within the conventional EEG bands: Delta (2.0 Hz), Theta (5.5 Hz), Alpha (10.0 Hz), Beta (21.5 Hz), and Gamma (65.0 Hz) (see Figure 1). These values are used as prescribed optimization targets rather than experimentally estimated subject-specific frequencies and therefore represent reference points within broad and state-dependent EEG frequency ranges. In this reduction, the system can be interpreted as a neuromechanical network composed of five coupled units (effective masses) and a corresponding number of elastic interactions, as schematically illustrated in Figure 2, preserving the validity of the eigenvalue problem associated with the dynamic system and allowing a direct comparison between the model's eigenfrequencies and the reference neural frequencies.

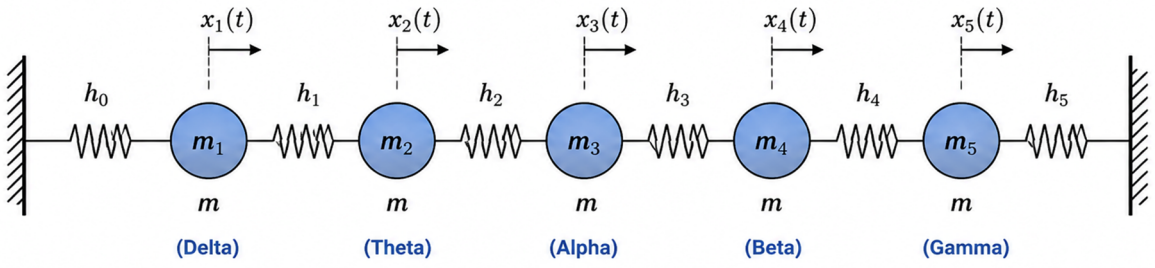


Figure 2. Schematic representation of the five-degree-of-freedom coupled system. The model consists of five identical effective masses arranged in a linear configuration and connected through linear elastic interactions characterized by the stiffness parameters (h_1)–(h_5). The generalized coordinates ($x_i(t)$) describe the dynamical variables associated with the five degrees of freedom. The five model eigenfrequencies are optimized to reproduce the prescribed representative frequencies of the conventional Delta, Theta, Alpha, Beta, and Gamma EEG bands. The masses, displacements, and

stiffness parameters are effective quantities of the mathematical model and should not be interpreted as direct physical representations of neurons, neuronal populations, synaptic strengths, or anatomical connectivity.

Each mass is characterised by a constant value, $m = 5 \times 10^{-15}$ Kg, chosen as the effective mass parameter within the mechanical model. Importantly, this quantity is not intended to represent the physical mass of an individual neuron or neuronal population. Rather, together with the stiffness parameters, it sets the frequency scale of the second-order dynamical system. Consequently, no direct biological meaning is assigned to its absolute numerical value. The masses are coupled by linear elastic interactions described by a set of stiffness constants $\{h_1, h_2, h_3, h_4, h_5\}$, which model the intensity of the coupling between consecutive units along the chain. Here, the stiffness constants are effective coupling parameters of the analogue model and should not be interpreted as direct measures of synaptic strength or anatomical connectivity. Their role is to determine, together with the effective mass, the eigenvalue spectrum of the coupled system. In the model considered, the five masses are arranged in a linear configuration and coupled via elastic interactions that define the topology of the chain. The first mass is connected to a fixed support by a spring with stiffness h_1, which introduces an elastic constraint at the left end of the system. The successive masses are coupled sequentially by springs with stiffness h_2, h_3 and h_4, which connect the adjacent pairs of masses (1–2), (2–3) and (3–4) respectively. Finally, the last elastic interaction, described by the constant h_5, couples the fourth and fifth masses. The elastic boundary conditions adopted at the ends of the chain are not intended to represent rigid physical constraints, but provide an effective dynamic closure of the system, necessary to define a discrete eigenvalue spectrum within the chosen minimal topology. With these conventions, the coupling matrix H is tridiagonal:

$$H = \begin{bmatrix} h_1 + h_2 & -h_2 & 0 & 0 & 0 \\ -h_2 & h_2 + h_3 & -h_3 & 0 & 0 \\ 0 & -h_3 & h_3 + h_4 & -h_4 & 0 \\ 0 & 0 & -h_4 & h_4 + h_5 & 0 \\ 0 & 0 & 0 & -h_5 & h_5 \end{bmatrix}$$

To obtain a system whose natural frequencies correspond to the main time scales observed in the selected EEG reference frequencies, the elastic coupling parameters $\{h_i\}_{i=1}^{5}$ are adjusted so the five model eigenfrequencies

$$\nu_n(h) = \frac{1}{2\pi}\sqrt{\lambda_n(h)} \qquad n = 1,\ldots\ldots,5 \tag{5}$$

where $\lambda_n(h)$ are the eigenvalues of the generalised problem

$$(h)u = \lambda_n(h)Mu_n \qquad M = mI_5, \quad m = 5 \times 10^{-15} \text{ kg} \tag{6}$$

are aligned with a set of target neural frequencies representative of EEG bands:

$$\nu_{target} = (\nu_\Delta, \nu_\Xi, \nu_A, \nu_B, \nu_\Gamma) = (2.0;\ 5.5;\ 10.0; 21.5; 65.0) H_Z$$

The alignment is formulated as an optimization problem, minimising an objective function based on the sum of the squares of the deviations between the simulated frequencies and the target frequencies:

$$\varepsilon(h) = \sum_{n=1}^{5} [\nu_n^{sim} - \nu_n^{target}]^2 \tag{7}$$

where ν_n^{sim} are the eigenfrequencies of the system, obtained by solving the eigenvalue problem associated with the stiffness matrix $H(h)$, and ν_n^{target} are the target frequencies corresponding to the EEG bands considered.

To ensure the positivity of the elastic parameters and improve the numerical stability of the optimization, the coupling parameters h_i were parameterised in log space, setting $h_i = exp(\theta_i)$, and the optimization was conducted with respect to the variables θ_i,with appropriate lower and upper bounds. These bounds constrain the numerical search space of the model and should not be interpreted as physiological ranges of neuronal coupling parameters. The alignment between the model's eigenfrequencies and the reference EEG frequencies is therefore formulated as a non-linear optimization problem, the solution strategy for which is described in the following section.

2.1. Optimization Strategy

The alignment between the model's natural frequencies and the reference EEG frequencies is formulated as a non-linear optimization problem in the parameter space θ = (θ1, …, θ5). The objective function is defined as:

$$\varepsilon(h) = \sum_{n=1}^{5} [\nu_n(\theta) - \nu_n^{target}]^2 \tag{8}$$

where $\nu_n(\theta)$ are the eigenvalues of the system, obtained from the eigenvalues λ_n of the generalised problem.

$$H(\theta)u_n = \lambda_n M u_n, \qquad \nu_n = \frac{1}{2\pi}\sqrt{\lambda_n} \tag{9}$$

The parameter space is explored using the Basin-hopping algorithm [20,21], which combines a global stochastic search with a sequence of local minimisations. At each iteration, the parameter vector is randomly perturbed according to

$$\theta' = \theta + \eta \tag{10}$$

where η is a vector of random perturbations extracted from a uniform distribution within a predetermined interval. The new point is then relaxed towards a local minimum of the objective function using a deterministic optimization algorithm. Local minimisation is performed using Powell's algorithm [22], which does not require the calculation of the derivatives of the objective function and is particularly suitable for non-linear problems with a moderate number of parameters. Powell's algorithm proceeds iteratively along conjugate directions in the parameter space, updating the vector θ until convergence towards a local minimum. The acceptance of moves in Basin-hopping follows a Metropolis-type criterion, based on the variation of the objective function, allowing energy barriers to be overcome and reducing the risk of convergence towards local rather than global minima. The optimization procedure is considered convergent when the relative variation of ε between successive iterations falls below a predefined threshold and the simulated frequencies are aligned with the target EEG frequencies within the prescribed numerical tolerance. The final optimized values of the elastic constants h_i were (see Table 1):

Table 1. Optimized values of elastic constants. Values of elastic constants hi obtained through spectral optimization based on Basin-hopping (global search) and Powell (local refinement), with mass set at m = 5 × 10^{-15} kg. The parameters were determined by minimising the discrepancy between the five eigenfrequencies of the model and the target EEG frequencies (Delta, Theta, Alpha, Beta and Gamma).

Spring Constant	Optimized Value (N/m)
h_1	8.30069378 × 10^{-10}
h_2	3.89291273 × 10^{-12}
h_3	4.18643016 × 10^{-11}
h_4	9.64165671 × 10^{-12}
h_5	5.42941618 × 10^{-12}

The following simulated natural frequencies are obtained from the model (see Table 2):

Table 2. Comparison between target EEG frequencies and simulated eigenfrequencies. Comparison between the reference neural frequencies associated with the main EEG bands (Delta, Theta, Alpha, Beta, and Gamma) and the corresponding eigenfrequencies obtained from the coupled model after optimization of the elastic parameters. The "Deviation" column shows the difference $\nu_n^{sim} - \nu_n^{target}$, demonstrating alignment within the numerical tolerance of the optimization algorithm and broadly compatible with the intrinsic variability of the EEG bands.

Brain Wave Mode	Target Frequency (Hz)	Simulated Frequency (Hz)	Deviation (Hz)
Delta	2.0	1.999	−0.0002
Theta	5.5	5.501	+0.0006
Alpha	10.0	9.999	−0.0011
Beta	21.5	21.503	+0.0030
Gamma	65.0	64.970	−0.0300

The optimized natural frequencies closely correspond to the target EEG frequencies within the numerical tolerance (see Figure 3).

To assess whether the optimized solution depends on the initial parameter configuration and to evaluate its reproducibility, the optimization procedure was subsequently extended to a multi-start analysis, as described in Section 2.2.

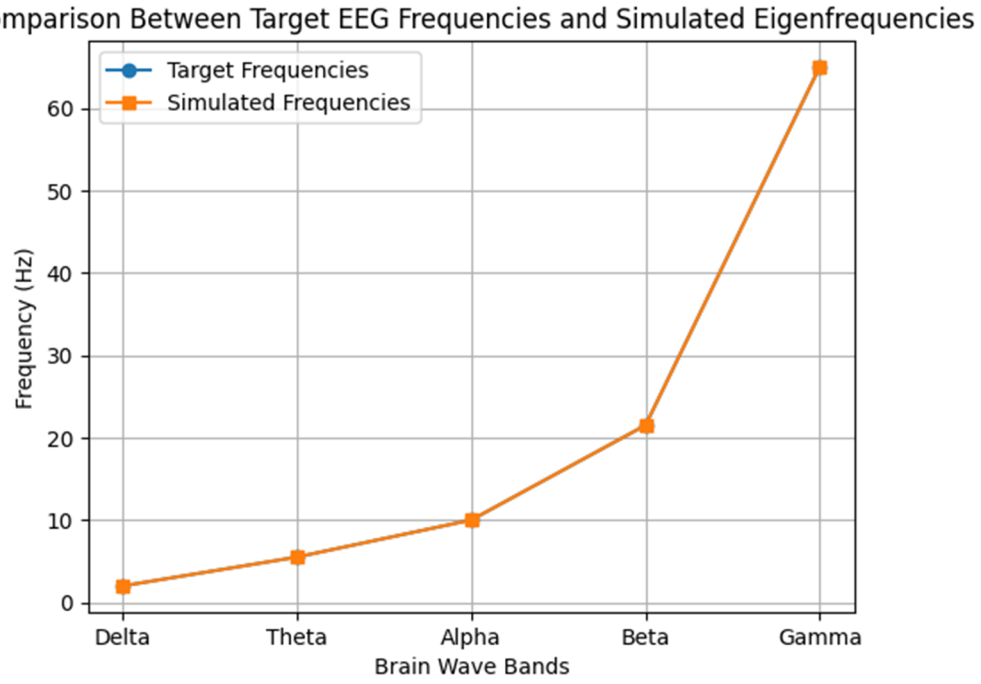


Figure 3. Comparison between target EEG frequencies and simulated eigenfrequencies. The figure shows the comparison between the reference EEG frequencies associated with Delta, Theta, Alpha, Beta, and Gamma bands and the corresponding eigenfrequencies obtained from the optimized model. The two curves are both plotted but almost completely overlap because the simulated eigenfrequencies reproduce the target values within numerical tolerance.

2.2. Robustness and Sensitivity Analysis

To evaluate the robustness of the inverse spectral solution beyond a single optimization run, two complementary analyses were performed. First, the complete Basin-hopping/Powell procedure was repeated over 50 independent runs using independently sampled initial conditions in logarithmic parameter space. For each run, the final spectral error and the corresponding optimized stiffness parameters were recorded. This analysis was used to assess the reproducibility of the reconstructed frequency spectrum and to investigate whether different parameter sets could provide comparably accurate spectral solutions. Second, a local sensitivity analysis was performed around the reference optimized solution by independently perturbing each stiffness parameter h_1–h_5 by ±5% and ±10%, while keeping the remaining parameters fixed. For each perturbation, the five eigenfrequencies were recalculated and their relative percentage changes with respect to the reference solution were determined. This analysis was used to quantify the local robustness of the reconstructed spectrum and to identify mode-dependent spectral sensitivity to individual coupling parameters.

3. Results and Discussion

It is useful to place this approach in the context of previous spectral models inspired by neural dynamics. In particular, in the "*Neuromechanical Network Model*" [19], EEG frequencies were already used as references in the process of optimising coupling parameters, through a local algorithm (Nelder-Mead) and with the introduction of additional constraints on the structure of the elastic parameters. In that context, however, the agreement between simulated eigenfrequencies and neural frequencies was mainly qualitative, making it necessary to resort to subsequent refinement procedures, such as adaptation using an exponential law and the introduction of a corrective scale constant. In this work, the adoption of a global-local optimization strategy based on Basin-hopping and Powell allows for a more effective exploration of the parameter space, enabling direct alignment of the eigenfrequencies with the reference EEG frequencies within numerical tolerance. This result renders the additional empirical corrections adopted in the previous model unnecessary. However, since the EEG reference frequencies are explicitly included as targets in the objective function, the resulting spectral agreement should be interpreted as evidence of the numerical effectiveness of the optimization procedure rather than as independent validation of a mechanism underlying neural oscillations. In this sense, the present approach represents a methodological advance over the previous "*Neuromechanical Network Model*" [19], while maintaining conceptual continuity with the idea of describing selected spectral properties associated with neural oscillations in terms of collective modes

and eigenfrequencies. To assess whether the high spectral accuracy obtained by the optimization was associated with a unique parameter configuration, the multi-start analysis was performed over 50 independent optimization runs. The reconstructed spectra remained highly consistent across runs, with the sum of squared errors (SSE) ranging from 3.17×10^{-25} to 3.18×10^{-9} Hz^2 and a median value of 1.61×10^{-15} Hz^2. Thus, the prescribed frequency hierarchy could be reproduced with negligible spectral error from different initial conditions. In contrast to this spectral stability, the optimized stiffness parameters showed substantial variability across the 50 solutions. The maximum-to-minimum ratios were 169.63, 106.73, 130.59, 210.13, and 458.47 for h_1, h_2, h_3, h_4, and h_5, respectively. The corresponding coefficients of variation were 0.972, 0.992, 1.740, 3.143, and 3.446. As shown in Figure 4, the substantial variability of the optimized parameter configurations (Figure 4A) contrasts with the high reproducibility of the reconstructed spectra (Figure 4B), where several of the 50 curves overlap because of the extremely small frequency residuals. The maximum absolute frequency residual observed across all runs was 4.95×10^{-5} Hz. These results show that markedly different parameter configurations can reproduce essentially the same prescribed spectral hierarchy, providing numerical evidence of practical parameter non-identifiability when frequency information alone is used to constrain the model. Therefore, the individual optimized stiffness values reported in Table 1 should not be regarded as uniquely determined physical parameters, but as one possible parameter configuration capable of reproducing the prescribed spectrum. This finding is relevant to the interpretation of the inverse spectral problem. While the Basin-hopping/Powell strategy provides a highly accurate and reproducible reconstruction of the target frequencies, spectral accuracy does not imply uniqueness of the underlying coupling parameters. The multi-start analysis therefore distinguishes two aspects of the model that would otherwise remain conflated: robustness of the reconstructed spectrum and identifiability of the parameters. In the present system, the former is high, whereas the latter is limited. A complete structural identifiability analysis and an analytical characterization of possible isospectral parameter families are beyond the scope of the present study and represent possible directions for future investigation.

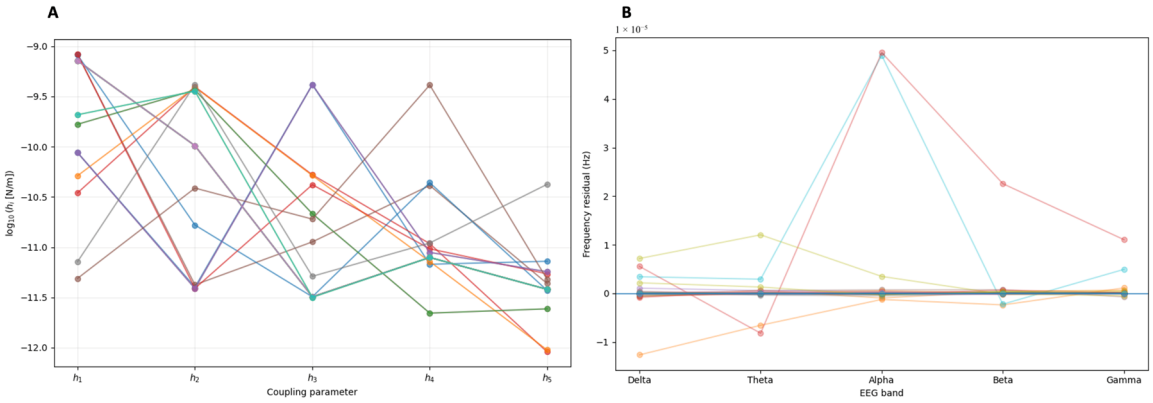


Figure 4. Multi-start analysis of parameter variability and spectral reproducibility. (**A**) Optimized stiffness configurations obtained from 50 independent Basin-hopping/Powell optimization runs starting from independently sampled initial parameter configurations. The stiffness parameters are expressed as $\log_{10}$ (hi [N/m]). The substantial dispersion of the parameter profiles shows that markedly different stiffness configurations can provide comparably accurate solutions to the inverse spectral problem. (**B**) Frequency residuals, defined as the difference between the reconstructed and prescribed target frequencies, for the same 50 optimization runs. Several curves overlap because of the extremely small differences among the reconstructed spectra. The maximum absolute frequency residual across the 50 runs was 4.95×10^{-5} Hz. The spectral SSE ranged from 3.17×10^{-25} to 3.18×10^{-9} Hz^2, with a median value of 1.61×10^{-15} Hz^2. These results indicate high spectral reproducibility while providing numerical evidence of practical non-identifiability of the individual coupling parameters from frequency constraints alone.

The local sensitivity analysis provides complementary information on the behaviour of the model around the reference optimized solution. Independent perturbations of each stiffness parameter by ±5% and ±10% produced moderate and mode-dependent changes in the eigenfrequencies. For the ±10% perturbations, the largest relative changes were observed between h_1 and the Gamma mode (−5.11% and +4.86%), h_2 and the Delta mode (−4.12% and +3.78%), h_3 and the Beta mode (−4.64% and +4.45%), h_4 and the Alpha mode (−2.83% and +2.77%), and h_5 and the Theta mode (−3.06% and +2.68%). The same mode-dependent pattern was observed for the ±5% perturbations, with correspondingly smaller frequency shifts. For clarity, Figure 5 summarizes the mean absolute relative frequency changes obtained from the ±10% perturbations. Weaker cross-mode effects were also observed,

indicating that the correspondence is not strictly one-to-one. Taken together, these results reveal a structured local spectral sensitivity: although substantially different global parameter configurations can reproduce the prescribed spectrum, small perturbations around a given optimized solution affect the eigenmodes selectively. In particular, h_1 predominantly influences the Gamma mode, h_2 the Delta mode, h_3 the Beta mode, h_4 the Alpha mode, and h_5 the Theta mode within the investigated local neighborhood. These associations should be interpreted as mathematical properties of the coupled system rather than as direct biological correspondences between individual stiffness parameters and specific EEG rhythms. More generally, oscillatory descriptions of neural activity should be interpreted with caution, since the presence of rhythmic structure in neural signals does not necessarily imply a single underlying oscillatory mechanism, and different dynamical processes may contribute to the observed spectral organization [23].

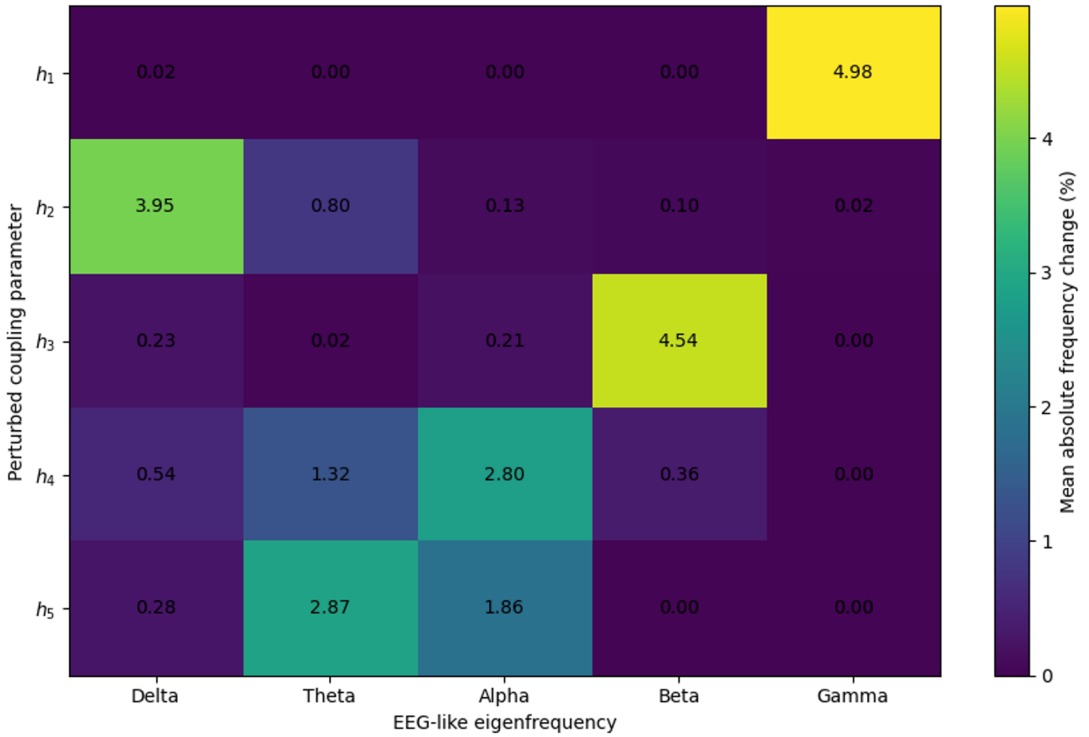


Figure 5. Local spectral sensitivity to coupling-parameter perturbations. Heatmap of the mean absolute relative changes in the five model eigenfrequencies following independent ±10% perturbations of each optimized stiffness parameter h_1–h_5, with all remaining parameters held fixed. Each cell represents the mean absolute percentage frequency change obtained from the corresponding −10% and +10% perturbations. The analysis reveals a mode-dependent sensitivity pattern, with the strongest responses observed for h_1-Gamma (4.98%), h_2-Delta (3.95%), h_3-Beta (4.54%), h_4-Alpha (2.80%), and h_5-Theta (2.87%). Additional cross-mode effects are present, particularly for h_4 and h_5, indicating that the sensitivity structure is not strictly one-to-one. These associations describe local mathematical sensitivities of the model and should not be interpreted as direct biological correspondences.

4. Conclusions

This paper presents a minimal physical-mathematical model for investigating selected spectral properties associated with brain oscillations, based on the analysis of a coupled discrete system. The system is described by a second-order linear dynamic model, whose collective dynamics are characterised by the generalised eigenvalue problem associated with the mass and coupling matrices. Through a global-local optimization procedure of the elastic parameters, the natural frequencies, obtained as roots of the eigenvalues of the system, are quantitatively aligned with five frequencies representative of the main EEG bands (Delta, Theta, Alpha, Beta and Gamma). Since these frequencies are prescribed as optimization targets, their close numerical agreement with the reconstructed eigenfrequencies should be interpreted as evidence of the effectiveness of the inverse spectral optimization procedure rather than as independent validation of a mechanism generating neural oscillations. The corresponding eigenmodes provide a natural description of the collective patterns associated with each timescale, within the mathematical structure of the model. The additional robustness analyses further clarify the properties and limitations of the inverse solution. The 50-run multi-start analysis showed that the prescribed spectral hierarchy

can be reconstructed with high reproducibility from different initial conditions, while substantially different stiffness configurations can produce essentially equivalent spectra. This result provides numerical evidence of practical parameter non-identifiability and indicates that the optimized stiffness values should not be assigned a unique physical or biological interpretation. Local sensitivity analysis further revealed a structured, mode-dependent spectral response to perturbations of the individual coupling parameters, while also showing cross-mode effects. The model does not aim to provide a biophysical description of neural tissue, but rather represents a conceptual framework for investigating how a prescribed hierarchy of timescales can be represented within the spectral structure of a minimal coupled system. It does not incorporate neuronal excitability, synaptic dynamics, nonlinear interactions, stochastic activity, time delays, spatial brain connectivity, or subject-specific experimental EEG data, and therefore should not be regarded as a mechanistic model of the generation of brain rhythms. Overall, the results indicate that a highly simplified coupled system can provide a mathematically transparent representation of selected EEG-related spectral timescales and can serve as a tractable framework for studying the relationship between coupling parameters, eigenfrequencies, and collective modes. Future developments should test the framework against independent experimental data and investigate whether the present spectral formulation can be extended to more realistic network topologies, heterogeneous coupling, stochastic effects, and nonlinear dynamics.

Author Contributions

G.P.: conceptualization, methodology, supervision; R.M.: conceptualization, methodology, investigation, formal analysis, writing—original draft preparation, visualization; P.P.: methodology, visualization, supervision. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

The data and computational results generated during this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

References

1. Niedermeyer, E.; Lopes da Silva, F.H. (Eds.) *Electroencephalography: Basic Principles, Clinical Applications, and Related Fields*, 5th ed.; Lippincott Williams & Wilkins: Philadelphia, PA, USA, 2005.
2. Buzsáki, G. *Rhythms of the Brain*; Oxford University Press: Oxford, UK, 2006. https://doi.org/10.1093/acprof:oso/9780195301069.001.0001.
3. Buzsáki, G.; Draguhn, A. Neuronal oscillations in cortical networks. *Science* **2004**, *304*, 1926–1929. https://doi.org/10.1126/science.1099745.
4. Fries, P. Rhythms for Cognition: Communication through Coherence. *Neuron* **2015**, *88*, 220–235. https://doi.org/10.1016/j.neuron.2015.09.034.

5. Başar, E.; Başar-Eroglu, C.; Karakaş, S.; et al. Gamma, alpha, delta, and theta oscillations govern cognitive processes. *Int. J. Psychophysiol.* **2001**, *39*, 241–248. https://doi.org/10.1016/S0167-8760(00)00145-8.
6. Wang, X.J. Neurophysiological and Computational Principles of Cortical Rhythms in Cognition. *Physiol. Rev.* **2010**, *90*, 1195–1268. https://doi.org/10.1152/physrev.00035.2008.
7. Strogatz, S.H. Exploring complex networks. *Nature* **2001**, *410*, 268–276. https://doi.org/10.1038/35065725.
8. Breakspear, M.; Heitmann, S.; Daffertshofer, A. Generative Models of Cortical Oscillations: Neurobiological Implications of the Kuramoto Model. *Front. Hum. Neurosci.* **2010**, *4*, 190. https://doi.org/10.3389/fnhum.2010.00190.
9. Uhlhaas, P.J.; Roux, F.; Rodriguez, E.; et al. Neural synchrony and the development of cortical networks. *Trends Cogn. Sci.* **2010**, *14*, 72–80. https://doi.org/10.1016/j.tics.2009.12.002.
10. Magazù, S.; Caccamo, M.T. Parametric resonance brain model. *Sci. Rep.* **2024**, *14*, 24657. https://doi.org/10.1038/s41598-024-76610-8.
11. Madadi Asl, M.; Valizadeh, A. Entrainment by transcranial alternating current stimulation: Insights from models of cortical oscillations and dynamical systems theory. *Phys. Life Rev.* **2025**, *53*, 147–176. https://doi.org/10.1016/j.plrev.2025.03.008.
12. Pikovsky, A.; Rosenblum, M.; Kurths, J. *Synchronization: A Universal Concept in Nonlinear Sciences*; Cambridge University Press: Cambridge, UK, 2001. https://doi.org/10.1017/CBO9780511755743.
13. Arenas, A.; Díaz-Guilera, A.; Kurths, J.; et al. Synchronization in complex networks. *Phys. Rep.* **2008**, *469*, 93–153. https://doi.org/10.1016/j.physrep.2008.09.002.
14. Atasoy, S.; Donnelly, I.; Pearson, J. Human brain networks function in connectome-specific harmonic waves. *Nat. Commun.* **2016**, *7*, 10340. https://doi.org/10.1038/ncomms10340.
15. Musotto, R.; Wanderlingh, U.; Pioggia, G. Ca^{2+} waves in astrocytes: Computational modeling and experimental data. *Front. Cell. Neurosci.* **2025**, *19*, 1536096. https://doi.org/10.3389/fncel.2025.1536096.
16. Musotto, R.; Wanderlingh, U.; D'Ascola, A.; et al. Dynamics of astrocytes Ca^{2+} signaling: A low-cost fluorescence customized system for 2D cultures. *Front. Cell Dev. Biol.* **2024**, *12*, 1320672. https://doi.org/10.3389/fcell.2024.1320672.
17. Hodgkin, A.L.; Huxley, A.F. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol.* **1952**, *117*, 500–544. https://doi.org/10.1113/jphysiol.1952.sp004764.
18. Breakspear, M. Dynamic models of large-scale brain activity. *Nat. Neurosci.* **2017**, *20*, 340–352. https://doi.org/10.1038/nn.4497.
19. Musotto, R.; Pioggia, G. Neuromechanical network model. *Front. Phys.* **2025**, *13*, 1656677. https://doi.org/10.3389/fphy.2025.1656677.
20. Wales, D.J.; Doye, J.P.K. Global optimization by basin-hopping and the lowest energy structures of Lennard-Jones clusters containing up to 110 atoms. *J. Phys. Chem. A* **1997**, *101*, 5111–5116. https://doi.org/10.1021/jp970984n.
21. Röder, K.; Wales, D.J. Mutational Basin-Hopping: Combined Structure and Sequence Optimization for Biomolecules. *J. Phys. Chem. Lett.* **2018**, *9*, 6169–6173. https://doi.org/10.1021/acs.jpclett.8b02839.
22. Powell, M.J.D. An efficient method for finding the minimum of a function of several variables without calculating derivatives. *Comput. J.* **1964**, *7*, 155–162. https://doi.org/10.1093/comjnl/7.2.155.
23. Doelling, K.B.; Assaneo, M.F. Neural oscillations are a start toward understanding brain activity rather than the end. *PLoS Biol.* **2021**, *19*, e3001234. https://doi.org/10.1371/journal.pbio.3001234.