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Resilient State and Fault Estimation for Stochastic Nonlinear Systems Under Dynamic Event-Triggered Schemes

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Abstract: This paper focuses on the joint non-fragile state and fault estimation issue for a class of stochastic nonlinear systems under the dynamic event-triggered transmission scheme (DETS). To better conform to practical engineering scenarios, we consider an additive fault whose second-order difference is piecewise zero. A zero-mean matrix with bounded covariance is adopted to characterize the phenomenon of random gain variation. The threshold parameter of the DETS is adjustable via a given dynamic equation, rather than being fixed. By extending the original state with the fault and its first-order difference, the original system is converted into a stochastic parameter one. Accordingly, the goal of this paper is to design a non-fragile filter, which ensures an upper bound (UB) for the filtering error covariance (FEC) represented by specific matrix difference equations; thereafter, the gain parameter is determined by minimizing the acquired UB. Subsequently, a sufficient condition is derived regarding the mean-square boundedness of the filtering error. Finally, a numerical example is given to confirm the effectiveness of our estimation algorithm.

Keywords: stochastic nonlinear systems; fault estimation; dynamic event-triggered transmission scheme (DETS); mean-square boundedness; matrix difference equations

1. Introduction

The fault diagnosis problem has recently received great attention mainly because of the need to enhance the reliability and safety of modern complex systems [1,2]. The faults are generally defined as unexpected deviation of characteristic properties or system parameters from standard condition, which includes incipient faults, intermittent faults, abrupt faults and so forth [3–10]. In most of the existing results, the faults can be modeled as constant faults. With the aim of better reflecting practical engineering, the faults can undergo dynamic changes, with constant faults treated as a special case, the randomly occurring faults were introduced in [11] modeled by Bernoulli distributed white sequences. Recently, ref. [12] has discussed an additive fault whose second-order difference was piecewise zero.

Compared with the fault detection and isolation method, the fault estimation can better provide us with the location and shape of the underlying fault, which are one of the main reasons for the prevalence of the fault estimation. As a result, the fault estimation issue has been explored for a family of stochastic systems, and fruitful achievements have been gained [13–18]. To mention a few, ref. [14] has designed a joint estimator to estimate the state and the actuator/sensor fault, and the finite-horizon H_∞ performance of the estimation error has been guaranteed by the derived sufficient conditions. Ref. [17] has studied fault estimation and sliding mode fault-tolerant control for networked control systems. Different from most of the existing fault estimation results, ref. [18] has addressed the state and fault estimation problem for two-dimensional discrete systems subject to deterministic/stochastic disturbances.

The filtering issue concerning stochastic nonlinear systems has drawn substantial attention in the research community, primarily due to its broad practical applications in numerous domains including navigation, aviation and fault diagnosis [19]. To date, the existing nonlinear models can be categorized into the Takagi-Sugeno fuzzy model [20], the randomly occurring nonlinearity described by Bernoulli random variables [21] and the sector-bounded nonlinear vector-valued function [22]. The basic idea of state estimation of the dynamic systems is to reconstruct the unknown system state based on available measurements. To date, a substantial body of literature has reported various filtering results for stochastic nonlinear systems (see [23–33] and the references therein). For example, ref. [21] has explored the dynamic event-triggered H_∞ filtering issue for delayed complex networks with randomly occurring nonlinearities, while analyzing the mean-square stability of the filtering error system. Ref. [28] has concentrated on analyzing the statistical distribution characteristics of sensor energy, and accordingly, an innovative finite-horizon filtering approach was established for nonlinear time-delay systems.

It is worth noting that the filter gain parameters in the majority of existing literature are generally assumed to be precisely known. Nevertheless, the possible deviations and fluctuations of the designed filter gains are inevitable in many practical applications owing to some reasons such as round-off errors of numerical computation, parameter biases, and aging of the electrical element. If not being properly handled, the performance of the developed filter can be degraded. Consequently, a new method, termed as the non-fragile filtering algorithm, has been designed to deal with the parameter drifts [5]. As is well known, implementation errors are typically modeled as norm-bounded uncertainties and addressed using linear matrix inequality techniques. Lately, stochastic implementation errors models have been characterized by the zero-mean and bounded covariance.

Recently, event-triggered communication strategies have been developed to alleviate data transmission burdens and enhance the efficiency of networked systems. Among them, static and dynamic event-triggered transmission scheme(DETS) have emerged as two typical frameworks and been extensively investigated [17, 34]. In such triggering architectures, signal transmission only occurs when a given triggering condition is fulfilled, which effectively reduce the overall amount of transmitted data packets. There is an implicit assumption with the static event-triggered scheme that the threshold is a fixed constant, while the threshold in the dynamic strategy can be adjusted in a dynamical way introducing an additional internal dynamic variable. Theoretical results have verified that the dynamic event-triggered mechanism exhibits superior performance in reducing triggering times compared with its static counterpart [35]. Motivated by these benefits, numerous dynamic event-triggered approaches for filtering design and fault estimation have been presented in existing works, as can be seen in representative studies such as [24,36–38]. Nevertheless, the issue of non-fragile state and fault estimation within a dynamic event-triggered framework has not received adequate research attention, especially for the scenario where the second-order derivative of the considered fault is zero piecewise.

Drawing on the above analysis, there exists an urgent need to propose a new joint state and fault estimation algorithm for an array of stochastic nonlinear systems under the DETS. The stochastic perturbation described by mean and covariance is introduced in the state and measurement equation. The second-order difference of the fault under consideration is assumed to be zero piecewise, and the DETS is adopted here to improve the utilization of the network resources. By augmenting the state vector, the fault and its first-order difference, we construct a new augmented state and measurement equation. On this basis, a non-fragile filter is proposed and the corresponding minimized upper bound (UB) is established with the help of difference equations, and the mean-square boundedness of the filtering error is analyzed. The primary difficulties we encountered are: (1) How to obtain the UB of the filtering error covariance (FEC) in the presence of the above-mention factors? (2) How to reveal the relationship between these parameters and filtering error system? The core contributions of this paper are listed below: *(1) We make the first endeavor to develop a recursive non-fragile estimator for the simultaneous estimation of state and fault, considering both the DETS and stochastic gain fluctuations; (2) Through the design of appropriate filter gains, a minimized UB of the FEC is obtained; (3) A sufficient condition is established to ensure that the filtering error remains bounded in the mean-square sense.*

2. Problem Formulation

The stochastic nonlinear systems under investigation are given as follows:

$$\begin{cases} x_{\iota+1} = \Phi_\iota x_\iota + S(x_\iota, \alpha_\iota) + M_\iota f_\iota + C_\iota w_\iota \\ y_\iota = H_\iota x_\iota + T(x_\iota, \beta_\iota) + D_\iota v_\iota \end{cases} \tag{1}$$

where $x_\iota \in \mathbb{R}^{n_x}$ denotes the system state, $y_\iota \in \mathbb{R}^{n_y}$ is the measurement output and $f_\iota \in \mathbb{R}^{n_f}$ denotes an additive fault whose second-order difference is zero piecewise. $w_\iota \in \mathbb{R}^{n_w}$ and $v_\iota \in \mathbb{R}^{n_v}$ denote the zero-mean process noise and measurement noise, respectively, with their covariance matrices being Q_ι and R_ι. Meanwhile, α_ι and β_ι stand for the zero-mean multiplicative noises. Φ_ι, H_ι, M_ι, C_ι, and D_ι are known time-varying matrices with appropriate dimensions.

The nonlinear functions $S(x_\iota, \alpha_\iota)$ and $T(x_\iota, \beta_\iota)$ satisfy the following conditions:

$$\begin{aligned} & S(0, \alpha_\iota) = 0, T(0, \beta_\iota) = 0, \\ & \mathbb{E}\left\{ \begin{bmatrix} S(x_\iota, \alpha_\iota) \\ T(x_\iota, \beta_\iota) \end{bmatrix} \Big| x_\iota \right\} = 0, \end{aligned} \tag{2}$$

and

$$\begin{aligned}&\mathbb{E}\Big\{\begin{bmatrix}S(x_\iota,\alpha_\iota)\\T(x_k,\beta_k)\end{bmatrix}\begin{bmatrix}S(x_\iota,\alpha_\iota)\\T(x_k,\beta_k)\end{bmatrix}^T\Big|x_\iota\Big\}\\&=\sum_{i=1}^{l}\mathcal{A}_{i,\iota}x_\iota^T B_\iota x_\iota\delta_{\iota,k}\end{aligned}\tag{3}$$

where $\mathcal{A}_{i,\iota}\triangleq\text{diag}\{\mathcal{A}_{i1,\iota},\mathcal{A}_{i2,\iota}\}$, $\mathcal{A}_{i1,\iota}$, $\mathcal{A}_{i2,\iota}$, and B_ι are known matrices. If $\iota=k$, $\delta_{\iota,k}=1$ and otherwise $\delta_{\iota,k}=0$.

Assumption 1. *The initial state x_0, α_ι, β_ι, w_ι, and v_ι are supposed to be mutually independent. $\bar{x}_0$ and P_0 are the expectation and covariance of x_0, respectively.*

Assumption 2. *The fault f_ι satisfies $\triangle^2 f_\iota=\triangle f_{\iota+1}-\triangle f_\iota=0$, with $\triangle f_\iota=f_{\iota+1}-f_\iota$.*

To alleviate the communication load, a DETS is adopted to decide whether the current measurement ought to be sent to the filter. The event instants are generated according to the following condition:

$$\iota_{t+1}=\inf\{\iota\in N|\iota>\iota_t,\frac{\gamma_\iota}{\nu}+\tau-\|y_\iota-\tilde{y}_\iota\|\leq 0\}\tag{4}$$

where the variable γ_ι satisfies

$$\gamma_{\iota+1}=\psi\gamma_\iota+\tau-\|y_\iota-\tilde{y}_\iota\|\tag{5}$$

with $0<\psi<1$ and $\tau>0$.

Consequently, the real measurement that is received by the filter is expressed as

$$\tilde{y}_\iota=y_{\iota_t},\iota\in\{\iota_t,\iota_t+1,\iota_t+2,\cdots,\iota_{t+1}-1\}.\tag{6}$$

Let $\tilde{X}_{\iota+1}=[x_{\iota+1}^T,f_{\iota+1}^T,\triangle f_{\iota+1}^T]^T$, then the original system (1) is transformed into

$$\begin{cases}\tilde{X}_{\iota+1}=\tilde{\Phi}_\iota\tilde{X}_\iota+\tilde{S}(\tilde{X}_\iota,\alpha_\iota)+\tilde{C}_\iota w_\iota\\ y_\iota=\tilde{H}_\iota\tilde{X}_\iota+\tilde{T}(\tilde{X}_\iota,\beta_\iota)+D_\iota v_\iota\end{cases}\tag{7}$$

where

$$\tilde{\Phi}_\iota\triangleq\begin{bmatrix}\Phi_\iota&M_\iota&0\\0&I&I\\0&0&I\end{bmatrix},\tilde{C}_\iota\triangleq\begin{bmatrix}C_\iota\\0\\0\end{bmatrix},\tilde{H}_\iota\triangleq\begin{bmatrix}H_\iota^T\\0\\0\end{bmatrix}^T,$$

$$\tilde{S}(\tilde{X}_\iota,\alpha_\iota)\triangleq\begin{bmatrix}S(x_\iota,\alpha_\iota)\\0\\0\end{bmatrix},\tilde{T}(\tilde{X}_\iota,\beta_\iota)\triangleq T(x_\iota,\beta_\iota).$$

In light of the available measurements $\tilde{y}_\iota$, we design a non-fragile filter for system (7) as follows

$$\begin{cases}\hat{\tilde{X}}_{\iota+1|\iota}=\tilde{\Phi}_\iota\hat{\tilde{X}}_{\iota|\iota}\\ \hat{\tilde{X}}_{\iota+1|\iota+1}=\hat{\tilde{X}}_{\iota+1|\iota}+(G_{\iota+1}+\triangle G_{\iota+1})\\ \qquad\times(\tilde{y}_{\iota+1}-\tilde{H}_{\iota+1}\hat{\tilde{X}}_{\iota+1|\iota})\end{cases}\tag{8}$$

where $\hat{\tilde{X}}_{\iota+1|\iota}$ and $\hat{\tilde{X}}_{\iota+1|\iota+1}$ serve as the one-step predictor and the filter, respectively. $G_{\iota+1}$ represents the gain matrix that remains to be determined, and $\triangle G_{\iota+1}$ satisfies

$$\mathbb{E}\{\triangle G_{\iota+1}\}=0,\ \ \mathbb{E}\{\triangle G_{\iota+1}\triangle G_{\iota+1}^T\}\leq gI.$$

To facilitate subsequent discussion, the prediction error and the filtering error are respectively denoted as

$$\Psi_{\iota+1|\iota} \triangleq \tilde{X}_{\iota+1} - \hat{\tilde{X}}_{\iota+1|\iota}$$
$$\Psi_{\iota+1|\iota+1} \triangleq \tilde{X}_{\iota+1} - \hat{\tilde{X}}_{\iota+1|\iota+1}$$

and the corresponding error covariance matrices are expressed as

$$\wp_{\iota+1|\iota} \triangleq \mathbb{E}\{\Psi_{\iota+1|\iota}\Psi^T_{\iota+1|\iota}\}$$
$$\wp_{\iota+1|\iota+1} \triangleq \mathbb{E}\{\Psi_{\iota+1|\iota+1}\Psi^T_{\iota+1|\iota+1}\}$$

The core goal of this paper is to construct a non-fragile filter (8) such that: (1) there exists a set of matrices $\bar{\wp}_{\iota+1|\iota+1}$ which satisfies the condition $\wp_{\iota+1|\iota+1} \leq \bar{\wp}_{\iota+1|\iota+1}$; (2) the acquired upper bound $\bar{\wp}_{\iota+1|\iota+1}$ is minimized through the appropriate selection of the gain parameter $G_{\iota+1}$.

Remark 1. *Assumption 2 has provided us with the characteristic of the fault involved in this paper. In comparison with the constant fault $f_{\iota+1} = f_\iota$ and dynamic fault $f_{\iota+1} = H_{f,\iota} f_\iota$, this kind of faults (e.g. the ramp fault and the abrupt fault) has been investigated in some existing references. To illustrate, a novel fault estimation approach has been proposed in [12] for both linear and nonlinear systems, while the resilient fault diagnosis issue for a group of complex networks has been addressed in [5]. In the latter work, a random bias modeled by Gaussian white noise with a large covariance was introduced to characterize the unknown fault occurrence time. It is important to highlight that the main emphasis of this paper lies in the design of a dynamic event-triggered non-fragile state and fault estimator for a certain category of stochastic nonlinear systems.*

3. Main Results

3.1. Non-Fragile State and Fault Estimator Design

In this section, we will design the non-fragile state and fault estimator by means of the stochastic analysis method. Prior to stating the main results, we first introduce several lemmas.

Lemma 1. *Given known matrices A and B, and $\varepsilon > 0$, then we have*

$$AB^T + BA^T \leq \varepsilon AA^T + \varepsilon^{-1}BB^T \tag{9}$$

Lemma 2. *The state covariance matrix $O_{\iota+1} \triangleq \mathbb{E}\{\tilde{X}_{\iota+1}\tilde{X}^T_{\iota+1}\}$ of system (7) satisfies*

$$O_{\iota+1} = \tilde{\Phi}_\iota O_\iota \tilde{\Phi}^T_\iota + Q_{s,\iota} + \tilde{C}_\iota Q_{w_\iota} \tilde{C}^T_\iota \tag{10}$$

where

$$O_0 \triangleq \begin{bmatrix} P_0 + \bar{x}_0\bar{x}_0^T & 0 \\ 0 & 0 \end{bmatrix}, F \triangleq [I, 0, 0],$$

and

$$Q_{s,h} \triangleq \begin{bmatrix} \sum_{i=1}^{l} \mathcal{A}_{i1,\iota}\mathrm{tr}(O_\iota F^T B_\iota F) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Proof. Utilizing the definition of $O_\iota = \mathbb{E}\{\tilde{X}_\iota\tilde{X}^T_\iota\}$, one has

$$\begin{aligned} O_{\iota+1} =& \tilde{\Phi}_\iota O_\iota \tilde{\Phi}^T_\iota + \tilde{C}_\iota Q_{W_\iota} \tilde{C}^T_\iota \\ &+ \mathbb{E}\{\tilde{S}(\tilde{X}_\iota, \alpha_\iota)\tilde{S}^T(\tilde{X}_\iota, \alpha_\iota)\} \end{aligned} \tag{11}$$

From (3), we further obtain that

$$\begin{aligned}&\mathbb{E}\{S(x_\iota,\alpha_\iota)S^T(x_\iota,\alpha_\iota)\}\\=&\mathbb{E}\{\sum_{i=1}^{l}\mathcal{A}_{i1,\iota}\tilde{X}_\iota^T F^T B_\iota F\tilde{X}_\iota\}\\=&\sum_{i=1}^{l}\mathcal{A}_{i1,\iota}\mathrm{tr}(O_\iota F^T B_\iota F)\end{aligned}\tag{12}$$

which implies (10) holds. □

Lemma 3. *Let $\varepsilon_{1,\iota}$ and $\varepsilon_{2,\iota}$ be the positive numbers, then $U_\iota \triangleq \mathbb{E}\{\gamma_\iota^2\}$ has an upper bound $\bar{U}_\iota$ which satisfies*

$$\begin{aligned}\bar{U}_{\iota+1}=&\big[(1+\varepsilon_{1,\iota})(1+\varepsilon_{2,\iota})\psi^2+\frac{(1+\varepsilon_{1,\iota}^{-1})(1+\nu)}{\nu^2}\big]\bar{U}_\iota\\&+\big[(1+\varepsilon_{1,\iota})(1+\varepsilon_{2,\iota}^{-1})+(1+\varepsilon_{1,\iota}^{-1})(1+\nu^{-1})\big]\tau^2\end{aligned}\tag{13}$$

with $\bar{U}_0=\gamma_0^2$.

Proof. Utilizing (5) and Lemma 1 leads to

$$\begin{aligned}U_{\iota+1}\leq&(1+\varepsilon_{1,\iota})\mathbb{E}\{(\psi\gamma_\iota+\tau)^2\}+(1+\varepsilon_{1,\iota}^{-1})\mathbb{E}\{\|y_\iota-\tilde{y}_\iota\|^2\}\\\leq&(1+\varepsilon_{1,\iota})\Big[(1+\varepsilon_{2,\iota})\psi^2U_\iota+(1+\varepsilon_{2,\iota}^{-1})\tau^2\Big]\\&+(1+\varepsilon_{1,\iota}^{-1})\mathbb{E}\{\|y_\iota-\tilde{y}_\iota\|^2\}\end{aligned}\tag{14}$$

On the other hand, we have

$$\|y_\iota-\tilde{y}_\iota\|^2\leq(\frac{\gamma_\iota}{\nu}+\tau)^2\leq(1+\nu^{-1})\tau^2+\frac{(1+\nu)\gamma_\iota^2}{\nu^2}\tag{15}$$

Substituting (15) into (14) yields (13). □

Theorem 1. *The recursion of the prediction error covariance $\wp_{\iota+1|\iota}$ satisfies:*

$$\wp_{\iota+1|\iota}=\tilde{\Phi}_\iota\wp_{\iota|\iota}\tilde{\Phi}_\iota^T+Q_{s,\iota}+\tilde{C}_\iota Q_{w_\iota}\tilde{C}_\iota^T\tag{16}$$

Proof. From the definition of the prediction error $\Psi_{\iota+1|\iota}$, one arrives at

$$\Psi_{\iota+1|\iota}=\tilde{\Phi}_\iota\Psi_{\iota|\iota}+\tilde{S}(\tilde{X}_\iota,\alpha_\iota)+\tilde{C}_\iota w_\iota\tag{17}$$

Substituting (17) into $\wp_{\iota+1|\iota}$ yields

$$\begin{aligned}\wp_{\iota+1|\iota}=&\mathbb{E}\{\tilde{\Phi}_\iota\Psi_{\iota|\iota}\Psi_{\iota|\iota}^T\tilde{\Phi}_\iota^T\}+\mathbb{E}\{\tilde{C}_\iota w_\iota w_\iota^T\tilde{C}_\iota^T\}\\&+\mathbb{E}\{\tilde{S}(\tilde{X}_\iota,\alpha_\iota)\tilde{S}^T(\tilde{X}_\iota,\alpha_\iota)\}\end{aligned}\tag{18}$$

A combination of $Q_{s,\iota}$ and (18) results in (16). □

Theorem 2. *Let $\varepsilon_{3,\iota+1}$ and $\varepsilon_{4,\iota+1}$ be given positive scalars. Assume that $\bar{\wp}_{\iota+1|\iota}$ and $\bar{\wp}_{\iota+1|\iota+1}$ are the solutions to the following equations:*

$$\bar{\wp}_{\iota+1|\iota}=\tilde{\Phi}_\iota\bar{\wp}_{\iota|\iota}\tilde{\Phi}_\iota^T+Q_{s,\iota}+\tilde{C}_\iota Q_{w_\iota}\tilde{C}_\iota^T\tag{19}$$

and

$$\begin{aligned}\bar{\wp}_{\iota+1|\iota+1} =&(1+\varepsilon_{3,\iota+1})(I-G_{\iota+1}\tilde{H}_{\iota+1})\wp_{\iota+1|\iota}\\&\times(I-G_{\iota+1}\tilde{H}_{\iota+1})^T+(1+\varepsilon_{3,\iota+1}^{-1}+\varepsilon_{4,\iota+1})\\&\times G_{\iota+1}[\frac{1+\nu}{\nu^2}\bar{U}_{\iota+1}+(1+\nu^{-1})\tau^2]IG_{\iota+1}^T\\&+G_{\iota+1}\sum_{i=1}^{l}\mathcal{A}_{i2,\iota+1}\mathrm{tr}(O_{\iota+1}F^TB_{\iota+1}F)G_{\iota+1}^T\\&+(1+\varepsilon_{4,\iota+1}^{-1})G_{\iota+1}D_{\iota+1}Q_{v_{\iota+1}}D_{\iota+1}^TG_{\iota+1}^T\\&+\lambda_{\max}\Big[(1+\varepsilon_{3,\iota+1})\tilde{H}_{\iota+1}\wp_{\iota+1|\iota}\tilde{H}_{\iota+1}^T+(1+\\&\varepsilon_{3,\iota+1}^{-1}+\varepsilon_{4,\iota+1})[\frac{1+\nu}{\nu^2}\bar{U}_{\iota+1}+(1+\nu^{-1})\tau^2]I\\&+\sum_{i=1}^{l}\mathcal{A}_{i2,\iota+1}\mathrm{tr}(O_{\iota+1}F^TB_{\iota+1}F)\\&+(1+\varepsilon_{4,\iota+1}^{-1})D_{\iota+1}Q_{v_{\iota+1}}D_{\iota+1}^T\Big]gI\end{aligned}\tag{20}$$

with the initial condition $\wp_{0|0}=\bar{\wp}_{0|0}$*, then* $\bar{\wp}_{\iota+1|\iota}$ *and* $\bar{\wp}_{\iota+1|\iota+1}$ *are upper bounds of* $\wp_{\iota+1|\iota+1}$ *and* $\wp_{\iota+1|\iota+1}$*, respectively, i.e.,*

$$\wp_{\iota+1|\iota}\le\bar{\wp}_{\iota+1|\iota},\ \ \wp_{\iota+1|\iota+1}\le\bar{\wp}_{\iota+1|\iota+1}.$$

Proof. The proof of this theorem is accomplished through mathematical induction. Obviously, we have $\wp_{0|0}\le\bar{\wp}_{0|0}$. Assume that $\wp_{\iota|\iota}\le\bar{\wp}_{\iota|\iota}$ holds, then we can easily prove $\wp_{\iota+1|\iota}\le\bar{\wp}_{\iota+1|\iota}$.

In what follows, what we need to prove is $\wp_{\iota+1|\iota+1}\le\bar{\wp}_{\iota+1|\iota+1}$.

Similar to (17), we have

$$\begin{aligned}\Psi_{\iota+1|\iota+1} =&\Psi_{\iota+1|\iota}-(G_{\iota+1}+\triangle G_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1}\\&+y_{\iota+1}-\tilde{H}_{\iota+1}\hat{X}_{\iota+1|\iota})\\=&[I-(G_{\iota+1}+\triangle G_{\iota+1})\tilde{H}_{\iota+1}]\Psi_{\iota+1|\iota}\\&-(G_{\iota+1}+\triangle G_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1})\\&-(G_{\iota+1}+\triangle G_{\iota+1})\tilde{T}(\tilde{X}_{\iota+1},\beta_{\iota+1})\\&-(G_{\iota+1}+\triangle G_{\iota+1})D_{\iota+1}v_{\iota+1}\end{aligned}\tag{21}$$

Furthermore, $\wp_{\iota+1|\iota+1}$ is computed by

$$\begin{aligned}\wp_{\iota+1|\iota+1} =&(I-G_{\iota+1}\tilde{H}_{\iota+1})\wp_{\iota+1|\iota}(I-G_{\iota+1}\tilde{H}_{\iota+1})^T\\&+G_{\iota+1}\mathbb{E}\{(\tilde{y}_{\iota+1}-y_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1})^T\}G_{\iota+1}^T\\&+G_{\iota+1}\mathbb{E}\{\tilde{T}(\tilde{X}_{\iota+1},\beta_{\iota+1})\tilde{T}^T(\tilde{X}_{\iota+1},\beta_{\iota+1})\}G_{\iota+1}^T\\&+G_{\iota+1}\mathbb{E}\{D_{\iota+1}v_{\iota+1}v_{\iota+1}^TD_{\iota+1}^T\}G_{\iota+1}^T\\&+\mathbb{E}\Big\{\triangle G_{\iota+1}[\tilde{H}_{\iota+1}\Psi_{\iota+1|\iota}\Psi_{\iota+1|\iota}^T\tilde{H}_{\iota+1}^T\\&+(\tilde{y}_{\iota+1}-y_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1})^T\\&+\tilde{T}(\tilde{X}_{\iota+1},\beta_{\iota+1})\tilde{T}^T(\tilde{X}_{\iota+1},\beta_{\iota+1})\\&+D_{\iota+1}v_{\iota+1}v_{\iota+1}^TD_{\iota+1}^T]\triangle G_{\iota+1}^T\Big\}\\&+\Gamma_{1,\iota+1}+\Gamma_{1,\iota+1}^T+\Gamma_{2,\iota+1}+\Gamma_{2,\iota+1}^T\end{aligned}\tag{22}$$

where

$$
\begin{aligned}
\Gamma_{1,\iota+1} \triangleq &\mathbb{E}\Big\{-[I-(G_{\iota+1}+\triangle G_{\iota+1})\tilde{H}_{\iota+1}]\Psi_{\iota+1|\iota} \\
&\times(\tilde{y}_{\iota+1}-y_{\iota+1})^T(G_{\iota+1}+\triangle G_{\iota+1})^T\Big\}, \\
\Gamma_{2,\iota+1} \triangleq &\mathbb{E}\Big\{(G_{\iota+1}+\triangle G_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1}) \\
&\times v_{\iota+1}^T D_{\iota+1}^T(G_{\iota+1}+\triangle G_{\iota+1})^T\Big\}.
\end{aligned}
$$

By means of Lemma 1, the terms $\Gamma_{1,\iota+1}$ and $\Gamma_{2,\iota+1}$ can be computed as follows

$$
\begin{aligned}
&\Gamma_{1,\iota+1}+\Gamma_{1,\iota+1}^T \\
\leq&\varepsilon_{3,\iota+1}\mathbb{E}\Big\{[I-(G_{\iota+1}+\triangle G_{\iota+1})\tilde{H}_{\iota+1}] \\
&\times\Psi_{\iota+1|\iota}\Psi_{\iota+1|\iota}^T[I-(G_{\iota+1}+\triangle G_{\iota+1})\tilde{H}_{\iota+1}]^T\Big\} \\
&+\varepsilon_{3,\iota+1}^{-1}\mathbb{E}\{(G_{\iota+1}+\triangle G_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1}) \\
&\times(\tilde{y}_{\iota+1}-y_{\iota+1})^T(G_{\iota+1}+\triangle G_{\iota+1})^T\}
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
&\Gamma_{2,\iota+1}+\Gamma_{2,\iota+1}^T \\
\leq&\varepsilon_{4,\iota+1}\mathbb{E}\{(G_{\iota+1}+\triangle G_{\iota+1})(\tilde{y}_{\iota+1}-y_{\iota+1}) \\
&\times(\tilde{y}_{\iota+1}-y_{\iota+1})^T(G_{\iota+1}+\triangle G_{\iota+1})^T\} \\
&+\varepsilon_{4,\iota+1}^{-1}\mathbb{E}\{(G_{\iota+1}+\triangle G_{\iota+1})D_{\iota+1}v_{\iota+1}v_{\iota+1}^T D_{\iota+1}^T \\
&\times(G_{\iota+1}+\triangle G_{\iota+1})^T\}
\end{aligned}
\tag{24}
$$

Combining (15) with (22)–(24), we obtain $\wp_{\iota+1|\iota+1}\leq\bar{\wp}_{\iota+1|\iota+1}$. □

Theorem 3. *The upper bound on the FEC $\bar{\wp}_{\iota+1|\iota+1}$ is minimal with the following gain parameter:*

$$
G_{\iota+1}=(1+\varepsilon_{3,\iota+1})\bar{\wp}_{\iota+1|\iota}\tilde{H}_{\iota+1}^T\Xi_{\iota+1}^{-1}
\tag{25}
$$

where

$$
\begin{aligned}
\Xi_{\iota+1}\triangleq&(1+\varepsilon_{3,\iota+1})\tilde{H}_{\iota+1}\bar{\wp}_{\iota+1|\iota}\tilde{H}_{\iota+1}^T+(1+\varepsilon_{3,\iota+1}^{-1} \\
&+\varepsilon_{4,\iota+1})(\frac{1+\nu}{\nu^2}\bar{U}_{\iota+1}+(1+\nu^{-1})\tau^2)I \\
&+\sum_{i=1}^{l}\mathcal{A}_{i2,\iota+1}\mathrm{tr}(O_{\iota+1}F^TB_{\iota+1}F) \\
&+(1+\varepsilon_{4,\iota+1}^{-1})D_{\iota+1}Q_{v_{\iota+1}}D_{\iota+1}^T.
\end{aligned}
\tag{26}
$$

Proof. Reorganizing the upper bound $\bar{\wp}_{\iota+1|\iota+1}$ yields

$$
\begin{aligned}
\bar{\wp}_{\iota+1|\iota+1}=&(1+\varepsilon_{3,\iota+1})\bar{\wp}_{\iota+1|\iota}+\lambda_{\max}(\Xi_{\iota+1})gI \\
&-(1+\varepsilon_{3,\iota+1})G_{\iota+1}\tilde{H}_{\iota+1}\bar{\wp}_{\iota+1|\iota} \\
&-(1+\varepsilon_{3,\iota+1})\bar{\wp}_{\iota+1|\iota}\tilde{H}_{\iota+1}^TG_{\iota+1}^T \\
&+G_{\iota+1}\Xi_{\iota+1}G_{\iota+1}^T
\end{aligned}
\tag{27}
$$

Taking the partial derivative of $\mathrm{tr}(\bar{\wp}_{\iota+1|\iota+1})$ with respect to $G_{\iota+1}$ results in

$$
\begin{aligned}
&\frac{\partial(\mathrm{tr}(\bar{\wp}_{\iota+1|\iota+1}))}{\partial(G_{\iota+1})} \\
&=-2(1+\varepsilon_{3,\iota+1})\bar{\wp}_{\iota+1|\iota}\tilde{H}_{\iota+1}^T+2G_{\iota+1}\Xi_{\iota+1}
\end{aligned}
\tag{28}
$$

Setting $\frac{\partial(\mathrm{tr}(\bar{\wp}_{\iota+1|\iota+1}))}{\partial(G_{\iota+1})} = 0$ yields

$$G_{\iota+1} = (1+\varepsilon_{3,\iota+1})\bar{\wp}_{\iota+1|\iota}\tilde{H}^T_{\iota+1}\Xi^{-1}_{\iota+1} \tag{29}$$

which is (25). □

Remark 2. *In fact, it is challenging to acquire the exact expression of $\wp_{\iota+1|\iota+1}$ mainly because of the unknown terms $\tilde{y}_{\iota+1} - y_{\iota+1}$ and $\triangle G_{\iota+1}$. In this case, it makes sense to find the corresponding UB of $\wp_{\iota+1|\iota+1}$ by means of the Lemma 1. As a result, many efforts have been made to cope with this difficulty and the main results have obtained in Theorem 2. Clearly, the aforementioned factors (i.e., stochastic nonlinearities, the DETS, and the gain variation) have all considered in the design of joint state and fault estimator. To be specific, ν, τ^2, and $\bar{U}_{\iota+1}$ represent the influence from the dynamic event-triggered scheme, gI quantify the phenomenon of the gain variation, and $\sum_{i=1}^{l}\mathcal{A}_{i2,\iota+1}\mathrm{tr}(O_{\iota+1}F^TB_{\iota+1}F)$ describe the random disturbance.*

3.2. Performance Analysis

In this section, we will address the boundedness of the filtering error via the following theorem.

Lemma 4 ([39]). *Let $\kappa > 0$, $0 < \delta < 1$, and $\mu > 0$ be scalars satisfying the condition stated below:*

$$\mathbb{E}\{|\chi_i|^2\} \le \kappa\mathbb{E}\{|\chi_0|^2\}\delta^i + \mu \tag{30}$$

then the stochastic process χ_i is mean-square exponentially bounded.

Assumption 3. *Suppose there exist positive real numbers $\bar{\phi}$, $\bar{c}$, a_1, $\bar{o}$, $\bar{b}$, q_w, $\underline{\varepsilon}_i$, and $\bar{\varepsilon}_i$ $(i = 3, 4)$ such that the following holds:*

$$\|\tilde{\Phi}_\iota\| \le \bar{\phi}, \|\tilde{C}_\iota\| \le \bar{c}, \|\mathcal{A}_{i1,\iota}\| \le a_1, \mathrm{tr}(O_\iota) \le \bar{o},$$
$$\|B_{i,\iota}\| \le \bar{b}, \|Q_{w_\iota}\| \le q_w, \underline{\varepsilon}_i \le \varepsilon_i \le \bar{\varepsilon}_i (i = 3, 4).$$

Relying on Lemma 2 together with Assumption 3, we now proceed to examine the boundedness of the estimation error.

Theorem 4. *Under Assumption 3, the filtering error attains mean-square exponential boundedness, provided that the subsequent inequality is fulfilled:*

$$\varphi_1 \triangleq (1+\bar{\varepsilon}_3)\bar{\phi}^2 < 1 \tag{31}$$

Proof. Computing the norm for each side of (19) leads to

$$\|\bar{\wp}_{\iota+1|\iota}\| \le \|\tilde{\Phi}_\iota\|^2\|\bar{\wp}_{\iota|\iota}\| + \|Q_{s,\iota}\| + \|\tilde{C}_\iota\|^2\|Q_{w_\iota}\| \tag{32}$$

in which $\|Q_{s,\iota}\|$ satisfies

$$\|Q_{s,\iota}\| = \|\sum_{i=1}^{l} A_{i1,\iota}\mathrm{tr}(O_\iota F^T B_\iota F)\| \le la_1\bar{o}\bar{b}n_x \tag{33}$$

It follows from Assumption 3 and (33) that

$$\|\bar{\wp}_{\iota+1|\iota}\| \le \bar{\phi}^2\|\bar{\wp}_{\iota|\iota}\| + la_1\bar{o}\bar{b}n_x + \bar{c}^2 q_w \tag{34}$$

According to Theorem 3, we further know that

$$\begin{aligned}\bar{\wp}_{\iota+1|\iota+1} =& (1+\bar{\varepsilon}_3)\bar{\wp}_{\iota+1|\iota} + \lambda_{max}(\Xi_{\iota+1})gI \\ &- G_{\iota+1}\Xi_{\iota+1}G^T_{\iota+1}\end{aligned} \tag{35}$$

thus

$$\|\bar{\wp}_{\iota+1|\iota+1}\| \leq (1+\bar{\varepsilon}_3)\|\bar{\wp}_{\iota+1|\iota}\| + \lambda_{max}(\Xi_{\iota+1})g \tag{36}$$

Substituting (34) into (36) leads to

$$\begin{aligned}
\|\bar{\wp}_{\iota+1|\iota+1}\| \leq & (1+\bar{\varepsilon}_3)\bar{\phi}^2\|\bar{\wp}_{\iota|\iota}\| + \lambda_{max}(\Xi_{\iota+1})g \\
& + (1+\bar{\varepsilon}_3)(la_1\bar{o}\bar{b}n_x + \bar{c}^2 q_w) \\
= & \varphi_1\|\bar{\wp}_{\iota|\iota}\| + \varphi_2 \\
& \cdots \\
= & \varphi_1^{\iota+1}\|\bar{\wp}_{0|0}\| + \frac{\varphi_2(1-\varphi_1^{\iota+1})}{1-\varphi_1} \\
\leq & \varphi_1^{\iota+1}\|\bar{\wp}_{0|0}\| + \frac{\varphi_2}{1-\varphi_1}
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
\varphi_1 & \triangleq (1+\bar{\varepsilon}_3)\bar{\phi}^2 \\
\varphi_2 & \triangleq (1+\bar{\varepsilon}_3)(la_1\bar{o}\bar{b}n_x + \bar{c}^2 q_w) + \lambda_{max}(\Xi_{\iota+1})g
\end{aligned} \tag{38}$$

If $\varphi_1 < 1$, then $\|\bar{\wp}_{\iota+1|\iota+1}\|$ converges eventually which implies the filtering error is mean-square exponentially bounded. □

Remark 3. *Thus far, the joint state and fault estimation issue has been systematically explored in this paper. In contrast to conventional state and fault estimation algorithms, our findings possess three distinctive features: (1) We make the first attempts to estimate faults with zero second-order difference, accounting for stochastic nonlinearities, DETS, and gain variations simultaneously; (2) A novel augmented system model is developed by incorporating the original state, the fault, and its first-order difference. An UB of the FEC is subsequently derived and minimized through appropriate gain parameter selection; (3) A theoretical analysis is conducted to establish the mean-square boundedness of the filtering error.*

4. Illustrative Examples

In this section, a simulation example is offered to show the usefulness and performance of the developed estimation algorithm.

Example: Let us introduce a stochastic nonlinear system, with the associated matrices specified below.

$$\begin{aligned}
\Phi_\iota &= \begin{bmatrix} 0.95 & 1.2 \\ -0.5 & 0.92\mathrm{sin}(0.5\iota) \end{bmatrix}, \\
H_\iota &= \begin{bmatrix} 0.92 + 0.05\mathrm{cos}(0.2\iota) & 0.82 \end{bmatrix}, \\
M_\iota &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, C_\iota = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}, D_\iota = 0.01,
\end{aligned}$$

and the stochastic functions satisfy

$$\begin{aligned}
S(x_\iota, \alpha_\iota) = & \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} (0.3\mathrm{sign}(x_{1,\iota})x_{1,\iota}\alpha_{1,\iota} \\
& + 0.4\mathrm{sign}(x_{2,\iota})x_{2,\iota}\alpha_{2,\iota}), \\
T(x_\iota, \beta_\iota) = & 0.3(0.3\mathrm{sign}(x_{1,\iota})x_{1,\iota}\beta_{1,\iota} + 0.4\mathrm{sign}(x_{2,\iota})x_{2,\iota}\beta_{2,\iota})
\end{aligned}$$

where $x_{i,\iota}$, $\alpha_{i,\iota}$, and $\beta_{i,\iota}(i=1,2)$ are the ith elements of x_ι, α_ι, and β_ι. The random sequences w_ι, v_ι, α_ι and β_ι obey the Gaussian distribution, and $Q_{w_\iota} = 0.1$, $Q_{v_\iota} = 0.1$, $Q_{\alpha_{1,\iota}} = 0.1$, $Q_{\alpha_{2,\iota}} = 0.1$, $Q_{\beta_{1,\iota}} = 0.1$, $Q_{\beta_{2,\iota}} = 0.1$. The corresponding matrices are $A_{11,\iota} = [0.1, 0.2]^T[0.1, 0.2]$, $A_{12,\iota} = 0.09$, and $B = \mathrm{diag}\{0.09, 0.16\}$ after simple calculation.

The considered fault in this paper can be assumed as

$$f_\iota = \begin{cases} 0, & \iota < 20, \\ 0.03\iota - 0.57, & 20 \leq \iota \leq 120 \\ 0.3, & \iota > 120 \end{cases}$$

In addition, other parameters are set as $\varepsilon_{1,\iota} = 1$, $\varepsilon_{2,\iota} = 1$, $\varepsilon_{3,\iota} = 0.01$, $\varepsilon_{4,\iota} = 1$, $\psi = 0.2$, $\tau = 0.2$, $\nu = 5$, $\gamma_0 = 0.15$, $g = 0.01$, $x_0 = [0.3, 0.1]^T$, and $\wp_{0|0} = 10^{-4} I_4$.

Based on the aforementioned parameters in conjunction with Theorems 1–3, $\bar{\wp}_{\iota+1|\iota+1}$ and the gain parameter G_ι are computed recursively. Figures 1–3 show the trajectories of x_ι, f_ι, and their estimates, revealing excellent filtering performance. Figures 4–6 present the mean square error (MSE) along with its corresponding UB. As clearly shown, the UB curves consistently exceed the MSE curves, which demonstrates the effectiveness of the proposed estimation method.

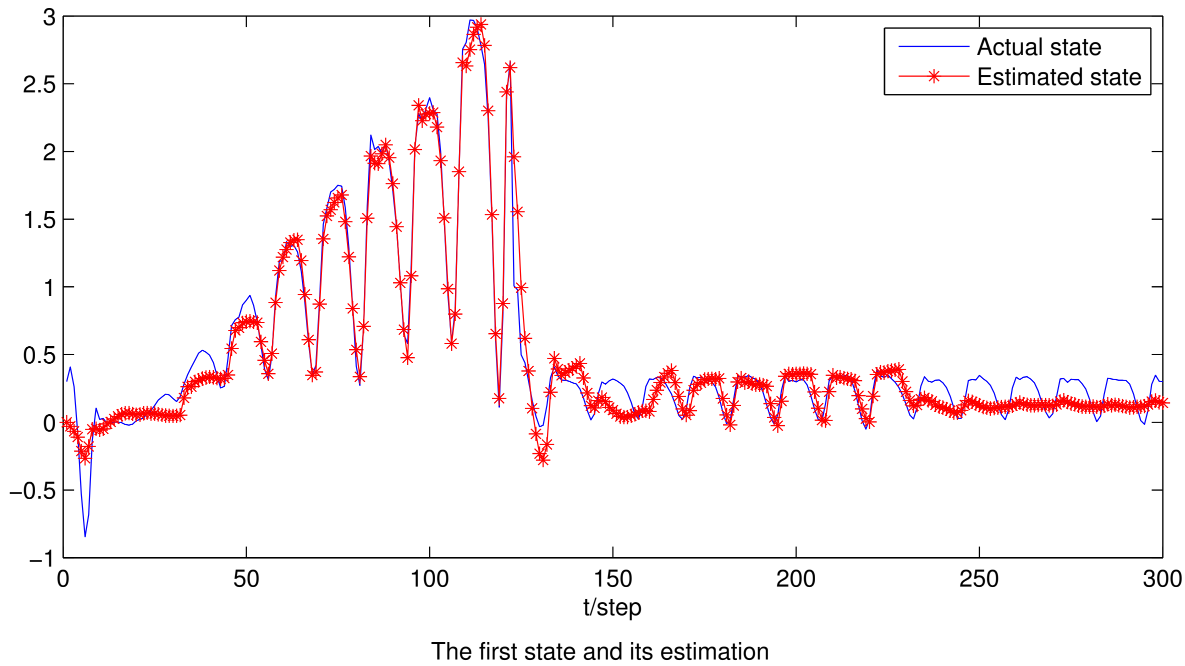

Figure 1. The curves of $x_{1,\iota}$ and $\hat{x}_{1,\iota}$.

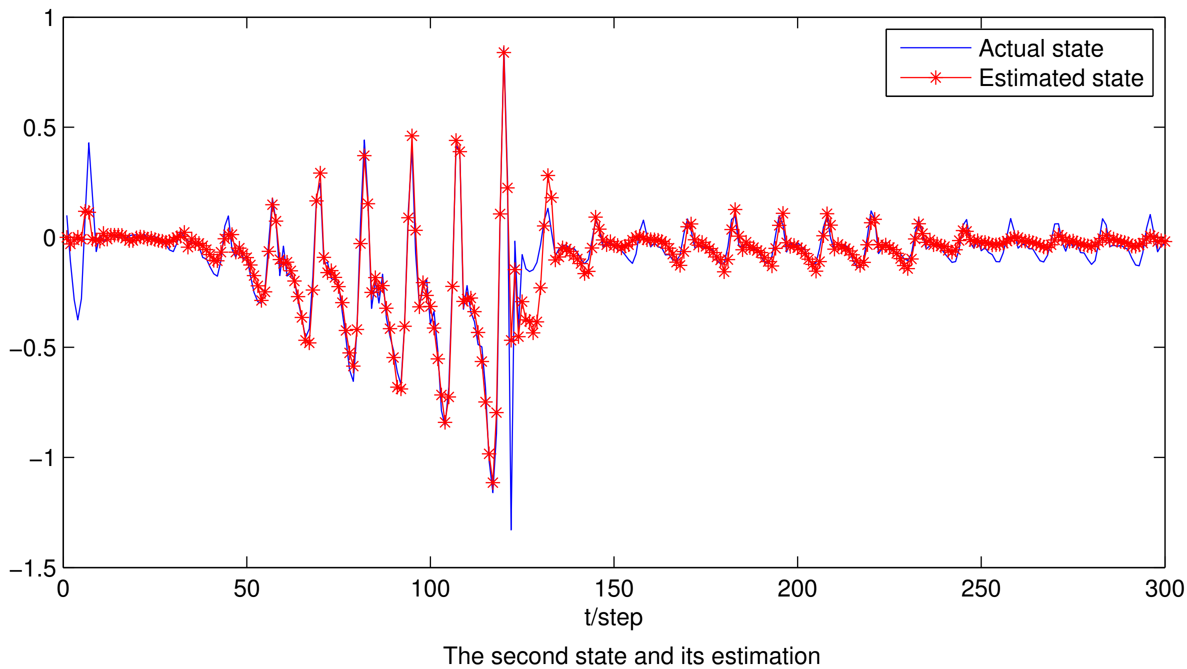

Figure 2. The curves of $x_{2,\iota}$ and $\hat{x}_{2,\iota}$.

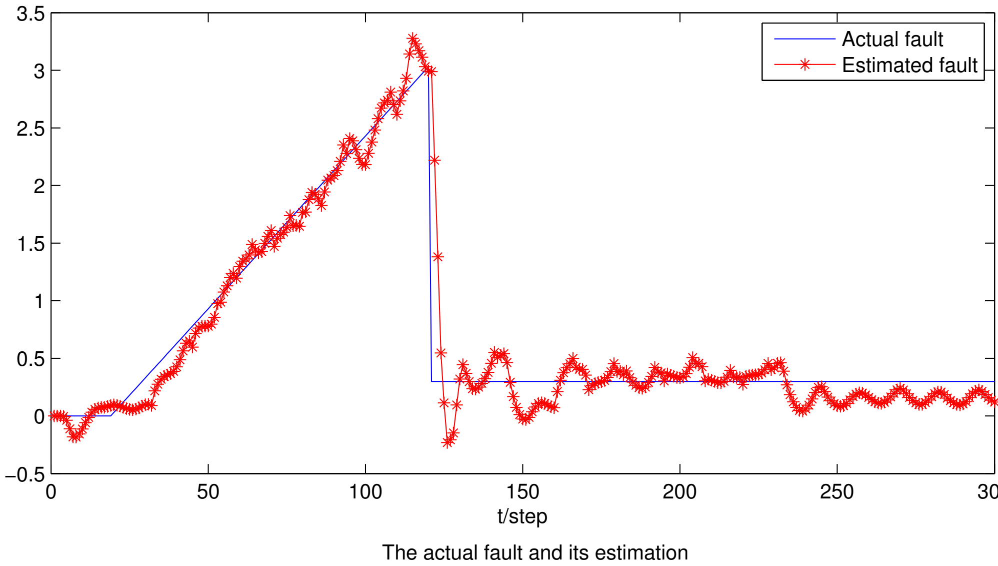

Figure 3. The curves of f_ι and its estimation.

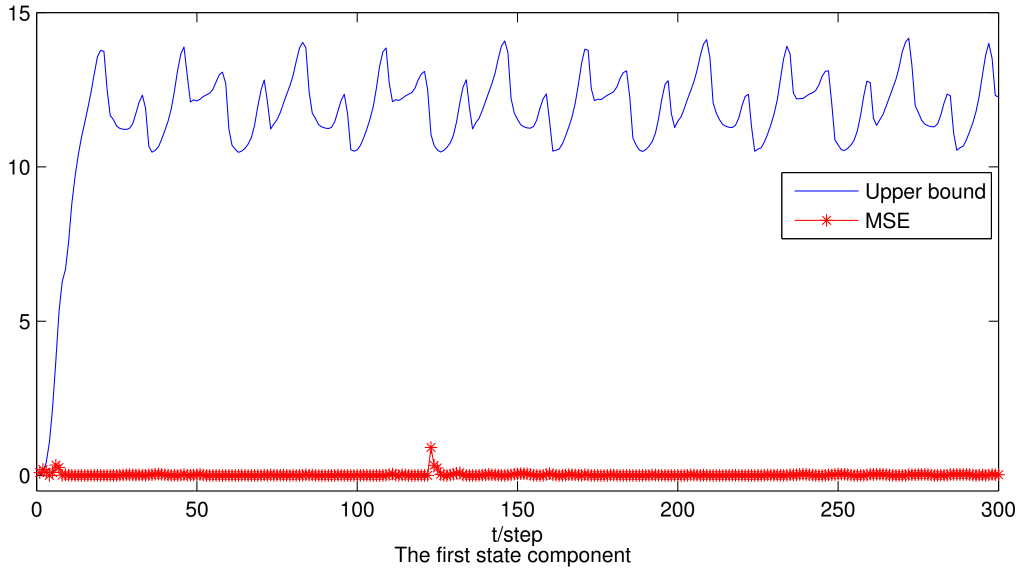

Figure 4. The UB of $x_{1,\iota}$ and its MSE.

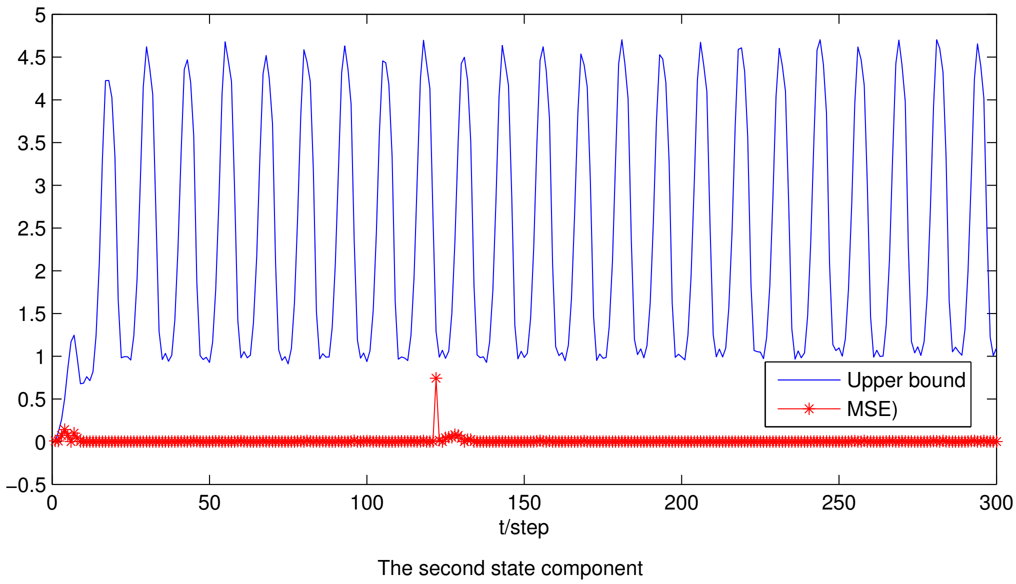

Figure 5. The UB of $x_{2,\iota}$ and its MSE.

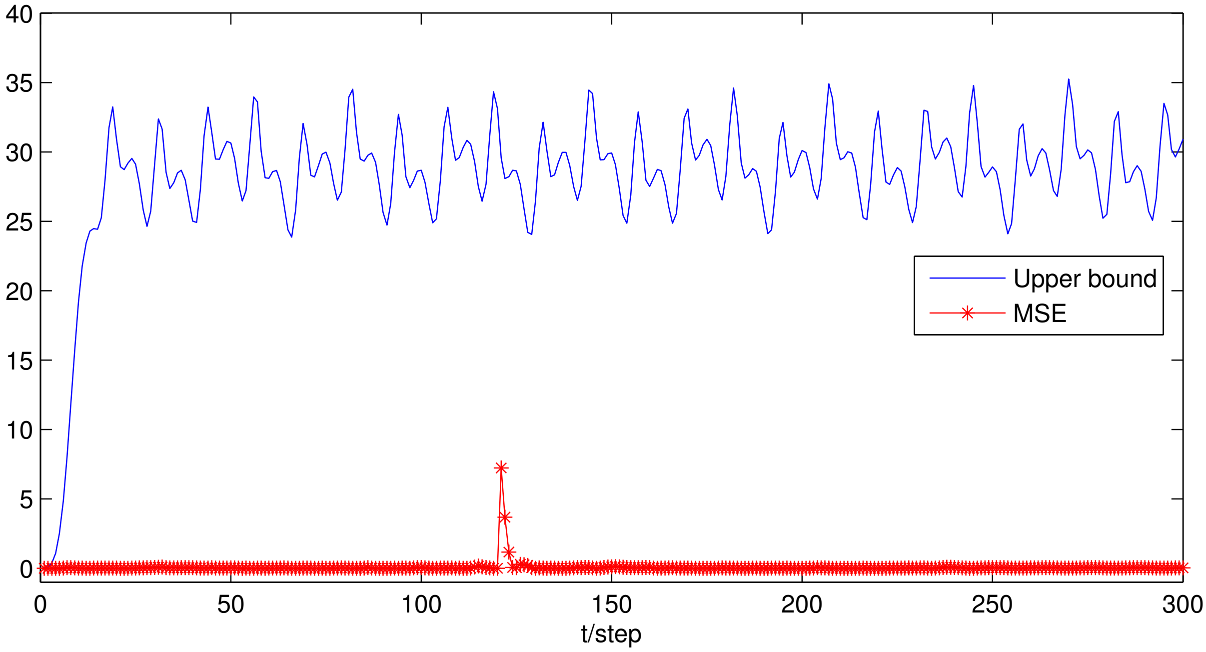

Figure 6. The UB of f_ι and its MSE.

5. Conclusions

In this paper, the non-fragile dynamic event-triggered state and fault estimation issue has been studied for stochastic nonlinear systems. Faults characterized by zero second-order difference have been considered, and the dynamic event-triggered scheme has been adopted to improve network utilization efficiency. Using state augmentation, an augmented stochastic parameter system has been constructed. In light of the difficulty in obtaining an exact expression for the filtering error covariance, a novel recursive filter has been designed for the augmented system. An UB of the actual FEC, expressed in terms of matrix difference equations, has been derived and subsequently minimized through appropriate gain selection. Moreover, a sufficient condition has been derived to guarantee the mean-square boundedness of the non-fragile filter. To conclude, some numerical results have been offered to illustrate the utility of the proposed estimation algorithm. Future research will focus on extending the obtained results to more general systems.

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Notations

$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{E}\{*\}$	The expectation of random variable "$*$"
$\mathrm{tr}(A)$	The trace of the matrix A
A^T	The transpose of the matrix A
$\|A\|$	The norm of the matrix A
$\lambda_{max}(A)$	The maximum eigenvalue of the matrix A

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