

A Fuzzy Multi-Criteria Decision-Making Framework for Optimal Mining Method Selection in Complex Geological Conditions

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Abstract: Mining Method Selection (MMS) is a complex, multi-criteria decision-making problem influenced by geological, technical, economic, environmental, and safety factors. Traditional methods, such as Nicholas, University of British Columbia (UBC), and reformed UBC approaches, rely on expert judgment and fixed decision matrices, often failing to account for uncertainty, interdependencies, and conflicting criteria. To address these limitations, this study proposes the Flexible Mining Method Selection (FMMS) framework, which integrates Fuzzy Entropy (FE), Fuzzy Analytic Network Process (FANP), and Modified Fuzzy TOPSIS. FMMS dynamically adjusts criteria weights based on entropy-driven uncertainty measures, enabling adaptive, data-driven decisions while capturing interdependencies among selection criteria. Applied to the Gol Gohar iron ore deposit in Iran, FMMS identified Top Slicing as the most reliable alternative, achieving the highest crisp score (0.1066) with the lowest variance (0.000052) and standard deviation (0.0063), compared to the lowest-ranked score of 0.0932 for Stopping and Pillar. This contrasts with conventional Nicholas, UBC, and reformed UBC evaluations, which favored Cut and Fill but produced inconsistent rankings across weighting adjustments. Sensitivity analysis over a weighting parameter range of $p = 0$ –10 confirmed stable rankings under varying weight scenarios, with Block Caving, Sublevel Stopping, and Sublevel Caving showing particularly robust behavior once internal criterion relationships were incorporated. These results demonstrate that FMMS effectively balances objective factors (geology, cost, safety, environmental impact) with subjective expert evaluations, offering a flexible, scalable, and systematic approach that enhances efficiency, accuracy, and sustainability in mining method selection. Furthermore, it lays a foundation for future AI-based decision-support systems in mining.

Keywords: mining method selection (MMS); fuzzy multi-criteria decision-making (MCDM); flexible mining method selection (FMMS); fuzzy entropy

1. Introduction

Mining method selection is one of the most critical decisions in mine design and planning, as it directly influences the technical, economic, safety, and environmental outcomes of an operation. The complexity of this



decision stems from the wide range of geological, geotechnical, economic, and operational factors that must be considered simultaneously. Conventional approaches often struggle to handle the inherent uncertainty, vagueness, and subjectivity associated with such multi-criteria evaluations. In this context, optimization algorithms and predictive methods play a pivotal role by improving efficiency, cost-effectiveness, and reliability, thereby enhancing decision-making in various operational processes of mining, including mine planning and extraction strategies [1–3]. A recent scientometric review of over 1500 MCDM applications in mining confirmed the widespread and growing use of fuzzy-based decision-support methods across the sector [4].

The selection of a suitable mining method is influenced by geological, technical, economic, environmental, and safety factors. Early studies employed qualitative classification schemes [5], followed by selection charts [6] and methods based on rock-mass classification [7]. With the advent of numerical ranking systems [8], deposit geometry-based classification models [9], and the UBC Mining Method Selection Process, which iteratively improved upon Nicholas' initial system [10], the methodology advanced further. However, existing selection methods based on expert judgment are inadequate under conditions of uncertainty, imprecise data, and multiple conflicting criteria.

Recent advances in fuzzy logic and hybrid decision-making models have provided powerful tools for addressing the ambiguity and imprecision inherent in mining data and expert judgments. By integrating fuzzy sets with multi-criteria decision-making (MCDM) techniques, these models enable a structured and systematic evaluation of competing mining method alternatives under uncertain conditions [11]. Fuzzy Set Theory, a generalization of classical set theory, offers a mathematical framework to model and process incomplete or gradual information, making it particularly useful for handling uncertainties in decision-making [12]. Additionally, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has been employed for post-processing. The hybrid metaheuristic fleet-design framework developed for the Zarshuran mine provides Pareto-optimal solutions, further demonstrating its effectiveness in ranking competing alternatives in complex mining decision problems [13].

Fuzziness, unlike classical (crisp) sets where an element is either fully included (1) or excluded (0), allows for partial membership in fuzzy sets. This characteristic is crucial for more accurately modeling uncertainty. A fuzzy membership function maps an element x in the universe U to a membership degree within the interval $[0, 1]$. The two most common types of fuzzy membership functions are triangular and trapezoidal functions, as well as Gaussian functions, which are selected based on the type of uncertainty [14].

Fuzzy Decision Support Systems (DSS) incorporate heuristics and data-driven decision-making processes, often evaluating trade-offs among cost, safety, environmental impact, and engineering constraints. Enhanced weight assignment of criteria can be achieved through techniques such as the Fuzzy Analytic Network Process (FANP) and Fuzzy Entropy (FE), while a modified TOPSIS method improves ranking accuracy [15]. Additionally, Fuzzy AHP has been applied to rank open-pit rehabilitation alternatives at the Zarshuran gold mine, highlighting the relevance of fuzzy decision-making frameworks in mine planning and closure decisions [16].

Fuzzy MCDM models have also been applied to other high-stakes mining decisions, such as open-pit equipment selection, where they outperformed conventional expert-judgment approaches by incorporating both tangible and intangible criteria [17]. Specifically, these systems can automatically manage conflicting selection criteria (e.g., ore grade, environmental impact, and operational cost) to generate more reliable and flexible decisions [18]. Such approaches not only enhance the robustness of decision outcomes but also facilitate the alignment of operational strategies with long-term economic and safety objectives.

The latest advancements leverage machine learning (ML) and real-time data analysis to enhance adaptability to fluctuating mining conditions, including ore quality, equipment reliability, and environmental constraints [19]. When fuzzy logic is combined with artificial intelligence-powered operational models, it provides more accurate predictions and serves as a robust tool for mining operations. Furthermore, the social and environmental impacts of mining, such as fuel consumption, carbon footprint, and ecological effects, are increasingly incorporated into decision-making models. Future research is particularly focused on integrating big data, AI, and automated monitoring systems into real-time decision-support frameworks by combining hybrid decision support systems (DSS) and soft computing techniques to improve sustainability and mitigate risks [20–22].

Decision-making approaches in mining problems have been widely studied. Yager's method has been applied to coal transportation system selection [23], underground mining method selection [24], and open-pit equipment selection [25]. It has also been combined with fuzzy dominance methods and the Analytic Hierarchy Process (AHP) to improve decision accuracy [26]. PROMETHEE has been utilized for selecting ore transport systems [27], while AHP-based models have been employed in drilling technology investments, support design, and mine planning [28]. Additionally, AHP has been used to select sites for cement plants [29] and to choose tunneling roadheaders [30,31].

There are also software developments based on Yager's method that focus on computerized decision-support systems, mining, and equipment selection [32]. The application of the fuzzy TOPSIS method has confirmed the necessity of hybrid fuzzy models in mining decision-making [33].

A review of MCDM applications to choice problems in mining and mineral processing found that hybrid methods are increasingly favored over single-technique approaches, reflecting a broader shift toward integrated decision models [34].

The FMMS approach is exceptional among these methods as it combines fuzzy analytic network process (FANP) with fuzzy extension (FE) and modified Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and encompasses interdependencies and uncertainties of alternatives in decision-making specifically in regards to factors of mining method selection.

Through the integration of fuzzy logic and MCDM approaches, the proposed model systematically evaluates geological, geotechnical, economic, and environmental criteria, offering a reliable framework for selecting the optimal mining method. The results provide valuable insights into the applicability of hybrid fuzzy models in mining engineering decision-making and contribute to improving the efficiency and sustainability of mining operations.

The remainder of this article is organized as follows: Section 2 presents the methodology underlying the proposed framework, integrating fuzzy entropy, FANP, and modified fuzzy TOPSIS; Section 3 compares FMMS with the Nicholas, UBC, and reformed UBC methods through a case study of the Gol Gohar iron ore deposit; Section 4 introduces FMMS as a decision-support tool and details its computational steps; Section 5 presents a sensitivity analysis of the resulting rankings; Section 6 analyzes the standard deviation of scores for each alternative; and the final section concludes the study. Despite recent progress in fuzzy multi-criteria decision-making applications to mining engineering, existing approaches generally apply entropy-based weighting, network-based interdependency analysis, or fuzzy TOPSIS ranking in isolation, and the robustness of their rankings is rarely validated across varying weighting conditions, leaving a gap in current knowledge regarding the joint treatment of uncertainty, interdependency, and ranking robustness. This gap is addressed by the FMMS framework proposed in this study, which advances the state of the art by integrating Fuzzy Entropy, FANP, and Modified Fuzzy TOPSIS into a single model that combines objective and subjective weighting while explicitly capturing interdependencies among selection criteria. The available body of knowledge is extended by demonstrating, through application to the Gol Gohar iron ore deposit and accompanying sensitivity analysis, that accounting for criteria interdependencies yields more stable and reliable mining method rankings than those produced by conventional selection approaches, thereby providing a validated and extensible foundation for future AI-based decision-support systems in mining.

Research Objective

The primary scientific objective of this study is to develop a mathematically rigorous, computationally adaptive decision-making framework that surpasses the limitations of static, deterministic models in complex deep-mining environments. Instead of simply applying existing fuzzy tools in isolation, this research seeks to design a comprehensive framework that fundamentally redefines the quantification and resolution of coupled uncertainties and interdependent criteria in mine planning.

Specifically, the research objectives are organized around the following core scientific goals:

- (1) **Formulation of a Unified Uncertainty-Dependency Matrix:** This study aims to mathematically model the complex, nonlinear interdependencies among geological, geotechnical, economic, and environmental criteria. By integrating Fuzzy Entropy (FE) to objectively quantify information disorder with the Fuzzy Analytic Network Process (FANP) to map mutual dependencies, it establishes a novel theoretical framework for managing multidimensional uncertainties in mining systems.
- (2) **Design of a Dynamic Weight-Adjustment Mechanism:** Develop an adaptive weighting protocol that mitigates the inherent biases of fixed-weight matrices (e.g., Nicholas or UBC methods). The objective is to ensure that the evaluation of mining alternatives remains mathematically robust against subjective fluctuations in expert judgments, thereby guaranteeing ranking stability.
- (3) **Establishing a Quantifiable Reliability Metric for Mining Alternatives:** This involves introducing a standardized evaluation of score dispersion, specifically using the variance and standard deviation of fuzzy scores, as a definitive scientific metric to assess the operational reliability of selected mining methods. This approach shifts the analytical focus from simply identifying the “highest score” to ensuring “long-term operational stability” under fuzzy conditions.
- (4) **Translation of Abstract Fuzzy Logic into a Scalable Mine Planning Architecture:** This work aims to bridge the gap between theoretical fuzzy set mathematics and practical, large-scale mine design. The objective is to develop a computationally scalable framework that can be seamlessly integrated into modern, AI-driven autonomous decision-support systems within the mining industry.

The scientific contribution of this work lies in the paradigm shift from isolated, single-technique evaluations to an interdependency-aware fuzzy architecture. While conventional literature treats criteria weighting and alternative ranking as sequential, independent mathematical steps, this research demonstrates that mapping the internal network of criteria fundamentally alters the stability and validity of decision outcomes. Furthermore, the introduction of dispersion analysis, evaluating the standard deviation of fuzzy scores, provides the mining science community with a novel, quantitative measure for risk assessment in mine design. This contribution extends the theoretical boundaries of Multi-Criteria Decision-Making (MCDM) in mining by demonstrating that robustness is not merely an inherent property of the alternatives but a dynamic outcome of how criteria interdependencies are mathematically modeled and integrated.

2. Methodology

In order to make an optimal selection, a hybrid fuzzy decision-making framework, combining the use of Fuzzy Entropy (FE), Fuzzy Analytic Network Process (FANP), and Modified Fuzzy TOPSIS, is adopted for mining method selection. The combination of objective weight determination and expert-driven interdependency analysis creates a process for selection that is both accurate and adaptable [35].

Weighting of Handling Various Criteria

Fuzzy Entropy, as well as FANP-based pairwise comparisons, are employed for assigning criteria weights.

Fuzzy Entropy Weights: Entropy quantifies disorder, allowing us to assign greater weights to criteria that exhibit high variability. Entropy for quantification of uncertainty was introduced by Shannon in 1948 [36]. Later on, Zadeh (1968) reformulated the idea defining fuzzy probability measures [37], while DeLuca & Termini (1972) proposed the definition of the non-probabilistic entropy of fuzzy sets [38]. Entropy of the j -th criterion is calculated, as shown in Equation (1):

$$E_j = -k \sum_{i=1}^m p_{ij} \ln(p_{ij}) \quad (1)$$

where

$$k = \frac{1}{\ln(m)}$$

where p_{ij} is the normalized value of alternative i for criterion j , and m is the total number of alternatives. The entropy weight is determined by Equation (2):

$$w_j^E = \frac{1-E_j}{\sum_{j=1}^n (1-E_j)} \quad (2)$$

This means that less discriminative criteria are not weighted as highly in the decision process.

Fuzzy ANP Weights: While the AHP assumes independence between criteria, the FANP captures mutual dependences of the criteria [39]. By using the triangular fuzzy numbers to make pairwise comparisons, a comparison matrix can be expressed. The local weight vector is produced, as expressed in Equation (3) [40]:

$$W = \left(\prod_{j=1}^n a_{ij} \right)^{\frac{1}{n}} \quad (3)$$

where a_{ij} is the fuzzy pairwise comparison value between criteria i and j .

Modified Fuzzy TOPSIS for Mining Method Selection: Fuzzy TOPSIS takes into account different types of uncertainty, while TOPSIS can effectively rank the alternatives based on how close they are to an ideal solution [35].

The fuzzy decision matrix is normalized using Equation (4):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (4)$$

The weighted normalization is calculated using Equation (5):

$$v_{ij} = w_j r_{ij} \quad (5)$$

where:

$$w_j = p w_j^{FANP} + (1-p) w_j^E,$$

Defuzzification is performed via the center-of-area method, as shown in Equation (6):

$$D(A) = \frac{a_l + a_m + a_u}{3} \quad (6)$$

where a_l , a_m , a_u are the 1st, 2nd, and 3rd quartiles of the fuzzy number [41].

The positive and negative ideal solutions are defined in Equation (7):

$$A^+ = \{\max_i(v_{ij})\}, A^- = \{\min_i(v_{ij})\} \quad (7)$$

The separation distances are computed using Equation (8):

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - A_j^+)^2}, D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - A_j^-)^2} \quad (8)$$

The relative closeness (RC_i) is calculated and used for ranking, as shown in Equation (9):

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad (9)$$

Figure 1 illustrates the step-by-step methodology.

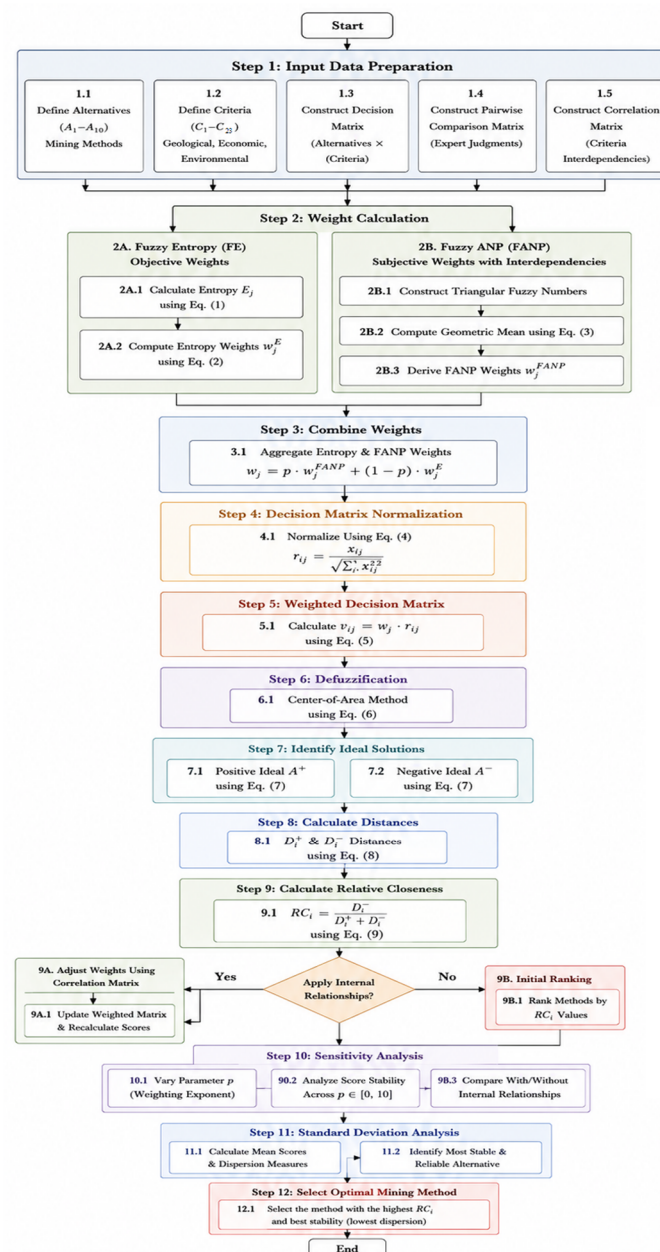


Figure 1. Overall workflow of the proposed Flexible Mining Method Selection (FMMS) framework.

The mining method with the highest is regarded as the best method of mining [42,43]. This framework combines fuzzy entropy for optimal weight assignment, FANP for modeling dependencies, and fuzzy TOPSIS for alternative evaluation and ranking, thus providing systematic and reliable decision-making for mine method selection [44,45].

3. Comparison of FMMS Conventional MMS Approaches

A case study was conducted on the Gol Gohar deposit in southern Iran to evaluate the performance of FMMS compared to other well-known mining method selection techniques. This region contains six iron ore deposits, with underground mining proposed for Deposit No. 1, as illustrated in Figure 2. Given the depth and geological richness of the deposit, selecting an optimal mining method is essential to maximize resource recovery while minimizing operational risks and costs.

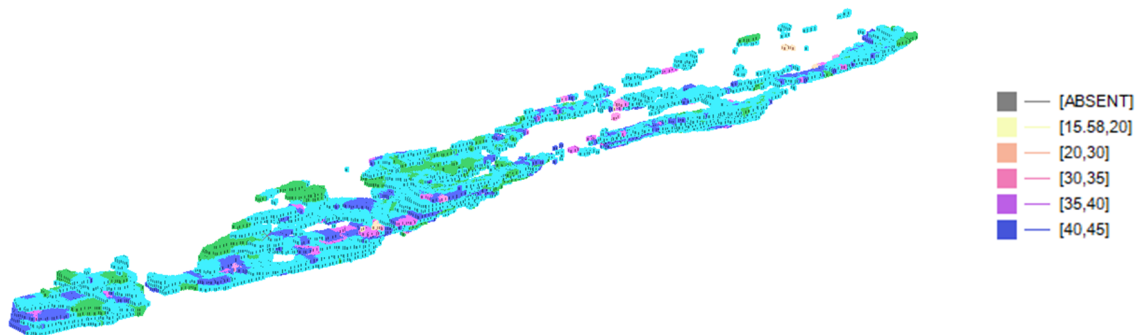


Figure 2. The View of Case Study Deposits.

Several geotechnical, economic, and environmental criteria were considered to select an appropriate mining method. These criteria include shape, dip, grade distribution, RMR, thickness, depth, mining cost, environmental impact, and reserve tonnage, as summarized in Table 1. The alternative mining methods proposed for the study are outlined in Table 2.

Table 1. Criteria used in case study.

	Hanging Wall	Ore	Foot Wall
Shape (C ₁)	-	layered	-
Grade Distribution (C ₂)	-	disseminated	-
Dip (C ₃)	-	23	-
Thickness (C ₄)	-	70 m	-
Depth (C ₅)	-	1000 m	-
RMR (C ₆ , C ₇ , C ₈)	Good (60–80)	Good (60–80)	Good (60–80)
RSS (C ₉ , C ₁₀ , C ₁₁)	6	8.9	6.5
Recovery (C ₁₂)			
Domestic employment (C ₁₃)			
Output per man shift (C ₁₄)			
Mining Cost (C ₁₅)			
Environmental impact (C ₁₆)			
Tonnage of reserve (C ₁₇)	-	197 million tones	-
Joint Condition (C ₁₈ , C ₁₉ , C ₂₀)	Filling is hard	None filling, roughness	Filling is hard
RQD (C ₂₁ , C ₂₂ , C ₂₃)	50%	75%	50%

Table 2. Options (output parameters) used in this case study.

Open Pit (OP)	Options
Block Caving (BC)	
Sublevel Stopping (SS)	
Sublevel Caving (SC)	
Long wall (LW)	
Room and Pillar (RP)	
Stopping and pillar (SP)	

Table 2. Cont.

Shrinkage Stopping (SH)	Options
Top Slicing (TS)	
Square Set Stopping (SQ)	
Cut and Fill (CF)	
Open Pit (OP)	

3.1. Nicholas Method

Nicholas [8] utilized a numerical ranking scale to assess various mining methods based on weighted input parameters. A total numerical rating is then assigned to each mining method by summing the scores from the relevant parameters. A significant disadvantage of this methodology is the equal weighting of the selection criteria, which can lead to inaccurate recommendations. Furthermore, Nicholas's method is less effective for deep mines, as depth is not explicitly considered.

Table 3 presents the results obtained using the Nicholas method, where Open Pit (OP) achieved the highest score of 45 points. However, open-pit mining at a depth of 1000 m is impractical due to the excessive stripping ratio and the large volume of overburden that would need to be removed. Sublevel Caving (SC), Top Slicing (TS), and Shrinkage Stopping (SH) each scored 33 points, ranking second, while Cut and Fill (CF) scored 32 points, placing third. Notably, the Nicholas method [46] has inherent limitations, as it does not account for depth-related constraints or cost considerations, which can significantly affect the feasibility of deep mining operations.

Table 3. Rating results with the Nicholas method.

Options	C ₁	C ₄	C ₃	C ₂	Foot Wall			ORE			Hanging Wall			Sum	Rate
					C ₂₀	C ₂₃	C ₁₁	C ₁₉	C ₂₂	C ₁₀	C ₁₈	C ₂₁	C ₉		
OP	2	4	3	3	4	4	3	3	4	4	4	4	4	45	1
SC	4	4	1	2	4	3	0	2	4	3	0	3	3	33	2
SH	2	4	1	2	3	3	2	2	4	3	0	3	4	33	2
TS	3	3	1	2	3	3	2	2	4	3	0	3	4	33	2
CF	4	0	3	3	2	2	4	3	2	2	2	2	3	32	3
SS	2	4	1	3	4	2	0	2	4	3	4	1	−49	30	4
SQ	2	1	3	3	2	2	4	3	1	1	2	2	3	29	5
BC	2	2	2	2	3	3	2	3	0	1	0	3	4	27	6
RP	4	−49	1	3	3	3	0	2	4	3	4	2	0	−20	7
LW	4	−49	0	2	3	4	2	3	0	1	0	3	4	−23	8
SP	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non

3.2. UBC Method

The University of British Columbia (UBC) method is an updated version of the Nicholas method, placing greater emphasis on stopping methods, making it more applicable to Canadian mining conditions. However, the UBC approach does not address the significance of multivariable selection criteria, nor does it account for uncertainties in geotechnical properties.

Both results for the UBC method, shown in Table 4, have negative scores, which disqualifies the Open Pit (OP) method. Although SH performs well in the Nicholas method, it is excluded from consideration due to the presence of a very thick ore body. Consequently, SH is also eliminated. The final ranking was CF > SS > SC. These ratings differ significantly from those produced by the Nicholas method, indicating considerable variation in the selection process depending on the weightings applied [46].

Table 4. Rating results with the UBC method.

Option	C ₁	C ₄	C ₃	C ₅	C ₂	Foot Wall		ORE		Hanging Wall		Sum	Rate
						C ₈	C ₁₁	C ₇	C ₁₀	C ₆	C ₉		
CF	4	1	3	4	3	2	3	3	1	4	5	33	1
SS	4	4	1	2	4	2	1	4	2	3	1	28	2
SC	4	4	1	2	2	3	2	1	3	3	3	28	2
BC	2	3	2	3	2	3	3	0	2	3	3	26	3
TS	2	2	2	1	1	1	2	1	2	2	2	18	4
SQ	1	0	3	2	1	0	2	0	3	1	2	15	5
OP	2	4	3	−49	3	4	3	3	3	4	3	−17	−

Table 4. Cont.

Option	C ₁	C ₄	C ₃	C ₅	C ₂	Foot Wall		ORE		Hanging Wall		Sum	Rate
						C ₈	C ₁₁	C ₇	C ₁₀	C ₆	C ₉		
LW	4	−49	0	3	1	0	0	2	5	4	5	−25	−
SH	4	−49	0	2	2	2	2	3	1	2	1	−30	−
RP	4	−49	0	2	2	0	0	5	0	3	0	−33	−
SP	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non

3.3. Reformed UBC Method

To achieve this, the proposed UBC method was refined based on previous approaches by other authors, incorporating adjustments to the weighting of geotechnical parameters such as ore dip, Rock Strength (RSS), and Rock Mass Rating (RMR). As shown in Table 5, the results reaffirm the Cut and Fill (CF) method as the most preferred mining method, followed by Sublevel Stopping (SS) and Sublevel Caving (SC). However, the ranking of the methods varies significantly with minor changes in the weighting approach, indicating a lack of robustness in the selection framework.

The Nicholas, UBC, and reformed UBC methods have a significant drawback: their strictly geotechnical selection criteria underestimate economic, environmental, and social constraints. Furthermore, these approaches do not account for dependencies among the selection parameters, which may result in inappropriate recommendations.

Table 5. Rating results with the reformed UBC method.

Option	C ₁	C ₄	C ₃	C ₅	C ₂	Foot Wall		Ore		Hanging Wall		Sum	Rate
						C ₈	C ₁₁	C ₇	C ₁₀	C ₆	C ₉		
CF	4	1	2.6	4	3	2	2.6	2.7	1.6	4	4.4	31.9	1
SC	4	4	1	2	2	3	1.8	1.6	3	3	3.3	28.7	2
SS	4	4	1.2	2	4	2	0.8	4	2.6	3	0.7	28.3	2
BC	2	3	2.2	3	2	3	3.2	0.6	1.7	3	3.3	27	3
TS	2	2	2.4	1	1	1	2	1	1.7	2	2.3	18.4	4
SQ	1	0	2.8	2	1	0	2.2	0.3	2.4	1	2.6	15.3	5
OP	2	4	3	−49	3	4	3	3	3	4	3	−17	−
LW	4	−49	0.8	3	1	0	0	2.6	4.2	4	5.3	−24.1	−
RP	4	−49	0.8	2	2	0	0	4.4	0.8	3	0	−32	−
SH	4	−49	−9.8	2	2	2	1.6	3	1.6	2	0.7	−39.9	−
SP	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non	Non

3.4. FMMS: A New Tool for Selecting Mining Methods

In FMMS, a hybrid method is employed to classify options. First, the options and criteria (input parameters) are specified to construct various matrices, including the Decision Matrix, Confidence Matrix, Pairwise Comparison Matrix, and Correlation Matrix, all aligned with the options and criteria. The final weights of the options are determined based on these matrices. The criteria involved in this selection process include C1 through C16, as shown in the table below. These parameters are input as criteria into the model.

In Table 6, the input parameters for FMMS are given.

Table 6. Input Parameters and Their Values for FMMS.

Criteria	Description	Values
C ₁	Shape	[0.03, 0.08, 0.08, 0.11, 0.11, 0.10, 0.08, 0.06, 0.03, 0.06]
C ₂	Grade Distribution	[0.04, 0.10, 0.04, 0.07, 0.06, 0.07, 0.10, 0.04, 0.07, 0.10]
C ₃	Dip	[0.06, 0.13, 0.09, 0.02, 0.02, 0.09, 0.09, 0.02, 0.02, 0.13]
C ₄	Thickness	[0.09, 0.16, 0.06, 0.02, 0.07, 0.02, 0.07, 0.02, 0.06, 0.16]
C ₅	Depth	[0.07, 0.12, 0.07, 0.07, 0.15, 0.14, 0.07, 0.07, 0.02, 0.12]
C ₆	Rock Mass Rating (RMR)	[0.06, 0.11, 0.05, 0.04, 0.08, 0.05, 0.04, 0.02, 0.04, 0.11]
C ₇	Rock Strength (RSS)	[0.02, 0.05, 0.02, 0.07, 0.07, 0.07, 0.06, 0.02, 0.02, 0.07]
C ₈	Joint Condition	[0.04, 0.10, 0.04, 0.05, 0.14, 0.04, 0.04, 0.02, 0.04, 0.10]
C ₉	Recovery	[0.10, 0.10, 0.11, 0.10, 0.07, 0.10, 0.07, 0.10, 0.10, 0.10]
C ₁₀	Domestic Employment	[0.05, 0.08, 0.04, 0.04, 0.15, 0.08, 0.04, 0.02, 0.04, 0.08]
C ₁₁	Output per Man Shift	[0.04, 0.11, 0.09, 0.07, 0.07, 0.12, 0.09, 0.04, 0.02, 0.08]
C ₁₂	Mining Cost	[0.09, 0.10, 0.08, 0.09, 0.12, 0.12, 0.08, 0.07, 0.02, 0.08]
C ₁₃	Environmental Impact	[0.03, 0.06, 0.03, 0.14, 0.12, 0.03, 0.03, 0.02, 0.07]

Table 6. Cont.

Criteria	Description	Values
C ₁₄	Tonnage of Reserve	[0.03, 0.06, 0.02, 0.02, 0.02, 0.04, 0.02, 0.02, 0.06]
C ₁₅	RQD	[0.03, 0.06, 0.02, 0.02, 0.02, 0.04, 0.02, 0.02, 0.06]
C ₁₆	Geotechnical Conditions	[0.11, 0.11, 0.12, 0.08, 0.11, 0.11, 0.08, 0.09, 0.12, 0.08]

Using a hybrid multi-criteria decision-making (MCDM) algorithm, the process completes the necessary steps to determine the final ranking of the mining methods. We can now analyze the different components of the code and explain the output step by step, as illustrated in Algorithm 1.

Step 1: Load the Data

First, the normalized decision matrix, entropy results, and pairwise comparison matrix need to be loaded. The C1–C16 criteria do not have the same values across all alternatives; rather, they vary only across the A1–A10 alternatives. This process is automated, and the matrices automatically populate the corresponding values.

Step 2: Calculate Weights from the Pairwise Comparison

Weights Calculation from the Pairwise Comparison Matrix Using the Geometric Mean Method. After calculating the geometric mean of each row, the average of these means is computed to assign weights to each criterion.

Step 3: Combine Entropy Weights with Pairwise Comparison Weights

The entropy weights are normalized and combined with the pairwise comparison weights. For simplicity, equal importance was assumed, so the final weights are calculated as the average of both sets of weights.

Step 4: Compute the Final Scores for Each Method

The normalized decision matrix is then multiplied by the final weights to obtain the weighted decision matrix. Subsequently, the scores for each method are calculated by summing the weighted values across all criteria.

Step 5: Rank the Mining Methods

The methods are ranked according to their scores, with the best method achieving the highest ranking.

Step 6: Calculate the Final Weights Considering Internal Relationships

Final weights are adjusted using Matrix B by applying an internal relationship matrix that accounts for the dependencies between criteria. These weights are then updated and normalized.

Step 7: Calculate the Final Scores Using the Updated Weights

Finally, the normalized decision matrix is multiplied by the updated weights to calculate the final scores.

Step 8: Rank the Mining Methods According to Final Scores

The mining methods are re-ranked based on the final scores after considering their internal relationships.

The proposed FMMS framework was implemented in the Python programming environment (Python 3.12.6). All fuzzy computations, entropy-based weighting, FANP calculations, modified fuzzy TOPSIS ranking, and sensitivity analysis were developed using Python libraries, including NumPy for numerical matrix operations, Pandas for data management, SciPy for mathematical computations, and scikit-fuzzy for fuzzy logic operations. Sensitivity analysis was conducted by systematically varying the weighting parameter p within the interval $[0, 1]$ and recalculating the final ranking results for each scenario. The stability of the obtained rankings was assessed by comparing changes in the relative closeness coefficients (RC_i) and the rank positions of alternatives.

Algorithm 1. FMMS Framework (Fuzzy Entropy–FANP–Modified Fuzzy TOPSIS)

Input: Alternatives $A = \{A_1, \dots, A_m\}$, criteria $C = \{C_1, \dots, C_n\}$,
linguistic judgments, weighting parameter $p \in [0, 1]$

Output: Ranked list of alternatives and crisp scores RC_i

```

1:    // Step 1: Problem Definition
2:    Define  $A$ ,  $C$  and the hierarchical structure of criteria
3:
4:    // Step 2: Fuzzy Pairwise Comparisons
5:    for each pair of elements in the hierarchy do
6:        Elicit expert judgment using linguistic terms
7:        Convert to triangular fuzzy numbers
8:        Construct fuzzy pairwise comparison matrix  $\tilde{A}$ 
9:    end for

```

```

10:
11: // Step 3: Fuzzy Entropy Weighting (FE)
12: for each criterion  $C_j$  do
13:     Normalize decision values  $\rightarrow p_{ij}$ 
14:     Compute entropy  $e_j$  using Eq. (1)
15:     Compute diversification  $d_j = 1 - e_j$ 
16: end for
17: Compute entropy weights  $w_j^E = \frac{d_j}{\sum_{j=1}^n d_k}$ 
18:
19: // Step 4: FANP Weighting (with Internal Relationships)
20: Perform FM extent analysis to obtain local weights
21: Build the supermatrix  $W$  and raise to limiting powers
22: Obtain global weights  $w_j^{FANP}$ 
23:
24: // Step 5: Integrated Weight Calculation
25: for each criterion  $C_j$  do
26:      $w_j = p w_j^{FANP} + (1 - p)w_j^E$ 
27: end for
28:
29: // Step 6: Fuzzy Decision Matrix
30: for each alternative  $A_i$  do
31:     for each criterion  $C_j$  do
32:         Evaluate using fuzzy numbers  $\tilde{x}_{ij}$ 
33:     end for
34: end for
35:
36: // Step 7: Modified Fuzzy TOPSIS
37: Normalize  $\tilde{X}$  to obtain  $\tilde{R}$  using Eq. (4)
38: Compute weighted matrix  $\tilde{V} = [v_{ij}]$  using Eq. (5)
39: Determine FPIS ( $A^+$ ) and FNIS ( $A^-$ ) using Eq. (7)
40: for each alternative  $A_i$  do
41:     Compute distances  $D_i^+$  and  $D_i^-$  using Eq. (8)
42:     Compute relative closeness
43:      $RC_i = \frac{D_i^-}{D_i^+ + D_i^-}$  using Eq. (9)
44: end for
45:
46: // Step 8: Ranking & Sensitivity Analysis
47: Sort alternatives in descending order of  $RC_i$ 
48: for  $p$  from 0 to 1 with step  $\Delta p$  do
49:     Repeat Steps 4–7
50:     Record rankings and analyze stability
51: end for
52: return Ranked list of alternatives and crisp scores  $RC_i$ 
53: end

```

The notations used in Algorithm 1 are defined below:

m : number of alternatives

n : number of criteria

p : weighting parameter ($0 \leq p \leq 1$)

w_j^{FANP} : FANP global weight

w_j^E : entropy weight

w_j : integrated weight

A^+ : fuzzy positive ideal solution

A^- : fuzzy negative ideal solution

D_i^+, D_i^- : distances to A^+ and A^-

RC_i : relative closeness of A_i .

4. Sensitivity Analysis

The sensitivity analysis method checks how the mining methods' final scores change for various values of the weighting parameter p . Here, p is a power exponent applied to the normalized criterion weights prior to renormalization: each weight w_j is raised to the power p and the resulting values are renormalized so that they sum to one. When $p = 1$, the original weight vector is recovered; values of $p > 1$ progressively amplify the influence of criteria that already carry higher weights while suppressing the influence of lower-weighted criteria, and values of $p < 1$ flatten the weight distribution, moving the criteria weights toward equal importance. The weighted decision matrix is recomputed for each value of p , and the resulting scores are plotted to assess the robustness of the mining method rankings to the emphasis placed on criteria importance.

4.1. Interpretation of the Output

The results of the sensitivity analysis are illustrated in Figure 3. The plot displays the final score on the y-axis and the parameter p , ranging from 0 to 10, on the x-axis for each mining method (A1–A10). This provides a clearer perspective on the robustness of the ranking with respect to the weighting parameter. The figure represents the sensitivity analysis results for different mining methods in the absence of internal relationships. The y-axis indicates the score, while the x-axis shows various p values. Each line corresponds to a different mining method (BC, SS, SC, LW, RP, SH, CF, TS, SQ, SP).

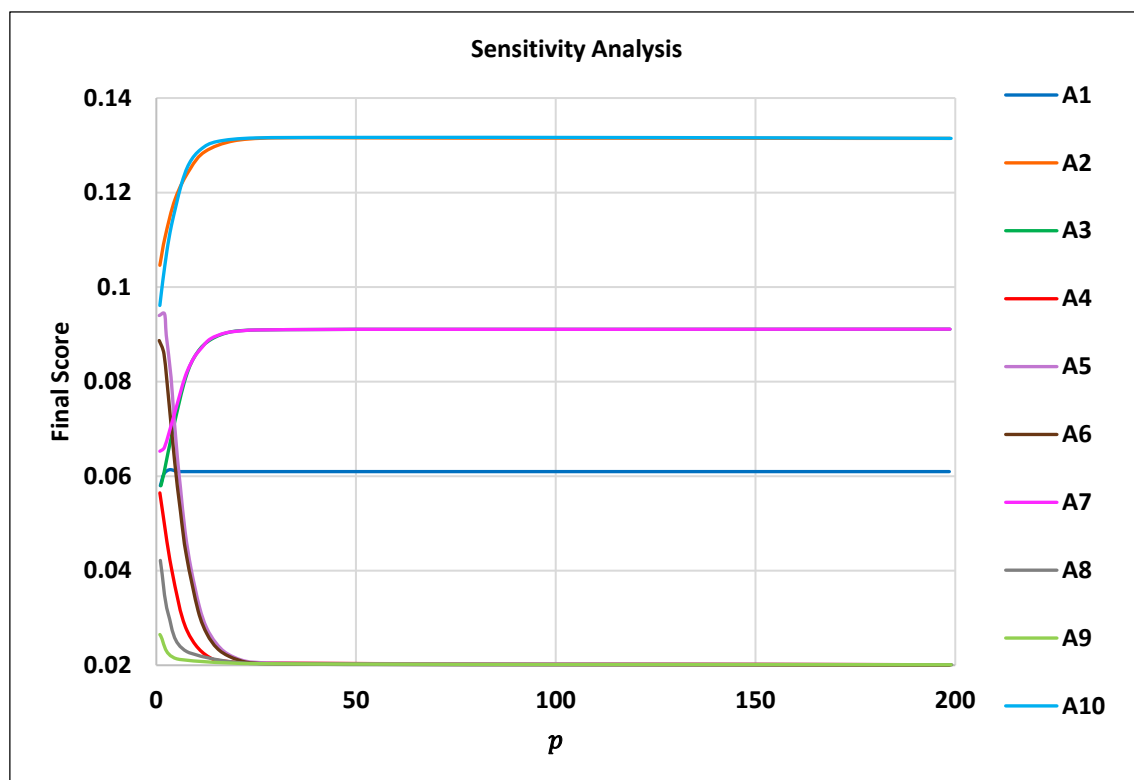


Figure 3. Sensitivity Analysis of Final Scores for Mining Methods.

4.2. Analysis of the Output

Stable methods: The scores of certain methods, specifically BC, SS, and SC, are stable across the p range, indicating that their ranking is fairly robust to the choice of weighting parameter. The stability indicates that these methods are not particularly sensitive to changes in the relative importance of criteria. **Sensitive Method:** Other methods like RP exhibit large differences in scores with p increasing. This sensitivity implies that orderings of these methods can be very different if the weights of the criterion are changed. These techniques are more evidence-weight sensitive. **Fluctuations in Initial Cells:** On small p values, there are clear fluctuations in method scores. This means that the initial weights matter a lot to the scores and rankings. These variance fluctuations stabilize as p is increased, indicating a tendency for a stable overall ranking due to the reduced impact of the

original weights. Converging Scores: Certain methods can start out with (LW, RP) dramatic alterations in their resulting scores, but then converge to a stable value as p increases. This shows that although the methods are sensitive for the basic weights, they become much more stable when the criterion is weighted 0.0, based on a sensitivity analysis performed on the weighting parameter p . This sensitivity analysis illustrates both the stability of various mining methods in response to variations in the weighting parameter p , as well as the richest mechanism is able to present consistently the best score throughout all variations of the p value. The different mining methods are less sensitive to the choice of the criterion importance, while methods that experience more changes to their scores can be considered less stable and more sensitive to changes in weights. On the other hand, methods that yield large score variations are more sensitive and may require subtle adjustments to the weights on criteria to maintain the integrity of rankings. The final scores of the mining methods under varying weighting conditions are further illustrated in Figure 4.

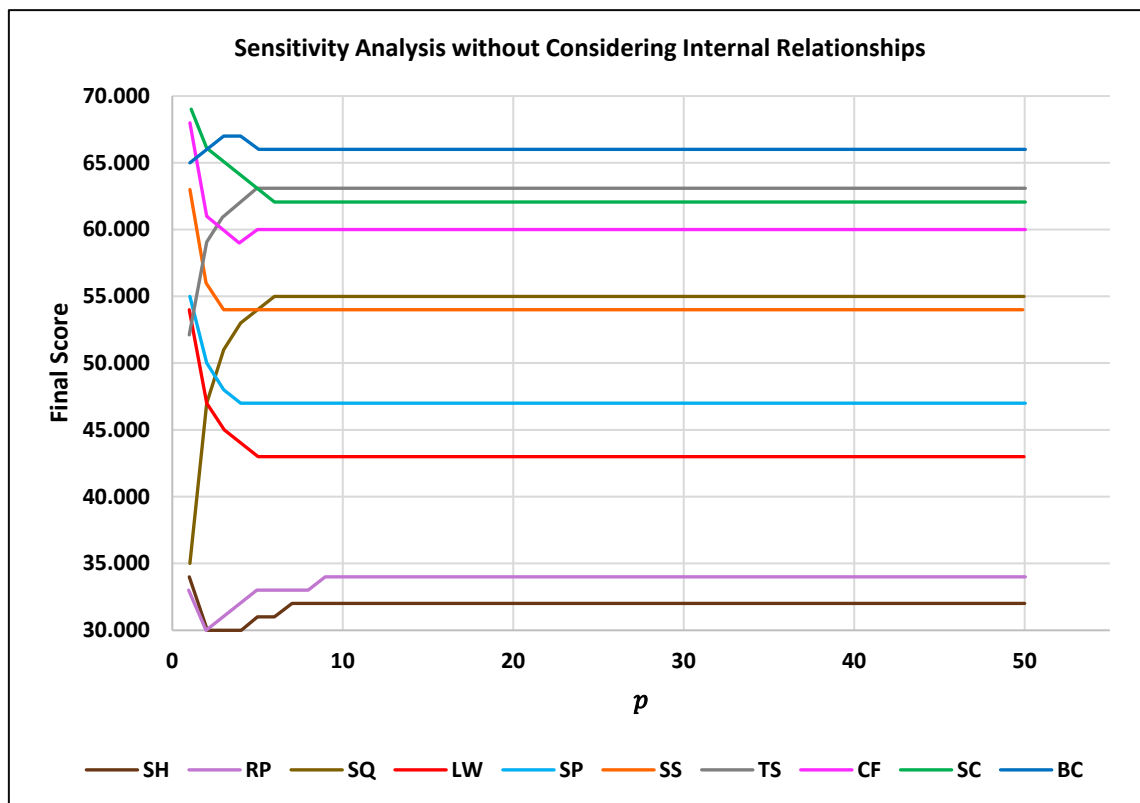


Figure 4. Sensitivity Analysis of Final Scores for Mining Methods.

4.3. Sensitivity Analysis of Mining Methods with and without Internal Relationships

A mining methods sensitivity analysis was performed to find whether internal relationships between criteria are significant enough to affect the final position of different mining methods. We experimented with assigning different values to the parameter p while generating the weights and observed the scores without and with internal relationships. The results are then provided in the form of comparative plots for every mining method. The final scores for each mining method, with and without internal relationships, are compared in Figure 5.

Plots contrasting the sensitivity of final scores for each mining method (BC, SS, SC, LW, RP, SH, CF, TS, SQ, SP) with and without internal relationships. The x-axis here corresponds to different values of parameter p , while the y-axis shows the resulting scores/ranks for each method. The comparative rankings of each mining method, with and without internal relationships, are summarized in Table 7.

The reason behind this inconsistency is learned from the sensitivity analysis in light of affiliation to internal relationships within the criterion, which helps us to stabilize and steady the outcome of different mining methods. Accordingly, mining methods BC and CF present more stability and include internal relations that can make a more accurate ranking. In contrast, SS and TS show large initial variability, indicating that their ranking is more dependent on the choice of the weighting parameter p .

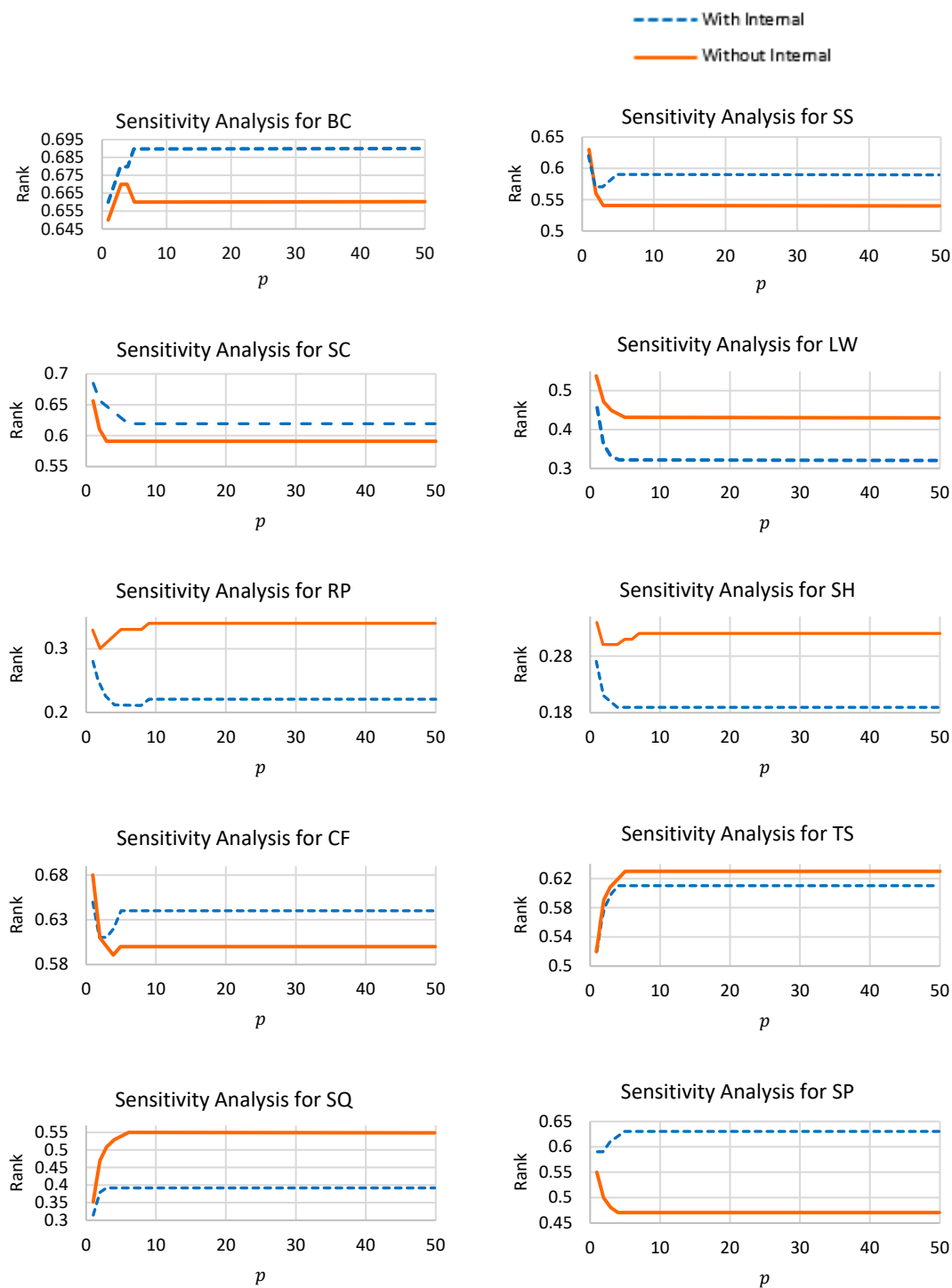


Figure 5. Sensitivity Analysis of Final Scores for Mining Methods with and without Considering Internal Relationships.

Table 7. Comparison of Mining Method Rankings with and without Internal Relationships.

Mining Method	With Internal Relationships	Without Internal Relationships
Block Caving	Stable around 0.67	Initial fluctuations, then stabilizes slightly below 0.67
Sublevel Stopping	Stable around 0.60	Significant initial fluctuations, then stabilizes slightly above 0.55
Sublevel Caving	Stable around 0.62	Remains stable around 0.61, showing less sensitivity to changes in p .
Longwall Mining	Stable around 0.45	Remains relatively stable around 0.40
Room and Pillar Mining	Stable around 0.33	Remains relatively stable around 0.32
Shrinkage Stopping	Stable around 0.32	Minor fluctuations, and stabilizes around 0.33
Cut and Fill Mining	Stable around 0.64	Remains relatively stable around 0.62

Table 7. Cont.

Mining Method	With Internal Relationships	Without Internal Relationships
Top Slicing	Stable around 0.52	More fluctuations, then stabilizes around 0.55
Square Set Mining	Stable around 0.45	More fluctuations, then stabilizes around 0.42
Solution Mining	Stable around 0.55	Remains relatively stable around 0.53

Overall, adding internal relationships results in more consistent and stable rankings, especially for methods that otherwise are sensitive to variations in the weight of the criteria. This emphasizes the need for taking internal dependencies between the criteria into account in the decision-making for the selection of the optimum mining methods.

5. Analysis of Standard Deviation of Scores for Each Alternative

Table 8 and Figure 6 summarize the crisp scores and dispersion measures for each alternative. A8 achieved the highest crisp score (0.106563) along with the lowest standard deviation (0.006268) and variance (0.000052), confirming it as the most stable and reliable option under fuzzy reliability assessment. In contrast, A10 recorded the lowest crisp score (0.093229), identifying it as the least suitable alternative. A2 exhibited the largest standard deviation (0.007990) and variance (0.000079), indicating considerable variability and reduced robustness despite its relatively high mean score. Overall, alternatives with lower dispersion values, such as A8 and A7, offer more consistent performance, while high-variability options like A2 should be approached with caution. These results emphasize the importance of evaluating both average performance and stability when selecting optimal mining methods. In the context of the Gol Gohar iron ore deposit, the findings highlight A8 as the most appropriate and dependable mining method, offering a balanced combination of efficiency and robustness for long-term operations.

Table 8. Standard Deviation Scores.

Alternative	Crisp Score	Variance	Standard Deviation
A1	0.099613	0.000086	0.007661
A2	0.101686	0.000079	0.007990
A3	0.096333	0.000055	0.006858
A4	0.095192	0.000063	0.006969
A5	0.101165	0.000060	0.005947
A6	0.103827	0.000074	0.007256
A7	0.104745	0.000052	0.006437
A8	0.106563	0.000061	0.006268
A9	0.097647	0.000052	0.006367
A10	0.093229	0.000066	0.006832

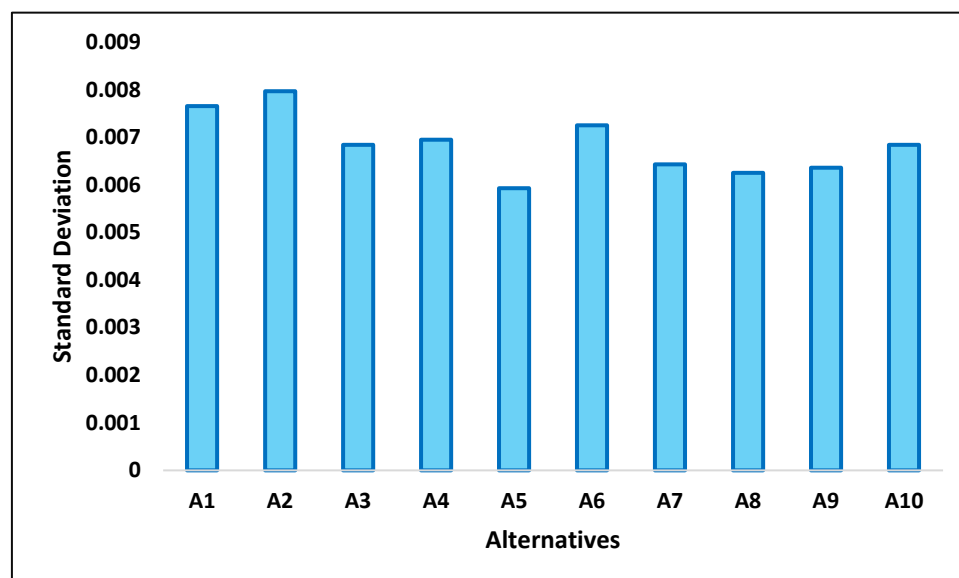


Figure 6. Standard Deviation of Scores for Each Alternative.

Despite the promising results obtained from the proposed FMMS framework, several limitations must be acknowledged. First, although fuzzy logic reduces uncertainty in expert judgments, the framework still relies on

subjective evaluations during the construction of fuzzy pairwise comparison matrices. Consequently, the quality of the final ranking depends on the expertise and consistency of the decision-makers. Second, the FANP component requires specifying interrelationships among criteria, which can become increasingly complex and computationally demanding as the number of criteria grows. Third, the proposed methodology was validated using a single mining case study; therefore, the generalizability of the findings to other geological settings, mining methods, and operational conditions requires further investigation. Additionally, the weighting parameter p , which controls the contribution of entropy-based and FANP-based weights, was analyzed through sensitivity analysis; however, its optimal value may vary across different applications. Finally, the current framework evaluates alternatives under static conditions and does not explicitly consider temporal variations in economic, environmental, or operational factors that may influence mining method selection over the life of a mine.

Future research could enhance the FMMS framework by incorporating dynamic, real-time decision-making capabilities using machine learning and artificial intelligence techniques. Integrating interval type-2 fuzzy sets, intuitionistic fuzzy sets, or Z-numbers may further improve the representation of uncertainty. Additionally, applying the framework to multiple deposits and mining projects would offer a more comprehensive evaluation of its robustness and transferability. Future studies could also explore automated methods for identifying criterion interdependencies and determining the weighting parameter p , thereby reducing reliance on expert judgment and increasing the objectivity of the decision-making process.

6. Practical Application

The proposed Flexible Mining Method Selection (FMMS) framework serves as a practical decision-support tool for mine planning and feasibility studies. It utilizes data commonly available during mineral deposit evaluation, including geological, geotechnical, economic, and environmental parameters, along with expert judgments represented by triangular fuzzy numbers. Objective and subjective criteria weights are combined to generate adaptive weights, which are then used to rank mining alternatives. Sensitivity analysis and standard deviation analysis are subsequently performed to assess the robustness and reliability of the final ranking.

In practical applications, FMMS can be integrated into mine planning software and digital decision-support systems to facilitate transparent and consistent selection of mining methods. Compared to conventional approaches, it reduces subjectivity by combining data-driven weighting with expert knowledge while explicitly accounting for the interdependencies among evaluation criteria. Additionally, the framework enables decision-makers to evaluate the stability of ranking results under various weighting scenarios, thereby minimizing decision-making uncertainty.

Although demonstrated using the Gol Gohar iron ore deposit, the proposed methodology is readily applicable to other underground mining projects. Moreover, its modular structure allows for extension to related mining engineering problems, including equipment selection, mine design, and rehabilitation planning. Overall, FMMS provides a reliable, scalable, and practical framework to support robust mining method selection under complex geological and operational conditions.

7. Conclusions

Mining method selection (MMS) is a complex decision-making process that requires balancing technical, economic, environmental, and operational factors. Traditional approaches, such as the Nicholas and UBC methods, depend on fixed weightings and static decision matrices, which limit their capacity to address uncertainty, interdependencies, and the dynamic nature of mining environments.

This study introduces a Flexible Mining Method Selection (FMMS) framework based on fuzzy multi-criteria decision-making (MCDM), integrating fuzzy entropy (FE), fuzzy analytic network process (FANP), and a modified fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). By applying fuzzy logic combined with entropy-based weighting, FMMS dynamically adjusts criteria weights, reduces uncertainty, and captures interdependencies among geological and economic parameters. Application to the Gol Gohar iron ore deposit demonstrates that FMMS produces more accurate, flexible, and robust rankings than conventional MMS methods, with sensitivity analysis confirming stability under varying conditions.

FMMS represents a significant advancement in MMS by providing a data-driven, adaptive, and scalable solution. Beyond mining method selection, it can be extended to other decision-making challenges in mining, such as equipment selection, design optimization, and risk analysis. With further integration of machine learning and artificial intelligence, FMMS has the potential to support real-time, adaptive decision-making, thereby enhancing efficiency, accuracy, and sustainability in mining projects.

Author Contributions

R.P.: developed the conceptual framework, conducted the analysis, and prepared the original manuscript draft. M.R.: contributed to methodology development, data processing, and model implementation. A.B.: assisted in data collection, validation, and technical analysis. M.M.: guided study design, interpreted results, and made critical revisions to the manuscript. M.O.: contributed to conceptual development and reviewed and edited the manuscript. M.K.: supervised the research, contributed to conceptual development, and reviewed and edited the manuscript. All authors contributed to the discussion of results and approved the final version of the manuscript.

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The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

During the preparation of this work, the authors used ChatGPT 5.6 to grammatical checking. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

List of Abbreviations

Abbreviation	Full Term
MMS	Mining Method Selection
MCDM	Multi-Criteria Decision-Making
FMMS	Flexible Mining Method Selection
FE	Fuzzy Entropy
FANP	Fuzzy Analytic Network Process
AHP	Analytic Hierarchy Process
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
DSS	Decision Support System(s)
ML	Machine Learning
AI	Artificial Intelligence
UBC	University of British Columbia (mining method selection process)
RMR	Rock Mass Rating
RSS	Rock Strength
RQD	Rock Quality Designation

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