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Cryptocurrencies on the Balance Sheet: Insights from Strategy—Bitcoin Interactions

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Abstract: This paper investigates the evolving link between cryptocurrency and equity markets in the context of the recent wave of corporate Bitcoin (BTC) treasury strategies. We assemble a dataset of 39 publicly listed firms holding BTC, from their first acquisition through April 2025. Using daily logarithmic returns, we first document significant positive co-movements via Pearson correlations and single factor model regressions, discovering an average BTC beta of 0.62, and isolating 12 companies, including *Strategy* (formerly *MicroStrategy*, MSTR), exhibiting a beta exceeding 1. We then classify firms into three groups reflecting their exposure to BTC, liquidity, and return co-movements. We use transfer entropy (TE) to capture the direction of information flow over time. Transfer entropy analysis consistently identifies BTC as the dominant information driver, with brief, announcement-driven feedback from stocks to BTC during major financial events. Our results highlight the critical need for dynamic hedging ratios that adapt to shifting information flows. These findings provide important insights for investors and managers regarding risk management and portfolio diversification in a period of growing integration of digital assets into corporate treasuries.

Keywords: bitcoin beta; single factor model; directional dependence; corporate treasury

1. Introduction

Bitcoin (BTC-USD pair, hereafter BTC) emerged as the top-performing asset in 2024, a phenomenon largely attributable to the launch of spot exchange-traded funds (ETFs) and growing optimism regarding potential regulatory easing under the newly inaugurated U.S. administration. Over the course of 2024, BTC delivered a return exceeding 113%, significantly outperforming all major traditional asset classes. In comparison, the S&P 500 (SPY) returned 23.7%, gold (GLD) 28.7%, government bonds (GOVT) -2.18%, and real estate (VNQ) -0.93% [1].

Despite this remarkable performance, BTC remains a highly volatile asset. By the end of 2024, its price had more than doubled from approximately $\$40,000$ to nearly $\$94,000$. The most pronounced acceleration occurred following the U.S. presidential election, with BTC surpassing the $\$108,000$ threshold by mid-December. This surge was widely interpreted as a market response to expectations that President Donald Trump's victory would have increased institutional participation in the cryptocurrency market.

In addition to these exogenous factors, a key structural feature influencing BTC price dynamics is the halving mechanism embedded in the BTC protocol [2,3]. Approximately every four years, or after every $210,000$ blocks, the reward granted to miners for validating transactions and appending new blocks is reduced by half. The primary purpose of this mechanism is to regulate the supply of new BTCs, ultimately capping the total number of coins at 21 million. Historical evidence shows the economic significance of these events: following the first halving on 28 November 2012, when the block reward decreased from 50 to 25 BTC, a substantial price escalation ensued. The most recent halving occurred on 20 April 2024, at block $840,000$, reducing the reward from 6.25 BTC to 3.125 BTC. Empirical studies suggest that such supply contractions contribute to increased scarcity, reinforcing BTC's role as a digital store of value, frequently compared to gold, and fostering investors' interest and trading activity [4].

Broader trends in global investment strategies demonstrate the increasing integration of BTC into traditional financial markets. By the end of 2024, the value of globally invested assets surpassed \$200 trillion, with cryptocurrencies representing 1.5% of the total, with approximately \$3 trillion [5]. Institutional investors substantially increased their exposure to spot BTC ETFs in the fourth quarter of 2024, with reported holdings reaching \$38.7 billion, more than three times the \$12.4 billion recorded in the previous quarter [6]. This growing institutional involvement has reinforced the dual role of BTC as both a benchmark for the broader cryptocurrency market, and an indicator of the global risk appetite. Its finite supply and decentralized governance contrast sharply with traditional financial instruments, positioning BTC as an alternative growth asset within a changing economic environment. Discussions regarding the potential classification of BTC as a strategic reserve asset in the U.S., in a period of rising public debt and increased borrowing costs, further underline the cryptocurrency's evolving role in the global financial system.

An episode happened in April 2025 on Wall Street highlights how firmly digital assets have integrated into mainstream financial markets. Following the circulation of a false headline suggesting that the U.S. President Donald Trump would have delayed planned tariffs by 90 days, the S&P 500 temporarily gained approximately \$2.5 trillion in market value, only to reverse those gains once the information was disproved. BTC and other major cryptocurrencies mirrored this sharp rise and subsequent fall, reacting to macroeconomic rumors and exhibiting the same risk-on/risk-off behavior traditionally associated with equity markets [7]. Such dynamics confirm that cryptocurrencies now participate in the global feedback loops that govern conventional financial assets, marking their transition from speculative instruments to a fully fledged asset class.

During the Federal Reserve's 2022–2024 tightening cycle, BTC increasingly behaved like other speculative growth assets, particularly the so-called high-beta technology stocks [8]. Recent data support this view. The 90-day rolling correlation between BTC and the NASDAQ-100 index reached 0.46 in May 2024 (the highest since August 2023), and had previously spiked above 0.80 in early 2022 after the Fed's initial rate hikes [8]. These trends challenge the traditional view of BTC as a store of value immune to macroeconomic swings. Its sharp price fluctuations, combined with the launch and rapid growth of U.S. spot BTC ETFs, have broadened its investor base and further linked its returns to overall market risk sentiment. BTC now behaves more as a traditional growth asset, reflecting its evolving role in institutional portfolios. Consistent with this view, recent multiscale evidence by Wątorek et al. [9] shows that, since the COVID-19 shock, BTC and Ethereum have become coupled to traditional financial markets, and to US equities in particular, with this coupling becoming stronger around macroeconomic announcements and in periods of market stress. The corporate treasury firms we study operate within this regime of macro level integration.

Parallel to the increasing institutional adoption of BTC, a small but growing set of listed firms have begun to re-orient their corporate treasuries around direct cryptocurrency holdings. The most prominent case is *Strategy Incorporated* (formerly *MicroStrategy Incorporated*, hereafter MSTR), that offers AI powered enterprise analytics software and services. Its Executive Chairman Michael Saylor has articulated a business model that treats BTC as the firm's primary reserve asset. Since August 2020 *Strategy* has financed successive purchases of BTC through a mix of follow-on equity offerings, senior secured notes, and convertible bonds, supplementing residual cash flows from its legacy software division [10]. As on 27 April 2025, the company held $553,555$ BTC, the most prominent corporate position worldwide, and Mr. Saylor ranked as the top percentage gainer on Forbes' 2024 list of crypto billionaires [11]. Although the underlying software business has shown only modest growth, the equity has attracted substantial retail and institutional demand precisely because of the balance sheet's aggressive exposure to BTC [12].

Imitative strategies have emerged in other jurisdictions. *Metaplanet Inc.*, often described as the "Japanese *Strategy*", sold most of its hospitality real estate assets in 2024 and redirected the proceeds, together with borrowed funds, into BTC; its flagship property in Tokyo has been re-branded "The Bitcoin Hotel". Over the twelve months to April 2025, *Metaplanet*'s share price rose roughly 4800%, the most significant gain among Japanese equities and one of the highest globally [13]. In the United States, *Fold Holdings Inc.* is the first publicly traded financial services company built entirely around BTC, offering rewards, savings, and payment solutions denominated in BTC. On 24 July 2024, *Fold* executed its inaugural BTC purchase of 1000 BTC [14]. Thereafter, the company issued a \$46.3 million convertible note secured by 500 BTC and added 475 BTC and a further 10 BTC to its treasury, placing it among the top ten U.S. public firms by crypto holdings. Despite this aggressive accumulation strategy, *Fold*'s share price has fallen by 62.3% over the past year [15].

These and similar convertibles have become attractive to hedge funds seeking volatility-driven arbitrage. Large investors restricted from holding BTC directly likewise use such equities as indirect proxies [12]. Smaller issuers have adopted analogous tactics with other tokens: *Upexi Inc.* raised \$100 million in April 2025 to acquire Solana (SOL), lifting its share price by more than 325% on the announcement date, while *Janover Inc.* (a real estate finance company controlled by a consortium of former Kraken executives) disclosed a \$10.5 million purchase of SOL earlier the same month [16].

Despite their appeal, these "crypto treasury" models entail significant financial risk. Digital asset price volatility can translate into earnings variability and elevated refinancing risk, the effects of which are amplified when acquisitions are debt-financed. These concerns are especially acute for smaller firms, whose core operating businesses are only tangentially related to cryptocurrency markets, raising questions about their long-run solvency [16]. These examples highlight not only the growing corporate appetite for direct crypto exposure, but also the critical importance of robust risk management and disclosure practices when adopting such balance sheet strategies.

This study therefore investigates the evolving relationship between Bitcoin (BTC) and equity markets, focusing on firms that have adopted BTC as part of their corporate treasury strategy. Using a dataset of 39 publicly listed BTC-holding companies from 2017 to 2025, the paper applies correlation analysis, single factor return models, and transfer entropy methods to quantify both linear and nonlinear dependencies. We will show that BTC consistently acts as the dominant information driver, particularly during major market events, while feedback from equities to BTC remains rare and event-specific. By illustrating these directional and time-varying patterns, the study provides investors, risk managers, and policymakers useful information for managing the growing integration of digital assets into conventional financial markets.

Our contribution is fourfold. First, we assemble a manually curated sample of every publicly traded firm holding BTC on its balance sheet as of April 2025, sourced from *Bitcoin Treasuries* [14]. Second, we propose a two-step empirical framework that combines a static cross sectional taxonomy based on BTC sensitivity, balance sheet intensity and trading liquidity with a rolling transfer entropy measure of directional information flow, giving a unified view of exposure, liquidity and lead-lag dynamics for the same set of firms. Third, we quantify the BTC to equity information channel for "crypto treasury" stocks, isolating an asymmetric and time varying pattern that is not covered in prior work. Fourth, we draw implications for the hedging of crypto treasury equities, showing that static hedge ratios systematically misprice the dependence on BTC.

The remainder of the paper is organized as follows. Section 2 reviews the literature on entropy based techniques in financial research. Section 3 describes the dataset and provides a detailed analysis of MSTR, the specific stock of interest. Section 4 outlines the empirical methodology, while Section 5 presents the results from Pearson correlations, single factor model regressions, and transfer entropy analyses. Finally, Section 6 concludes the paper and discusses remaining open challenges.

2. Literature Review

Early empirical research on market interconnectedness has heavily relied on the correlation function to quantify co-movements among asset returns [17–24]. While convenient, correlation analysis suffers from two well-known shortcomings [25]. First, it captures only linear associations, thereby overlooking the nonlinear dependencies characterizing financial data. Second, correlation is intrinsically symmetric and, therefore, incapable of indicating which series, if any, exerts influence over the other. Mutual Information (MI) provides an entropy-based measure of dependence sensitive to nonlinear structures, but, like correlation, remains directionless [26]. Directionality can be recovered via Transfer Entropy (TE) [27,28], an extension of Wiener-Granger causality [29–31] that quantifies the incremental information flow from one time series to another [32].

Marschinski and Kantz (2002) [33] were among the first to apply TE in a financial context. They quantified the information flow between the Dow Jones and DAX index, identifying a clear unidirectional transfer from the U.S. market to the German one. Building on this foundation, Baek et al. (2005) [34] employ TE to map pairwise information flows within the U.S. equity market, identifying energy sector firms, particularly those engaged in oil, gas, and electricity, as net transmitters of information to the broader market. Their results suggest that TE not only captures nonlinear dependencies but also helps to isolate market-leading institutions. Dimpfl et al. (2013) [35] use TE firstly to examine the importance of the credit default swap market relative to the corporate bond market for the pricing of credit risk, and secondly to analyze the dynamic relation between market risk and credit risk. Sandoval Jr. (2014) [36] analyses daily returns for the 197 largest global financial companies from 2003 to 2012. One-day lagged TE is used to map causal links, revealing which firms dominate others within the financial sub-sectors (banks, diversified financial services, savings and loans, insurance, private equity funds, real estate investment companies, and real estate trust funds). He et al. (2017) [37] analyze the relationships among 9 stock indices from the U.S., Europe, and China over the period 1995 to 2015 using various forms of TE. They find that the U.S. holds the leading position in long term lagged relationships, whereas China emerges as the most influential market at shorter lags.

Dimpfl et al. (2019) [38] introduce Effective Group transfer entropy (EGTE) as a tool for identifying informational leadership. They devise a bootstrap method for confidence intervals, demonstrate that linear techniques miss many information flows, and apply EGTE to intraday cryptocurrency data, uncovering predominantly nonlinear dependencies. Peng et al. (2022) [39] compare simple Pearson correlations with TE for Chinese stocks using rolling windows and lead-lag shifts. Neto et al. (2023) [40] use the same information measure to build a directed, weighted network of Brazilian equities on a 32 year period in which each edge quantifies information flow from one stock's returns to another. Applying network centrality measures, the authors successfully identify the most influential and the most influenced stocks at each point in time. Mungo et al. (2024) [41] demonstrate that investment structures themselves can shape market dynamics, as correlations in token returns tend to reflect the underlying patterns of crypto co-investment across investors.

Ma (2025) [42] examines the volatility dynamics of BTC and MSTR from September 2019 to September 2024 using the GARCH model [43]. They show that MSTR's volatility has closely followed the footsteps of BTC, especially during the 2024 rally in that market, including all its shocks and recoveries. Krause (2025) [44] finds that, while *Strategy* has generated substantial returns, its debt- and equity-financed accumulation of BTC raises concerns about sustainability and speculative risks. Because the business model depends on continued capital inflows and a rising BTC price, several commentators liken it to a Ponzi-style structure. Krause evaluates the risk return profile of this strategy, compares the firm's performance against BTC itself, and weighs the relative merits of holding MSTR shares versus owning the underlying cryptocurrency directly.

Wątorek et al. (2023) [9] use detrended cross-correlation analysis and the q-dependent coefficient ρ_q on high-frequency data to document a structural change in the coupling between crypto and equity markets around the COVID-19 shock, and report that BTC reacts to U.S. macroeconomic news such as CPI releases. Their results support the conclusion that BTC has lost its traditional role as a hedge or safe-haven asset, which we reach from a different angle in this paper, through the corporate treasury equity channel and the rolling TE evidence presented in Section 5.3.

3. Data

Although no official registry exists, the public database *Bitcoin Treasuries* [14] reports that 66 publicly listed companies worldwide, and an additional 12 private firms, hold BTC on their balance sheets, including many whose core businesses were previously unrelated to cryptocurrencies. In this paper, we construct a dataset of BTC holders, including corporate and government entities that hold this crypto asset directly or through custodial arrangements, based on data from [14], updated as of 11 April 2025. Our universe is the set of every publicly traded firm that discloses direct BTC custody on its balance sheet as of 1 April 2025, sourced from *Bitcoin Treasuries* [14]. From this universe of disclosed BTC holders, we restrict attention to economically relevant firms by requiring that, in addition to holding BTC, the firm satisfies at least one of the following further criteria (for convenience, we use the terms 'companies', 'equities', and 'stocks' interchangeably throughout the paper, although we primarily refer to the publicly traded shares of these companies): (i) they hold more than 700 BTC, or (ii) they have a market capitalization exceeding $\$1.00B$, or (iii) their BTC holdings represent more than 50.00% of that market capitalization. These thresholds ensure that each firm's BTC exposure is economically significant and will likely have a measurable influence on its share price dynamics. This filter narrows the universe to the 39 firms studied throughout the paper. The complete list of companies is presented in Table A1, Appendix A.

To assess the robustness of our sample selection to the three inclusion thresholds, we ran a univariate sensitivity sweep, holding two thresholds fixed and varying the third over a wide grid, and a joint OR-sweep that moves all three thresholds together over six scenarios (Tables A2 and A3 in Appendix B). The main findings (average BTC single factor sensitivity between 0.6 and 0.7, around one third of the firms with sensitivity above 1, and more than 80% of contemporaneous correlations significant at the 5% level) hold across the entire grid. As expected, tightening only the absolute BTC holding criterion (for example to $>$5000 BTC) mechanically raises the mean exposure, since the stricter cutoffs retain firms with structurally higher BTC exposure. We also verify, in Figure 1, that the chosen cutoffs lie close to the lower quartile of the empirical distribution of each criterion: this is a natural break that keeps the tail of materially exposed firms while excluding tokenistic positions (for example *BlackRock, Inc.* with 6.15 BTC, or *Intesa Sanpaolo* with 11 BTC) that would otherwise dilute the signal of interest. Figure 2 reports the univariate sweep for the main statistics.

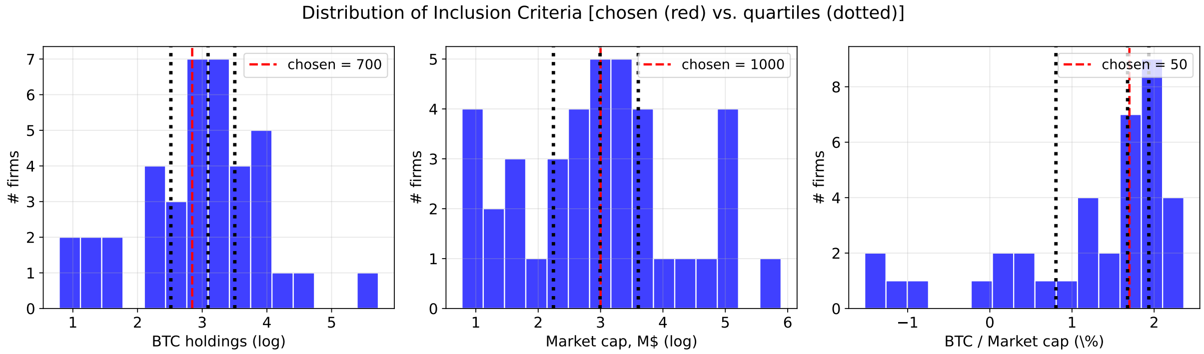

Figure 1. Empirical distribution of the three inclusion criteria over the universe of publicly traded BTC holders in log space. Dashed red lines mark the chosen cutoffs; dotted black lines mark the 25th, 50th and 75th percentiles of the corresponding empirical distribution. The chosen cutoffs lie close to the lower quartile of each criterion, separating the tail of materially exposed firms from the bulk of tokenistic positions.

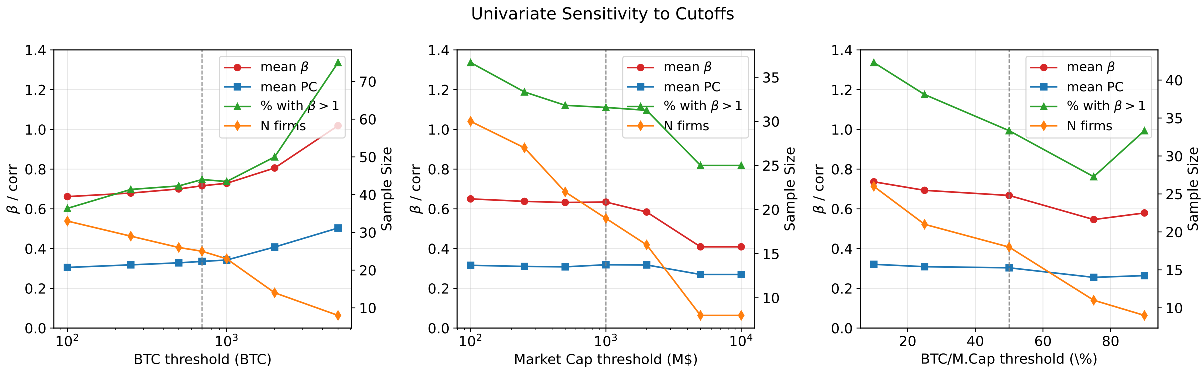

Figure 2. Univariate sensitivity of the main statistics to each of the three inclusion cutoffs. In each panel one threshold is varied over a wide grid while the other two are held fixed at the chosen values; the panel reports the mean BTC single factor sensitivity (red dots), the mean Pearson correlation with BTC (blue squares), the share of firms with sensitivity larger than 1 (green triangles) and the resulting sample size (orange diamonds, right axis). Dotted vertical lines mark the chosen cutoffs. Note: β represents the sensitivity of a firm's equity returns to BTC returns; it will be formally introduced in Section 5.2. We report it here, ahead of its definition, only to summarise the robustness of the sample selection.

For each qualifying holder, we collect their BTC acquisition balance sheet data and match it with the firm's market capitalization on the corresponding dates, as reported by the LSEG Refinitiv Eikon platform. We also retrieve daily closing share prices from the date of each firm's initial BTC purchase through 1 April 2025 (daily closing prices are obtained with the Python package `yfinance`). The descriptive statistics of daily log returns for the 39 companies of our sample is presented in Table A4, Appendix C.

We complement the moments reported in Table A4 with formal normality tests on the daily log returns of every series in our sample. We compute the Jarque-Bera, Shapiro-Wilk, Anderson-Darling and D'Agostino K^2 statistics: the null of normality is rejected at the 5% level under all four tests for every series, including BTC itself. The corresponding Q-Q plots for BTC and MSTR (see Figure 3) show the heavy tailed deviation typical of return data in both tails. This motivates two methodological choices that we adopt in Section 5: in Section 5.2 we report regression coefficients with Newey-West HAC standard errors [45] that are robust to both heteroscedasticity and serial correlation, and in Section 5.3 we exploit the distribution free nature of transfer entropy to capture the non linear dependencies that a Gaussian linear measure would miss. The full set of test statistics is reported in Table A5 in Appendix D.

Figure 4 offers an overview of the temporal distribution of institutional BTC acquisitions. Figure 4a details the timing of initial acquisitions, revealing that early institutional involvement between 2018 and 2022 was both limited and sporadic, likely reflecting cautious exploratory behavior in a context of regulatory uncertainty and market volatility. A discernible increase in first time acquisitions begins in 2021, coinciding with the post-pandemic bull market, heightened media attention, and a growing recognition of BTC as a potential hedge against inflation and currency debasement. The most pronounced surge, however, emerges in 2023, followed by a secondary peak in early 2025. This acceleration suggests successive waves of institutional entry, potentially catalyzed by the approval

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of spot BTC exchange-traded funds, the stabilization of global monetary policy, and a post U.S. election market rally in late 2024. Figure 4b, which depicts the distribution of final acquisition dates, corroborates this trajectory. While earlier years exhibit fragmented activity, the data show a clear concentration of last acquisitions in 2024 and early 2025. This pattern indicates sustained institutional accumulation, reflecting strategic long term positioning and growing confidence in BTC's market legitimacy. Contributing factors likely include improved regulatory clarity, custodial and trading infrastructure maturation, and evolving macroeconomic conditions favoring alternative assets. Figure 4c aggregates all acquisition dates into a unified histogram, providing a view of the temporal dynamics of BTC purchases. The frequency of acquisitions increased markedly beginning in early 2024, peaking in 2025. Whereas earlier institutional activity was sparse and opportunistic, the recent surge emphasise a broad-based shift toward BTC adoption. This shift is driven not only by favorable regulatory developments, such as ETF approvals, but also by institutional portfolio diversification strategies, increased liquidity, and the perceived resilience of BTC among global financial uncertainties.

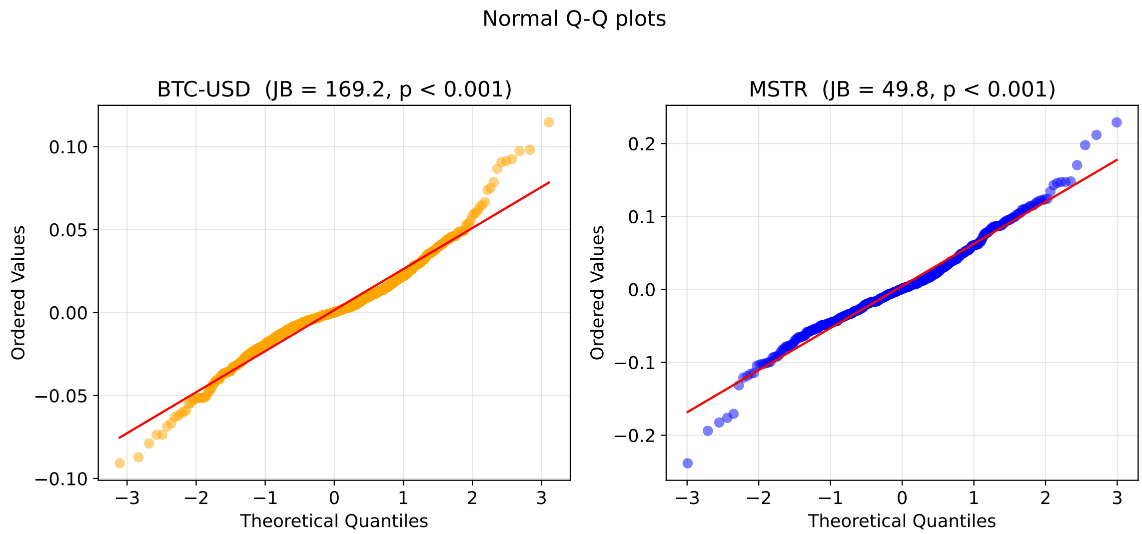

Figure 3. Normal Q-Q plots of the daily log returns of BTC-USD (left) and MSTR (right) over the period 1 April 2023 to 1 April 2025. The red line is the Gaussian reference; the deviation in the two tails is the typical sign of heavy tailed return distributions.

The red vertical lines in Figure 4 represent significant events in the history of BTC markets, happened in the period spanning from December 2018 to April 2025:

(a) **6 February 2018 – BTC Price Correction**: After reaching a peak of approximately $\$19,000$ in November 2017, BTC's price experienced a sharp decline, falling below $\$6,000$ by early February 2018. From 6 January 2018, to 6 February 2018, the cryptocurrency lost roughly 65% of its value, marking one of the most severe drawdowns in its history following the unprecedented 2017 bull market [46].

(b) **16 December 2020—Post COVID Rally**: Following a low of around $\$3600$ in March 2020, triggered by the global market sell-off coinciding with the COVID-19 pandemic, BTC's price surged by over 400% throughout the year. On December 16, 2020, it surpassed $\$20,000$ for the first time, reaching a new all-time high and marking the beginning of another major bull cycle [47].

(c) **15 June 2022—Historic Monthly Loss**: BTC experienced its most severe monthly decline in over a decade, falling by 37.3% in June 2022. This drop represented the largest monthly loss since 2011 and occurred in correspondence of a broader market downturn [48].

(d) **10 January 2024—SEC Approves BTC ETFs**: The U.S. SEC approved 11 BTC ETF applications from major financial institutions, including BlackRock, Fidelity, and VanEck. This regulatory breakthrough expanded institutional access to BTC via traditional investment channels [49].

(e) **5 December 2024—Post U.S. Election Rally**: Following the outcome of the U.S. presidential election, BTC's price surged past $\$100,000$ for the first time. The rally was driven by investor expectations of more favorable regulatory conditions under the new administration [50].

(f) **6 March 2025—The White House commitment**: U.S. President Donald Trump signed an executive order establishing a Strategic Bitcoin Reserve and a U.S. Digital Asset Stockpile. The move framed BTC as a sovereign reserve asset [51].

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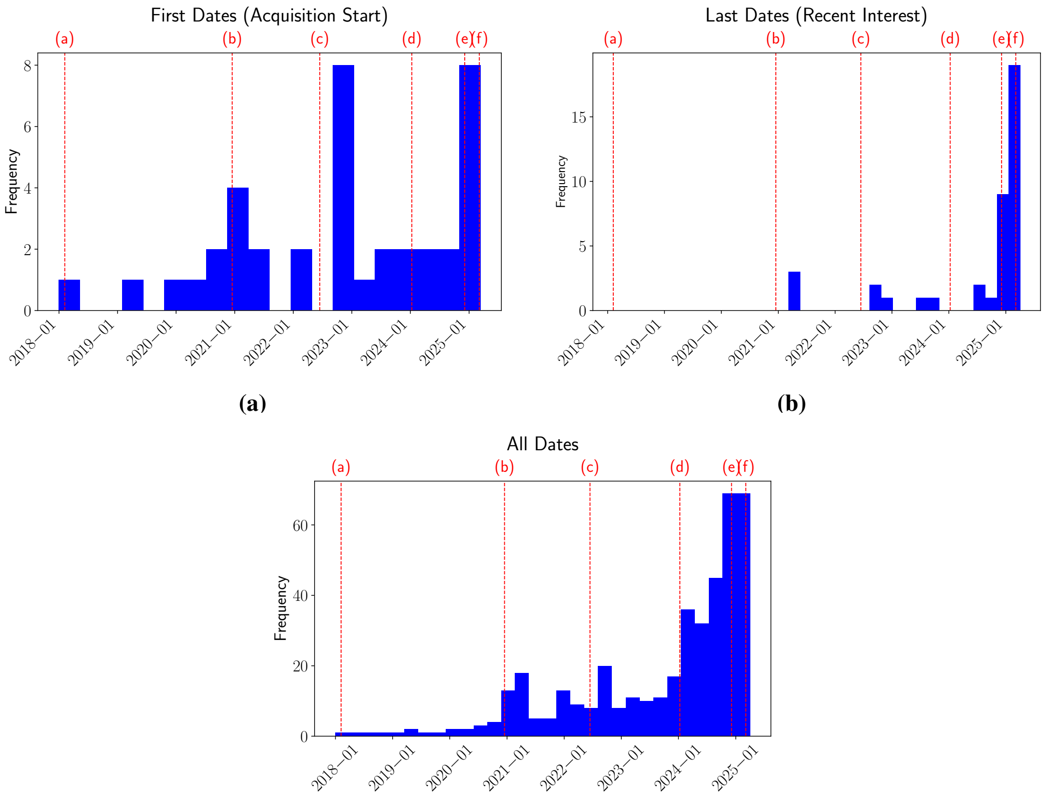

(a) **(b)**

(c)

Figure 4. Red vertical lines mark key events in the BTC market timeline. (**a**): distribution of first acquisition dates across companies in the dataset, highlighting the initial waves of institutional adoption. (**b**): distribution of last acquisition dates, capturing more recent activity and assessing whether interest is currently accelerating. (**c**): overall distribution of BTC acquisition dates for all recorded transactions in the dataset. There is a clear acceleration in acquisition activity starting in early 2024, peaking in 2025. Earliest first date: 31 December 2017 (*Hut 8 Mining Corp*); latest last date: 3 April 2025 (*MARA Holdings, Inc.* and *Riot Platforms, Inc.*). Data are updated as of 11 April 2025.

Figure 5 summarises the intensity of BTC acquisition activity across the 39 companies in our sample: 5 of them executed only a single purchase, and 13 conducted two to four discrete transactions. The majority (21 of them), including several crypto-native miners, undertook five or more separate purchases, signalling an active approach to balance sheet accumulation. Figure 6 complements this by illustrating the distribution of BTC holdings via a box plot. The lower and upper edges of the box represent the 25th (228 BTC) and 75th (3185 BTC) percentiles, respectively, with the central line marking the median holding (1200 BTC). Whiskers extend to the minimum (6.15 BTC held by *BlackRock, Inc.*) and maximum (528, 185 BTC held by *Strategy*) values, and diamonds denote outliers beyond these bounds. The pronounced skew of the distribution highlights that a small number of firms, most notably *Strategy*, hold large BTC positions compared to the rest of the sample.

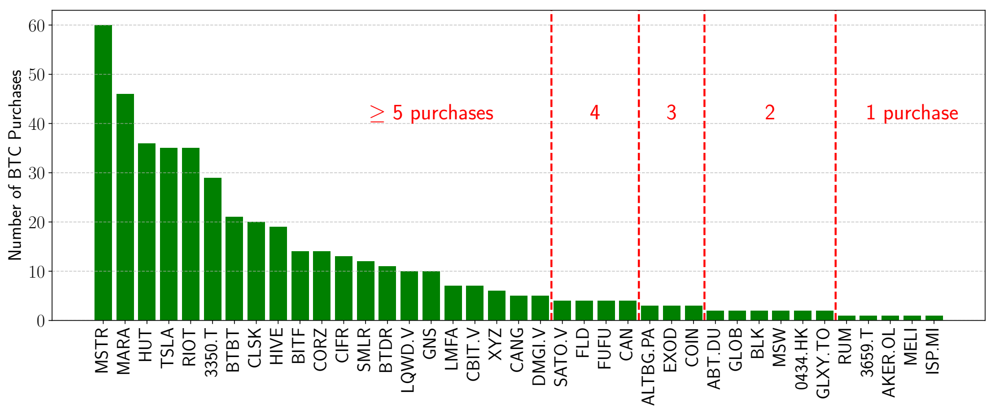

Figure 5. Frequency of BTC purchased by firm.

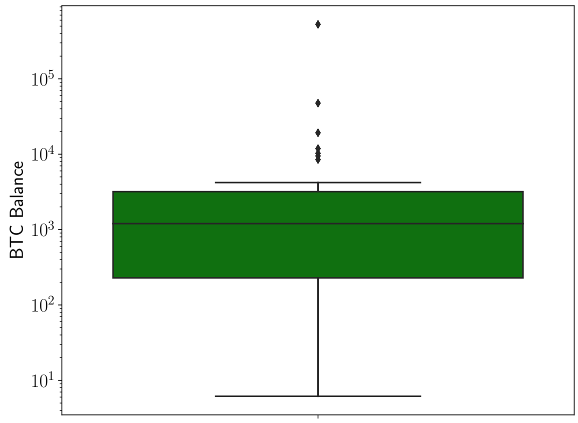

Figure 6. The box plot displays the distribution of BTC holdings among the 39 companies in our dataset. The lower and upper edges of the box correspond to the first (25%) and third (75%) quartiles, respectively; the line within the box marks the median (50%). Whiskers extend to the minimum and maximum values within the distribution, and small diamond markers beyond the whisker indicate outlier firms with exceptionally large BTC positions.

Strategy (MSTR)

Since this paper centers on *Strategy*, the largest corporate holder of BTC, we confine our detailed analysis exclusively to this company.

Following its most recent purchase on 31 March 2025, the company's cumulative BTC holdings reached 528, 185 BTC. Figure 7 illustrates the relationship between BTC acquisitions and daily market dynamics. As of April 2025, *Strategy*'s equity has delivered a 21% total return over the previous three months and a 229% return over the past year. Its implied volatility is 72%, indicating that market participants expect future price fluctuations of this magnitude throughout the life of the outstanding MSTR options; indeed, the 30 day historical volatility, calculated as the standard deviation of daily log returns over the last month, measures 59%. Table 1 reproduces only the rows from Table A4 (in Appendix C) that pertain to MSTR and BTC, presenting a descriptive statistics of daily log returns from 1 April 2023 to 1 April 2025.

Table 1. Descriptive statistics of daily log returns for BTC and MSTR, from 1 April 2023 to 1 April 2025.

Ticker	Mean	Median	Std	Skewness	Kurtosis	Min.	Max.
BTC-USD	0.0015	0.0002	0.0252	0.3056	2.3004	−0.0908	0.1146
MSTR	0.0047	0.0007	0.0582	0.1576	1.5407	−0.2384	0.2290

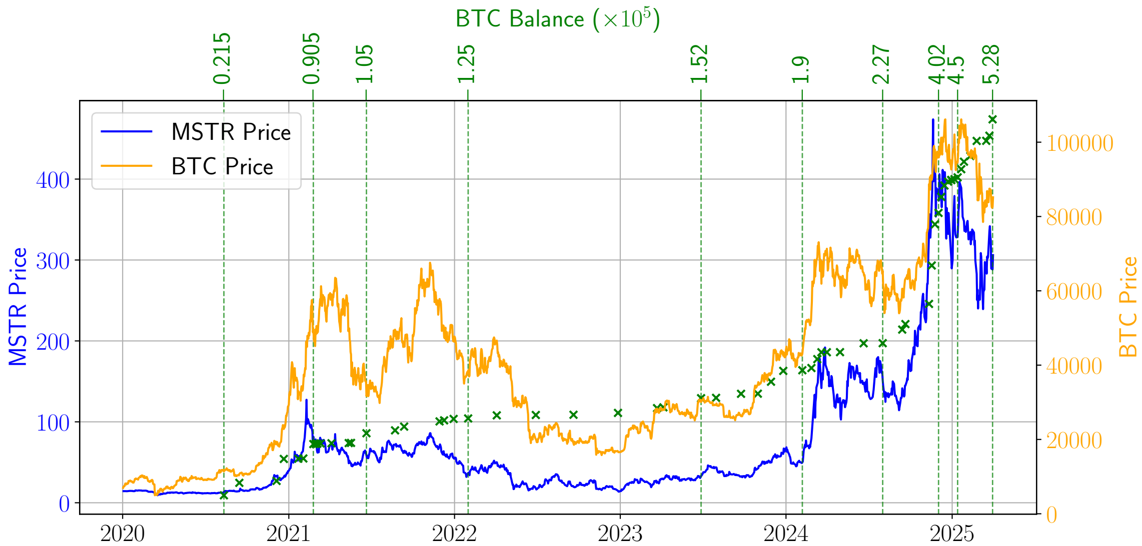

Figure 7. BTC price (in orange), MSTR price (in blue), and cumulative BTC holdings (in green).

From 2021 through 2024, MSTR stock price significantly outperformed BTC, growing $7.5\times$ versus $3.2\times$. Between late August and late October 2024, MSTR's share price nearly doubled, whereas BTC rose by only about 24%. The divergence is especially pronounced over 5 to 7 October 2024, when MSTR gained 18% while BTC advanced by just 1%. According to Phillips & Pohl (2024) [10], this decoupling stems from MSTR evolving strategy, notably its rebranding as a "Bitcoin Treasury Company". This shift involved aggressive capital raising, in the form of equity and debt issuance, to fund substantial BTC acquisitions, thereby enhancing its leverage and altering its capital structure. Choueifaty et al. (2025) [52] provide a decomposition of the MSTR performance into three sources: (i) monetization of a significant portion of the pre-existing premium into book value, (ii) leveraging BTC's performance through increased exposure, and (iii) fluctuations in the BTC price premium. Notice that the public disclosure of *Strategy*'s BTC acquisitions is almost immediate, as the firm typically files a Form 8-K and issues a press release within hours, usually by the next trading session, of completing each purchase [53].

4. Methodology

To probe the directional interdependencies between daily BTC returns and the equity returns of the firms in our sample, we estimate the Transfer Entropy (TE), an information measure that captures lagged, conditional dependence between time series. Let

$$\mathbf{X}^-_{t,\boldsymbol{\tau}} = \left(X_{t-\tau_1}, \ldots, X_{t-\tau_j}\right), \qquad \mathbf{Y}^-_{t,\boldsymbol{\lambda}} = \left(Y_{t-\lambda_1}, \ldots, Y_{t-\lambda_k}\right) \tag{1}$$

denote, respectively, the vectors containing the past j lags of the BTC return series X_t and the past k lags of a given firm's return series Y_t. Under the Wiener-Granger causality framework [29], X_t is said to *cause* Y_t at prediction horizon h if the history of X_t improves forecasts of Y_{t+h} beyond what is achievable using only the past of Y_t. As stated in Equation 2, the TE from X to Y at horizon h is defined as the reduction in uncertainty, quantified by Shannon entropy $H(\cdot)$, about Y_{t+h} when including the history of X_t in the conditioning set.

$$T_{\mathbf{X}\to\mathbf{Y}} = H\left(Y_{t+h} \mid \mathbf{Y}^-_{t,\boldsymbol{\lambda}}\right) - H\left(Y_{t+h} \mid \mathbf{X}^-_{t,\boldsymbol{\tau}}, \mathbf{Y}^-_{t,\boldsymbol{\lambda}}\right) \tag{2}$$

By applying the definition for the conditional mutual information measure (i.e., $I(A;B|C) = H(A|C) - H(A|B,C)$), Equation (2) can be written as per in Equation (3).

$$T_{\mathbf{X}\to\mathbf{Y}} = I\left(Y_{t+h}; \mathbf{X}^-_{t,\boldsymbol{\tau}} \mid \mathbf{Y}^-_{t,\boldsymbol{\lambda}}\right) \tag{3}$$

Formally, the conditional mutual information can be expressed as an expectation over the joint distribution of the relevant variables:

$$T_{\mathbf{X}\to\mathbf{Y}} = \mathbb{E}\left[\log \frac{P(Y_{t+h} \mid \mathbf{X}^-_{t,\boldsymbol{\tau}}, \mathbf{Y}^-_{t,\boldsymbol{\lambda}})}{P(Y_{t+h} \mid \mathbf{Y}^-_{t,\boldsymbol{\lambda}})}\right], \tag{4}$$

where $P(\cdot)$ denotes the appropriate conditional probability distributions. This highlights that transfer entropy measures the expected information gain, in bits, from incorporating the history of X_t into predictions of Y_{t+h}.

By construction, $T_{\mathbf{X}\to\mathbf{Y}} \geq 0$, with $T_{\mathbf{X}\to\mathbf{Y}} = 0$ indicating no causal influence from X to Y at the specified lags. To assess whether an observed value of transfer entropy reflects a genuine directional dependency rather than random fluctuations, a non-parametric hypothesis testing procedure is employed. The null hypothesis H_0 assumes no directional influence from X to Y, meaning any observed transfer entropy arises purely by chance. To approximate the distribution of $T_{\mathbf{X}\to\mathbf{Y}}$ under H_0, surrogate datasets are generated by independently reshuffling the time indices of the driver series X_t. This procedure preserves the marginal distribution of X_t but destroys its temporal and structural dependencies with Y_t. For each shuffled sample $i = 1, \ldots, N^{\text{shuffle}}$, the corresponding transfer entropy value T_i^{shuffle} is computed, yielding an empirical null distribution. The significance of the observed transfer entropy $T_{\mathbf{X}\to\mathbf{Y}}$ is evaluated by calculating the proportion of shuffled values that exceed or equal it:

$$p = \frac{1}{N^{\text{shuffle}}} \sum_{i=1}^{N^{\text{shuffle}}} \mathbb{I}\left(T_i^{\text{shuffle}} \geq T_{\mathbf{X}\to\mathbf{Y}}\right), \tag{5}$$

where $\mathbb{I}(\cdot)$ is the characteristic function. This empirical p-value estimates the probability of observing a transfer entropy as large as or larger than the measured value under the null hypothesis. A small p-value indicates that the observed directional dependency is unlikely to have occurred by chance, providing evidence against H_0 and supporting the presence of a meaningful causal relationship.

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5. Results

Guided by the temporal trend shown in Figure 4c, and supported by the empirical analyses presented in Sections 5.1 and 5.2, we focus on the most recent two-year period, from 1 April 2023 to 1 April 2025 (∼500 trading days). The code for reproducing all experiments is available at https://github.com/FinancialComputingUCL/Crypto_Balance_Sheets (accessed on 17 June 2026).

5.1. Global Measures of Information

In our preliminary analysis, we compute three Pearson correlation (PC) coefficients to examine both same-day and lagged one-day return dependencies between BTC and every stock X in the sample:

1. $\rho(BTC_t, X_t)$: the same-day correlation between BTC and equity returns;
2. $\rho(BTC_{t-1}, X_t)$: the correlation capturing the extent to which BTC returns anticipate equity returns by one day;
3. $\rho(BTC_t, X_{t-1})$: the correlation assessing whether equity returns precede BTC returns by one day.

Analyzing lagged correlations helps identify potential lead-lag dynamics that may arise from information transmission delays, market microstructure effects, or behavioral asymmetries. Such dynamics are closely related to the concept of directional dependence, which we further explore using TE (TE estimates are obtained with the R package `RTransferEntropy` [54]). Indeed, to quantify the direction and magnitude of information flow, we integrate these correlation measures with two TE estimates:

4. $TE_{BTC(t-1)\to X(t)}$: the information transfer from BTC returns at lag 1 to current stock X returns;
5. $TE_{X(t-1)\to BTC(t)}$: the information transfer from stock X returns at lag 1 to current BTC returns.

A concise summary is provided in Table 2. The mean $\rho(BTC_t, X_t)$ is moderately positive (i.e., 0.29), indicating that, on average, stocks co-move with BTC, likely reflecting shared exposure to the digital asset ecosystem. The relatively low standard deviation (i.e., 0.21) and mild skewness (i.e., 0.16) suggest a fairly symmetric distribution centered around positive values. The negative kurtosis reflects a light tailed distribution, implying few extreme correlation values. In contrast, $\rho(BTC_t, X_{t-1})$ and $\rho(BTC_{t-1}, X_t)$ are centered near 0, with means of 0.018 and -0.018, respectively. This indicates that no consistent linear lead-lag pattern emerges across the sample. However, their skewness values diverge: BTC-leading correlations are negatively skewed (i.e., -1.11), suggesting that a subset of stocks exhibit pronounced negative lagged correlation with future BTC returns; while equity-leading correlations are positively skewed (i.e., 1.78), with significant excess kurtosis (i.e., 4.91), pointing to heavy tails. This hints at asymmetric or nonlinear relationships, where a few stocks might carry signal that anticipate BTC, even if the overall mean effect is muted. These patterns motivate the use of transfer entropy (TE), which can capture nonlinear and directional information flows. The TE results show low mean values (0.0151 from BTC to stocks, 0.0144 in the reverse direction), but with positive skewness and mild kurtosis, especially for $TE_{X(t-1)\to BTC(t)}$. Consistently with isolated lead-lag relationships observed in the correlation tails, this suggests that while the average information transfer is limited, a subset of equities might sporadically transmit information to BTC.

Table 2. Summary statistics of the dependence measures between BTC and the 39 stocks. The table reports the mean, median, standard deviation, skewness, and kurtosis for each of the five global information metrics discussed earlier in the section.

Statistic	PC: BTC(t) ⇄ X(t)	PC: BTC(t − 1) ⇄ X(t)	PC: BTC(t) ⇄ X(t − 1)	TE: BTC(t − 1) → X(t)	TE: X(t − 1) → BTC(t)
Mean	0.292	−0.018	0.018	0.0151	0.0144
Median	0.254	−0.014	0.004	0.0140	0.0140
Std	0.209	0.040	0.0782	0.0066	0.0060
Skewness	0.164	−1.105	1.780	0.176	0.787
Kurtosis	−1.363	0.893	4.906	0.170	1.186

While these global measures offer valuable insights, they also smooth over temporal variation that may be crucial for understanding dynamic relationships among financial assets. The asymmetric distribution of lagged correlations and the episodic nature of significant transfer entropy values suggest that informational dependencies between BTC and equities are highly unstable over time. As such, relying on full-sample estimates may obscure localized periods of directional influence or structural shifts in market behavior. To uncover these dynamics, in Section 5.3, we proceed with a rolling estimation of the TE, which enables us to capture time-varying information flow and identify windows where BTC or equity returns exert a statistically significant influence on one another.

This local analysis is especially pertinent for real-time forecasting, trading strategies, and market surveillance, where detecting regime shifts and lead-lag transitions is more informative than long-run averages.

5.2. Single Factor Model: Exposure Driven BTC β

To assess the sensitivity of equities to BTC price fluctuations, we estimate a single factor model [55] for each firm i in the sample:

$$r_{i,t} = \alpha_i + \beta_i r_t^{\mathrm{BTC}} + \varepsilon_{i,t}, \quad t = 1, \dots, T \tag{6}$$

where $r_{i,t}$ and r_t^{BTC} denote the daily log returns of stock i and BTC, respectively; α_i is the firm-specific intercept; and β_i represents the stock's sensitivity to BTC returns, that is, the percentage change in the stock's return associated with a 1% change in BTC on the same trading day. Companies with $\beta > 1$ are characterized by leveraged exposure to BTC price changes, while those with $\beta \leq 1$ exhibit proportional or lower sensitivity. We report the regression coefficients for the whole dataset in Table A6 in Appendix E, highlighting that 12 firms exhibit a $\beta > 1$, while the average beta is $\hat{\beta} = 0.62$.

We estimate this single factor model for MSTR daily log returns on BTC over the period spanning from 1 April 2023 to 1 April 2025. The estimated factor loading ($\beta = 1.37$) implies that a 1% increase in BTC returns is associated with an average 1.37% increase in MSTR returns, consistent with a high degree of systematic exposure to BTC price risk. The model explains a substantial proportion of return variation, with an R^2 of 0.44, indicating that BTC returns account for 44% of the variance in MSTR's daily returns. The fitted regression, together with its confidence band, is shown in Figure 8. Since financial return series typically exhibit residual heteroscedasticity and serial correlation (see also the normality evidence in Section 3), the OLS confidence intervals may be unreliable. We therefore re-estimate Equation (6) for every firm in the sample using OLS with Newey-West HAC standard errors [45], choosing the lag length L via the automatic data-driven rule $L = \lfloor 4(T/100)^{2/9} \rfloor$ proposed in [45], which for our sample size of $T = 500$ trading days returns $L = 5$. For each firm we also run the Breusch-Godfrey test for serial correlation up to lag L and the Breusch-Pagan test for heteroscedasticity. The diagnostics reject the null of homoscedastic and uncorrelated residuals for 12 of the 39 firms in each case (including MSTR), so the HAC correction matters for a meaningful subset of the sample. For MSTR, the HAC standard error of $\hat{\beta}$ is about 1.3 times the OLS one, which lowers the t-statistic but leaves it far above the 5% critical value. All central empirical conclusions are preserved: 31 of 39 firms keep a significant $\hat{\beta}$ at the 5% level under HAC (versus 32 under OLS), the average $\hat{\beta}$ remains 0.62, and all 12 firms with $\hat{\beta} > 1$ keep their classification. Figure 8 shows the MSTR regression with both the OLS and the HAC-implied 99% confidence bands. The complete HAC table is reported in Table A7 in Appendix F.

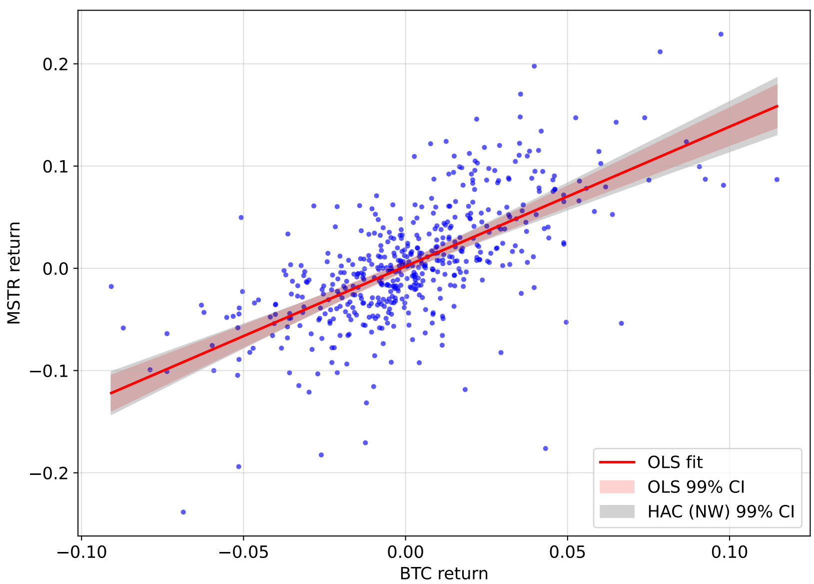

Figure 8. Single factor model of MSTR daily returns on BTC (1 April 2023 to 1 April 2025), with Newey-West HAC standard errors [45]. The grey band is the HAC-implied 99% confidence interval; the pale-red band is the corresponding OLS band shown for comparison. Estimated values of $\hat{\beta}$, OLS and HAC standard errors, t-statistics, R^2, sample size N and the Newey-West lag length are reported in the inset.

Figure 9 combines two global measures: the Pearson correlation (discussed in Section 5.1) and the estimated betas. The x-axis reports each firm's BTC intensity γ, measured as BTC holdings expressed as a percentage of market capitalization, while the y-axis shows the Pearson correlation between the firm's daily log returns and BTC returns. Firms with $\beta > 1$ are labeled in red, indicating high sensitivity to BTC price movements, whereas those with $\beta \leq 1$ are labeled in black. Marker size and color gradient are proportional to the Amihud illiquidity ratio δ [56] in Equation (7), defined as the daily ratio of a stock's absolute return to its dollar trading volume. This ratio reflects the daily price response to one dollar of trading volume and serves as an approximate measure of price impact.

$$\delta = \frac{1}{N}\sum_{t=1}^{N} \frac{|\, r_t \,|}{\text{DollarVolume}_t}, \tag{7}$$

The original Amihud ratio δ (where $0 < \delta < 1$) measures price impact: a high δ implies that even small trade volumes cause large price moves (low liquidity), whereas a low δ indicates that larger volumes are required to move the price (high liquidity). To invert this scale and align larger values with greater liquidity, we use $|\log(\delta)|$. Under this transformation, high values of $|\log(\delta)|$ correspond to original $\delta \sim 0$, that is more liquid stocks; while low $|\log(\delta)|$ values coincide with illiquid stocks. Consequently, in our plot, darker and larger markers (high $|\log(\delta)|$) denote highly liquid equities, whereas lighter and smaller markers indicate lower liquidity. It is worth emphasizing that the Amihud illiquidity ratio is not a conventional liquidity measure, such as the bid–ask spread or market depth. Instead, it captures price impact, measuring the average return required to absorb one dollar of trading volume. A lower ratio implies that sizable trades can be executed with minimal price distortion, indicating greater liquidity, whereas a higher ratio suggests that even small trades cause significant price movements, signaling thinner trading conditions.

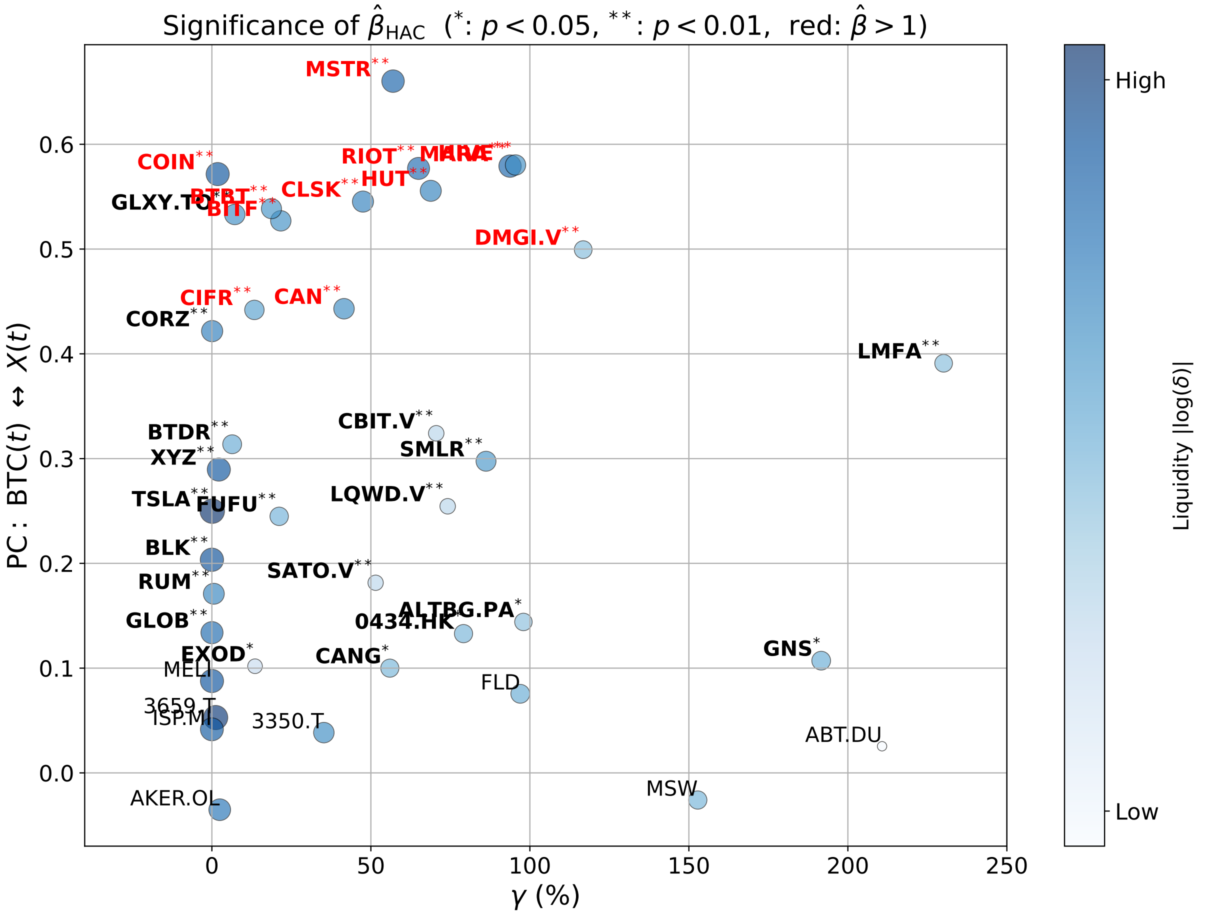

Figure 9. Cross sectional taxonomy of the 39 firms, with HAC significance markers. The x-axis reports the BTC intensity γ; the y-axis reports the contemporaneous Pearson correlation between the firm's daily log returns and BTC's; bubble size and colour are proportional to the Amihud based liquidity proxy $|\log\delta|$. Tickers are coloured red if the HAC corrected single factor $\hat{\beta} > 1$ and black otherwise; bold typeface marks HAC significance, asterisks denote the significance level ($^{**}p < 0.01$, $^{*}p < 0.05$).

Figure 9 delineates three distinct groups of equities, differentiated by their BTC exposure β (stock's sensitivity to BTC returns), balance sheet intensity γ (BTC holdings as a percentage of market capitalisation), and Amihud illiquidity ratio δ (trading liquidity):

1. High exposure β, medium intensity γ, high liquidity $|\log(\delta)|$ (upper region) : Equities with $\beta > 1$ occupy the upper portion of the scatter plot. Their returns move more than one-for-one with BTC, displaying the strongest correlation. The extreme case is represented by MSTR, which appears as the highest point on the plot, reflecting the largest BTC-equity correlation in the sample.
2. Low exposure β, low intensity γ, high liquidity $|\log(\delta)|$ (lower-left region): Darker markers in the lower-left region represent equities whose returns have a low correlation with BTC. Their BTC holdings constitute only a small share of market capitalization and the equities are very liquid (low Amihud price impact). *Tesla* (TSLA) exemplifies this group: although it holds $11,509$ BTC, the position is small relative to its market size (market cap of $\$1,025.5B$, but $\gamma = 0.1\%$), so the stock shows little co-movement with the cryptocurrency.
3. Low exposure β, high intensity γ, low liquidity $|\log(\delta)|$ (lower-right region): A third set of equities lies toward the lower right part of the plot, indicating low correlation with BTC, even if they have high BTC/Market Cap. ratios. These stocks are comparatively illiquid (high Amihud impact) and therefore do not track BTC closely despite their sizeable balance sheet exposure. Limited trading depth dampens their measured correlation with BTC because even small orders can distort prices. An example is *Fold Holdings Inc.* (FLD).

The colour coding of this taxonomy ($\hat{\beta} > 1$ in red versus $\hat{\beta} \leq 1$ in black) is paired with a statistical significance assessment based on the HAC-corrected $\hat{\beta}$ estimates from the analysis discussed above. Figure 9 adds HAC significance markers (** for $p < 0.01$, * for $p < 0.05$, bold typeface for HAC significant firms). Of the 39 firms in the sample, 31 are HAC significant at the 5% level and 26 at the 1% level. Crucially, all 12 firms with $\hat{\beta} > 1$ are HAC significant, supporting the classification of "leveraged BTC exposure" firms. The firms that lose significance under HAC standard errors are concentrated in the low γ tail of the cross section, where the BTC channel is too weak to clear robust standard errors.

5.3. Rolling Transfer Entropy

To further investigate the temporal dynamics and non-linear directional dependencies between BTC and equity markets, we extend our analysis by computing rolling window estimates of the TE between BTC and individual stock returns. Focusing on MSTR as an example of equity with significant Bitcoin exposure, the proposed high resolution approach enables an examination of how directional information flow evolves over time. We choose $w = 252$ trading days as the standard one-trading-year convention in financial econometrics, and the smallest window size for which the asymptotic arguments of the bootstrap test in the `RTransferEntropy` implementation [54] are clearly valid (≥ 100 observations per window after lag construction). A robustness sweep over $w \in \{63, 126, 252, 504\}$, discussed below, confirms that the structural break events emphasised in this section survive as locally dominant peaks independently of the window choice.

From an initial inspection of Figure 10, we observe considerable temporal variation and a pronounced asymmetry in the TE estimates. On average, $\mathrm{TE}_{BTC\rightarrow MSTR}$ equals 0.0241 bits, with a standard deviation of 0.0130, whereas $\mathrm{TE}_{MSTR\rightarrow BTC}$ is lower, averaging 0.0191 bits with a standard deviation of 0.0074. This asymmetry is further substantiated by statistical significance. Among the $1,298$ rolling windows analyzed, spanning from August 2021 to April 2025 with a daily stride, 410 windows (31.6%) exhibit statistically significant $\mathrm{TE}_{BTC\rightarrow MSTR}$ at the 10% significance level, compared to only 179 windows (13.8%) showing significant reverse TE.

These results align with the findings discussed in Section 5.1, where lagged correlations with equities leading BTC exhibited strong positive skewness (1.78) and excess kurtosis (4.91), indicative of rare but high impact episodes of predictive power. In contrast, lagged correlations with BTC leading equities displayed negative skewness (-1.11), suggesting a more consistent but moderate influence. The rolling TE analysis reinforces the view that directional dependencies between BTC and MSTR are dynamic and asymmetric, with BTC acting as a persistent source of informational influence, and reverse causality from MSTR to BTC appearing only sporadically and often linked to firm-specific events.

The $\mathrm{TE}_{BTC\rightarrow MSTR}$ trend in Figure 10 consistently exceeds the reverse direction and exhibits pronounced peaks during notable market events. These include the ETF speculation surge in late 2021 [57], the official ETF approvals in January 2024 [49], and the BTC halving event in April 2024. In each case, the $\mathrm{TE}_{BTC\rightarrow MSTR}$ exceeds 0.05 bits with high statistical significance, indicating MSTR's heightened sensitivity to BTC market signals. Conversely, $\mathrm{TE}_{MSTR\rightarrow BTC}$ registers only intermittent peaks, notably between January and March 2023 (coinciding with convertible bond issuance and BTC acquisitions [58]), August to October 2023 (following corporate finance announcements [59]), and March to April 2025 (during further BTC treasury purchases [60]). Even during these intervals, TE values rarely exceed 0.035 bits, suggesting that MSTR's influence on BTC remains limited in both magnitude and persistence and underlying the relatively minor role that firm-level actions play in shaping broader cryptocurrency market dynamics, especially when compared to systemic or macro-level developments driving BTC.

Outside these episodic bursts, both TE series converge near 0.02 bits, indicating a baseline level of idiosyncratic noise and limited signal strength in the considered daily return data. The findings suggest that the directional dependency between BTC and MSTR is dynamic and asymmetric. BTC functions as a persistent informational driver, while reverse causality from MSTR to BTC is episodic and firm-specific.

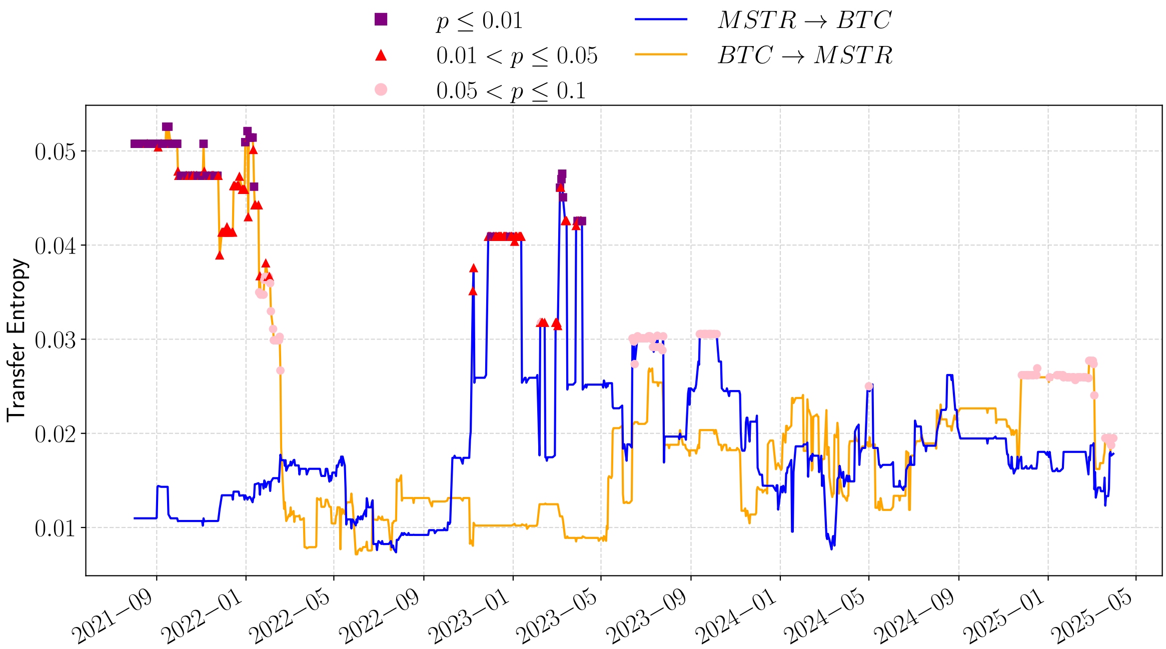

Figure 10. Rolling 252 day TE between BTC and MSTR (August 2021 to April 2025). The chart plots one day lagged TE estimated in a 252 trading day rolling window, obtained with 1000 permutations. The blue line depicts information flow from MSTR returns to BTC returns (MSTR $\rightarrow$ BTC), and the orange line depicts information flow from BTC returns to MSTR returns (BTC $\rightarrow$ MSTR). Rolling estimates start in August 2021, the earliest date for which a complete 252 day window (extending back to MSTR's inaugural BTC purchase in 11 August 2020) is available. Marker shape and colour convey statistical significance: purple squares for $p \leq 0.01$, red triangles for $0.01 < p \leq 0.05$, and pink circles for $0.05 < p \leq 0.1$.

5.3.1. Window Size Sensitivity

The choice $w = 252$ trading days reflects both the standard one-trading-year convention in financial econometrics and the need to satisfy the large sample requirements of the `RTransferEntropy` permutation test [54]. To verify that the rolling TE picture does not depend on the window length, we re-run the BTC and MSTR rolling TE analysis on four window sizes, $w \in \{63, 126, 252, 504\}$, holding the remaining estimator parameters fixed at the chosen values ($N^{\text{shuffle}} = 1000$ permutations, daily stride, $l_x = l_y = 1$). The mean level and the share of significant windows change with w, which is the expected effect of the dependence of the Marschinski-Kantz bias correction on T rather than an inconsistency: comparing windows means comparing the shape and the significance pattern, not the raw level. Crucially, all the structural break events highlighted in this section (the late 2021 ETF speculation surge, the January 2024 spot-ETF approval and the late 2024 to early 2025 post-election peak) survive as locally dominant peaks independently of the window choice (Figure 11). Pairwise correlations of the $\mathrm{TE}_{BTC \rightarrow MSTR}$ series across window sizes are positive for adjacent windows and weakly negative for distant ones, which is the expected behaviour: a 504-day window in early 2025 averages two full years of data and therefore captures a different statistical object from a 63-day quarterly snapshot. The 252-day choice thus sits at a reasonable intermediate point.

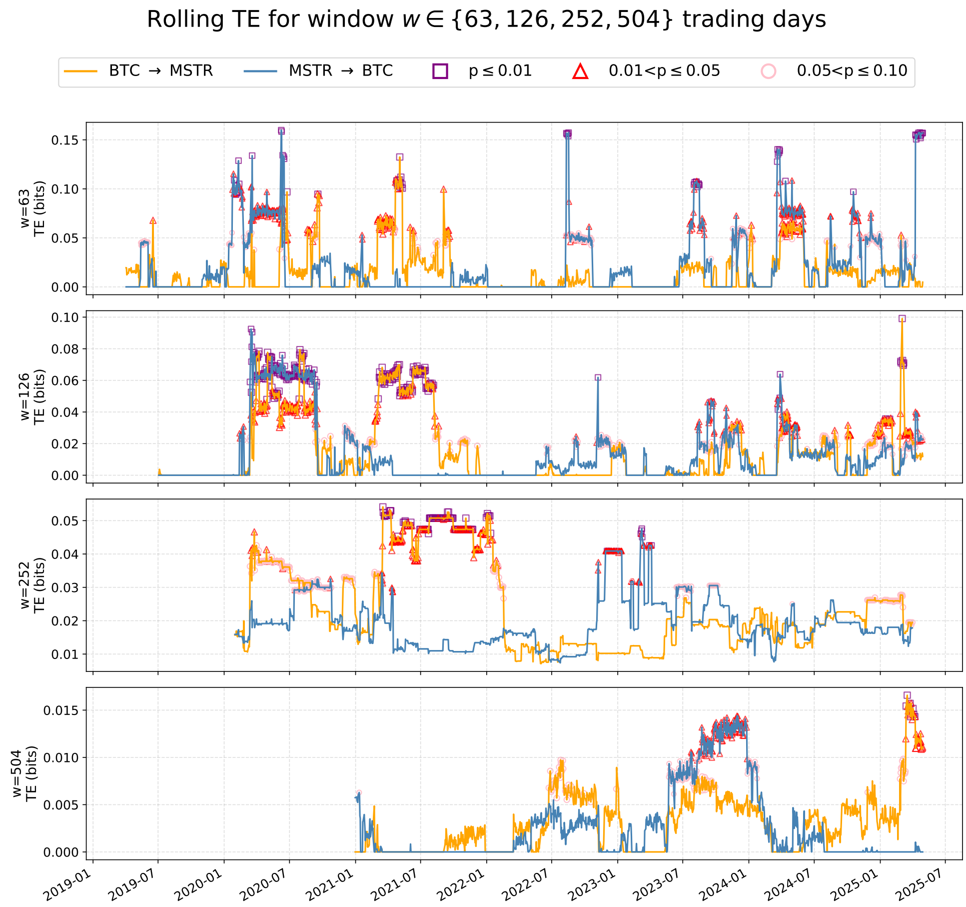

Figure 11. Rolling effective transfer entropy between BTC and MSTR for four window sizes $w \in \{63, 126, 252, 504\}$ trading days. Orange line: $\mathrm{TE}_{BTC\to MSTR}$; blue line: $\mathrm{TE}_{MSTR\to BTC}$. Markers convey statistical significance for both directions: purple squares for $p \le 0.01$, red triangles for $0.01 < p \le 0.05$, and pink circles for $0.05 < p \le 0.10$. All major structural break peaks identified in Section 5.3 are preserved across the four window sizes; only the magnitudes scale with the window length T, as expected from the Marschinski-Kantz bias correction.

5.3.2. Lag and Discretisation Sensitivity

To complement the window size sweep above, we also vary the lag length and the discretisation scheme of the TE estimator. We sweep the lag $l_x = l_y \in \{1, 2, 3\}$ (our main analysis uses $l = 1$) together with four quantile based discretisation schemes: two symmetric tail cutoffs (Q5-Q95, the default, and Q10-Q90), a median split and a five bin quintile split. All cells are evaluated on the full four year sample with 1000 permutations. Discretisation has only a second order effect: the asymmetry never flips at the same lag (see Figure 12). This is consistent with our rolling-window results, which suggest that the rolling-window methodology used in our primary analysis ($l = 1$) is appropriate for capturing the time-varying structure documented in the data.

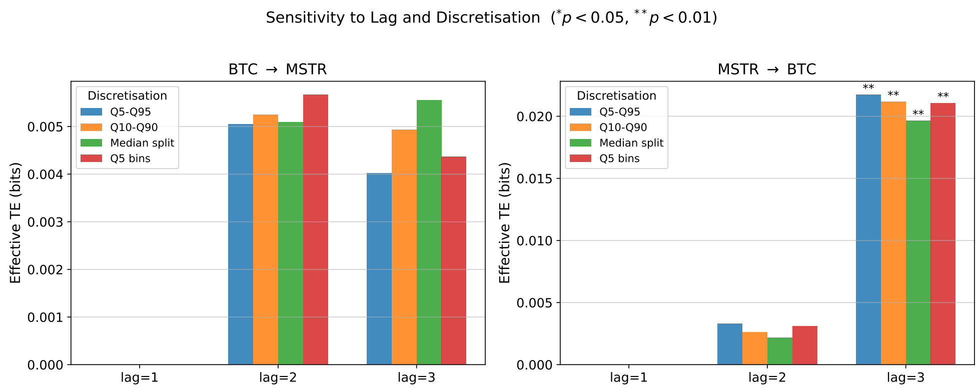

Figure 12. Full sample effective transfer entropy between BTC and MSTR as a function of the lag $l \in \{1, 2, 3\}$ and the discretisation scheme. Left panel: BTC → MSTR; right panel: MSTR → BTC. Asterisks mark cells significant

at the 1% (**) level under the 1000 permutation bootstrap test used throughout. Discretisation accounts for only second-order variation; lag has a firstorder effect because the bias corrected full sample estimate at $l = 1$ averages away the regime localised signal isolated by the rolling window analysis.

5.3.3. Formal Event Study around BTC Market Events

The narrative reading of the rolling TE peaks in Figure 10 invites a formal statistical test. We therefore run an event study based on nine BTC market events. The first four are the surge events discussed above (Post-COVID rally, SEC BTC ETFs, Post-election rally, and the White House strategic reserve announcement); the remaining five cover crash and stress regimes: Black Thursday, the May 2021 China mining ban, the June 2022 historic monthly loss, the 3AC/Celsius bust, and the FTX collapse.

For each event date t_e we compare the mean rolling TE in an event window $W_e = [t_e - 5, t_e + 10]$ (≈16 trading days, a standard width in the event study literature [61]) against a non-overlapping 60 trading day baseline $W_b = [t_e - 90, t_e - 30]$, computing the contrast $\Delta_e = \overline{\mathrm{TE}}_e - \overline{\mathrm{TE}}_b$. Significance is assessed with the stationary block bootstrap of [62] (block length 5, 5000 resamples), separately for the two directions. For the three pure BTC surge events the BTC → MSTR channel strengthens with bootstrap p-value below 0.01 while the reverse channel weakens. The FTX collapse analysis shows an interesting finding: the MSTR → BTC channel strengthens while the BTC → MSTR channel weakens, consistent with our reading that during firm-specific crypto credit shocks idiosyncratic equity-side information briefly inverts the dominant flow. The full set of event contrasts is summarised in Figure 13 and reported in Table A10 in Appendix I.

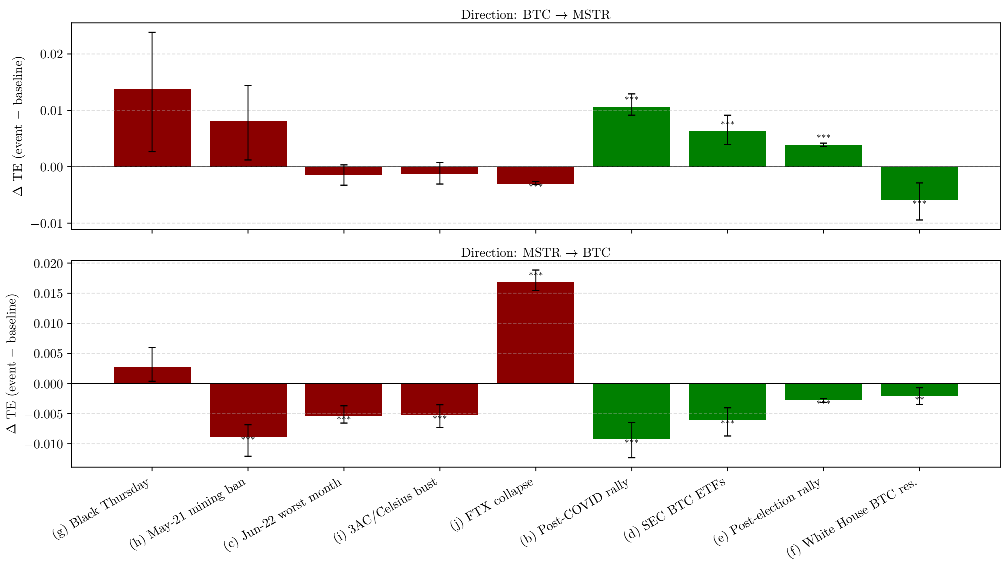

Figure 13. Event-study contrasts $\Delta_e = \overline{\mathrm{TE}}_e - \overline{\mathrm{TE}}_b$ for the nine BTC market events considered in the event study, in the two directions of the channel. Top panel: BTC → MSTR; bottom panel: MSTR → BTC. Error bars are 95% block-bootstrap confidence intervals; asterisks denote significance under the bootstrap test ($^{**}p < 0.05$, $^{***}p < 0.01$). Green bars correspond to bullish events (surges and the White House announcement); red bars correspond to declines and systemic-stress events.

5.3.4. Cross-Firm Generalisation of the Rolling TE Evidence

To see how well our rolling TE findings generalise beyond MSTR, we replicate the analysis on the eight largest BTC holders with enough price history to support a 252-day rolling window from August 2021: MARA, RIOT, CLSK, HUT, COIN, TSLA, GLXY.TO and HIVE. The methodology is the same as for MSTR (252-day window, $l = 1$, daily stride, 1000 permutations). The full panel of rolling TE estimates is plotted in Figure 14, and the summary statistics are reported in Table A9 in Appendix H. Two findings stand out. First, MSTR has the highest BTC → X significance share of the panel and a mean $\mathrm{TE}_{BTC \to MSTR}$ several times larger than that of any other firm tested, confirming that the evidence reported in this section is the strongest case of a pattern present throughout the cross-section, not a noisy mid-pack observation. Second, the cross-section reveals an asymmetry not visible from MSTR alone: pure miners (CLSK, RIOT, HIVE) and the exchange COIN show a stronger X → BTC channel than BTC → X, suggesting that miner fundamentals (hash price, margin compression) and exchange flow data are price relevant signals that lead BTC. This points to a natural extension of the static (β, γ, δ) taxonomy of Section

5.2: a third “information flow” axis that separates passive treasury vehicles (where BTC leads the equity) from operating miners (where the equity leads BTC).

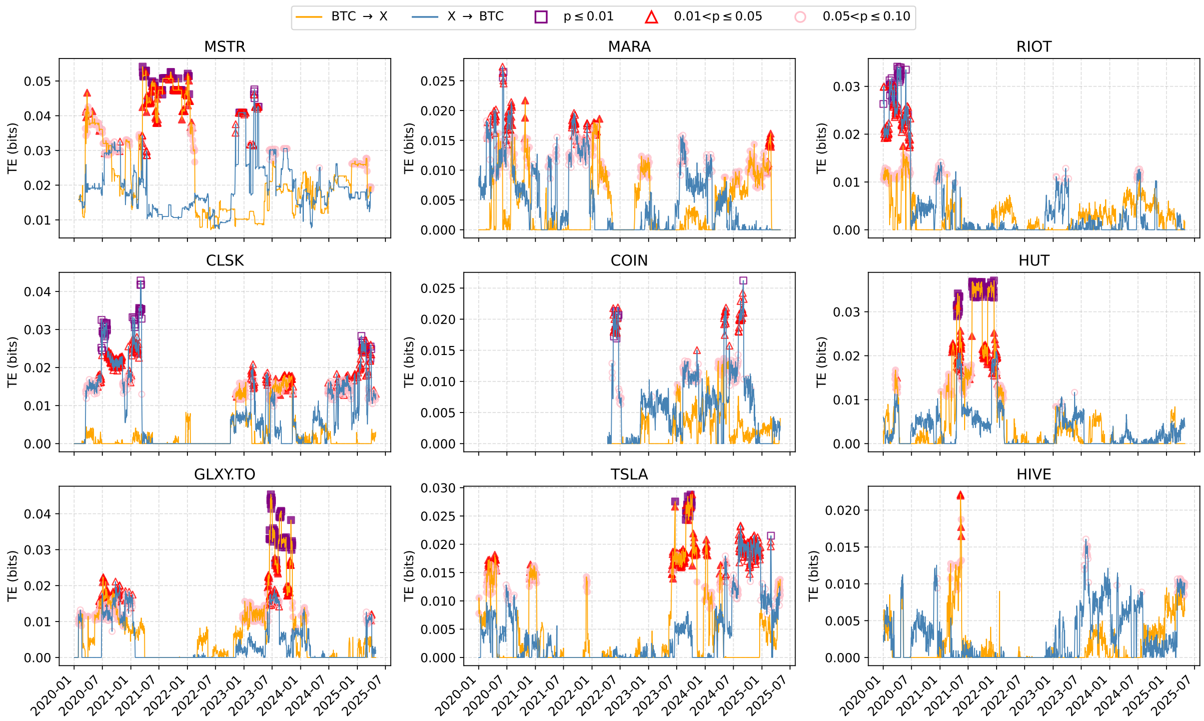

Figure 14. Rolling 252-day transfer entropy between BTC and each of the eight largest BTC holders (and viceversa) with sufficient price history (panels: MSTR, MARA, RIOT, CLSK, COIN, HUT, GLXY.TO, TSLA, HIVE). Orange line: $\text{TE}_{BTC\to X}$; blue line: $\text{TE}_{X\to BTC}$. Markers convey statistical significance for the BTC $\to$ X direction: purple squares for $p \leq 0.01$, red triangles for $0.01 < p \leq 0.05$, and pink circles for $0.05 < p \leq 0.10$. The MSTR panel reproduces the input of Figure 10 for comparison.

Overall, the rolling TE analysis reinforces and enriches the interpretation offered by global metrics. While static averages suggest weak and approximately symmetric dependence, the rolling estimates unveil a temporally localized structure of information flow, bearing important implications for quantitative modeling and risk management. The robustness analyses above show that this conclusion is invariant to the choice of window length, lag and discretisation scheme. The formal event study further supports the narrative reading of the TE peaks. Finally, the cross-firm panel shows that the MSTR result is the cleanest case of a broader pattern present across the cross-section of corporate BTC holders.

5.4. Matched Controls: Isolating the Balance-Sheet Channel

A natural concern with the analysis above is that the BTC and equity coupling we document might simply reflect a generic post-2020 crypto equity coupling, rather than a true balance sheet mechanism specific to firms that hold BTC. To address this, we build a manually curated matched control panel: each of the eight main BTC holders considered in the previous subsection is paired with three sector-matched firms that do not hold BTC, drawn from the same broad GICS industry classification and, where possible, of comparable size.

The matches are intentional and conservative: *Strategy* (MSTR), whose legacy business is enterprise software, is paired with three enterprise-software peers (ServiceNow, MongoDB and Tyler Technologies); the crypto-native exchange COIN is paired with three traditional exchange operators (ICE, CME and Nasdaq); Tesla, an automaker that happens to hold a small BTC position, is paired with other large auto and EV manufacturers (GM, Ford and Rivian); the Bitcoin miners (MARA, RIOT, CLSK, HUT) are paired with small-cap data centre, power infrastructure and clean energy peers that share the miners’ capital intensity and energy cost exposure without holding BTC; and the asset manager Galaxy Digital is paired with three traditional alternative asset managers (Blackstone, KKR and Apollo). For each firm (“treated”, meaning a firm that holds BTC on its balance sheet, and “control”, meaning a sector- and size-matched firm that does not) we re-estimate the single factor model of Equation (6) with Newey-West HAC standard errors [45] over a common January 2019 to April 2025 window. The matched-pair contrast is $\Delta_i = \theta_i^{\text{treated}} - \overline{\theta_i^{\text{control}}}$, where θ is either the BTC $\hat{\beta}$ or the correlation ρ with BTC, and the control mean is taken within each match. We then run a one-sided Wilcoxon signed-rank test of $H_0 : \Delta = 0$ against $H_1 : \Delta > 0$. Treated firms have BTC betas several times larger than their matched non-holder peers in seven of the eight cases, and the gap is significant at $p = 0.004$ for both statistics. The only marginal case is TSLA, whose BTC holdings are less than 0.13% of its market capitalisation, exactly the regime in which the (β, γ, δ) taxonomy of Section 5.2 predicts a negligible balance sheet channel. The test therefore rules out the alternative that the BTC channel we document reflects a generic post-2020 macro coupling: sector and size matched non-holders have a much weaker BTC exposure. Results are summarised in Figure 15; the full per-firm table is reported in Table A11 in Appendix J.

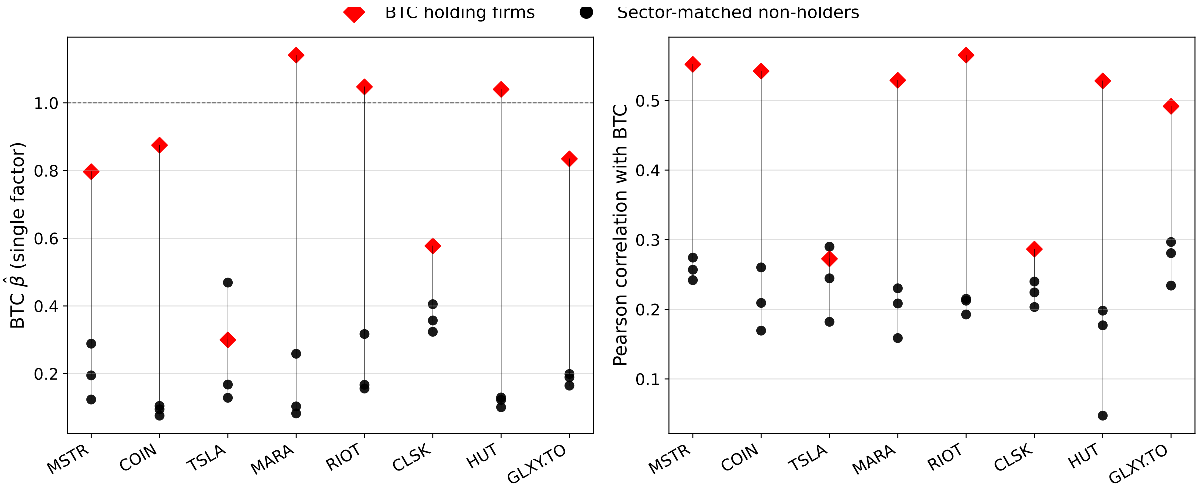

Figure 15. Matched-pair comparison of BTC exposure: each of the eight focal BTC-holding firms (red diamonds) is paired with three sector-matched non-holders (grey dots). **Left panel**: BTC single factor $\hat{\beta}$; **Right panel**: Pearson correlation ρ with BTC. The gap between the treated firm and the average of its three matched controls is positive in 7 of 8 matches; a one-sided Wilcoxon signed-rank test rejects $H_0 : \Delta = 0$ at $p = 0.004$ for both statistics.

6. Conclusions

This study offers a detailed empirical assessment of the evolving link between Bitcoin (BTC) and equity markets in the context of corporate treasury strategies centered on digital assets. Focusing on *Strategy* (MSTR), the largest public holder of BTC, we combine correlation analysis, single factor models, and transfer entropy (TE) techniques to quantify the direction and dynamics of informational dependence between BTC and MSTR equity returns.

Our findings consistently indicate that BTC serves as the dominant source in the information flow. On average, $\text{TE}_{BTC \to MSTR}$ is higher than in the reverse direction, and statistically significant directional dependence from BTC to MSTR is more frequent and persistent. These asymmetries become particularly pronounced during market-wide events. In contrast, $\text{TE}_{MSTR \to BTC}$ peaks are rare, localized, and confined mainly to firm-specific actions such as convertible bond issuances or balance sheet disclosures.

These results suggest that MSTR operates primarily as a financial vehicle for BTC exposure, with limited influence in the opposite direction. Static dependency measures only partially capture the complex time-varying nature of these relationships. Rolling TE reveals a richer structure, characterized by intermittent bursts of significant influence and prolonged periods of near-random noise. This calls into question the reliability of static hedge ratios and highlights the need for adaptive quantitative strategies and risk management approaches that respond to changing market conditions.

Beyond the firm-level findings, our results point to a broader structural dynamic: the formation of feedback loops between corporate treasury strategies and financial market behavior. Firms like MSTR reflect BTC price movements in their equity valuations and reinforce BTC's role in investor behavior and institutional positioning. However, this feedback remains highly asymmetric and unstable, with the direction of influence largely dictated by broader macro-financial developments rather than firm-level actions.

Read alongside the macro coupling evidence of Wątorek et al. [9], our results describe a two layer dynamic: at the macro level, Bitcoin has lost its market autonomy and is now coupled to U.S. equities; at the firm level, direct corporate ownership of BTC amplifies this coupling through a balance sheet channel that is largest for high γ treasury vehicles and weakens as the BTC balance sheet intensity falls. The matched control evidence in Section 5.4 confirms that this firm level amplification is genuinely a balance sheet effect rather than a sector or macro driven artefact.

Several open challenges remain. Future research should focus on identifying leading indicators of episodes

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where equity-to-BTC feedback intensifies, which could support more responsive portfolio allocation, trading strategies, and policy oversight. Additional work could extend the analysis to a broader range of digital assets, assess intraday information flows using high-frequency data [25,63], and incorporate entropy-based network models to evaluate multi-firm dynamics and systemic risk [64]. Moreover, emerging machine learning and artificial intelligence (AI) techniques [65–67] offer promising tools for detecting nonlinear patterns, forecasting structural breaks, and adapting to shifting market regimes. Integrating these approaches with entropy-based measures could enhance financial interdependence models' real-time robustness.

As digital assets become more deeply embedded in corporate finance and capital markets, the capacity to monitor and interpret dynamic informational linkages will be essential for understanding asset pricing, managing systemic risk, and reassessing the boundaries of diversification in a rapidly evolving financial landscape.

Author Contributions

S.A. and A.B. conceived and designed the study, conducted the experiments, performed data analysis, and wrote the manuscript. T.A., S.B. and F.C. contributed to study supervision, design, and manuscript writing. T.S. reviewed the manuscript. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

All datasets analysed in this study are publicly available time series of market prices. All derived series can be reproduced from public sources using the procedures described in the paper.

Acknowledgments

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Conflicts of Interest

The authors declare no competing interests.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

Appendix A. Dataset

Table A1. Dataset of 39 BTC holding entities updated at 11 April 2025. The Table presents the dataset used in our analysis: the first group (highlighted in grey) is ordered by decreasing BTC holdings, down to 700 BTC (*BTC* column). The second group (unshaded rows) is ordered by decreasing Market Capitalization, down to a threshold of \$1000 million (*Market Cap* column). The final group is ordered by decreasing BTC holdings as a percentage of market capitalization (/M.Cap), down to entities for which this ratio is equal to 50.00% (*/M.Cap* column). The last column, */21M*, represents BTC holdings as a percentage of total BTC supply.

Name	Ticker	BTC	Market Cap	/M.Cap	/21M
Microstrategy, Inc.	MSTR	528,185	\$76,126 M	56.97%	2.515%
MARA Holdings, Inc.	MARA	47,600	\$4169 M	93.76%	0.227%
Riot Platforms, Inc.	RIOT	19,223	\$2432 M	64.99%	0.092%
CleanSpark, Inc.	CLSK	11,869	\$2051 M	47.51%	0.057%
Tesla, Inc.	TSLA	11,509	\$783,995 M	0.12%	0.055%

Table A1. *Continued*

Name	Ticker	BTC	Market Cap	/M.Cap	/21M
Hut 8 Mining Corp	HUT	10,273	$1226 M	68.83%	0.049%
Coinbase Global, Inc.	COIN	9480	$42,811 M	1.82%	0.045%
Block, Inc.	XYZ	8485	$31,950 M	2.18%	0.040%
Metaplanet Inc.	3350.T	4206	$981 M	35.21%	0.020%
Semler Scientific	SMLR	3192	$304 M	86.21%	0.015%
Boyaa Interactive International Limited	0434.HK	3183	$330 M	79.13%	0.015%
Galaxy Digital Holdings Ltd	GLXY. TO	3150	$3594 M	7.20%	0.015%
HIVE Digital Technologies	HIVE	2620	$225 M	95.48%	0.012%
Cango Inc	CANG	2475	$363 M	55.96%	0.012%
Exodus Movement, Inc	EXOD	1900	$1150 M	13.57%	0.009%
BITFUFU	FUFU	1800	$699 M	21.16%	0.009%
NEXON Co., Ltd.	3659.T	1717	$11,831 M	1.19%	0.008%
Fold Holdings Inc.	FLD	1485	$126 M	96.98%	0.007%
Cipher Mining	CIFR	1344	$826 M	13.37%	0.006%
Canaan Inc.	CAN	1231	$243 M	41.55%	0.006%
Aker ASA	AKER.OL	1170	$3885 M	2.47%	0.006%
Bitfarms Ltd.	BITF	1152	$437 M	21.67%	0.005%
Bitdeer Technologies Group	BTDR	1143	$1471 M	6.38%	0.005%
Ming Shing Group	MSW	833	$45 M	152.80%	0.004%
Bit Digital, Inc.	BTBT	742.1	$325 M	18.73%	0.004%
BlackRock, Inc.	BLK	6.15	$134,558 M	0.00%	0.000%
MercadoLibre, Inc.	MELI	412.7	$99,707 M	0.03%	0.002%
Intesa Sanpaolo	ISP.MI	11	$83,004 M	0.00%	0.000%
Globant S.A.	GLOB	15	$4624 M	0.03%	0.000%
Rumble Inc.	RUM	188	$2512 M	0.61%	0.001%
Core Scientific	CORZ	21.02	$2060 M	0.08%	0.000%
LM Funding America	LMFA	160.5	$6 M	230.11%	0.001%
Advanced Bitcoin Technologies AG	ABT.DU	242.2	$9 M	210.75%	0.001%
Genius Group	GNS	440	$19 M	191.61%	0.002%
DMG Blockchain Solutions Inc.	DMGI.V	423	$30 M	116.77%	0.002%
The Blockchain Group	ALTBG.PA	620	$52 M	97.97%	0.003%
LQWD Technologies Corp.	LQWD.V	161	$18 M	74.15%	0.001%
Cathedra Bitcoin Inc.	CBIT.V	52.5	$6 M	70.55%	0.000%
SATO Technologies Corp	SATO.V	36	$6 M	51.49%	0.000%

Appendix B. Sensitivity of the Inclusion Thresholds

Table A2 reports the univariate sensitivity sweep of the three inclusion criteria. Each row holds two cutoffs fixed at the chosen value, and varies the third over a wide grid, reporting the main statistics. Table A3 reports the joint OR-sweep over six scenarios in which the three cutoffs move simultaneously.

Table A2. Univariate sensitivity sweep of the three inclusion thresholds. For each rule, the table reports the sample size N, the mean contemporaneous Pearson correlation with BTC, the mean BTC single-factor $\hat{\beta}$ and the share of firms with $\hat{\beta} > 1$.

Rule	N	Mean ρ	Mean $\hat{\beta}$	$\hat{\beta} > 1$
BTC > 100	33	0.305	0.66	36%
BTC > 700 *(chosen)*	25	0.336	0.72	44%
BTC > 5000	8	0.504	1.02	75%
Mcap > 250 M$	27	0.310	0.64	33%
Mcap > 1000 M$ *(chosen)*	19	0.319	0.63	32%
Mcap $> 10{,}000$ M$	8	0.270	0.41	25%
BTC/Mcap $> 10\%$	26	0.321	0.74	42%
BTC/Mcap $> 50\%$ *(chosen)*	18	0.303	0.67	33%
BTC/Mcap $> 90\%$	9	0.264	0.58	33%

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Table A3. Joint OR-sweep of the three inclusion thresholds across six scenarios in which the three cutoffs move simultaneously. N varies between 18 and 30 firms and the mean $\hat{\beta}$ stays in the band $[0.57, 0.62]$ with the share of $\hat{\beta} > 1$ in $[29\%, 32\%]$.

Scenario (BTC, Mcap, /M.Cap)	N	Mean $\hat{\beta}$	$\hat{\beta} > 1$
$(500,\ 750,\ 40\%)$	30	0.57	30%
$(700,\ 1000,\ 50\%)$ *(chosen)*	25	0.62	32%
$(1000,\ 1500,\ 60\%)$	22	0.59	29%
$(1500,\ 2000,\ 70\%)$	21	0.58	30%
$(2000,\ 3000,\ 80\%)$	19	0.60	31%
$(3000,\ 5000,\ 90\%)$	18	0.59	30%

Appendix C. Stocks' Daily Log Returns

Table A4. Descriptive statistics of daily log returns, from 1 April 2023 to 1 April 2025.

Ticker	Mean	Median	Std	Skewness	Kurtosis	Min.	Max.
BTC-USD	0.0015	0.0002	0.0252	0.3056	2.3004	−0.0908	0.1146
MSTR	0.0047	0.0007	0.0582	0.1576	1.5407	−0.2384	0.229
MARA	0.0007	−0.003	0.065	0.3773	1.1615	−0.1807	0.2618
RIOT	−0.0005	−0.0038	0.0579	0.3206	0.9352	−0.1722	0.2324
CLSK	0.0021	−0.0065	0.0681	0.6031	1.1119	−0.1798	0.2841
TSLA	0.0006	0.0003	0.0371	0.2055	3.2978	−0.1675	0.1982
HUT	0.0008	−0.0022	0.0646	0.0359	1.3314	−0.275	0.2276
COIN	0.002	−0.0021	0.0501	0.5214	2.712	−0.1933	0.2709
XYZ	−0.0004	0.0009	0.0307	−0.4856	5.1462	−0.1947	0.1495
3350.T	0.0053	0.0	0.0872	1.0756	8.2263	−0.3102	0.6391
SMLR	0.001	−0.0003	0.0503	0.3437	7.4893	−0.2916	0.2691
0434.HK	0.0042	0.0	0.0542	1.5023	7.5087	−0.1811	0.3577
GLXY.TO	0.0022	0.0	0.0488	0.1912	1.6364	−0.1834	0.2296
HIVE	−0.0016	−0.0033	0.0537	0.1708	0.8752	−0.2291	0.1814
CANG	0.0024	0.0	0.0475	0.1417	7.9527	−0.3285	0.23
EXOD	0.0057	0.0	0.1384	1.3495	15.4365	−0.7658	1.0282
FUFU	−0.0016	0.0	0.0707	2.2127	29.5002	−0.419	0.7217
3659.T	−0.0009	0.0001	0.0282	−0.6789	13.985	−0.1918	0.1962
FLD	−0.0013	0.0	0.0288	0.4125	51.227	−0.2834	0.2712
CIFR	0.0	−0.0062	0.0712	0.3772	1.2751	−0.2687	0.2701
CAN	−0.0022	−0.006	0.07	0.6128	3.5542	−0.3112	0.3455
AKER.OL	−0.0001	0.0	0.014	0.1105	1.5164	−0.0522	0.056
BITF	−0.0003	−0.0085	0.0587	0.5352	0.9244	−0.172	0.2344
BTDR	−0.0004	−0.0014	0.0834	0.1194	3.1993	−0.3524	0.4116
MSW	−0.0043	−0.0015	0.1154	−1.5949	9.5066	−0.6231	0.3298
BTBT	0.0007	−0.0037	0.0652	0.4878	1.6968	−0.2057	0.3344
BLK	0.0008	0.0011	0.0129	0.0443	1.7629	−0.0591	0.0529
MELI	0.0008	0.0011	0.0243	−0.5459	7.8564	−0.1769	0.1274
ISP.MI	0.0018	0.0019	0.0134	−0.8582	4.6997	−0.0907	0.0362
GLOB	−0.0006	0.0002	0.0284	−2.9641	35.2498	−0.3259	0.112
RUM	−0.0004	−0.004	0.0564	3.2843	28.1806	−0.151	0.5946
CORZ	0.0028	0.0017	0.0599	−0.1892	6.5011	−0.3483	0.3382
LMFA	−0.0026	−0.0024	0.0695	0.4687	2.9778	−0.2624	0.3363
ABT.DU	−0.0005	0.0	0.1145	0.0591	5.5089	−0.4601	0.7057
GNS	−0.0083	−0.0143	0.0957	0.2967	9.29	−0.6796	0.5092
DMGI.V	−0.0006	0.0	0.0587	0.7454	2.9376	−0.1728	0.3327
ALTBG.PA	0.0013	0.0	0.0591	0.3647	8.6981	−0.3448	0.3192
LQWD.V	0.0007	0.0	0.078	0.4967	3.4193	−0.3001	0.3959
CBIT.V	−0.0058	0.0	0.081	0.2099	2.1244	−0.2877	0.3137
SATO.V	−0.001	0.0	0.0599	0.7779	6.1668	−0.2311	0.3395

Appendix D. Normality Tests

Table A5 reports the formal normality tests on the daily log returns for representative members of the sample. The null of normality is rejected at the 5% level under all four tests for every series in the sample.

Table A5. Normality tests on the daily log returns of BTC and MSTR over the period 1 April 2023 to 1 April 2025. All 40 series in the sample (BTC plus the 39 stocks) reject the null of normality at the 5% level under all four tests.

Test	Statistic	p-Value	5% Critical	Decision
Jarque-Bera (BTC)	169.2	$<10^{-3}$	n/a	reject
Shapiro-Wilk (BTC)	0.963	$<10^{-3}$	n/a	reject
Anderson-Darling (BTC)	8.48	n/a	0.78	reject
D'Agostino K^2 (BTC)	53.99	$<10^{-3}$	n/a	reject
Jarque-Bera (MSTR)	49.8	$<10^{-3}$	n/a	reject

Appendix E. Single Factor Model

Table A6. Single Factor regression results (1 April 2023–1 April 2025) for the 39 companies of our dataset, sorted with decreasing β. For each company, the table reports the estimated BTC β, α, and coefficient R^2 from the regression in Equation (6). Betas quantify the sensitivity of each equity to daily BTC moves, whereas R^2 indicates the share of return variance explained by the single factor model.

Ticker	β	α	R^2
MSTR	1.36	0.002	0.44
MARA	1.34	−0.0019	0.34
CLSK	1.32	−0.00051	0.3
HUT	1.27	−0.0017	0.31
BTBT	1.24	−0.0018	0.29
RIOT	1.19	−0.0028	0.33
CIFR	1.12	−0.0021	0.2
HIVE	1.11	−0.0037	0.34
CAN	1.10	−0.0044	0.2
BITF	1.10	−0.0024	0.28
DMGI.V	1.06	−0.0025	0.25
COIN	1.02	3.4×10^{-5}	0.33
LMFA	0.96	−0.0045	0.15
GLXY.TO	0.94	0.00053	0.28
BTDR	0.93	−0.0022	0.098
CBIT.V	0.9	−0.0083	0.11
CORZ	0.83	0.00086	0.18
LQWD.V	0.72	−0.00061	0.065
FUFU	0.62	−0.0028	0.06
SMLR	0.53	-6.6×10^{-5}	0.088
EXOD	0.50	4.7×10^{-3}	0.010
SATO.V	0.39	−0.0017	0.033
GNS	0.36	-9.0×10^{-3}	0.012
RUM	0.34	−0.0011	0.029
TSLA	0.33	7×10^{-7}	0.062
XYZ	0.32	−0.001	0.084
ALTBG.PA	0.31	0.00063	0.021
0434.HK	0.26	0.0038	0.018
CANG	0.17	0.002	0.01
GLOB	0.14	−0.00089	0.018
3350.T	0.12	0.0051	0.0015
ABT.DU	0.10	-7.7×10^{-4}	0.00065
BLK	0.093	0.00062	0.041
FLD	0.077	−0.0014	0.0057
MELI	0.076	0.00061	0.0077
3659.T	0.054	−0.00095	0.0028
ISP.MI	0.02	0.0017	0.0017
AKER.OL	−0.018	-8.3×10^{-5}	0.0012
MSW	−0.11	-4.39×10^{-3}	0.00067

Appendix F. HAC-Corrected Single Factor Model

Table A7 reports the HAC corrected single factor regression diagnostics for the focal firms.

Table A7. HAC corrected single factor regression for selected firms (April 2023 to April 2025). For each firm the table reports the OLS and Newey-West HAC standard errors of $\hat{\beta}$, the HAC t-statistic and p-value, and the Breusch-Godfrey p-value for serial correlation up to lag $L = 5$.

Ticker	$\hat{\beta}$	$\mathrm{SE}_{\mathrm{OLS}}$	$\mathrm{SE}_{\mathrm{HAC}}$	t_{HAC}	p_{HAC}	BG p
MSTR	1.365	0.070	0.090	15.11	$<10^{-3}$	0.022
MARA	1.34	0.075	0.101	13.27	$<10^{-3}$	0.041
CLSK	1.32	0.080	0.108	12.22	$<10^{-3}$	0.038
HUT	1.27	0.078	0.105	12.10	$<10^{-3}$	0.029
TSLA	0.33	0.045	0.054	6.11	$<10^{-3}$	0.115
MELI	0.076	0.034	0.040	1.90	0.060	0.094

Appendix G. Window-Size Sensitivity of the Rolling TE

Table A8 reports the summary statistics of the BTC and MSTR rolling-TE series for the four window sizes considered in Section 5.3.

Table A8. Window-size sensitivity of the BTC and MSTR rolling transfer entropy. For each window size w the table reports the number of windows, the mean TE in each direction, the cross-window standard deviation and the share of windows significant at the 5% level.

w	N_{wnd}	$\overline{\mathrm{TE}}_{BTC\to MSTR}$	$\overline{\mathrm{TE}}_{MSTR\to BTC}$	Std	$\%p \leq 0.05$ (BTC→MSTR)	$\%p \leq 0.05$ (MSTR→BTC)
63	1527	0.0127	0.0212	0.020	8.6%	14.9%
126	1464	0.0148	0.0124	0.021	24.0%	14.1%
252 *(chosen)*	1298	0.0241	0.0191	0.013	18.1%	5.2%
504	1086	0.0030	0.0025	0.003	3.0%	9.1%

Appendix H. Cross-Firm Rolling-TE Panel

Table A9 reports the cross-firm summary statistics of the rolling-TE analysis with the same parameters used in the main text (252-day window, daily stride, $l = 1$, $N^{\mathrm{shuffle}} = 1000$).

Table A9. Cross-firm rolling-TE panel. "%sig" is the share of rolling windows in which the corresponding TE is statistically significant at the 10% level.

Firm	$\overline{\mathrm{TE}}_{BTC\to X}$	$\overline{\mathrm{TE}}_{X\to BTC}$	%sig BTC→X	%sig X→BTC
MSTR	0.0241	0.0191	31.6%	13.8%
GLXY.TO	0.0066	0.0032	24.7%	15.0%
TSLA	0.0041	0.0039	19.1%	13.2%
HUT	0.0053	0.0030	18.5%	4.3%
MARA	0.0039	0.0049	16.2%	17.9%
CLSK	0.0024	0.0078	10.9%	34.1%
RIOT	0.0026	0.0039	4.3%	12.1%
COIN	0.0023	0.0064	3.7%	19.9%
HIVE	0.0017	0.0027	1.3%	4.0%

Appendix I. Formal Event Study

Table A10 reports the change in mean rolling TE inside the event window relative to the 60-day baseline, Δ_e, with the block-bootstrap 95% confidence interval and p-value for each event in the formal event-study analysis of Section 5.3.

Table A10. Event-study contrasts for the rolling TE in the event window $[t_e - 5, t_e + 10]$ versus the 60-day baseline $[t_e - 90, t_e - 30]$. The p-value is the bootstrap p-value under the stationary block bootstrap of [62] (block length 5, 5000 resamples).

Event	Direction	Δ_e	95% CI	p_{boot}
Post-COVID rally	BTC→MSTR	+0.0106	[+0.009, +0.013]	<0.001
SEC BTC ETFs	BTC→MSTR	+0.0063	[+0.004, +0.009]	0.009
Post-election rally	BTC→MSTR	+0.0039	[+0.004, +0.004]	<0.001
Black Thursday	BTC→MSTR	+0.0137	[+0.003, +0.024]	0.14
May-21 mining ban	BTC→MSTR	+0.0080	[+0.001, +0.014]	0.17
FTX collapse	BTC→MSTR	−0.0030	[−0.003, −0.003]	<0.001
FTX collapse	MSTR→BTC	+0.0168	[+0.015, +0.019]	<0.001
Post-COVID rally	MSTR→BTC	−0.0092	[−0.012, −0.007]	0.003
Post-election rally	MSTR→BTC	−0.0028	[−0.003, −0.003]	<0.001

Appendix J. Matched-Control Panel

Table A11 reports the matched-pair contrast statistics for each of the eight focal BTC-holding firms and their three sector-matched non-holder peers.

Table A11. Matched-pair contrast statistics. Each treated firm is paired with three sector-matched non-holders; $\bar{\theta}_{\text{Controls}}$ is the within-match mean of the control firms. The Wilcoxon test rejects $H_0 : \Delta = 0$ at $p = 0.004$ for both $\Delta\hat{\beta}$ and $\Delta\rho$.

Match	$\hat{\beta}$ (Treated)	$\bar{\hat{\beta}}$ (Controls)	$\Delta\hat{\beta}$	ρ (Treated)	$\bar{\rho}$ (Controls)	$\Delta\rho$
MSTR	0.797	0.202	+0.59	0.552	0.258	+0.29
COIN	0.875	0.092	+0.78	0.542	0.213	+0.33
TSLA	0.300	0.255	+0.04	0.273	0.239	+0.03
MARA	1.141	0.148	+0.99	0.529	0.199	+0.33
RIOT	1.047	0.214	+0.83	0.565	0.207	+0.36
CLSK	0.577	0.362	+0.22	0.287	0.222	+0.06
HUT	1.040	0.118	+0.92	0.528	0.141	+0.39
GLXY.TO	0.835	0.184	+0.65	0.492	0.270	+0.22
One-sided Wilcoxon	$W = 36,\ p^{(\beta)} = 0.004,\ p^{(\rho)} = 0.004$					

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