

Review

Nonlinear Dynamics of Bio-Inspired X-Structures: Progress Review

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Abstract: Nonlinearities are critical in engineering systems, making their intentional utilization vital for structural design and dynamic analysis. Inspired by animal limb skeletons, X-structures offer a passive, low-cost, and highly adjustable approach to achieving beneficial and controllable nonlinearity in terms of stiffness, damping and inertia, also featured by elegant static and dynamic models for nonlinear analysis, design and prediction with less burden in a trial and error cycle. Since the establishment of foundational principles, relevant studies have been expanded dramatically. This review synthesizes recent advances, tracing the evolution of X-structures from vibration isolation to versatile platforms with complex designs and interdisciplinary integration. In nonlinear stiffness, QZS ranges exceed 85% of total stroke with resonant frequencies below 1 Hz, enabled by contact mechanisms, magnetic compensation, and active modulation. Nonlinear damping has advanced via integration with viscoelastic materials, friction elements, and passive magnetorheological dampers, achieving motion-dependent dissipation that suppresses resonance without deteriorating high-frequency isolation. Nonlinear inertia including several benchmark nonlinear forms is revealed as a third dimension for achieving desired system dynamic response. Architecturally, X-structures have evolved from single-axis isolators to 3-DOF units, 6-DOF Stewart platforms, and hierarchical designs. Interdisciplinary integration has yielded active control with 85% energy savings, metamaterials with 98% energy absorption, and self-powered sensing interfaces. Applications now venture into robotics, spacecraft on-orbit capture, and energy harvesting. This review concludes by identifying persistent challenges including high-dimensional nonlinear modeling, manufacturing precision, and extreme-condition reliability. Future directions focus on digital/intelligent X-structures, cross-scale, and sustainable X-structures. This scalable approach to beneficial nonlinearity will advance safer, greener and more efficient engineering systems.

Keywords: X-structure; geometric nonlinearity; multifunctional integration; vibration isolation



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1. Introduction

Nonlinear phenomena are often treated within frameworks where linear system approximations remain applicable. Traditionally, nonlinear behaviors have been associated with the primary causes of structural instability, degraded controllability, and even catastrophic failure. However, advances over recent decades have demonstrated that nonlinear dynamics can play an important—indeed, sometimes essential—role in enhancing system performance and enabling new functionalities tailored to specific engineering applications [1–23]. Consequently, the development of novel nonlinear structures designed to exploit distinctive dynamic responses and mechanical functionalities has become a major research frontier. This trend is evident across areas such as vibration control [17,21,23–28], energy harvesting [18,29–35], intelligent sensing [29,36–42], robotic systems [41,43–48], metamaterials [49–53], and related disciplines. Across these efforts, rigorous nonlinear analysis and design remain fundamental challenges that must be systematically addressed.

Nonlinear stiffness design offers an effective means of reconciling the conflicting requirements of high static load-bearing capacity and low dynamic stiffness in vibration control—an issue that is inherently challenging within linear system theory. To address this, a variety of approaches have been developed, including the classical three-spring mechanism [54–57], pneumatic techniques [58–62], magnetic methods [21,33,63–65], buckling- or elastic beam-based strategies [26,30,66,67], and, more recently, metamaterial-based designs employing origami-inspired folding structures and related mechanisms [68–73]. Notwithstanding these advances, it is important to emphasize that nonlinearities in engineering systems are generally required to be controllable, designable, and tunable, ensuring that the resulting nonlinear behaviors are predictable, manufacturable, and readily implementable in practice [12,17–19]. Such characteristics are essential across the aforementioned application domains, including vibration control, energy harvesting, and sensing, where reliability and practical feasibility are critical.

Nonlinear damping constitutes another important topic in both the literature and engineering applications. The limitations of linear damping have long been recognized [9,74–76], particularly in that increasing linear damping improves vibration suppression near resonance but often leads to degraded performance at higher frequencies [77–81]. Nonlinear damping mechanisms can effectively address this trade-off. As a result, a wide range of nonlinear damping strategies have been reported, including approaches based on magnetorheological and electrorheological (MR/ER) fluids [82–84], electromagnetic coil systems [80,85–87], advanced damping materials [88–90], active control techniques [84,91–94], geometric design optimizations [9,28,95,96], and acoustic black hole concepts [97–101].

Nonlinear inertia has recently emerged as a promising and rapidly developing research area. Conventionally, the inertia or equivalent mass of a vibration system is assumed to be constant, and extensive studies have been devoted to linear inertial elements, often referred to as inerters, along with their diverse applications [102–108]. However, recent investigations, as highlighted in [12,15,109,110], demonstrate that inertial elements can be deliberately designed to exhibit beneficial nonlinear characteristics, thereby enabling enhanced vibration control performance. This emerging direction opens up significant opportunities for further exploration and development in nonlinear dynamic systems.

Biological limbs exhibit remarkable vibration control capabilities [111–113]. An early investigation inspired by animal locomotion systems [8] demonstrated that advantageous nonlinear stiffness characteristics constitute a key underlying mechanism for such performance. This insight has led to the development of the X-structure or X-mechanism approach [12,15]. The X-shaped structure, as the most fundamental and simplest unit, can actually be designed in different size with various variants in its shape and spring arrangement methods [12] to cover diverse demands in engineering systems (e.g., resonant frequency, payload, space, weight, materials etc.) for nonlinearity reshaping in stiffness, damping and/or inertia. Building upon this foundation, a substantial body of research has emerged, reporting diverse bio-inspired vibration control strategies and nonlinear design methodologies [19,114–117].

Although a wide range of studies have been reported under bio-inspired approaches for vibration control and energy harvesting [118–122], for practical engineering applications, several key considerations must be addressed, including but not limited to:

- (a) Ease of implementation: the system should be readily deployable in real engineering environments;
- (b) Manufacturability: the design should facilitate straightforward fabrication and integration;
- (c) Simplicity and compactness: the structure should remain simple and spatially efficient, avoiding unnecessary complexity;
- (d) Stability: the system should exhibit robust and reliable stability characteristics rather than susceptibility to destabilization;
- (e) Adjustability and adaptability: the design should allow for convenient in situ tuning and adaptation to varying operating conditions and parameter requirements.

Overall, nonlinearity has been shown to play an important (or sometimes critical) role in engineering systems, influencing structural design [1,123,124], dynamic response [125–127], and parameter selection [128–130]. In applications such as vibration control [131–133], energy harvesting [134–136], sensor systems [137–140], and robotics [141–143], the deliberate exploitation of beneficial nonlinearity has become increasingly essential for achieving superior performance. Existing methods for stiffness and damping manipulation, such as metamaterials [144–146], shape-memory polymers [147–149], and origami structures [150–152], often suffer from complex modeling, limited adjustability, and difficulties in accurate analysis, which restrict their reliability and practical implementation. Inspired by animal leg and limb skeletons, this method leverages geometric nonlinearity to provide tunable and beneficial nonlinear stiffness (high-static–low-dynamic) [8,153,154], nonlinear damping (displacement- and frequency-dependent) [9,155], and nonlinear inertia [10,109,110,156] through simple, low-cost, and purely passive configurations. The X-structure can be realized in various forms, such as X-shape [11,114,157], quadrilateral [158,159], polygon [160], or their variants, while maintaining similar nonlinear characteristics, which offers exceptional design flexibility and ease of implementation. It can employ a regular, repeatable “X”-shaped unit, or take a single X-unit, its half (more like V) or many other irregular forms such as A, M, K, L, or else shapes, to deliberately integrate controllable nonlinear stiffness, damping, and inertia within the structure (typically a spring-linkage mechanism with diverse spring arrangements [12,161]), transforming it into a versatile mechanical platform or tool or unit that can achieve diverse nonlinear dynamics in stiffness, damping and/or inertial properties, demanded in engineering systems. The obvious advantages of the X-structure method lie in its controllability, adjustability and designability in practice to meet the challenging issues as mentioned above, which other methods may not.

A comprehensive review published in 2022 [12] established the foundational principles of this approach, systematically summarizing its bio-inspired origins, nonlinear properties, and initial applications. Since then, the field has expanded dramatically. Research has shifted from proof-of-principle demonstrations to pushing the performance limits of core nonlinear characteristics. It also involves developing complex configurational designs for multi-directional and multi degrees-of-freedom scenarios, integrating X-structures with advanced control, sensing, and metamaterial technologies, and expanding into multifunctional applications that combine vibration isolation with energy harvesting, robotics, and spacecraft protection. These developments have collectively transformed the X-structure from a specialized isolator into a versatile platform capable of addressing a wide range of engineering challenges.

Distinct from our previous 2022 review, this article provides a comprehensive update that specifically captures the rapid advancements and emerging trends developed thereafter. The scope includes four aspects: (i) deepened understanding of nonlinear stiffness, damping, and inertia, including expanded QZS ranges exceeding 85% of total stroke, resonant frequencies below 1 Hz, and the emergence of nonlinear inertia as a distinct design dimension; (ii) innovative configurational designs for multi-directional, hierarchical, and bio-inspired systems; (iii) integration with advanced control, sensing, and metamaterial technologies; and (iv) expansion into multifunctional applications spanning energy harvesting, robotics, and spacecraft on-orbit capture.

The article is organized as follows. Section 2 reviews advances in the core nonlinear characteristics, i.e., stiffness, damping, and inertia. Section 3 examines configurational innovations for multi-directional, hierarchical, and bio-inspired designs. Section 4 discusses the integration of X-structures with advanced control, sensing, and metamaterial technologies. Section 5 surveys application expansion into energy harvesting, robotics, and spacecraft on-orbit capture. Section 6 presents a critical discussion of open challenges and future perspectives, followed by concluding remarks in Section 7.

2. Nonlinear Characteristics and Core Mechanisms of X-Structures/Mechanisms

Following the establishment of fundamental principles in the pioneering 2022 review [12], research on X-structure/mechanism has entered a phase of area expansion, deep refinement and further discovery. This section summarizes key advances in enhancing the performance limits of core nonlinear properties and in revealing novel dynamic phenomena. Moving beyond basic derivations, the focus is on the sophisticated manipulation of nonlinear stiffness, inertia, and damping. This marks a transition from utilizing geometric nonlinearity to mastering the complex dynamics that it enables for next-generation applications.

2.1. Advancements in QZS Design and Tuning

QZS remains the cornerstone property of X-structures for ultra-low-frequency vibration isolation, yet recent research does more than just realize this characteristic. Current efforts focus on enhancing the practicality and robustness of QZS. This subsection reviews progress in three key directions: expanding the tunable QZS range,

introducing novel stiffness compensation mechanisms, and critically analyzing complex nonlinear dynamics like bifurcations under asymmetry and internal resonances.

2.1.1. Early Milestones in X-Structure QZS Design

The X-structure/mechanism approach to QZS vibration isolation originated from a series of foundational studies, which collectively established the core design principles and demonstrated practical feasibility. These early milestones, summarized below, lay the groundwork upon which the recent advances in subsequent subsections are built.

The first systematic introduction of the bio-inspired X-structure design was presented in [8], which proposed a limb-like structure (LLS) emulating animal musculoskeletal systems, as presented in Figure 1a. By combining rods of unequal length with horizontal and vertical springs, the LLS achieved flexibly tunable nonlinear stiffness with a resonant frequency as low as 1.32 Hz, establishing the fundamental design framework for X-structure-based QZS systems.

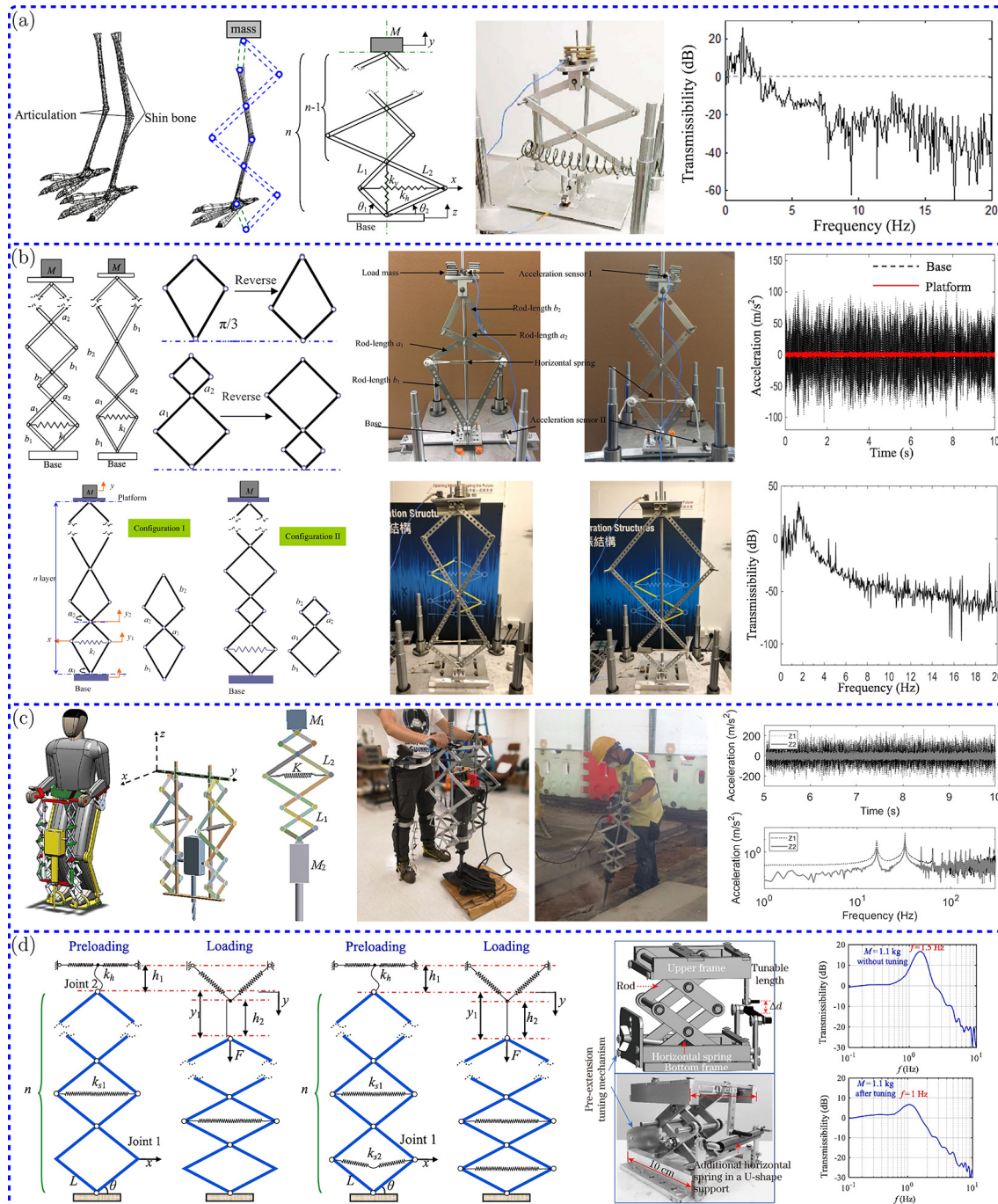


Figure 1. Early developments of typical X-structure-based QZS isolation systems, including their schematic, prototype and performance characteristics; (a) limb-like structure (LLS) [8], reprinted with permission from Copyright 2015, IOP Publishing Ltd.; (b) vertically asymmetric X-structure (vaX) [153,154], reprinted with

permission from Copyright 2019, Springer Nature; Copyright 2019, Elsevier Ltd.; (c) bio-inspired anti-vibration exoskeleton (BIAVE) for jackhammer operators [162,163], reprinted with permission from Copyright 2019, Elsevier Ltd.; (d) X-cube vibration mount and quasi-zero-gravity lifting platform [161], reprinted with permission from Copyright 2022, Elsevier Ltd.

Building on this foundation, systematic investigations into asymmetric X-structures [153,154] revealed additional design dimensions. The vertically asymmetric (Figure 1b) structures demonstrated that rod length ratio and assembly pattern significantly influence isolation performance, with optimized configurations achieving natural frequencies below 1.6 Hz and vibration suppression exceeding 75%. Notably, reverse assembly was found to reduce resonant peaks by up to 13.7%, revealing assembly pattern as an exploitable design parameter.

The practical potentials were clearly demonstrated through the bio-inspired anti-vibration exoskeleton (BIAVE) for jackhammer operators [162,163], as shown in Figure 1c. Its key innovation shows that greater downward force leads to lower dynamic stiffness. This design has effectively resolved the classic conflict between stiffness and isolation performance. Field tests confirmed 70–90% vibration reduction in the human-sensitive 4–50 Hz band, extending operator working time from 30 min to 5–8 h while improving efficiency by about 30%.

A comprehensive synthesis was reported in [161], which proposed a systematic stiffness manipulation method through a family of X-mechanism variants. As displayed in Figure 1d, the compact X-cube vibration mount (10 cm cubic structure) achieved a resonant frequency of 1 Hz, a QZS region exceeding 100 mm, and transmissibility decay exceeding –20 dB/Dec. A quasi-zero-gravity lifting platform reduced lifting force from 28 N to below 3.8 N, demonstrating significant energy-saving potential.

Overall, these milestones, including the foundational LLS [8], asymmetry insights [153,154], the transformative BIAVE application [162,163], and the systematic X-cube framework [161], established that beneficial nonlinearity can be designed, predicted, and tuned only through structural geometry. For a comprehensive treatment, the reader is directed to [13] and the 2022 review [12].

2.1.2. Enlarged and Tunable QZS Regions

The pursuit of a wider and more controllable QZS range is central to enhancing the practicality of QZS isolators. To this aim, recent advancements have strategically moved beyond the basic X-structure/mechanism, primarily through the integration of supplementary stiffness elements to compensate for and extend the native nonlinear force-displacement profile. These approaches can be broadly categorized into two groups: (i) intrinsic tuning mechanisms that modify the internal spring engagement of the X-structure itself, and (ii) coupled compensation mechanisms that combine the X-structure with external springs or linkage systems. This subsection reviews key developments in this area, highlighting the evolution from concept to various refined designs.

A landmark contribution in intrinsic tuning is the novel X-shaped anti-vibration structure with a tunable contact mechanism (NXSC) discussed in [161,164], which directly tackles the limitations of narrow QZS zones and limited load capacity. As illustrated in Figure 2a, the NXSC innovates through three distinct configurations (NXSC-I, II, and III). NXSC-I employs two horizontal springs where the bottom spring contacts a vertical fixed rod during compression, effectively transforming into an oblique spring to enhance vertical stiffness. NXSC-II features a relaxed bottom spring that activates only upon contact, avoiding preload-induced stiffness degradation. NXSC-III utilizes two pre-installed oblique springs connected to the fixed rod, eliminating contact-induced force discontinuities. The core innovation across all configurations is the dimensionless contact parameter α , which governs the engagement between the horizontal springs and the vertical rods. By tuning α , the QZS zone width, loading capacity, and resonant frequency can be continuously adjusted *in-situ* without compromising structural stability.

The performance of this tuning mechanism is quantifiably demonstrated through static and dynamic analyses. A novel QZS quality indicator $I_{qzs} = \sqrt{K/K_f}$ was introduced to bridge design parameters with practical isolation targets. For ultra-low-frequency suppression ($f_{qzs} \leq 1$ Hz), $I_{qzs} < 0.5$ is required. Static analysis confirms that tuning α and spring stiffness enables complete compensation of the negative stiffness inherent in classical X-structures, while expanding the load capacity by 2–3 times and the QZS zone width by 5–10 times. This results in a wide, flat low-stiffness plateau, as clearly shown in the force-displacement and stiffness-displacement curves of NXSC-I (Figure 2c,d). The shaded QZS region is substantially broader and more uniform compared to conventional isolators. Crucially, the NXSC avoids typical nonlinear instabilities such as frequency jumps and hardening/softening effects under large excitations, owing to its well-maintained weakly nonlinear stiffness near the zero-stiffness point.

Dynamic performance validation, utilizing the experimental prototype in Figure 2b, confirms these theoretical advantages. The absolute displacement transmissibility curves in Figure 2e provide a direct comparison

with a typical spring-mass-damper (SMD) system and an existing QZS isolator. The NXSC exhibits a lower resonant frequency (down to 1.10 Hz for NXSC-I), a significantly reduced resonant peak, and a wider effective isolation band. Moreover, it maintains a superior loading capacity (57.3 N vs. 40.4 N) and a broader QZS zone (0.10 m vs. 0.061 m) than the benchmarked QZS isolator. Experimental results align closely with theoretical predictions, showing effective vibration attenuation starting at 1.65 Hz and stable performance across a load range of 1.5–3.8 kg. This consistency highlights the NXSC's potential for demanding engineering applications, such as in aerospace precision equipment and heavy-duty machinery, where ultra-low-frequency vibration suppression is critical.

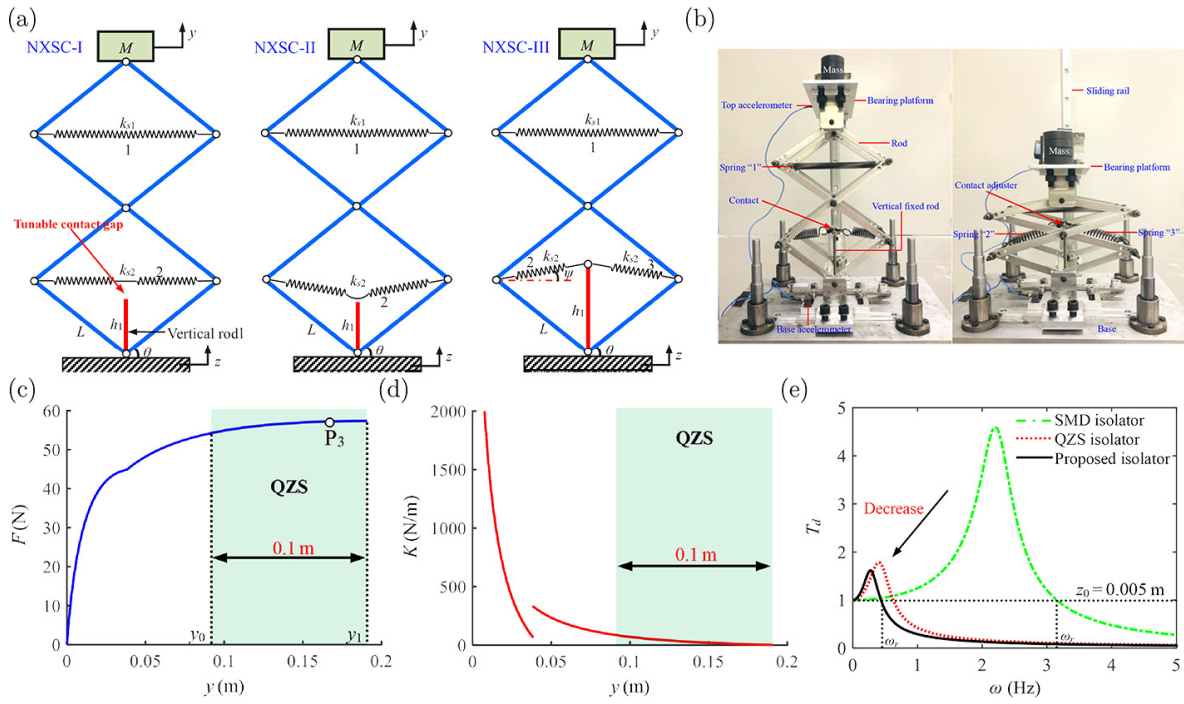


Figure 2. Novel X-shaped anti-vibration structure with tunable contact mechanism (NXSC) for enhanced QZS effect in [164], reprinted with permission from Copyright 2022, Elsevier Ltd. (a) schematic of three NXSC configurations, showing horizontal springs, vertical fixed rods, and tunable contact gaps; (b) experimental prototype of the NXSC-based platform; (c,d) force-displacement and stiffness-displacement curves of the NXSC structure, where shaded areas denote QZS zones; (e) absolute displacement transmissibility curves, demonstrating the NXSC's lower resonant frequency, wider isolation band, and reduced resonant peak compared to the SMD and existing QZS isolators.

Parallel to intrinsic tuning, significant efforts have focused on integrating the X-structure with complementary mechanisms to construct enhanced QZS systems. A representative sample of these innovative configurations is summarized in Figure 3. While diverse in form, they share a common philosophy, i.e., using the X-structure as a core nonlinear component whose properties are actively shaped by an additional element.

The auxiliary X-structure for functional decoupling is featured in an X-QZS system introduced in [165], where an additional X-shaped structure is attached to the isolation platform not as a primary load-bearing component, but as a dedicated tuning module. This design decouples the QZS-tuning function from the load-bearing function, which is handled by a central support spring, enabling independent optimization for broader QZS intervals and lower isolation frequencies.

In the integration of the X-structure with custom nonlinear springs [166] combines a concave X-shaped structure (CXSS) with custom variable-pitch springs (VPS), as shown in Figure 3a. The VPS provides a tailored nonlinear restoring force, effectively broadening the QZS range and avoiding issues associated with magnetic compensation methods, such as distance limitation and eddy current heating.

An optimized geometry for an improved stability and a broader QZS range is applied in the basic X-shaped isolator design reported in [167], integrates oblique springs that fully counterbalance the inherent negative stiffness. This not only ensures system stability but also expands the QZS range to cover a large portion of the stroke, all within a compact, low-friction design achieved by eliminating guiding sliders. Similar design components may also refer to [168].

Ref. [169] proposes an X-combined isolator that couples an X-structure with two oblique tension springs, see Figure 3b. This configuration successfully eliminates negative stiffness across the entire working range while significantly widening the QZS zone. The design simplifies assembly and improves stability by removing the need for additional guide rods. Similar design components and applications may also refer to [170].

A further variant design discussed in [167] is given in [115], which departs from purely nonlinear compensation and achieves linear compensation via polygonal linkages by presenting a Linkage Anti-Vibration Structure (LAVS). It employs a symmetric polygonal structure to generate a linear negative stiffness over a large stroke, which is then precisely compensated by a vertical linear spring. This linear-on-linear approach yields an exceptionally wide QZS interval (over 88% of the total stroke), demonstrating an alternative pathway to achieving high-static-low-dynamic stiffness (HSLDS).

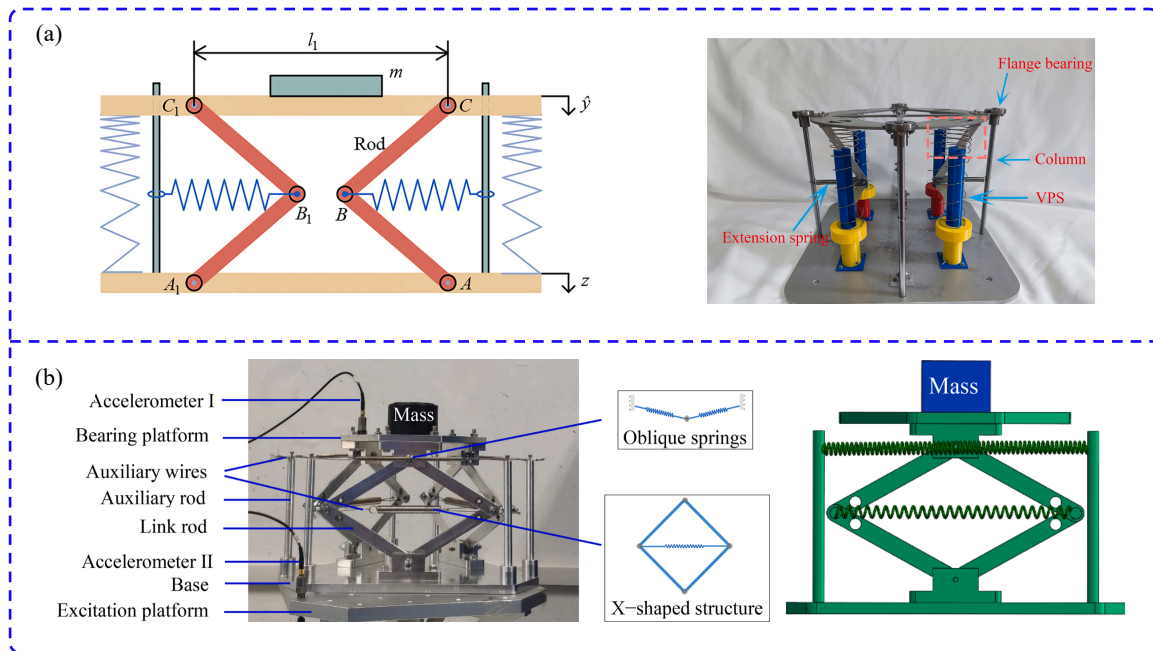


Figure 3. Illustrations and prototypes of recently developed X-structure variants for enhanced QZS performance; (a) a concave X-shaped structure combined with variable-pitch springs [166], reprinted with permission from Copyright 2024, Elsevier Ltd.; (b) an X-combined isolator coupled with oblique tension springs [169], reprinted with permission from Copyright 2022, Elsevier Ltd.

The collective progress reviewed above reveals a clear trend. The standalone basic X-shaped structure is evolving into a flexible component within more sophisticated hybrid systems. The approach reported in [161,164] represents a refinement of the internal mechanics, offering precise tunability. In contrast, the hybrid systems [115,165–167,169] leverage the nonlinearity of the basic X-structure as one part of an integrated system, often aiming for maximal QZS range or functional decoupling. A critical comparative insight is the inherent trade-off between tunability/complexity (as in NXSC) and robustness/simplicity (as in some hybrid designs). Future research directions may involve merging these approaches, for instance, incorporating tunable mechanisms into hybrid systems, or exploring the integration of smart materials for adaptive, real-time QZS adjustment. These advancements collectively move the field closer to practical, high-performance passive isolators for challenging ultra-low-frequency applications.

2.1.3. Novel Compensation Mechanisms

In contrast to the passive stiffness compensation methods mentioned above, the integration of magnetic elements has also emerged as a powerful and innovative approach for tailoring the nonlinear stiffness of X-structures. Magnetic forces, with their inherent nonlinearity and non-contact nature, offer unique advantages for constructing and tuning QZS regions, enabling designs that are more compact, tunable, and capable of novel dynamic phenomena. This subsection reviews recent advances in magnetically compensated X-structures, categorized by their fundamental compensation principle.

A foundational approach is illustrated by the nonlinear compensation method (NCM) proposed in Ref. [171]. This design pairs an X-structure (providing nonlinear negative stiffness) with a set of repulsive permanent magnets

(providing nonlinear hardening positive stiffness). Unlike linear compensation, where positive stiffness is constant, the magnetic force,

$$\Gamma_d(y_d) = \frac{P_8}{P_7 - y_d} - P_9, \quad (1)$$

where y_d is dimensionless displacement and P_i are magnetic coefficients, provides a geometrically increasing counterforce. This allows the resultant force,

$$F_c(y_d) = F_d(y_d) + \Gamma_d(y_d), \quad (2)$$

to achieve an accurately balanced, wide low-stiffness plateau. The parallel load-bearing of both components enhances stability, and the system demonstrates effective isolation starting near 4 Hz with an ultra-low resonant frequency (~ 2 Hz experimentally), validating the paradigm of using nonlinear positive stiffness to compensate nonlinear negative stiffness.

A further development of this nonlinear-on-nonlinear compensation approach is presented in [172], where a rhombus-type magnetic-levitation structure (RMLS) is introduced. This design similarly employs the hardening stiffness of repulsive magnets to compensate for the negative stiffness of a rhombic linkage (the basic X-structure). Its key innovation lies in a modular and tunable Maglev unit with two symmetric sets of fixed and moving permanent magnets, whose relative horizontal distance and height can be precisely adjusted via an L-shaped mechanism. This *in-situ* adjustability of the magnetic field configuration provides an additional, practical way for tuning the QZS range and resonant frequency after assembly, enhancing adaptability to varying load conditions. The RMLS show the trend toward hybrid designs where geometric and magnetic nonlinearities are designed together, creating a way to more adaptable and high-performance isolators.

Building on this, the work presented in [173,174] introduce a sophisticated torsional magnetic negative stiffness (TMNS) mechanism. Here, the compensation occurs at the rotational joints of the X-structure itself. Radially magnetized tile magnets generate a periodic torsional negative stiffness, which is compensated by traditional torsion springs. This configuration shifts the nonlinearity control from translational displacement to joint angle, enabling flexible placement of one or even multiple QZS regions anywhere within the stroke, which is a significant advance over fixed-interval designs. Furthermore, this work highlights a secondary benefit, i.e., the mass of the TMNS units introduces a nonlinear inertia effect, which manifests in dynamic tests as an anti-resonance phenomenon that further suppresses vibration transmission near certain frequencies.

An alternative bio-inspired strategy is presented in [175], which replaces the linear horizontal springs of a classic X-structure with a nonlinear magnetic spring assembly. This “muscle-like” magnetic device, consisting of coaxial magnetic rings and a linear spring, provides a tunable nonlinear positive stiffness. When combined with the X-structure’s geometric negative stiffness, it achieves a QZS region that is both wider and centered closer to the equilibrium point. This design offers practical tuning through geometric parameters (layer number n and angle θ) and magnetic configuration, yielding a dramatic 65.7% reduction in resonance peak compared to its non-magnetic counterpart.

The magnetically compensated designs reviewed above illustrate a clear development path in nonlinear stiffness adjustment. Early designs directly integrate repulsive magnets in parallel [171] and modular magnetic levitation units [172]. Later designs introduced torsional magnetic negative stiffness at the joints [173,174] and used coaxial magnetic components to replace horizontal springs in biomimetic structures [175]. Across these approaches, a shared idea emerges, i.e., employing magnetic nonlinearity to compensate for geometric nonlinearity provided by the X-structure. This strategy offers distinct advantages over conventional linear springs, including non-contact operation, compactness, adjustable performance after fabrication, and extra benefits such as nonlinear inertia (which will be further discussed in Section 2.3). Overall, these developments take the X-structure as a design component or the main structure of the hybrid system that combines material and geometric nonlinearities together. They greatly increase the performance range that QZS isolators can reach.

2.1.4. Exploration of Complex Nonlinear Dynamics

X-structures with variant designs provide diverse nonlinear dynamics (QZS, multi-stability, negative stiffness, and so on). Therefore, a deeper study of inherently complex nonlinear dynamics incurred by X-structures is indispensable. This is to recognize that phenomena such as internal resonance and bifurcation are not merely challenges to be overcome, but potential mechanisms that can be understood and leveraged for superior vibration control. This subsection reviews some results that systematically explores these complex dynamics, expanding the design approach for X-structure-based isolators.

An early attempt in this topic is given in [176], where ultra-subharmonics and subharmonics of an X-structure based isolator are studied, and the analytical formulations of frequency-responses, bifurcation sets and stability of the subharmonics of 1/2, 1/3, and 1/4 order, and ultra-subharmonic (2/3 order) responses are exhaustively derived. Further investigations are discussed in [177], which bridges the gap between X-structure design and nonlinear modal interactions by investigating a system intentionally tuned to a 3:1 internal resonance condition. The system integrates a primary X-structure isolator with a nonlinear absorber. By carefully selecting parameters like mass distribution and rod length, the first two natural frequencies can be locked into a 3:1 ratio, enabling strong nonlinear energy exchange between modes. Analysis via the method of multiple scales reveals rich dynamics, including amplitude-dependent frequency responses with characteristic jumps and multi-valued solutions. Parametric studies with frequency curves demonstrate that geometric parameters like rod length and assembly angle provide direct ways to adjust the strength of nonlinearity and the stability of the response. For instance, increasing the assembly angle increases nonlinearity and widens the multi-solution region, while adjusting rod length can reduce unstable behaviors. This work provides a foundational framework for utilizing internal resonance as a design tool in X-structures, moving beyond its traditional role as something only studied for analytical purposes.

In practical applications, isolators often operate under non-ideal conditions such as asymmetric manufacturing tolerances or off-design loads. The results in [178,179] present a substantial bifurcation and stability analysis of an X-shaped negative stiffness structure (XNSS) isolator, explicitly accounting for these realities. The study models three stiffness types (symmetric and two asymmetric) and three load conditions, offering a classification for real-world scenarios.

The core finding is that asymmetry and load mismatch may fundamentally alter the system's equilibrium distribution, causing saddle-node and transcritical bifurcations. A critical stability criterion is established, i.e., the system maintains a single, stable equilibrium within a specific range of the stiffness ratio parameter $\alpha_5 = K_{NS}/K_{PS}$, enhancing the design's stability margin. Dynamic analysis shows the performance effects. When the pre-bifurcation equilibrium lies on the "softer" side, the isolator achieves a lower onset frequency for isolation than similar linear systems. Furthermore, the study identifies parameter regions where chaotic motion may occur using Lyapunov exponents, providing crucial guidelines for avoiding undesirable behavior. This research changes the focus from idealized QZS design to robust design, offering quantitative metrics to ensure stable performance despite unavoidable practical imperfections.

The studies reviewed herein show a more developing phase in X-structure research. By systematically investigating internal resonance [177] and bifurcation dynamics under non-ideal conditions [178,179], the field is building an ability to predict complex nonlinear behaviors. This knowledge transforms these phenomena from potential failure modes into aspects that can be foreseen, controlled, or even used. For instance, the controlled energy transfer of internal resonance could be used for targeted vibration suppression, while bifurcation analysis provides a guide for robust design. Future work may focus on combining these findings, e.g., examining how internal resonance characteristics change with structural asymmetry, or on developing active/semi-active control laws that use these nonlinear features for real-time performance adaptation. This deeper study of nonlinear dynamics is essential for creating the next generation of intelligent, high-performance vibration isolation systems.

2.2. Advanced Damping and Dissipation Mechanisms

Foundational studies established that the geometric nonlinearity of X-structures inherently generates displacement-dependent nonlinear damping, which suppresses resonance without compromising high-frequency performance. These preliminary works focused more on theoretical mechanisms and did not integrate physical damping devices of different kinds. Recent studies have shifted to enhancing dissipation capacity by coupling X-structures with dedicated damping materials and devices. This section reviews two key directions, i.e., viscoelastic (VE) material integration for coupled nonlinear damping, and friction damping utilization. These developments transform the X-structure into a fully integrated nonlinear dissipation system, improving its engineering adaptability.

2.2.1. Foundational Contributions in Nonlinear Damping

Alongside the stiffness-focused developments, early research also established the fundamental understanding of nonlinear damping in X-structures. The presented in [9] systematically revealed that the limb-like or X-structure inherently generates a displacement- and frequency-dependent nonlinear damping with a simple modeling for the model in Figure 4a given by

$$\ddot{y}_1 + \frac{k}{M}y_1 + \left[\frac{c_1}{M} + \frac{c_2 n_x}{M} \left(\frac{\partial \varphi}{\partial y_1} \right)^2 + \frac{c_3}{M} \left(\frac{\partial x}{\partial y_1} \right)^2 \right] \dot{y}_1 + \ddot{z} = 0. \quad (3)$$

This damping is automatically strong near resonance to suppress peaks, yet low at other frequencies to preserve isolation performance. This ideal passive damping mechanism can be adjusted by parameters such as rod length, layer number, and assembly angle. It has been verified by experiments and performs better than traditional nonlinear damping systems. It provides a wider isolation band without reducing high-frequency transmissibility, as shown in Figure 4b.

Building on this insight, authors in [155] proposed the X-absorber, which is a nonlinear tuned mass damper that integrates the X-structure's beneficial nonlinear stiffness and damping with a traditional TMD, see Figure 4c. Through multi-variable optimization, the X-absorber achieves QZS and broadband damping, widening the suppression bandwidth by 3–4 times, eliminating instabilities such as bifurcation, and demonstrating superior robustness to parameter variations. Experimental validation confirmed its effectiveness under random, sinusoidal, and impulse excitations, with vibration attenuation speed 40% faster than conventional spring-mass dampers, as presented in Figure 4d.

Together, these studies established the theoretical and experimental basis for using nonlinear damping in X-structures. They show that desired nonlinear damping can be achieved purely passively through geometric design, which is a crucial complement to the stiffness-related milestones discussed earlier. For more details on nonlinear damping in X-structures, readers are referred to [13] and the review article [12].

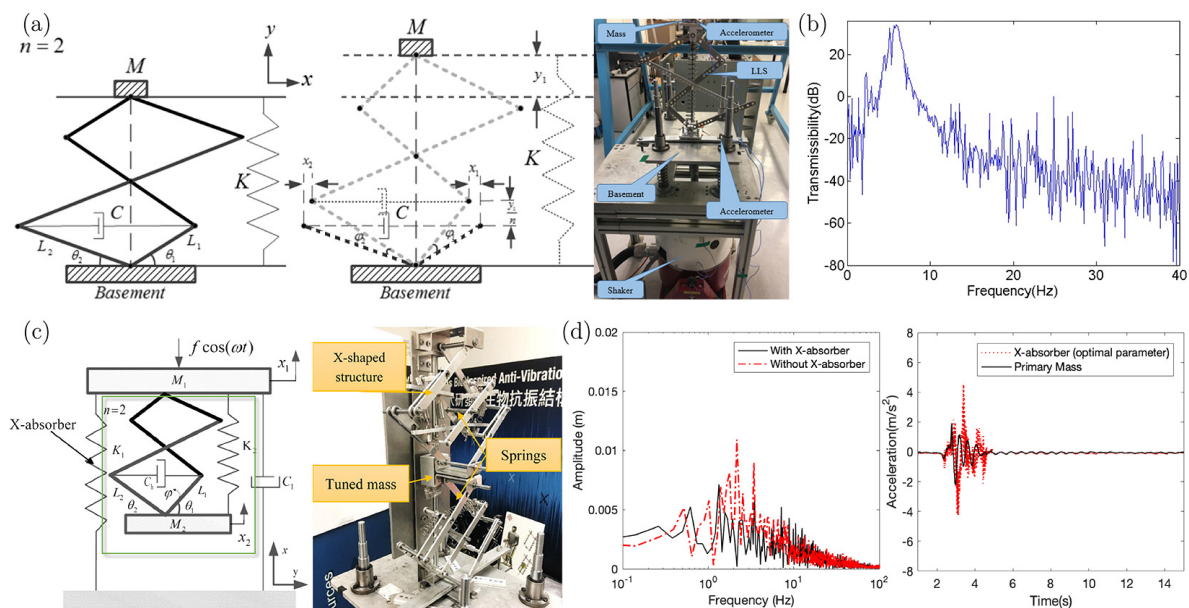


Figure 4. Nonlinear damping and X-absorber designs based on X-structures, including their schematic, prototype and performance curves; (a) X-structure nonlinear damping model [9]; (b) vibration isolation performance of the nonlinear damping system [9], reprinted with permission from Copyright 2019, Elsevier Ltd.; (c) X-absorber integrated with tuned mass damper [155]; (d) dynamic performance of the X-absorber system [155], reprinted with permission from Copyright 2021, Elsevier Ltd.

2.2.2. Viscoelastic and MR Damper-Integrated X-Structures

While the inherent geometric nonlinearity of X-structures provides beneficial displacement-dependent damping, full employment of such benefits is yet to be studied, targeting practical application requirements. Therefore, research studies focusing on integrating dedicated passive viscous damping devices or other smart materials into the X-structure framework are expected to transform X-structures from stiffness-tuning isolators into fully coupled nonlinear dissipation systems. Here, advances in two representative damping-device types are reviewed, i.e., viscoelastic (VE) dampers and passively operated magnetorheological (MR) dampers.

The one work comes from [180,181], which introduced the viscoelastic limb-like-structure (VE-LLS) device, as shown in Figure 5a. By embedding sandwich-type VE dampers into the joints of an X-shaped (limb-like) frame, the design achieves a synergistic coupling of material nonlinearity (from the VE polymer) and geometric nonlinearity (from the linkage kinematics), see Figure 5b. This dual-nonlinearity enhances energy dissipation capacity by over 30% compared to single-nonlinear designs, while maintaining broad-frequency effectiveness (0.1–4 Hz).

The study presented in [180] established the foundational mechanical model that clearly accounts for both viscoelastic forces and joint friction, and presents the expression for the equivalent damping C_e of the VE-LLS device as

$$C_e = Mc_v \frac{\Delta_h}{D} + 2(3N + 1)c_f \frac{\varphi}{D}, \quad (4)$$

where c_v denotes the viscoelastic damping coefficient of the VE damper and c_f denotes the friction damping coefficient of the joints. A key practical innovation was the groove-embedded bonding interface, which reduces debonding failure, the dominant failure mode of such hybrid devices. In [181] further developments of this work are presented by addressing a critical practical problem, i.e., the severe energy dissipation attenuation (up to 40%) of VE materials under concurrent high-temperature and low-frequency conditions, as illustrated in Figure 5c. Through systematic experiments, the study quantified the compensatory role of joint friction, revealing that friction contributes up to 160% of total dissipation at 0.1 Hz, see Figure 5d. This finding enabled development of a compensation strategy, i.e., by slightly increasing joint bolt preload, friction dissipation can make up for the reduced VE damping, stabilizing energy dissipation fluctuation from >40% to <8% across 0–40 °C and 0.1–2.0 Hz (Figure 5e). Together, these studies transform VE-LLS from a proof-of-concept into a reliable, stable, and practical dissipation technology.

The work discussed in [82] builds on the approach of integrating devices to include MR dampers, with a distinct feature, i.e., the MR damper is operated passively (zero current), relying only on its inherent viscous and hysteretic characteristics. The damping force F_{MR} is modeled as

$$F_{MR} = c_M \dot{x} + \alpha_M \tanh(e_1 \dot{x} + e_2 x), \quad (5)$$

where c_M denotes viscous damping coefficient, α_M denotes hysteresis loop scaling factor, e_1 and e_2 are scaling factors that determine the hysteretic slope and loop width, respectively. The core innovation comes from the geometric configuration, with six different MR damper installation arrangements in the scissor-like X-structure compared in detail. This geometric nonlinearity makes the passive MR damping displacement-dependent, a property typically requiring active control in conventional MR applications.

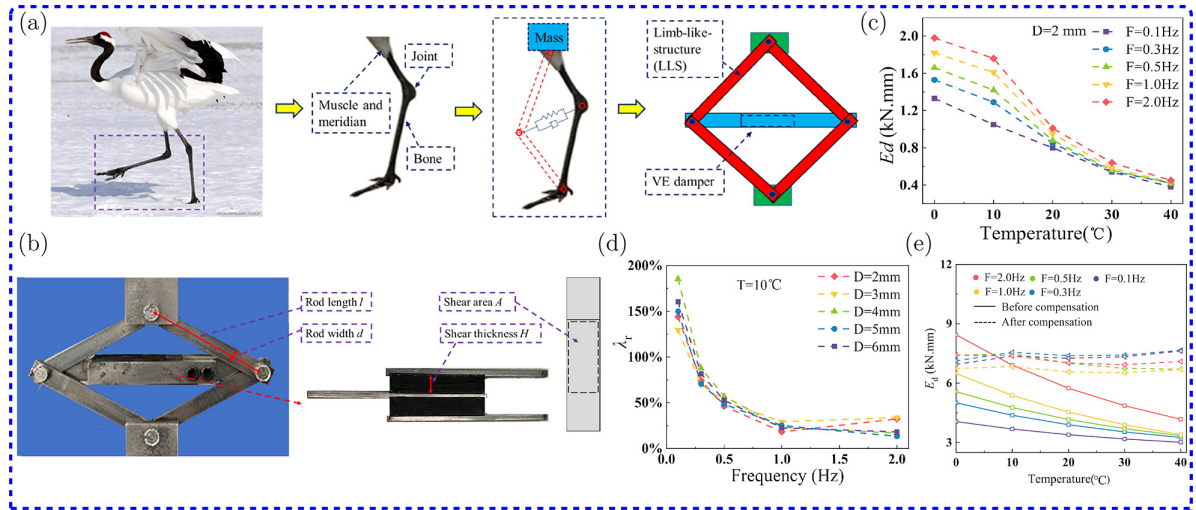


Figure 5. Integrated nonlinear damping systems based on X-structures by means of viscoelastic [180,181], reprinted with permission from Copyright 2022, Elsevier Ltd.; Copyright 2023, Taylor & Francis Group; (a) schematic of the viscoelastic limb-like-structure (VE-LLS) device; (b) coupling mechanism of material and geometric nonlinearities; (c) energy dissipation attenuation under high-temperature and low-frequency conditions; (d) contribution of joint friction to total energy dissipation; (e) Energy dissipation stability after friction compensation.

Parametric analysis identifies the optimal configurations, Type 1 and Type 3, which reduce isolation frequency by 34.8–75.4% as discussed in Ref. [82], and resonant peak by 8.1–43.7% compared with standard isolators. Notably, larger excitation amplitudes yield lower resonant peaks, an unexpected robustness beneficial for strong vibration scenarios. This work demonstrates that even without activating the MR effect, the geometric embedding itself can achieve superior nonlinear damping performance, expanding the design options for smart damper integration.

Both VE-LLS and passively configured MR dampers share a common design methodology, i.e., the X-structure acts as a mechanical amplifier and nonlinear modulator for otherwise traditional damping devices. The VE approach performs well in broad-frequency, high-capacity energy dissipation with stable and verified compensation; the MR approach provides layout flexibility and uses its inherent hysteresis. Collectively, these developments show the progress of X-structure damping research, shifting from studying the inherent joint friction to designing hybrid material–geometric dissipation systems made for specific working conditions.

2.2.3. Friction Damping Analysis and Utilization

A mechanically robust alternative for enhancing dissipation is the integration of friction damping elements into the X-structure framework [182] presents such an example, introducing an improved X-shaped isolator with plate-type friction damping elements installed at its vertical hinges.

The core contribution lies in the non-intrusive integration of friction damping. Ring-shaped friction plates are preloaded by bolts at the hinge joints, generating stable normal pressure and controllable frictional torque. The friction force provided by the plate-type friction damping element, which depends on the motion direction of the X-shaped isolator, is given by

$$F_x = \frac{8}{3} \frac{F_{kx} \mu (R^3 - r^3) \text{sign}(\dot{x})}{(R^2 - r^2) \sqrt{l^2 - (l \sin \theta + x/2)^2}}, \quad (6)$$

where F_{kx} is the preload force applied to the friction plate, μ is the friction coefficient, and the remaining symbols represent geometric parameters. Critically, this design preserves the original X-structure topology, because it alters neither rod lengths nor load-bearing paths, and thereby solves the classical conflict between high static stiffness and sufficient resonant damping. Damping level can be flexibly tuned in situ by adjusting bolt preload or selecting friction materials, offering good engineering practicality.

Comparative studies demonstrate that the integrated friction damping produces marked attenuation of the resonant peak and a wider isolation bandwidth, while the high static stiffness required for load capacity remains uncompromised. This directly addresses the potential limitation of excessive resonance amplification in classic X-shaped isolators without damping optimization. The study further establishes quantitative relationships between damping force and key design parameters (e.g., preload, friction coefficient, ring geometry), providing practical guidance for engineering optimization.

Compared with viscoelastic and passive MR damping in Section 2.2.2, the friction-based approach offers distinct advantages, such as velocity-independent dissipation, mechanical simplicity, and minimal structural modification. Its performance is stable across frequency and less affected by environmental factors such as temperature, although wear and preload maintenance requires lifecycle consideration. Collectively, these various integration methods provide the X-structure platform with a set of damping solutions that can be adapted to different applications.

2.3. Revelation and Utilization of Nonlinear Inertia Effects

While stiffness and damping have long been the primary focus for regulating nonlinear dynamics with X-structures, recent research has identified nonlinear inertia as an equally important third dimension for performance enhancement. Unlike the constant mass in linear systems, the effective inertia in certain X-mechanism configurations is amplitude-dependent, enabling unique dynamic phenomena and unprecedented design opportunities. This section reviews key progress in modeling, validating and utilizing nonlinear inertia effects, from theoretical foundations to hybrid designs and novel inertial nonlinearities. This research direction broadens the functional potential of the X-structure method, offering new strategies for ultra-low-frequency isolation and improved vibration control beyond pure stiffness optimization. For more details on nonlinear inertia effects in X-structures, readers can also be referred to [15].

2.3.1. Foundational Contributions in Nonlinear Inertia

Prior to the recent advances summarized in Section 2.3, early research provided the fundamental understanding of nonlinear inertia in X-structures. A series of pioneering studies showed how rotational motion coupling can produce useful inertial effects for vibration control.

The work in [10,156] demonstrated the potential of combining X-structure nonlinearity with lever-type mechanisms. By integrating an X-structure supporting structure with a two degrees-of-freedom lever system, Ref. [156] achieved a tunable band-stop vibration isolator, as illustrated in Figure 6a. Figure 6b shows that its stop-band

bandwidth is more than 3 times wider than that of optimized linear isolators, while strong loading capacity was maintained. Furthermore, it can be observed from Figure 6c that in [10] this concept was extended by systematically investigating both passive and semi-active control strategies. The results showed that the hybrid system can achieve consistently low transmissibility over the full frequency range. This represents an important improvement over traditional isolators with uneven performance. These studies confirmed that the geometric nonlinearity of X-structures can be effectively combined with lever mechanisms to achieve controllable low-frequency band-stop behavior without causing dynamic instability.

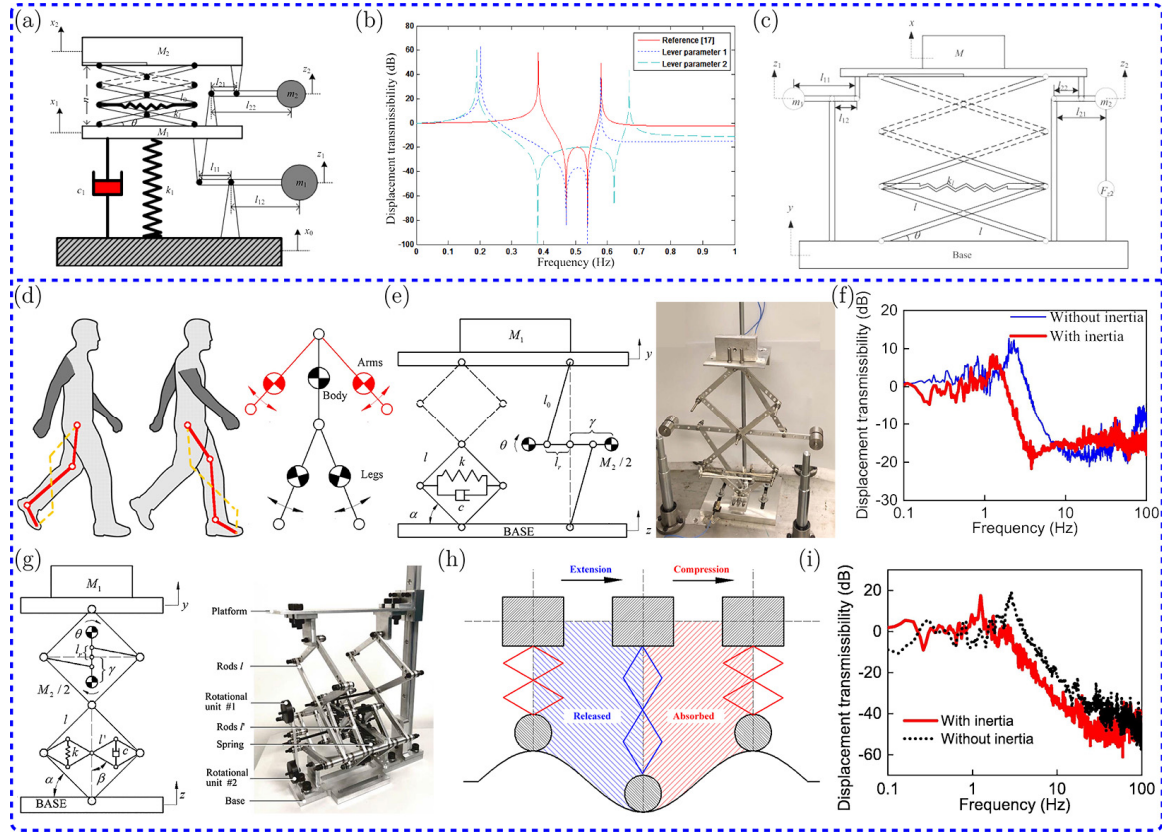


Figure 6. Fundamental developments of X-structure systems with nonlinear inertia; (a) tunable band-stop X-structure isolator with lever mechanism [156]; (b) stop-band performance of the lever-integrated X-structure isolator [156], reprinted with permission from Copyright 2015, Elsevier Ltd.; (c) passive and semi-active hybrid X-structure system [10], reprinted with permission from Copyright 2016, Elsevier Ltd.; (d) human locomotion inspired inertial adjustment mechanism; (e) human body inspired anti-vibration structure (HBIAVS) [109]; (f) vibration isolation performance of the HBIAVS system [109], reprinted with permission from Copyright 2019, Elsevier Ltd.; (g) HBIAVS-NI system with embedded X-structure and rotational inertia [110]; (h) nonlinear inertia-induced conservative force mechanism [110]; (i) dynamic performance of the HBIAVS-NI system [110], reprinted with permission from Copyright 2019, Elsevier Ltd.

Inspired by human movement as shown in Figure 6d, the studies reported in [109,110] introduced the integration of rotational inertia components with X-structure supporting structures. These designs imitate the inertial adjustment produced by arm swinging during walking. The work [109] proposed the human body inspired anti-vibration structure (HBIAVS), as shown in Figure 6e. For the first time, this system achieved designable nonlinear stiffness, nonlinear damping, and nonlinear inertia together in a single passive device. A key discovery was inertia-induced nonlinear damping. This mechanism dissipates energy during compression and provides support during extension. These two advantages were rarely achieved in earlier designs. Figure 6f reveals that the system reached an adjustable ultra-low resonant frequency as low as 1.3 Hz. It also has the useful feature that increasing damping reduces the resonance peak without weakening high-frequency isolation.

In [110] this approach with the HBIAVS-NI system was further developed, as shown in Figure 6g. This design incorporated a small embedded X-structure that imitates muscle-tendon behavior together with the rotational inertia unit. As depicted in Figure 6h, this work identified a new effect, i.e., nonlinear inertia-induced conservative force, which absorbed energy during compression and releases it during extension. It greatly reduces fluctuations in the supporting force and improves the stability of the center of mass. The system achieved resonant

frequencies as low as 1.25 Hz and high-frequency transmissibility as low as -60 dB, see Figure 6i. These results were verified experimentally under random and impact excitations.

In summary, these studies laid the theoretical and experimental foundation for research on nonlinear inertia in X-structures. They showed that rotational motion coupling can be used in fully passive systems to produce useful inertial effects. These effects include adjustable band-stop behavior, inertia-induced damping, and inertia-induced conservative forces. All these effects significantly improve vibration isolation performance. The parametric findings and design rules from these early works directly supported the later progress in nonlinear inertia modeling, hybrid design, and novel S-type behavior reviewed in the following sections.

2.3.2. Comprehensive Modeling and Validation

Relying on the early inspiration of utilizing rotational inertia inspired by bionics (as noted in [10,109,110,156]), the work developed in [183–185] represents a major breakthrough by establishing the first comprehensive theoretical and experimental framework for nonlinear inertia within X-structures. This series of studies moved beyond proof of concept to provide a predictive, quantifiable, and practically verifiable foundations.

Ref. [183] served as the cornerstone, introducing a novel nonlinear inertia unit and systematically combining it with an X-shaped anti-vibration mechanism, as shown in Figure 7a. Lagrange-based modeling and careful geometric analysis yield

$$\left(M_1 + \frac{1}{4}M_2\right)\ddot{y} + M_2\gamma^2\left(\frac{\partial\theta}{\partial\hat{y}}\right)^2\ddot{\hat{y}} + M_2\gamma^2\frac{\partial\theta}{\partial\hat{y}}\frac{\partial^2\theta}{\partial\hat{y}^2}\frac{\partial\hat{y}}{\partial y}\dot{\hat{y}}^2 + k(x - x_s)\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial y} + (M_1 + M_2)g = -c\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial t}. \quad (7)$$

The nonlinear inertia effects are introduced by two terms, i.e., nonlinear equivalent mass or nonlinear inertia $M_2\gamma^2\left(\frac{\partial\theta}{\partial\hat{y}}\right)^2$ and nonlinear inertia incurred conservative force $M_2\gamma^2\frac{\partial\theta}{\partial\hat{y}}\frac{\partial^2\theta}{\partial\hat{y}^2}\frac{\partial\hat{y}}{\partial y}$. This study did not just confirm the existence of nonlinear inertial effects but categorized them into three typical and tunable forms, i.e., high-compression-low-extension asymmetric (HCLE, Figure 7b), low-compression-high-extension asymmetric (LCHE, Figure 7c), and a U-shaped symmetric (HCHE, Figure 7d) type. Each type was linked to distinct performance profiles, e.g., HCHE for superior low-frequency isolation, and the asymmetric types for effective high-frequency suppression. An important discovery was the tunable band-suppress property (see Figure 7e,f), enabling precise adjustment of high-frequency transmissibility through structural parameters, which is a new control dimension in passive isolation. Furthermore, the work quantified how this designed nonlinear inertia expands the system's dynamic range, enabling significant reductions in resonant frequencies and allowing for larger operational displacements, as shown in Figure 7g.

The studies [184,185] finalized this foundational work by focusing on quantification and experimental verification, and further introduced a series of pivotal tools to facilitate its engineering application. Specifically, a design parameter defined as the Ratio of Inertia (RoI, $\zeta = l_0/l_r$) was proposed, which enables the quantitative design and switching among the three nonlinear inertia types, thus solving the critical problem of design selection. In addition, a new performance metric called the F-stability indicator with the form,

$$F_{\text{stability}} = \frac{\sum(F_n - F_0)^2}{\sum(F_l - F_0)^2}, \quad (8)$$

was established to quantify the improvement in dynamic force fluctuation compared with linear systems, providing a straightforward criterion for evaluating isolation performance, as shown in Figure 7i. The study also revealed and experimentally validated an unexpected but very useful robust phenomenon, i.e., the system demonstrates enhanced robustness under stronger external excitations, with its performance improving as the vibration amplitude increases, see Figure 7j. Furthermore, to address the challenges in relevant measurements, a practical experimental protocol employing Inertial Measurement Units (IMUs, Figure 7h) was developed. This method allows the indirect but accurate characterization of the nonlinear inertia coefficient and effectively bridges the gap between complex theoretical models and reproducible experimental validation. The subsequent fabrication and testing of a physical prototype (see Figure 7h) yielded conclusive verification results, demonstrating excellent consistency between theoretical predictions and experimental results over a wide range of parameters and excitation condition, as shown in Figure 7k.

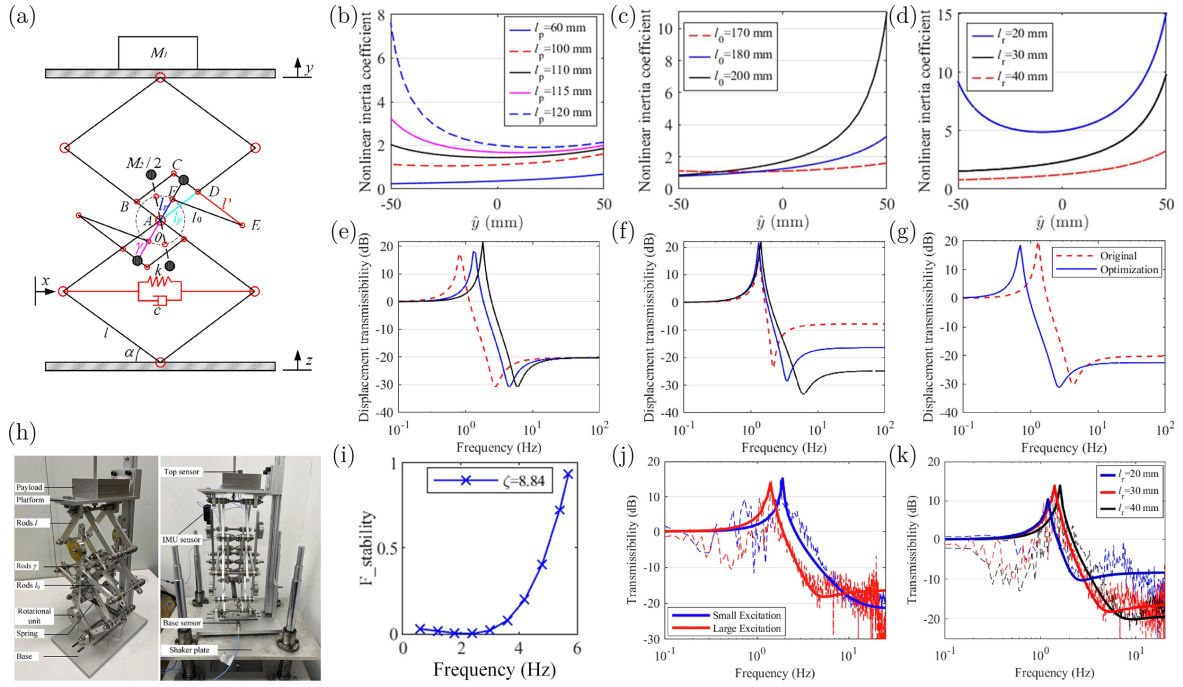


Figure 7. Nonlinear inertia characteristics, vibration isolation performance and experimental verification of the nonlinear X-inertia isolation systems [183–185], reprinted with permission from Copyright 2023, Elsevier Ltd.; Copyright 2024, Springer Nature. (a) schematic diagram of novel nonlinear inertial system; (b–d) high-compression-low-extension (HCLE) asymmetric, Low-compression-high-extension (LCHE) asymmetric and U-shaped (HCHE) symmetric nonlinear inertia feature; (e,f) tunable band-suppress performance; (g) isolation performance optimization of the system; (h) prototype and IMU-based experimental setup; (i) F-stability indicator for quantifying dynamic force fluctuation; (j) robustness performance under different excitation amplitudes; (k) effect of different rod length l_r on transmissibility.

The works presented [183–185] transformed the initial concept of “nonlinear inertia in X-structures” from a promising idea into an effective design approach. They provided researchers with a systematic understanding of its forms, the analytical tools to model it, the parameters to control it, and the methods to validate it. This comprehensive foundation is what subsequent studies, including novel hybrid designs and the exploration of new inertial nonlinearities, are based on.

2.3.3. Performance Enhancement via Hybrid Design

A distinct strategy for enhancing performance is to employ specially designed inertial elements within the nonlinear framework. An example could be referred to the Nonlinear Inertia-Stiffness Element (NISE) proposed in [186]. The work is about a direct integration of a spring and an inerter within a four-bar linkage (e.g., the basic X-shaped unit), creating a combined “negative stiffness-nonlinear inertia” mechanism. In particular, the spring force and inertial force can be expressed as

$$f_{\text{spring}} = k_h(l_s - 2l_{\text{rod}} \cos a), \quad (9)$$

$$f_{\text{inert}} = 2b_h l_{\text{rod}} (\ddot{a} \sin a + \dot{a}^2 \cos a). \quad (10)$$

This design achieves a critical functional decoupling, i.e., the X-structure is to present negative stiffness needed in the system dynamics, while the coupled inerter is to reduce dynamic stiffness near resonance. Notably, the inerter makes high-frequency transmissibility stay at a steady low level, which can also be referred to [183–185] for better understanding such phenomena with associated tuning methods. Evaluations across different system configurations confirm its effectiveness, particularly in mitigating low-frequency line-spectrum vibrations, and show application potential in areas such as marine vessel isolation.

This work provides an example of performance improvement for inertia applications through targeted hybrid component design. It demonstrates an alternative path to using inertial effects, complementary to rotational-mass-induced nonlinearity, by incorporating a two-terminal inerter, thus broadening the set of methods available for advanced vibration control based on the X-structure.

2.3.4. Exploration of Novel Inertial Nonlinearities

Pushing the development of nonlinear inertia, recent research has revealed completely new types of inertial phenomena beyond the established forms. An important contribution in this direction is [187], which describes the discovery and comprehensive characterization of novel S-type inertial nonlinearities within an X-mechanism coupled with a local rotating unit and a four-bar linkage, as presented in Figure 8a.

The study's key achievement is the identification of four different S-type variants, including one symmetric and three asymmetric (see Figure 8b), significantly expanding the classification and adjustable types of inertial effects. These S-type characteristics can be smoothly integrated with known forms (e.g., HCLE, LCHE and HCHE), offering designers a much wider range of options for adjusting dynamic responses. Furthermore, the work establishes a foundational equivalence principle, demonstrating that the complex nonlinear inertia generated by rotational components can be reproduced by a simpler, properly configured linear oscillator, as illustrated in Figure 8c. This principle greatly enhances practical feasibility by reducing mechanical complexity.

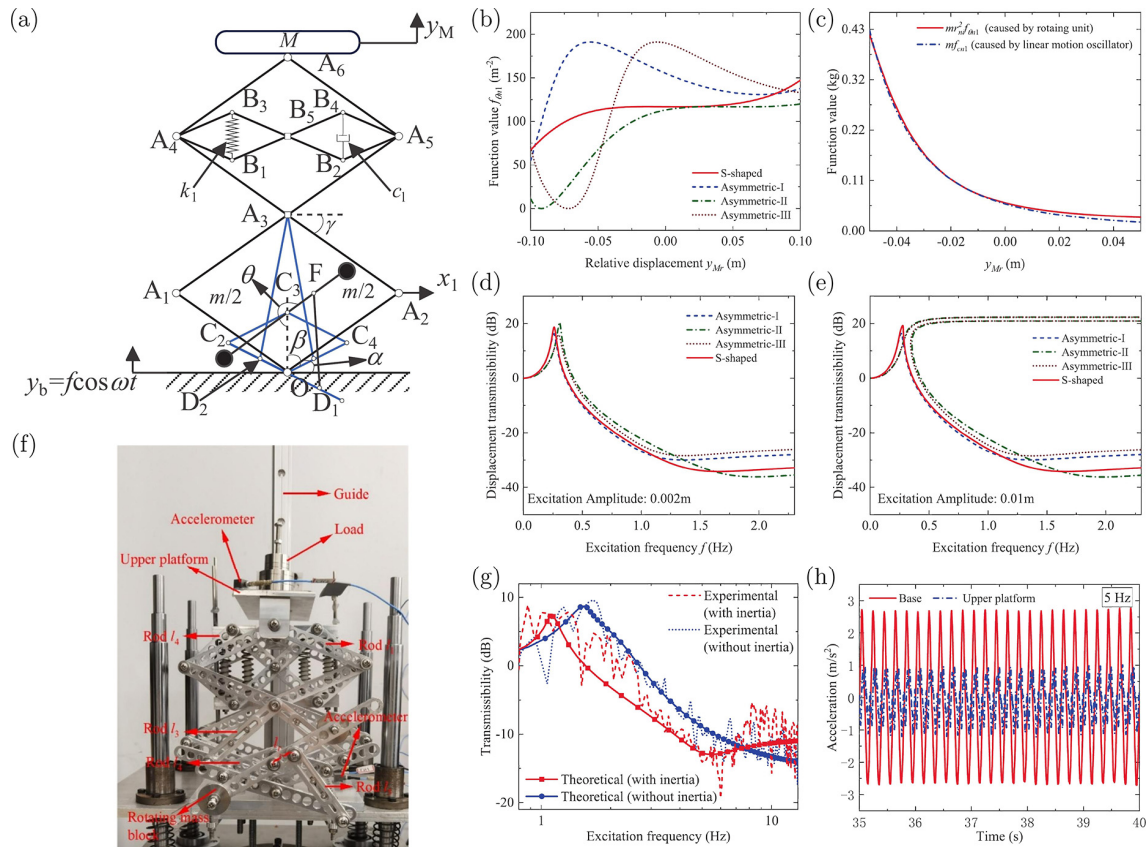


Figure 8. S-type inertial nonlinearities in the X-structure and their dynamic characteristics and validation [187], reprinted with permission from Copyright 2026, Elsevier Ltd.; (a) Schematic of the X-mechanism integrated with a rotating unit and a four-bar linkage; (b) four S-type inertial nonlinearity variants (one symmetric, three asymmetric); (c) equivalence principle between rotational nonlinear inertia and a linear oscillator; transmissibility curves illustrating (d) low-frequency resonance suppression and (e) system stability under large excitations for S-type nonlinearities; (f) experimental prototype; (g) measured displacement transmissibility under different conditions; (h) experimental verification under 5 Hz sine excitation.

The S-type nonlinearities provide excellent dynamic advantages. It can be observed from Figure 8d that they enable a marked reduction in resonant frequency (tunable to ~ 1 Hz) and peak transmissibility, while greatly enhancing the isolation decay rate (up to 20 dB/Hz). Crucially, they exhibit a strong stabilizing effect, significantly suppressing saddle-node bifurcations to maintain system stability under large excitations, as depicted in Figure 8e. This is a key advancement for robust performance. The symmetric variant demonstrates remarkable robustness across varying excitation levels, whereas the asymmetric types allow performance to be customized for specific frequency bands.

Experimental validation using a specially designed prototype depicted in Figure 8f confirmed both the existence of these theoretical phenomena and their superior isolation performance compared with typical systems, as shown in Figure 8g,h. By discovering new inertial forms, presenting a design equivalence principle, and

demonstrating their stabilizing benefits, this work opens a new direction in the fundamental understanding and application of nonlinear inertia with the X-structure, providing a route for developing next-generation, high-performance passive isolators.

3. Innovative Configurations and Structural Evolution for Complex Scenarios

Section 2 focused on improving X-structure/mechanism performance for single-direction (typically vertical) isolation in terms of stiffness, damping, and inertia, its inherent geometric flexibility allows many more applications beyond uniaxial systems. This section shifts from mechanism refinement to architectural innovation, reviewing how X-structures have been evolved and reconfigured for complex practical conditions. The advances are summarized in three parts: extension to multi-directional and MDOF isolation systems; development of multi-stage and hierarchical structures for better performance; and exploration of broader bio-inspired morphological designs that further expand the original X-structure concept. Collectively, these developments show the transition of X-structures from a specialized isolation mechanism to a flexible design platform that can handle multi-axial vibrations, space limits, and various engineering conditions.

3.1. Multi-Directional and Multi-Degree-of-Freedom Isolation

Real-world vibrations are rarely uniaxial, and thus X-structures have been extended to multi-directional and MDOF isolation. This subsection reviews advances in two areas: (i) enhanced multi-direction isolation, where X-units are arranged to suppress vibrations along multiple translational or rotational axes; and (ii) MDOF platform integration, where X-structures serve as legs in parallel mechanisms (e.g., Stewart platforms) to achieve coupled translational-rotational isolation. These developments demonstrate the evolution of X-structures from single-axis isolators to spatially adaptive vibration control systems.

3.1.1. Enhanced Multi-Direction Isolation

This subsection presents some results about how X-structures/mechanisms have been specifically modified and arranged to move beyond single-axis isolation. The emphasis is on architectural innovations, i.e., how X-units are configured, combined, or adjusted, rather than analytical approaches.

A common feature across these MDOF designs is to integrate multiple X-shaped linkages with special spring arrangements to achieve simultaneous isolation in horizontal, vertical, and rotational directions or more. As shown in Figure 9a, in [14] a 3-DOF passive isolation unit consisting of upper and lower X-shaped rods and five groups of springs was proposed, achieving a QZS range covering 75% of the total stroke with an ultra-low resonant frequency designable to zero Hz. The system exhibits stable equilibrium without jumping or bifurcation under large excitations, and its compact design (four bars, no guiding sliders) enables easy scalability. Restricting specific motions yields 1-DOF or 2-DOF variants, while combining two units can construct 6-DOF platforms. As a typical example, the decoupled Equations of Motion used in [14] can be expressed as

$$\begin{cases} M\ddot{x} + c_{1x}\dot{x} + 2c_{2x}f_{cx}(x)\dot{x} + f_{nx}(x) = -M\ddot{x}_b, \\ M\ddot{y} + c_{1y}\dot{y} + 2c_{2y}f_{cy}(y)\dot{y} + f_{ny}(y) = -M\ddot{y}_b, \\ J\ddot{\theta} + c_{1\theta}\dot{\theta} + 2c_{2\theta}f_{c\theta}(\theta)\dot{\theta} + f_{n\theta}(\theta) = -J\ddot{\theta}_b. \end{cases} \quad (11)$$

Similarly, as presented in [168], an alike 3-DOF unit integrating four central X-shaped rods, eight outer support rods, and seven springs, see Figure 9b. It achieves a 73% QZS range in the vertical direction with small resonant frequency designable to zero Hz. Parametric tunability allows flexible switching among positive, zero, quasi-zero, and negative stiffness in all directions, while maintaining high loading capacity and stability under large excitations.

Building on this similar concept, in [188] authors introduced a 3-DOF multi-directional vibration isolation system (MDVIS) featuring a detachable slider-hollow rod connection that enables flexible adjustment of key structural parameters without replacing springs or dampers, as depicted in Figure 9c. This design achieves a QZS range of up to 80% of the total stroke in the vertical direction, with measured resonant frequencies as low as 1 Hz, offering enhanced tunability and adaptability to varying operational conditions.

As another example, a 3-DOF system developed in [189] having the X-structure (in the early version referred to as scissor-like structure (SLS), see Chapter 2 of [13]). This design combines torsion springs (which provide positive stiffness and anti-overturning function) with linear springs (which provide negative stiffness). This hybrid stiffness mechanism yields a vertical QZS range significantly wider than the basic X-shaped isolators, with potential lower resonant frequency.

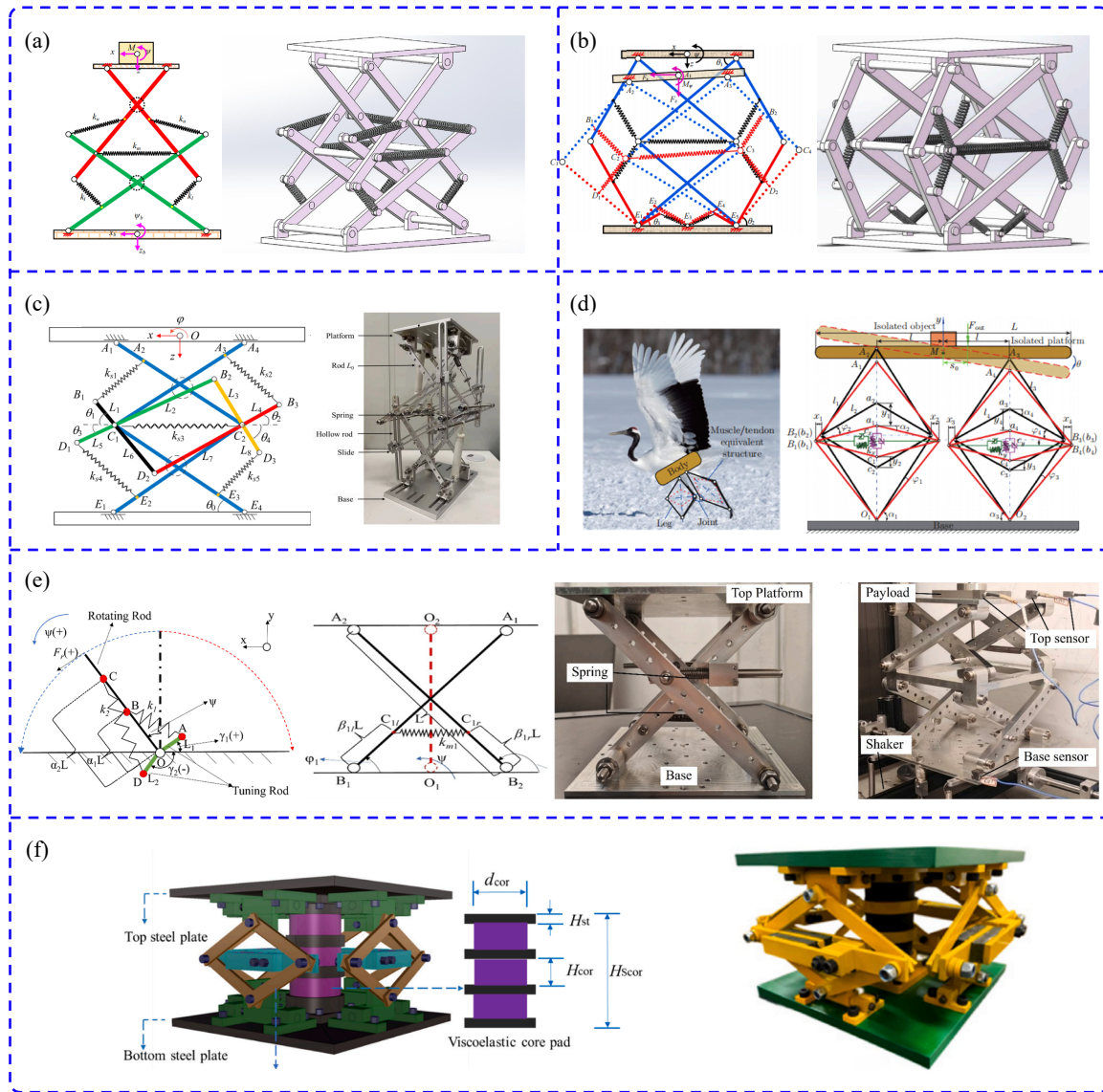


Figure 9. Typical X-structure-based multi-directional and MDOF isolation systems; (a) 3-DOF QZS isolation unit [14], reprinted with permission from Copyright 2022, Springer Nature; (b) 3-DOF multi-stiffness isolator [168], reprinted with permission from Copyright 2022, Elsevier Ltd.; (c) 3-DOF multi-directional vibration isolation system (MDVIS) [188], reprinted with permission from Copyright 2024, Elsevier Ltd.; (d) bilateral supported bio-inspired anti-vibration (BBAV) system [190] under the terms and conditions of the Creative Commons Attribution (CC BY 4.0) license; (e) rotational X-mechanism (RXM) for rotational isolation [191] under the terms and conditions of the Creative Commons Attribution (CC BY 4.0) license; (f) Viscoelastic bio-inspired multi-dimensional vibration isolation (VBM-VI) device [192–194], reprinted with permission from Copyright 2024, Taylor & Francis Group.

Beyond translational multi-direction isolation, recent work has extended the X-structure concept to rotational degrees-of-freedom. As shown in Figure 9e, authors in [191] proposed a rotational X-mechanism (RXM) that can achieve a full range of nonlinear rotational stiffness properties, including positive, negative, zero, quasi-zero, and multi-stable states. By integrating basic RXM units, 1-DOF and 2-DOF rotational platforms were constructed, with the QZS region covering almost the entire rotational stroke. Experimental validation confirmed ultra-low-frequency rotational isolation with a measured resonant frequency below 0.75 Hz, demonstrating that the geometric nonlinearity principles used in translational X-structures can be successfully transposed to rotational motion.

Considering decoupled vertical-rotational dynamics, authors of [190] studied a bilateral supported bio-inspired anti-vibration (BBAV) system with two symmetric X-shaped structures and a bearing platform, which imitates the motion of bird legs, Figure 9d. By considering link mass, moment of inertia, and vertical-rotational coupling, the system achieves tunable nonlinear properties (switching between softening and hardening types) and better loading capacity.

Extending the viscoelastic damping work reviewed in Section 2.2.1, authors of [192–194] systematically developed a viscoelastic bio-inspired multi-dimensional vibration isolation (VBM-VI) device. This design combines VE-LLS units into a multi-dimensional structure with upper and lower bearing plates, as illustrated in Figure 9f. Each VE-LLS unit includes geometric nonlinearity from the LLS and material nonlinearity from VE dampers. The collaborative load-bearing of multiple VE-LLS units improves vertical tensile resistance. This research series progressed from theoretical modeling [192] to experimental testing [193] and seismic performance verification [194]. These results show that the combined material and geometric nonlinearity can effectively improve energy dissipation, leading to lower resonant peaks and wider isolation bandwidth than traditional devices. Greater excitation amplitude reduces equivalent stiffness and improves energy dissipation at the same time, which is helpful for strong vibration environment. Shaking table tests under seismic waves of 55–220 cm/s² show dynamic amplification factors of 0.64–0.88. These results prove the effectiveness of the device in structural vibration control.

Collectively, these advances show the strong adaptability and structural flexibility of the X-structure method. From compact 3-DOF units achieving simultaneous multi-axis isolation, to rotational mechanisms extending nonlinear stiffness principles to new dimensions, and to multi-dimensional designs with hybrid damping, the X-structure has evolved from a single-axis isolator into a versatile platform for addressing complex, multi-directional vibration environments. The development of coupled dynamics and multi-material designs shows future directions where spatial adaptability and nonlinear performance can be further optimized.

3.1.2. From 3-DOF to 6-DOF Platforms

While the previous subsection focused on compact multi-directional isolators with several DOFs, real-world engineering often demands full spatial vibration control. This means suppressing translational motions along three axes and rotational motions around them at the same time. This subsection reviews the extension of X-structure principles to 6 degrees-of-freedom (6-DOF) isolation platforms. Most of them are realized through integration with the Stewart platform architecture.

The foundational work by Ref. [16] first demonstrated the feasibility of combining X-shaped structures with the Stewart configuration. As shown in Figure 10a, by equipping each of the six supporting legs with multi-layer X-shaped units and horizontal springs, the isolator achieved passive, tunable 6-DOF isolation without external energy supply. Static analysis revealed that key parameters, including rod length, initial assembly angle, layer number, and spring stiffness ratio, allow flexible switching among positive, zero, and negative stiffness. Experimental validation confirmed an ultra-low resonant frequency as low as 0.92 Hz, and the low-frequency isolation performance is much better than that of traditional linear-spring Stewart platforms.

Based on this design, authors of [195] introduced asymmetrical X-shaped structures (aXSS) as the leg units. It can be observed from Figure 10b that the three-layer asymmetric structure provides high-static-low-dynamic stiffness (HSLDS), which ensures high loading capacity and low dynamic stiffness to improve low-frequency isolation. After parameter optimization including spring stiffness, balance angle and rod length ratio, the resonant frequency is reduced by more than 50% compared with linear structures, and effective isolation starts from 1.8 Hz. This study proves that structural asymmetry can be used as an additional design method in the Stewart platform system.

An alternative design was presented in [196] by employing bio-inspired multi-joint X-structures as the supporting legs based on the multi-joint X-structure study [197]. The multi-layer structure includes quadrilateral joint units in each layer. This arrangement allows accurate control of nonlinear stiffness and increases loading capacity through symmetrical layer layout. Experimental results show that the minimum isolation frequency reaches 1.1 Hz. Differences in performance among different directions, with better performance in P_x/P_y directions than in P_z direction, offer guidance for optimization according to actual applications. The system maintained low dynamic stiffness even far from the equilibrium point, effectively suppressing large-displacement vibrations. This is an important advantage for payload isolation in spacecraft.

Figure 10c illustrates a distinct contribution of [198], which proposed an X-Stewart mechanism based on semi-X-units (SXUs), a half-variant of the full X-structure. While keeping the Stewart architecture, this design introduces translational-to-rotational motion coupling through three SXUs arranged around the z-axis. This coupling generates controllable nonlinear inertia and damping effects, providing new ways for performance optimization. Despite the MDOF platform architecture, the system works as a SDOF isolator. This is an intentional design choice that demonstrates how Stewart-based X-structures can be tailored for specific isolation requirements. The mechanism achieves a QZS region accounting for 64.9% of the total stroke (1.56 times that of typical X-shaped isolators) and an ultra-low resonant frequency of 0.5 Hz, with effective isolation starting at 0.72 Hz. Its

compact structure and high loading capacity make it particularly suitable for applications such as post-capture vibration suppression in spacecraft.

From the above overview, several observations can be made. Compact 3-DOF units [14,168,188,189] perform well in achieving ultra-low resonant frequencies and broad QZS ranges (up to 80% of stroke) within a single integrated structure. They are therefore suitable for applications with strict space limits. Rotation-focused designs such as RXM [191] achieve almost full-stroke QZS in the rotational direction ($\approx 100\%$), which is a special function rarely provided by traditional isolators. The viscoelastic-integrated VBM-VI device [192–194] focuses on energy dissipation and seismic resistance, with performance that emphasizes stability rather than the lowest possible frequency. At the highest complexity level, Stewart-based 6-DOF platforms [16,195,196] provide complete spatial vibration control, with similar translational and rotational performance. However, they have more complex structures and thus more coupled dynamics yet to be further investigated.

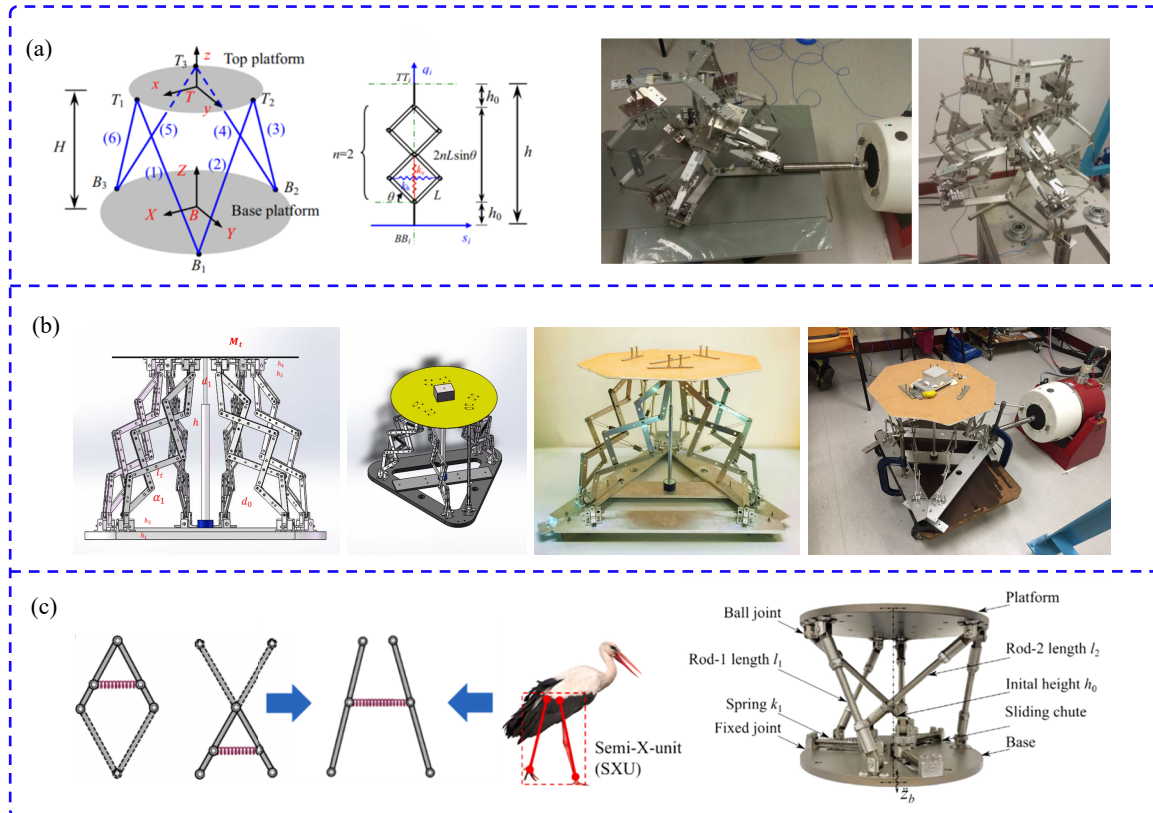


Figure 10. Typical X-structure-based 6-DOF Stewart isolation platforms; (a) 6-DOF Stewart isolator with multi-layer X-structure legs [16], reprinted with permission from Copyright 2016, Elsevier Ltd.; (b) asymmetrical X-structure (aXSS) based Stewart platform [195], reprinted with permission from Copyright 2018, Springer Nature; (c) X-Stewart mechanism with semi-X-units (SXUs) [198], reprinted with permission from Copyright 2025, Elsevier Ltd.

These studies show a clear development path, specifically the earliest Stewart-based platforms [16,195] laid the foundations for X-structure multi-DOF isolation designs. They proved that geometric nonlinearity can be applied to full spatial structures. Later compact 3-DOF units [14,168,188,189] further improved performance in specific directions. They achieved ultra-low resonant frequencies and wide QZS ranges in highly integrated structures.

At the same time, other studies developed specialized or extended functions. These include rotational isolation with the RXM [191], coupled dynamics with the BBAV [190], and combined material and geometric effects with the VBM-VI [192–194]. Most recently, motion-coupling designs such as the X-Stewart mechanism [198], suggest a future direction, where translational and rotational motions can be intentionally combined to achieve new levels of performance.

3.2. Multi-Stage and Hierarchical Designs

In addition to spatial designs for multi-directional isolation, another powerful design approach is structural hierarchy. This refers to the intentional arrangement of X-structures in series, parallel, or embedded configurations to achieve different functions and better performance. Unlike the performance-focused optimizations discussed in

Section 2 (e.g., contact mechanisms, magnetic compensation, material integration), the designs reviewed here use architectural complexity instead of material or component innovation. This subsection examines two different forms of hierarchical design, i.e., two-stage vibration isolators, where X-structures are combined with multi-platform systems to expand isolation bandwidth; and bilayer and layered structures, where inner and outer X-frameworks work together to simulate musculoskeletal coordination. These developments show how the X-structure design can be expanded and nested to meet increasingly complex vibration control needs.

3.2.1. Two-Stage Vibration Isolators

The concept of multi-stage isolation has long been employed to enhance high-frequency vibration attenuation. In such systems, the isolator includes multiple parts connected in series. Authors of [199] applied this idea to X-structures by proposing a two-stage vibration isolator that combines an X-shaped linkage mechanism with a two-platform structure. This design addresses a critical gap, i.e., traditional two-stage isolators usually have better high-frequency performance, but they are weak in low-frequency and cannot easily control nonlinear behavior. Such a design presents systematic unveiling and tuning of nonlinear characteristics within the two-stage framework. By establishing a 4 degrees-of-freedom dynamic model that accounts for both vertical and rotational displacements, the study reveals how the interaction between quadratic and cubic stiffness terms governs softening, hardening, and softening-hardening transitional behaviors. This finding is particularly valuable. Softening nonlinearity can shift the resonance peak to lower frequencies, significantly expanding the low-frequency isolation bandwidth, which is an advantage over traditional hardening-type isolators.

Experimental validation confirms that the proposed isolator achieves an initial isolation frequency as low as 2–5 Hz, with an acceleration attenuation rate of 71.3% under random excitation. Parameter studies further demonstrate that isolation performance can be adjusted by adjusting damping, mass ratio, and stiffness ratio, offering practical design flexibility. Compared with single-stage systems, the X-shaped two-stage isolator exhibits a lower isolation frequency, wider bandwidth, and faster reduction in transmissibility.

This work demonstrates that inherent nonlinear adjustability of X-mechanisms can be used effectively in hierarchical isolation systems. It offers new options for low-frequency vibration control in precision instruments and engineering structures that require high performance over a wide frequency range.

3.2.2. Bilayer and Layered Structures

Taking hierarchical design a step further, several studies have explored bilayer and layered X-structures that embed multiple X-frameworks within a single isolation unit.

Work [200] introduced the bio-inspired embedded X-shaped vibration isolation (BIE-XVI) structure, which integrates an outer X-shaped framework (simulating skeletons) and an inner embedded X-shaped structure (simulating muscles/tendons). This design simultaneously includes muscle/tendon contraction effects, joint rotational friction, and connecting rod mass in a unified dynamic model. The study reveals that softening and hardening nonlinearities can be changed into each other by adjusting the isolated mass and excitation amplitude. Based on this work, the study presented in [200] proposed a bilayer bio-inspired X-shaped vibration isolation (BBXVI) structure, which illustrates that an upper X-structure simulates the leg, while a lower X-structure represents the toe of a bird. Precise tunability of QZS characteristics can be achieved through initial assembly angle, rod length ratio, and lower leg length. The lower leg length enables mutual conversion between hardening and softening nonlinearity, offering an additional design method for performance optimization.

Throughout the studies above, the same idea is to use pure linear components within nested X-frameworks to generate beneficial nonlinear characteristics, while achieving different functions through structural hierarchy rather than material complexity. These developments expanded the design options of X-structures for creating high-performance, multi-functional vibration control systems.

3.3. Bio-Inspired Morphological Extensions

The X-structure/mechanism approach originated from the observation of animal leg/limb skeletons. This design idea has also encouraged the development of many other bio-inspired vibration isolation mechanisms. They have drawn inspiration from different biological species and proposed similar rod-spring configurations of the X-structure class that achieve beneficial nonlinear characteristics. This section aims to review representative contributions. They are not explicitly called “X-structure/mechanism”, but they share the same core methodology, i.e., constructing linkage mechanisms with embedded springs to realize QZS, HSLDS, or enhanced shock reduction, with an essential role taken by the basic X-structure components (quadrilateral, polygonal, V-shaped, or other variants). These studies, inspired by kangaroo legs, cockroach physiology, and secretary bird movement,

collectively expand the design space of bio-inspired isolators and demonstrate the broad applicability of the underlying X-structure/mechanism approach.

A series of studies has drawn inspiration from the powerful hind legs of kangaroos. These legs exhibit exceptional vibration isolation and posture stabilization during high-speed jumping. A design presented in [201] uses two main rods to imitate the calf and thigh bones, and uses linear springs to represent internal and external muscles. Similar design components can also be referred to [14,197]. This structure achieves HSLDS characteristics over a wide displacement range without reducing loading capacity. Experiments show that the isolator has an ultra-low effective starting frequency of 1.06 Hz. Its performance can be easily adjusted by changing springs or adjusting connection points on site.

Authors of [202] showed a difference example, which introduces spring connection within the X-unit to different positions. Parametric analysis shows that reducing the initial assembly angle, increasing the leg length, or enlarging isolated mass can significantly broaden the isolation bandwidth below 5 Hz. which actually shares similar nonlinear properties to those of the basic X-structure.

Work [203] proposed a cockroach-inspired structure (CIS), which uses ten rods, three springs, and hinge joints to mimics the insect's chamber, leg, and muscle configurations. The geometric nonlinearity of the rods provides negative stiffness, while the springs offer positive stiffness. Their combination results in QZS characteristics. The hardening effect of the CIS was identified as the core shock resistance mechanism. it can reduce the peak absolute acceleration by up to 91.5% under large-amplitude excitation. In [204] authors integrated the CIS with electromagnetic shunt damping (EMSD) to improve performance under complex excitation environments. By using the geometric nonlinearity of the CIS to adjust the electromagnetic coupling coefficient, the hybrid CIS-EMSD structure achieves better isolation performance at both low and high frequencies. The composite structure has a resonant frequency of 1.79 Hz and a vibration reduction rate of 61.9–88.3% in the 5–20 Hz range.

Authors of [205] proposed a secretary bird inspired vibration isolator (SBIVI). It combines an X-shaped stiffness adjustment mechanism that imitates leg bones and muscles with two independent pendulum-type vibration absorbers that imitate wings. This strongly coupled dynamic system can suppress low frequency vibrations and rapid shocks at the same time. This is a challenging combination for many traditional isolators. The study shows that increasing the inertial length ratio improves nonlinear inertia and damping effects. It can reduce the resonant frequency to as low as 1.14 Hz. It can be observed that anti-shock performance is strengthened with the attached mass design.

On the whole, these studies draw inspiration from kangaroos, cockroaches, and secretary birds. They demonstrate the generality and adaptability of the core design idea underlying X-structure/mechanism, i.e., constructing linkage mechanisms with embedded springs and use geometric nonlinearity to achieve beneficial vibration control. Rather than competing approaches, these contributions expand or extend the available methods for bio-inspired nonlinear isolation with the X-structure approach and show the value of biological inspiration and the continuous progress of nonlinear passive vibration control technologies.

4. Integration and Synergy with Advanced Technologies

Based on the deepened understanding of X-structure nonlinear characteristics in Section 2 and their extension to complex configurations in Section 3, this section focuses on how the X-structure design combines with and improves external technologies. Rather than treating the X-structure as an isolated component, the studies reviewed in this section explore its synergistic integration with active control systems, novel actuation/sensing mechanisms and metastructures. These developments represent a transition from passive exploitation of geometric nonlinearity to active and hybrid systems where the X-structure serves as a core nonlinear element within more comprehensive technical frameworks.

4.1. Synergy with Advanced Control Strategies

The inherent nonlinearity of X-structures, particularly their beneficial stiffness and damping characteristics, provides unique advantages for control system design. By incorporating X-structure dynamics as reference models or applying their nonlinear properties within control laws, researchers have achieved significant improvements in vibration suppression, energy efficiency, and robustness. This subsection reviews the evolution of these control strategies. It covers foundational studies that first established the methodology, approaches that use beneficial nonlinearity for performance enhancement, and modern frameworks based on advanced control theories.

4.1.1. Foundational Contributions in Control Exploiting X-Structure Nonlinearity

The integration of X-structure nonlinear dynamics into active control systems comes from a series of foundational studies. These studies established a new design approach. Rather than canceling suspension nonlinearities, which is the traditional method in ride comfort optimization, these works intentionally preserved and exploited beneficial nonlinearities to achieve better performance with significantly lower energy consumption.

As shown in Figure 11a, Ref. [206] first introduced this concept by adopting a bioinspired nonlinear reference model imitating the leg structure of *grus japonensis*, which is described by the following equation:

$$M\ddot{y}_r + h_1(y_r) + k_1\dot{y}_r + \mu_1\dot{y}_r + \mu_2n_xh_2(y_r)\dot{y}_r = -M\ddot{z}_u. \quad (12)$$

Through adaptive backstepping control, the active suspension was forced to track this model's output, as illustrated in Figure 11b. It used the beneficial nonlinear stiffness and damping instead of removing them. Comparative simulations demonstrated that this strategy achieved ride comfort like standard adaptive control, while reducing energy consumption by 52–76%, as shown in Figure 11c. This work established the basic idea that “working with nonlinearity, not against it” can improve both performance and efficiency.

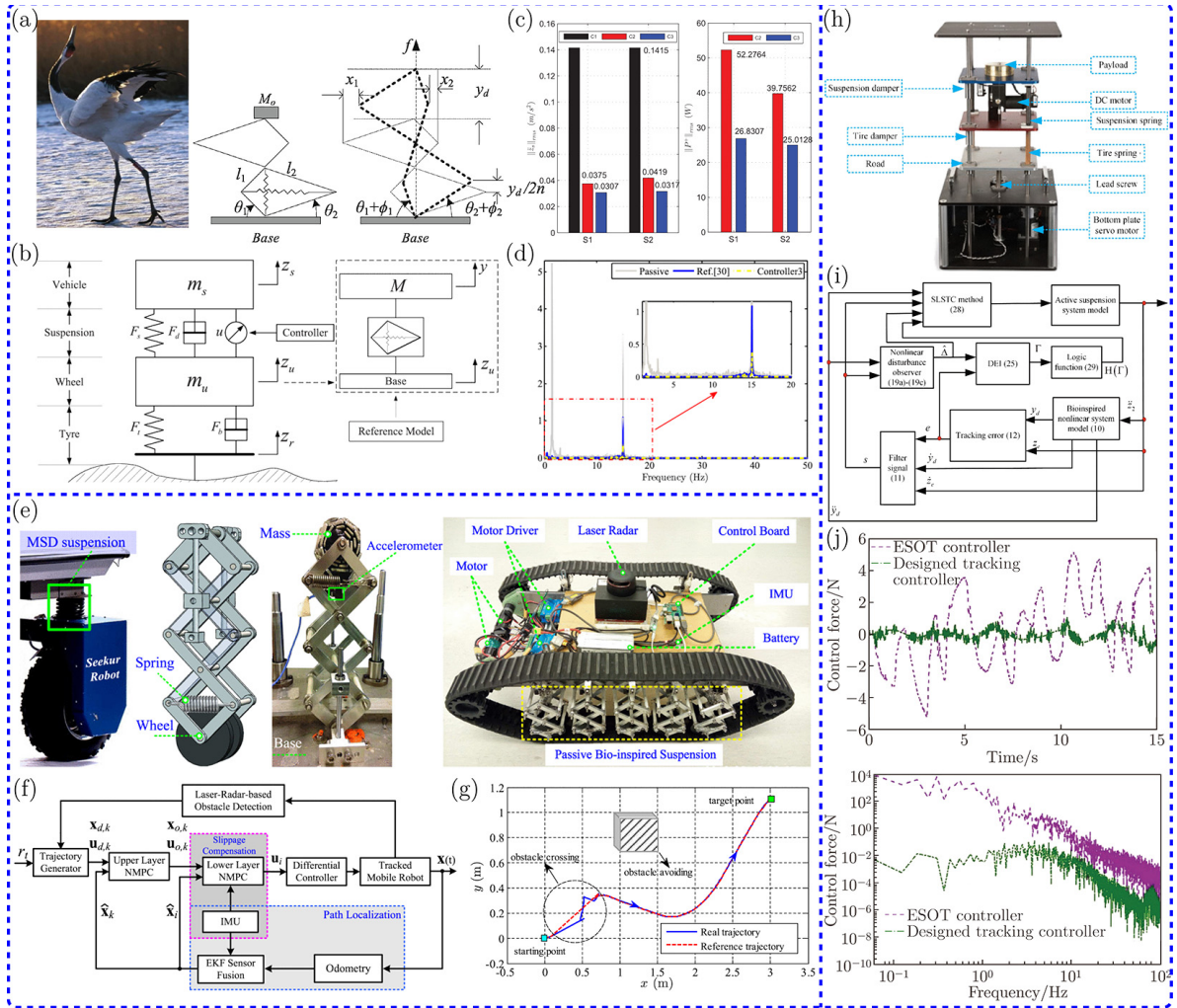


Figure 11. Foundational control strategies exploiting X-structure nonlinearity: models, control schemes and performance; (a) bioinspired reference model imitating *grus japonensis* leg structure [207], reprinted with permission from Copyright 2021, Elsevier Ltd.; (b) adaptive backstepping control for active suspension tracking [206]; (c) energy consumption and ride comfort performance [206], reprinted with permission from Copyright 2018, IEEE; (d) body acceleration RMS reduction effect [208], reprinted with permission from Copyright 2019, IEEE; (e) application of bio-inspired suspension to tracked mobile robots [209]; (f) double layer NMPC with slippage compensation [209]; (g) tracking error reduction on rough terrain [209], reprinted with permission from Copyright 2020, IEEE; (h) Quanser testbed for experimental validation [210], reprinted with permission from Copyright 2021, IEEE; (i) nonlinear disturbance observer and disturbance effect indicator (DEI) [207]; (j) control efforts of SLSTC scheme [207], reprinted with permission from Copyright 2021, Elsevier Ltd.

Authors in [208] further developed this framework by adding specially designed nonlinear damping to the bioinspired model. A quasi-band-pass filter structure provided frequency-dependent damping. This damping is strong near resonance but weak at other frequencies, which avoids the natural limitations of linear damping. From Figure 11d, it is shown that when combined with fuzzy adaptive control, this method reduced body acceleration RMS by more than 91% compared with passive suspensions. It also improved energy efficiency by 53–69% compared with standard fuzzy control. This shows that custom nonlinear damping can be effectively designed within the bioinspired reference model framework.

Figure 11e demonstrates that the work presented in [209] extended this approach to tracked mobile robots. It combined a multi-layer X-shaped passive suspension with a double-layer nonlinear model predictive control (NMPC) strategy, see Figure 11f. The bioinspired suspension provided high static stiffness to support a 10 kg load, while lowering the resonant frequency from 16 Hz to 6.3 Hz. The double-layer NMPC combines low-frequency global trajectory optimization at 5 Hz and high-frequency slippage compensation at 30 Hz. It reduced tracking errors by 38–62% on rough terrain, as presented in Figure 11g. This shows that bioinspired suspension design and advanced control can together solve both mechanical and control problems.

In [210,211] multi-source nonlinear couplings were systematically addressed whilst [211] integrated adaptive fuzzy sliding mode control with the bioinspired reference model. It achieved finite-time convergence while dealing with input dead zones and saturations. It reduced energy consumption by 44–49% compared with traditional SMC. The authors in [210] solved input delay and uncertain dynamics using adaptive neural network control. Experiments on a Quanser testbed (Figure 11h) show that this method reduced body acceleration RMS by 81% and energy consumption by 44–60% under a 20 ms delay. This was the first experiment to prove the combined effect of vibration suppression and energy saving.

In [212] it was proposed a fuzzy sliding mode control scheme that simultaneously addressed input saturation, dead zones, and external disturbances while maintaining model independence. Under C-grade random roads, it reduced vertical acceleration RMS by 91.89% and energy consumption by over 44% compared to passive suspensions. The method is simple to implement and has low computational cost. The authors of [207] introduced a new concept, i.e., disturbance classification and utilization. By designing a nonlinear disturbance observer and a disturbance effect indicator (DEI, Figure 11i), the switching logic-based saturated tracking control (SLSTC) method retained beneficial disturbances while removing harmful ones. As shown in Figure 11j, experiments on a Quanser testbed demonstrated that this approach reduced body acceleration RMS by more than 63% and energy consumption by 51–87% compared with extended state observer control. This established a new robust and green control framework.

Overall, these foundational studies established a comprehensive methodology for exploiting X-structure nonlinearities in active control systems. They showed that bioinspired nonlinear dynamics, when properly combined with adaptive control structures, can improve vibration suppression, reduce energy consumption, and enhance robustness against parameter changes, input limits, and external disturbances. These works provided the theoretical and experimental basis for the more advanced control strategies reviewed in the following subsections.

4.1.2. Beneficial Nonlinearity in Control

Following the basic idea of exploiting X-structure nonlinearities, a series of studies have further developed the concept of actively utilizing beneficial effects such as nonlinearity, disturbance, and state coupling. These works achieve synergistic optimization of vibration suppression, energy saving, and robustness.

The concept of disturbance classification and utilization was firstly proposed in [213] and it uses the DEI to divide disturbances into beneficial, harmful, and negligible groups according to their interaction with the sliding surface. Integrated with a bioinspired X-structured reference model shown in Figure 12a and a nonlinear disturbance observer, the energy-saving robust saturated controller maintained beneficial disturbances while canceling harmful ones. Experimental validation on a Quanser [214] testbed demonstrated that this intelligent disturbance processing approach reduced body acceleration RMS by 50–76% compared to passive suspensions, as shown in Figure 12b, and improved energy efficiency by 47–85% relative to extended state observer (ESOT) control.

A saturated PD-sliding mode control (PD-SMC) method was then proposed in [215] and it achieves a remarkable balance between performance and implementation simplicity. The method combines PD control for fast stabilization, SMC for robustness, and arctan saturation functions for actuator constraints. All modules are guided by a bioinspired X-structured reference model. The controller does not need accurate system parameters and only uses tracking error and its derivative. Experimental results showed body acceleration reductions of 51–91%, as presented in Figure 12c, and energy savings of 30–67% compared to passive suspensions and ESOT control, demonstrating that high performance can be achieved without high complexity.

Moreover, authors of [216] extended the method to realize finite-time convergence. It makes full use of beneficial state-coupling effects, disturbances, and nonlinearities. The method introduced two effect indicators for state coupling and disturbance, which enables the controller to retain beneficial effects while canceling harmful ones within a quantifiable finite time. Validated on a quarter-car testbed, this approach achieved body acceleration reductions of 61–92% and energy savings exceeding 60% compared to ESOT control, as depicted in Figure 12d. It also has a faster transient response than asymptotically stable methods.

The research reported in [214] represents the latest evolution, introducing the concept of effect transformation rather than simple retention or cancellation. Enhanced effect indicators with saturation functions allow precise evaluation of coupling and disturbance effects. These effects are then amplified if beneficial or converted into advantageous contributions if harmful. Combined with a fuzzy disturbance observer and bioinspired X-dynamics, this adaptive controller achieved body acceleration reductions of 36–61% and energy savings of 67–87% compared to ESOT control. This demonstrates that even inherently detrimental factors can be used to improve performance.

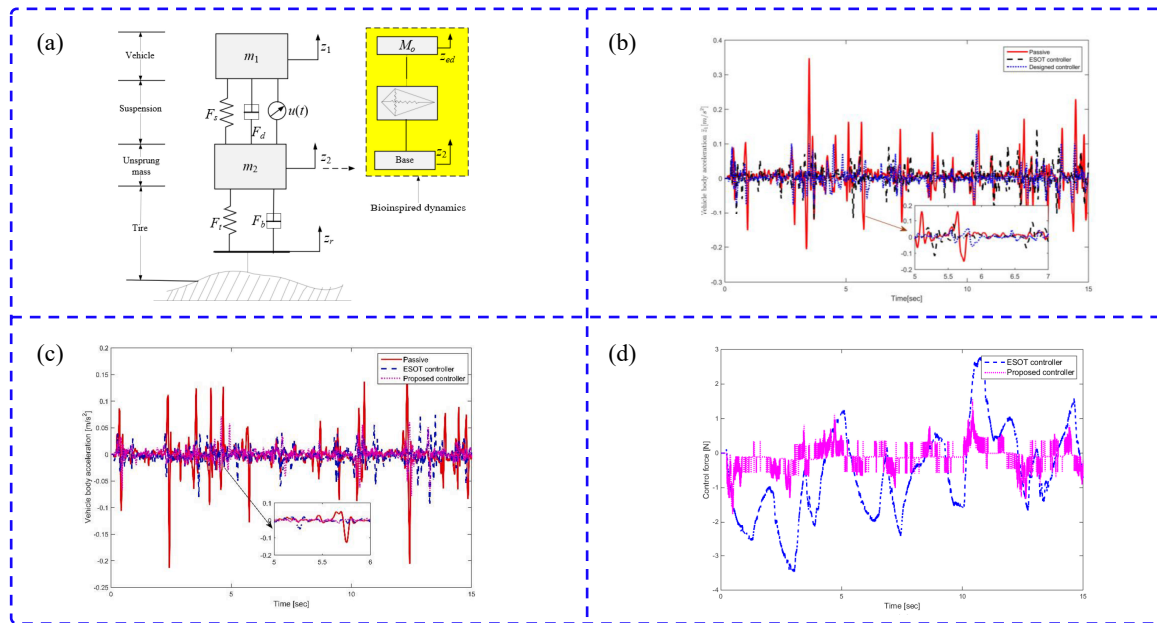


Figure 12. Advanced control strategies using beneficial nonlinearity of X-structures, including their design, implementation and performance; (a) bioinspired X-structured reference model [213]; (b) vibration reduction performance comparison [213], reprinted with permission from Copyright 2022, IEEE; (c) vibration suppression performance of PD-SMC scheme [215], reprinted with permission from Copyright 2022, Elsevier Ltd.; (d) energy saving performance of finite-time control [216], reprinted with permission from Copyright 2023, IEEE.

In general, these studies show a clear development path in the exploitation of X-structure nonlinearities within control systems. The progress starts from intelligent disturbance utilization [213], through simplicity-performance balance [215] and finite-time convergence [216], to effect transformation [214]. Throughout this evolution, the bioinspired X-structured reference model serves as the common foundation. It provides the beneficial nonlinear characteristics that support these advanced control strategies to achieve simultaneous improvements in vibration suppression, energy efficiency, and robustness. This work shows that active control systems can work together with inherent nonlinearities instead of fighting against them. It offers new directions for high-performance and energy-efficient vibration control.

4.1.3. Advanced Control Frameworks

Following the foundational exploitation of beneficial nonlinearities, new control strategies have emerged. These strategies integrate X-structure dynamics with advanced control theories to address increasingly complex engineering constraints. These frameworks move beyond traditional performance optimization to incorporate features such as fixed-time/predefined-time convergence, safety constraint satisfaction, fault tolerance, and output feedback control. All these features maintain the core principle of using X-structure nonlinearity for energy-efficient vibration suppression.

To address safety constraints and fixed-time convergence, authors in [217] proposed a fixed-time safe-by-design control method. It combines a bioinspired X-structure reference model with asymmetric time-varying barrier Lyapunov

functions (ATVBLFs) and a fixed-time auxiliary state system. The X-structure model provides the beneficial nonlinear trajectory, while ATVBLFs ensure strict compliance with time-varying displacement and velocity constraints. The auxiliary system compensates for input saturations, with fixed-time convergence independent of initial states. Experiments on a quarter-car testbed demonstrated a settling time as low as 0.304 s, 50.8% faster than nonlinear output finite-time control. It also reduces body acceleration reductions of 61–79%, as shown in Figure 13b, and saves 48–68% energy compared with passive suspensions, while strictly meeting safety constraints.

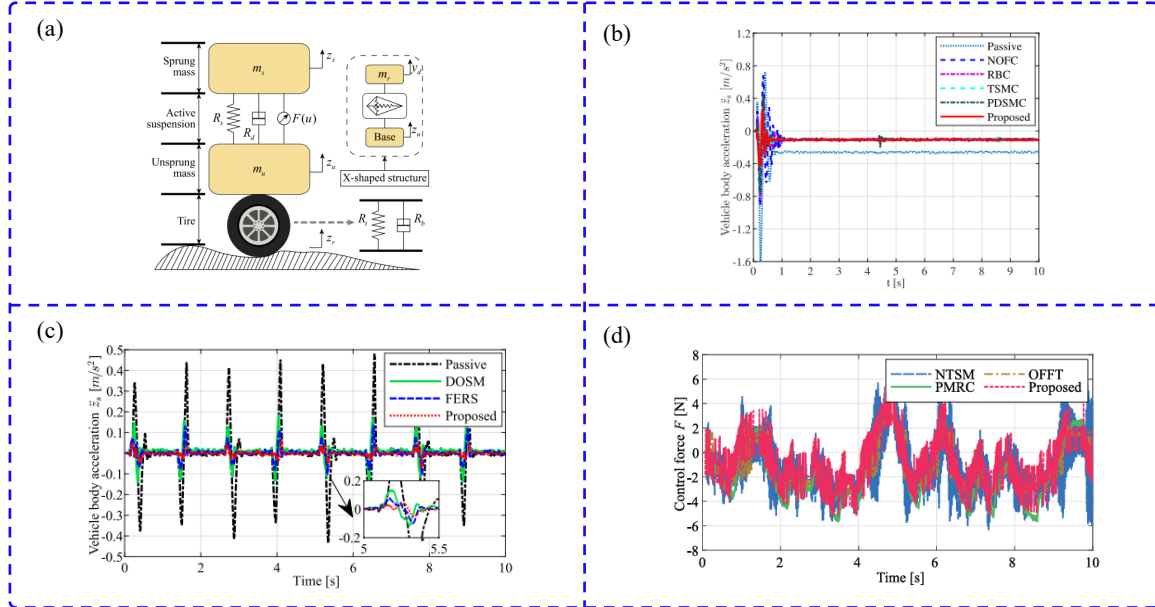


Figure 13. Modern control frameworks integrating X-structure nonlinearity: testbeds, dynamics and performance; (a) reference X-dynamics for predefined-time fault-tolerant control [218]; (b) vibration suppression performance [217], reprinted with permission from Copyright 2024, IEEE; (c) body acceleration reduction effect [218], reprinted with permission from Copyright 2024, IEEE; (d) predefined-time output feedback control force performance [219], reprinted with permission from Copyright 2025, IEEE.

Focusing on fault tolerance and predefined-time performance, work [218] introduced a predefined-time fault-tolerant control framework. Figure 13a reveals that it integrates reference X-dynamics with a predefined-time disturbance observer (PTDO) and conditional disturbance cancellation (CDC). The PTDO estimates actuator faults and disturbances within a user-set convergence time. The CDC classifies disturbances using a disturbance characteristic indicator (DCI), keeping beneficial components and canceling harmful ones. The controller achieves convergence within an arbitrarily set time, e.g., 0.5 s, that is independent of initial states. As presented in Figure 13c, it reduces body acceleration by 82–97% and saves 30–58% energy compared to passive suspensions, while eliminating singularities and mitigating chattering.

Aiming to handle system uncertainties and input dead zones, authors of [220] developed a neuroadaptive control method. It combines the bioinspired reference model with a fully actuated system transformation and radial basis function neural networks (RBFNNs). The fully actuated framework simplifies controller design and ensures asymptotic convergence to zero, while RBFNNs compensate for system uncertainties and input dead zones without requiring boundary information. Experimental results showed body acceleration reductions of 77–93% and energy savings of 32–39% compared to practical terminal sliding mode control, as illustrated in Figure 13b. It also produces continuous, chatter-free control signals suitable for real-world actuators.

To solve the practical problem of unmeasurable velocity states, authors of [219] proposed a predefined-time output feedback method. A predefined-time extended state observer (PT-ESO) estimates velocities and disturbances within a user-specified convergence time, while coupling and disturbance effect indicators (CEIs/DEIs) enable intelligent retention of beneficial components. Combined with a second-order predefined-time filter to avoid computational complexity, the controller achieves predefined-time convergence independent of initial states. It reduces body acceleration by 81–94% and saves 31–58% energy compared to nonsingular terminal sliding mode control, as depicted in Figure 13d, using only position measurements.

Overall, these control frameworks represent a major improvement in integrating X-structure nonlinearity with rigorous control theory. They share a common structure: a bioinspired X-structured reference model provides useful nonlinear dynamics. It is combined with advanced control techniques (fixed-time/predefined-time

convergence, constraint handling, fault tolerance, output feedback) to solve real-world engineering problems. All these studies have a consistent theme: X-structure nonlinearity serves as a foundation. On this foundation, complex control algorithms can achieve performance, efficiency, and robustness that traditional methods may not. This combination of mechanical nonlinear design and advanced control theory opens new possibilities for next-generation active vibration control systems. For more details on energy-saving robust control with X-structures, readers are also referred to [221].

4.1.4. Active Stiffness Modulation

The previous subsections focused on utilizing X-structure nonlinearity as a reference model for active control design. This subsection comes to physical integration, that is, active actuation mechanisms are embedded directly into X-structures to achieve real-time adaptive stiffness modulation. By combining the inherent geometric nonlinearity of X-mechanisms with electromagnetic or piezoelectric actuators, these hybrid systems overcome the fundamental limitation of passive isolators by enhancing the robustness, adjustability and adaptability with respect to external change or changing requirements such as payloads or excitation in an active and programmable manner.

A new strategy for real-time nonlinear stiffness control was proposed in [222], which introduced electromagnetically reshaped nonlinear stiffness. A two degrees-of-freedom (2-DoF) X-structured platform was developed for coupled vertical-rotational vibration isolation, as shown in Figure 14a. By releasing the central joint constraint of a traditional X-mechanism and integrating bilateral rhombic substructures with a custom Lorentz-force electromagnetic actuator, the platform achieves unified coupling of vertical and rotational motions while providing a compact structure for active stiffness modulation.

Figures 14b,c reveal that the core innovation lies in the real-time bidirectional reshaping of the nonlinear stiffness surface via electromagnetic field modulation. Unlike conventional multi-DOF X-structure studies that analyze vertical and rotational dynamics separately through decoupled models, this work presents a nonlinear electromechanical coupling model which is given by

$$\ddot{y}_e + \zeta_{y1}\ddot{a}_e + \left(\zeta_{y2} \cos \alpha_b + \zeta_{y3} \sin \alpha_b \right) \ddot{y}_b + \zeta_{y4}\ddot{a}_b + \kappa_y + \chi_{y1}\dot{y}_e + \chi_{y2}\dot{a}_e + \zeta_{y5}\dot{y}_e^2 + \zeta_{y6}\dot{a}_e^2 - \zeta_{y7}\dot{a}_b^2 + \zeta_{y8}\dot{y}_e\dot{a}_e + \zeta_{y9}\dot{a}_e\dot{a}_b + \zeta_{y3}g \sin \alpha_b + \zeta_{y2}g \cos \alpha_b = 0, \quad (13)$$

$$\ddot{a}_e + \zeta_{a1}\ddot{y}_e + \zeta_{a2} \sin \alpha_b \ddot{y}_b + \zeta_{a3}\ddot{a}_b + \kappa_a + \chi_{a1}\dot{y}_e + \chi_{a2}\dot{a}_e + \zeta_{a4}\dot{y}_e^2 + \zeta_{a5}\dot{a}_e^2 - \zeta_{a6}\dot{a}_b^2 + \zeta_{a7}\dot{y}_e\dot{a}_e - 2\zeta_{a2}\dot{y}_e\dot{a}_b + \zeta_{a2}g \sin \alpha_b = 0. \quad (14)$$

The model explicitly captures the intrinsic coupling between vertical and rotational motions caused by the X-mechanism geometry. This coupled modeling method enables coordinated multi-axis control with a single structural layer and reduces the number of actuators to one-third of that in Stewart platforms. Based on this model, a 2 DoF gain-scheduled control strategy combining model-free feedforward lookup with robust PID feedback achieves adaptive stiffness reshaping with sub-2 ms latency, as illustrated in Figure 14d. The custom Lorentz-force electromagnetic actuator has a linear thrust-current relationship and 40 mm stroke. It supports adjustable vertical load capacity up to 200% and rotational moment exceeding 1000%, and can generate multistable phenomena by regulating excitation current.

As shown in Figure 14e, experimental results confirm that the platform maintains a stable resonance frequency at 1.5 ± 0.5 Hz even when payload mass and moment of inertia both vary by 100%. It achieves vibration attenuation about -16 dB at 10 Hz under coupled excitations. Compared with passive QZS systems, this approach improves coupled-axis isolation performance by 60%, reduces computing cost by about 85% relative to conventional MIMO model predictive control, and consumes 85% less energy than MR semi-active systems. The integrated design also uses three times fewer actuators than Stewart platforms, offering a practical solution for dynamic environments where payload and vibration characteristics vary significantly.

Another effective method for active stiffness adjustment was developed in [223], which proposed an X-structure variable stiffness vibration isolator (X-VSVI). It integrates a piezoelectric actuator with a multi-layer X-shaped framework. The hybrid structure combines the beneficial nonlinear stiffness of the passive X-mechanism with an active variable stiffness unit. The unit includes a positive stiffness linear spring, a buckling-beam negative stiffness mechanism, and a piezoelectric actuator controlled by a proportional-integral (PI) controller. The preload of the negative stiffness mechanism is dynamically adjusted based on the displacement and velocity of the isolated object, enabling real-time adaptive stiffness control.

Theoretical analysis and experimental validation reveal that the X-VSVI achieves a 56.8% reduction in the first-order resonance peak and a 17.7% leftward shift in the initial isolation frequency [223]. These results are significantly better than those of the passive X-CSVI, which only achieves a 14.3% peak reduction. Parameter sensitivity analysis provides design guidelines. Increasing the layer number reduces natural frequency and

resonance peak. Decreasing the installation angle strongly lowers the first-order resonance. Adjusting PI control parameters can change the system response between periodic and quasi-periodic motion. Experiments agree well with theoretical results, which confirms that the active variable stiffness mechanism works effectively under complex vibration conditions.

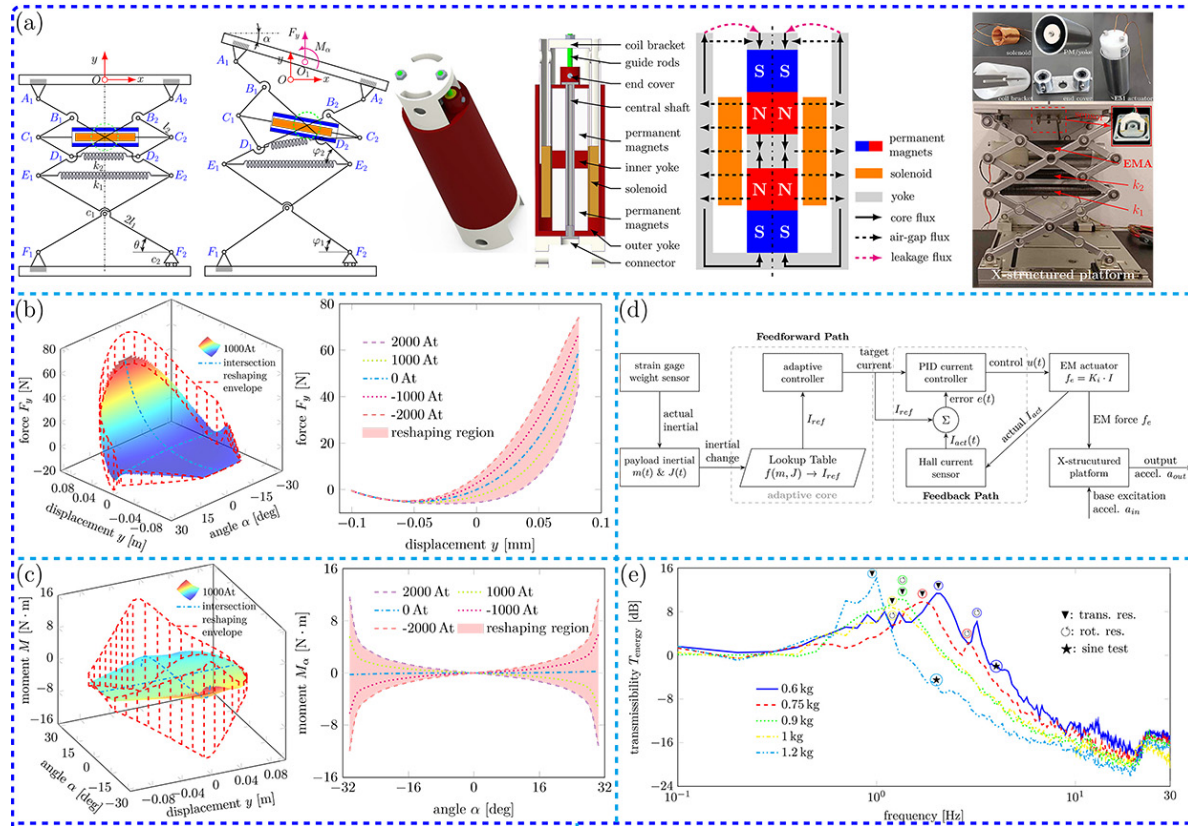


Figure 14. Active stiffness modulation in X-structure system, including design, principles, prototype and performance:

(a) 2-DoF X-structured platform with electromagnetic actuator [222], reprinted with permission from Copyright 2026, Elsevier Ltd.; (b) nonlinear vertical stiffness reshaping surface; (c) nonlinear rotational stiffness reshaping surface; (d) 2 DoF gain-scheduled control strategy; (e) vibration isolation performance under varying payloads.

Together, these studies present two complementary approaches to realize stiffness modulation in X-structures. The electromagnetic approach [222] performs well in multi-axis coupling and fast stiffness reshaping, achieving strong adaptability to payload and moment of inertia variations while maintaining compactness and energy efficiency. The piezoelectric method presented in [223] offers a simpler and more direct way to control stiffness. It effectively reduces resonance peaks and lowers isolation frequencies through real-time preload adjustment. Both approaches share a common idea, i.e., they use the X-structure's inherent geometric nonlinearity as a foundation and add active actuation to overcome the fixed characteristics of passive systems. This combination of mechanical nonlinear design with X-structures and appropriate embedded active control units supports the development of next-generation isolation systems capable of adapting to changing operational conditions without mechanical reconfiguration.

4.2. Fusion with Novel Actuation and Sensing Mechanisms

Beyond active control strategies that exploit X-structure nonlinearity as reference dynamics, another research direction focuses on physical integration. X-structures are embedded into novel actuation and sensing systems to create hybrid designs with enhanced functions. This section reviews progress in two important areas. One area is the combination of X-structures with series elastic actuators (SEAs) for precise motion control. The other is the integration of X-mechanisms with energy conversion devices for self-powered sensing and monitoring.

4.2.1. Series Elastic Actuation

The integration of X-structures with SEAs represents a novel design approach that exploits the beneficial nonlinearity of X-mechanisms for reference trajectory generation. Meanwhile, it takes advantage of the inherent

compliance and accurate torque output of SEAs to achieve precise motion control. A representative study shown in [224] demonstrates this concept on a stabilized platform for shipboard and mobile applications.

As presented in Figure 15a, the proposed system features a hybrid architecture where an X-structure serves as a virtual component to generate dynamic reference trajectories, replacing traditional fixed references. SEA provides flexible driving torque to maintain platform stability. This design is different from conventional stabilized platforms that either use rigid actuators with limited output accuracy or fail to exploit beneficial nonlinearities in their control objectives.

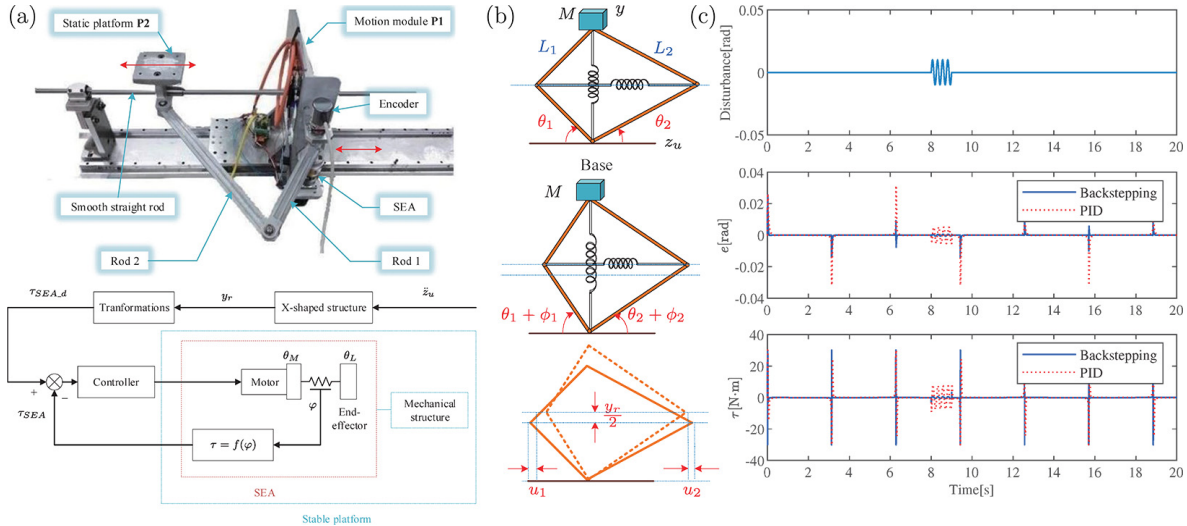


Figure 15. Integration of X-structure with series elastic actuation [224] under the terms and conditions of the Creative Commons Attribution (CC BY 4.0) license, with system design, dynamics and control performance; (a) hybrid stabilized platform with X-structure and series elastic actuator; (b) adaptive reference trajectory generated by X-structure dynamics; (c) trajectory tracking and disturbance rejection performance.

As illustrated in Figure 15b, the output of the X-structure, including displacement, velocity, and acceleration, serves as an adaptive tracking target for the platform. The system can thus use useful nonlinear stiffness and damping properties that are usually ignored in fixed-reference designs. A backstepping controller is designed to achieve high-precision trajectory tracking. Without relying on complex disturbance observers, the controller effectively deals with SEA nonlinearity, parameter uncertainties, and external disturbances while ensuring stable performance. As shown in Figure 15c, comparisons with PID control demonstrate significant advantages, i.e., settling time reduced by 85.2% from 0.135 s to 0.02 s, tracking error peaks reduced by 18.8–56.3%, and disturbance rejection improved by 81.3% under sinusoidal perturbations. The controller also remains strong robustness to load variations, with tracking error variation below 2% when payload halved.

This work introduces a useful and feasible combination between X-structure nonlinearity and SEA-based actuation. It provides an alternative insight into design methods for high-precision stabilization systems. By employing the X-structure as a virtual trajectory generator rather than a load-bearing component, it demonstrates that the beneficial characteristics of X-mechanisms can be exploited even when physical integration as a structural element is not feasible. The proposed framework is suitable for applications requiring stable control under dynamic disturbances and parameter variations, such as aerospace platforms, precision manufacturing equipment, and unmanned systems.

4.2.2. Emerging Sensing and Energy Conversion Interfaces

In addition to vibration isolation, X-structures also show unique potential as sensing platforms. Their QZS characteristics can be used to create “vibration-insensitive points” for absolute motion measurement. Early foundational work established the theoretical and experimental basis for X-structure-based sensors, while recent developments have moved toward multifunctional integration. These designs combine vibration isolation, energy harvesting, and self-powered monitoring within a single device.

The application of X-structures to absolute vibration displacement measurement originated from a series of studies exploiting their QZS properties. As presented in Figure 16a, authors in [225] proposed the first 3-D QZS sensor system. It achieves QZS behavior in three dimensions and allows indirect absolute motion measurement with errors below 0.01 under various excitations. Figure 16b shows that the work [226] advanced this concept

through prototype development. The device demonstrated measurement accuracy exceeding 80% in the 3–20 Hz band, with a natural frequency as low as 2 Hz, validating practical value of the design. Furthermore, it can be observed from Figure 16c that authors in [227] enhanced performance in noisy environments by integrating a square-root unscented Kalman filter (SR-UKF). The system achieves measurement accuracy above 85% at 8 Hz and reduces error by over 40% compared to direct measurement. This method effectively addresses the contradiction between vibration isolation and anti-noise performance. Collectively, these studies established the theoretical and experimental foundations for using X-structures as high-precision, low-cost absolute motion sensors. For more details on absolute vibration displacement sensor applications with X-structures, readers are referred to [228].

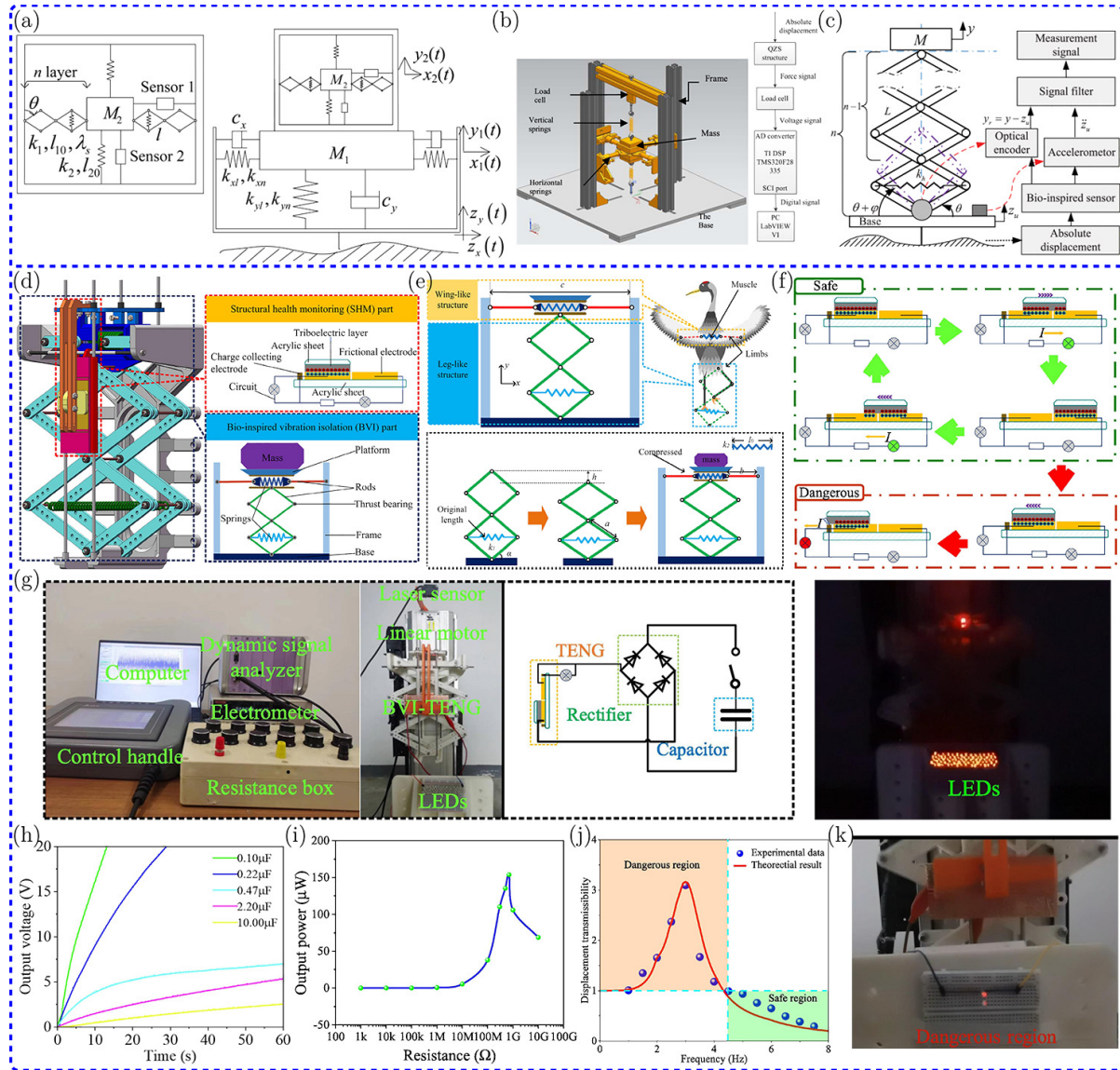


Figure 16. X-structure-based sensing and energy conversion systems, including their design, mechanisms, prototype and performance; (a) 3D QZS sensor system [225], reprinted with permission from Copyright 2015, IEEE; (b) prototype of 3DQZS motion sensor [226], reprinted with permission from Copyright 2016, IOP Publishing Ltd.; (c) bio-inspired sensor system structure [227], reprinted with permission from Copyright 2018, IEEE; (d) Bio-inspired vibration isolator integrated with TENG (BVI-TENG) [229], reprinted with permission from Copyright 2025, Elsevier Ltd.; (e) dual-structure design composed of wing-like and leg-like structure; (f) self-powered SHM signal switching mechanism of BVI-TENG; (g) experimental equipment, prototype, circuit and LED lights of the BVI-TENG; (h) capacitor charging curve of BVI-TENG; (i) peak output power of the TENG module; (j) vibration isolation performance of the BVI-TENG system; (k) LEDs lit under dangerous region.

After a period of focused development in pure X-structure sensing, the study [229] introduced a transformative concept, i.e., the bio-inspired vibration isolator integrated with a triboelectric nanogenerator (BVI-

TENG), as depicted in Figure 16d. This work represents an important change from single-function sensing to multifunctional integration, combining vibration isolation, energy harvesting, and self-powered structural health monitoring (SHM) within a single device inspired by bird limb postures.

As illustrated in Figure 16e, the system features a coupled “wing-like + leg-like” dual-structure. It achieves an enlarged QZS region for low-to-medium frequency vibration isolation, which works effectively above 4.5 Hz. At the same time, a TENG module collects vibration energy through the triboelectric effect. Critically, for sensing applications, Figure 16f shows that the SHM module supports real-time structural state monitoring through a novel signal switching mechanism. In the safe region, the TENG outputs a stable AC signal for synchronous energy collection. When vibration displacement exceeds the safe range, the signal switches to DC and triggers an alarm. This self-powered monitoring capability does not need external power supplies and enables autonomous structural health assessment.

As depicted in Figure 16g, experimental validation confirmed stable energy harvesting performance. The system charges a 0.10 μF capacitor to 20 V within 1 min (Figure 16h), with a peak power 160 μW (Figure 16i). It also provides reliable vibration isolation across 1.5–7.5 Hz, with displacement transmissibility below 1 above 4.5 Hz (Figure 16j). The integrated design demonstrates that X-structures can serve as platforms for multi-functional synergy. The same mechanical motion simultaneously enables isolation, energy conversion, and state monitoring, as illustrated in Figure 16k.

The evolution of X-structure-based sensing reflects a clear trend in this field, i.e., from specialized measurement devices exploiting QZS characteristics [225–227], to multifunctional systems that combine sensing with energy harvesting and self-powered operation [229]. The early sensor work established the theoretical and experimental foundations for using X-structures as absolute motion measurement platforms. They showed high accuracy, wide frequency range, and practical robustness. The recent integration with TENG technology represents an important change. It transforms the X-structure from a passive sensing component into an active, self-powered monitoring interface capable of real-time structural health assessment without external power. This combination of vibration control, energy harvesting, and self-powered sensing provides new opportunities for intelligent and autonomous management systems in mechanical engineering, civil engineering, ocean platforms, automation, and robots related to noise/vibration, energy harvesting, precision/process/stability control, structural health monitoring, sensing and detection etc.

4.3. Hybridization with Metastructures and Topological Design

Besides conventional vibration isolation devices, the X-structure design has recently extended into the field of metamaterials and topological design. By embedding the nonlinear mechanism of rigid X-mechanisms into flexible monolithic structures or periodic lattices, researchers have created lightweight and miniaturized systems with programmable bandgaps and QZS characteristics. These systems open new possibilities for micro-vibration control and wave manipulation.

Several preliminary attempts have been conducted in this emerging direction. As presented in Figure 17a, the authors of [230] introduced a transformative design approach. It replaces the rigid links and springs of a multi-stage X-mechanism with flexible arc beams and annular elastic elements to create a monolithic, coin-sized QZS isolator. Figure 17b reveals that the combined effect of buckling softening from vertical arc beams and tensile hardening from annular horizontal elements reproduces the beneficial nonlinearity of the X-mechanism. This design requires no rigid joints or separate springs. Finite element simulations and 3D-printed prototypes validated the QZS characteristic. The load capacity can be adjusted by changing the inner radius parameter, see Figure 17c. This work demonstrated that X-structure nonlinearity could be embedded into a single, miniaturized elastic structure, overcoming the size and complexity limitations of traditional QZS isolators.

Building on this foundation, the authors of [231] further developed the concept into programmable topological meta-structures, termed the X-QZS Meta-ring. As shown in Figure 17d, by topologically converting X-mechanism components into meta-beams and meta-rings, a single flexible monomer inherits the “positive-negative stiffness synergy” of the X-mechanism. It also achieves a weight only 2% of its rigid counterpart. A parameter mapping diagram relating centroid radius, beam thickness, and stiffness characteristics was established to allow customized QZS design and given by

$$M_\theta = F_v \cdot R_n (\cos \theta - \cos \theta_0) \text{ and } F_v = \frac{AE\epsilon_\theta(R_c - R_n)}{R_n(\cos \theta - \cos \theta_0)}. \quad (15)$$

As depicted in Figure 17e, single-layer, double-layer, and triple-layer meta-ring matrices were constructed through horizontal and vertical arraying. This design demonstrated linear load superposition and a progressively widening QZS interval with reducing resonance frequency from 3.09 Hz to 1.83 Hz for the triple-layer matrix, as

shown in Figure 17f. Dynamic tests confirmed an energy absorption ratio of 98.36% under random vibration. This establishes the X-QZS Meta-ring as a lightweight, high-performance platform for micro-vibration control.

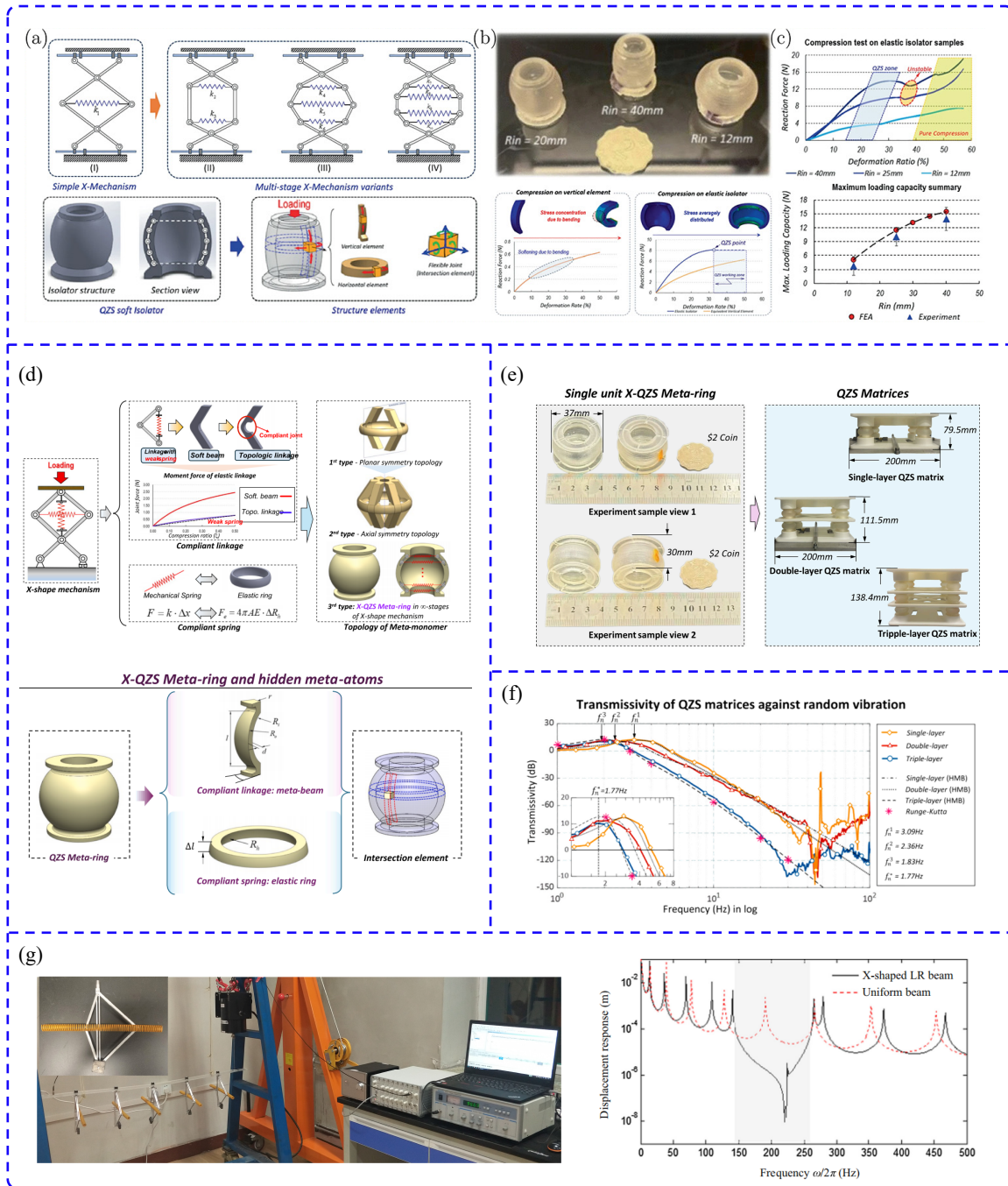


Figure 17. X-structure-inspired metastructures and topological designs, their configurations, mechanisms, prototypes and performance; (a) monolithic coin-sized QZS isolator with flexible arc beams [230]; (b) nonlinear stiffness mechanism of the flexible X-based isolator [230]; (c) load-bearing capacity tunability [230], reprinted with permission from Copyright 2024, Springer Nature; (d) X-QZS Meta-ring topological meta-structure [231], reprinted with permission from Copyright 2025, Elsevier Ltd.; (e) arrayed meta-ring matrices (single-layer to triple-layer); (f) resonance frequency of meta-ring matrices; (g) elastic metamaterial beam with X-shaped local resonators [232], reprinted with permission from Copyright 2019, Elsevier Ltd.

In parallel with these studies, other researchers have explored X-structure-inspired metamaterials with distinct mechanisms. As shown in Figure 17g, in [232] an elastic metamaterial beam with X-structure local resonators embedded periodically was proposed. By adjusting the rod length ratio and initial angles, the equivalent stiffness could be increased or reduced. This enables wide-range adjustment of bandgaps, such as a shift of more

than 100 Hz. Experiments on a 1.2 m prototype confirmed a clear bandgap in the 140–260 Hz range with vibration attenuation of several orders of magnitude.

In [233] single-stage and multi-stage X-shape inertial amplification (XIA) mechanisms integrated periodically into beam-type metastructures was introduced. The XIA mechanisms induce coupling between transverse and longitudinal waves, forming multiple bandgaps. Multi-stage designs reduced the initial frequency of the low-frequency bandgap by 33.6% compared to single-stage counterparts. Hybrid unit schemes achieved a bandgap coverage rate of 86.4%, demonstrating better attenuation efficiency for thin-walled structures.

It can be seen that in [234] the authors developed a spring-mass-truss (SMT) elastic metamaterial based on X-structures. Using the inertial amplification mechanism, horizontal input displacement is converted into amplified vertical motion of vertex masses. This expands low-frequency bandgaps without added weight. A 2DOF SMT chain achieved dual bandgaps with transmittance as low as 3.8%, significantly better than traditional locally resonant metamaterials.

Overall, these studies illustrate the strong combination between X-structural nonlinearity and metamaterial design. The studies [230,231] investigated the feasibility for transition from rigid mechanisms to flexible topological structures. They achieved miniaturization, programmability, and high load-bearing efficiency in a monolithic format. The research results in [232–234] used X-structure-inspired geometries for adjustable bandgaps, inertial amplification, and multi-stage wave attenuation. Together, they establish new design approaches, i.e., X-structures are no longer limited to discrete isolators but can serve as building blocks for scalable, lightweight metamaterials with designed wave propagation characteristics. Future directions may include multi-physics integration such as energy harvesting, adaptive topological optimization, and application in environments sensitive to micro vibration such as spaceborne instruments and MEMS devices.

5. Application Expansion: From Vibration Control to Multifunctional Systems

Previous sections focused on the understanding of X-structure nonlinear characteristics and their integration with other state-of-the-art technologies. This section is to discuss applications and application-driven expansion, related to how X-structures have been deployed beyond pure vibration control to address broader engineering challenges. The studies reviewed in this section show that the same geometric nonlinearity that enables excellent isolation can also be used for energy harvesting, robotic locomotion, and spacecraft protection etc. This transforms the X-structure from a specialized isolator into a versatile tool or design methodology for multifunctional systems with versatile nonlinear manipulation. This evolution reflects a dedicated field where fundamental principles are adapted to meet the diverse demands of real-world applications.

5.1. Energy Harvesting with Broadened Bandwidth and Enhanced Directionality

The inherent nonlinearity of X-structures, particularly their ability to achieve broadband, low-frequency resonance, makes them naturally suitable for vibration energy harvesting. Efficient conversion of ambient vibrations into electrical power requires matching the harvester's dynamics to the excitation spectrum. This subsection reviews the progressive integration of X-structure principles into energy harvesting systems. It covers foundational studies that established the core mechanisms, advanced modeling frameworks, multi-direction harvester designs, and the latest developments in self-powered multifunctional systems. The common feature across these works is the exploitation of X-structure nonlinearity to overcome the narrow bandwidth limitation of linear harvesters, enabling efficient energy extraction across wider frequency ranges and from multiple directions.

5.1.1. Foundational Contributions in X-Structure Energy Harvesting

The application of X-structures to vibration energy harvesting originated from a series of pioneering studies. These studies established the core principles of using geometric nonlinearity for broadband, low-frequency energy conversion. These foundational works systematically revealed how the tunable nonlinear stiffness and damping characteristics of X-mechanisms could overcome the narrow bandwidth limitation of linear harvesters while maintaining output stability.

The authors of [235] first introduced the concept of using X-structures for energy harvesting, proposing both single DOF and two DOF configurations, as shown in Figure 18a. Through rigorous dynamic modeling, the study showed that the geometric nonlinearity of X-structures provides low dynamic stiffness, significantly improving low-frequency harvesting efficiency. The 2DOF system uses translational-rotational motion coupling between two parallel X-structures to achieve broadband performance, with a maximum power peak of 88 dB. This performance is much better than that of linear harvester. Crucially, this work proved that nonlinear stiffness and damping could

be directly regulated through structural parameters such as assembly angle, layer number, and rod length, without introducing complex equilibrium control or output jump phenomena.

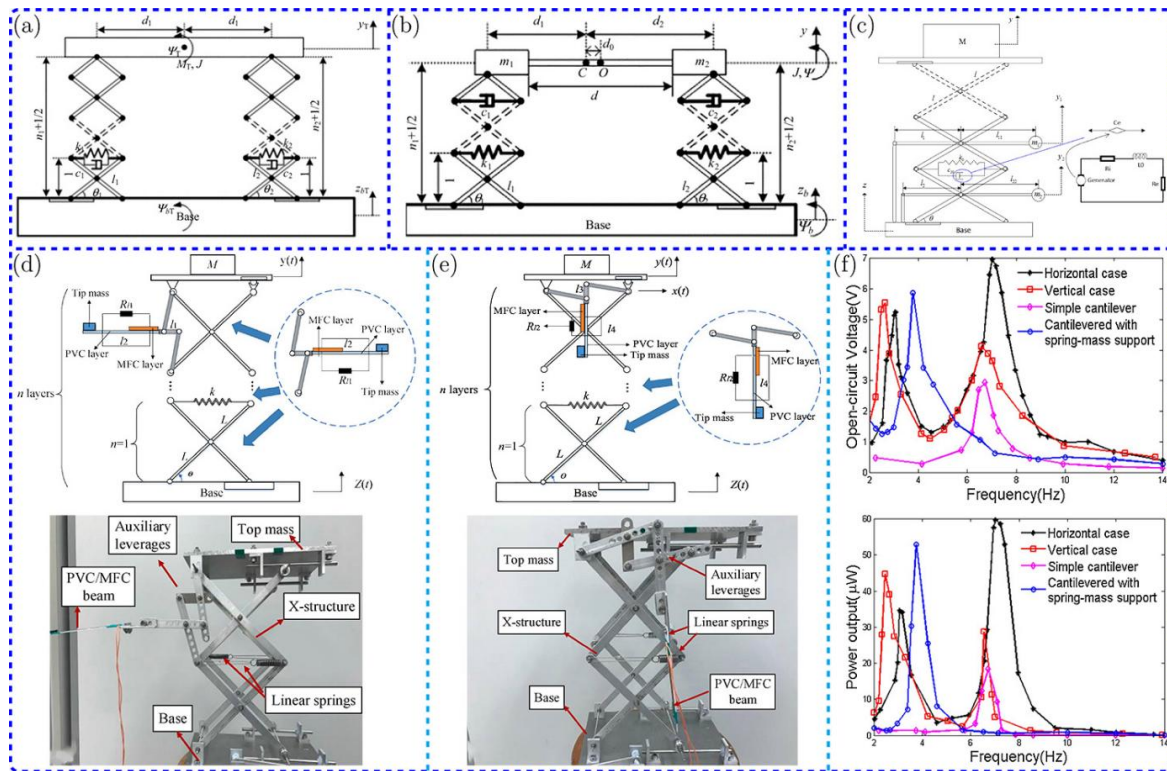


Figure 18. Foundational X-structure energy harvesters, including their configurations, mechanisms, prototypes and experimental performance; (a) 2DOF X-structured energy harvester configurations [235], reprinted with permission from Copyright 2016, Springer Nature; (b) 2DOF X-structured harvester with adjustable-inertia module [236], reprinted with permission from Copyright 2017, Springer Nature; (c) X-structured harvester integrated with lever amplification mechanism [237], reprinted with permission from Copyright 2017, Elsevier Ltd.; (d) Horizontal mounting configuration of X-structured piezoelectric harvester [238]; (e) vertical mounting configuration of X-structured piezoelectric harvester [238]; (f) experimental harvesting performance and output power [238], reprinted with permission from Copyright 2019, Elsevier Ltd.

Figure 18b reveals that the authors of [236] further developed this design by introducing an adjustable-inertia module into a 2DOF X-structured harvester. By exploiting asymmetric mass distribution, the system achieved enhanced mode coupling between translational and rotational motions. This enables simultaneous tuning of stiffness, damping, and inertia. This triple adjustability broadened the effective bandwidth by over 30% compared to uniform mass designs while maintaining stable output without bifurcation or chaos. The work demonstrated that inertia adjustment could independently optimize low-frequency performance and full-band power output, offering great design flexibility.

As shown in Figure 18c, the work reported in [237] further expanded the design scope by integrating a lever amplification mechanism with an X-structure supporting structure. The lever module amplified displacement input, while the X-structure provided displacement-dependent nonlinear damping, creating a synergistic dual-gain effect. This configuration achieved remarkable frequency adaptability, with resonance frequencies tunable from ultra-low (0.064 Hz) to high ranges by adjusting lever ratios and mass distributions. Compared to nonlinear systems without levers, power output in the low-frequency range was enhanced by up to 64.6 dB, while maintaining the weak nonlinearity essential for stable operation.

The implementation of theoretical analysis to practical application was reported in [238], which presented the first complete design-to-experiment validation of an X-structured piezoelectric energy harvester targeting ultralow-frequency bridge vibrations. Two mounting configurations (horizontal in Figure 18d and vertical in Figure 18e) were developed, both using the X-structure's ultra-low equivalent stiffness to achieve effective harvesting from 2 Hz to above 10 Hz. As shown in Figure 18f, experimental results confirmed three distinct harvesting peaks (2.42 Hz, 4.21 Hz, and 7.2 Hz), with a maximum output power of 84 μ W. This power is sufficient to supply micro-sensors. The prototype has a high static load capacity, supporting 0.8 kg with only 1/4 the mass

of conventional designs. Its tunable frequency response demonstrated that X-structure-based harvesters can be used in real-world applications such as bridge health monitoring.

In summary, these foundational studies provided the theoretical and experimental groundwork for X-structure energy harvesting. They showed that geometric nonlinearity can be systematically exploited to achieve broadband, low-frequency energy conversion with stable output. These capabilities cannot be achieved with linear harvester. The development process from 1DOF/2DOF concept validation [235], through multi-parameter tunability [236] and lever-enhanced amplification [237], to full experimental implementation [238], reflects a promising research trajectory. These early contributions not only validated the core mechanisms but also provided the design methodologies and parameter optimization guidelines. These form the basis for the more advanced harvesting systems reviewed in the following subsections.

5.1.2. Advanced Modeling for Harvesting

In addition to structural innovations, the advancement of X-structure energy harvesting has been significantly driven by sophisticated frequency domain modeling techniques. Drawing upon Volterra series theory, a key foundational contribution in the field, these methods establish explicit analytical relationships between structural parameters and energy output. They enable systematic optimization without the need for computationally intensive time-domain simulations. This subsection reviews the evolution of this modeling approach, from initial conceptual application to the fully developed nonlinear characteristic output spectrum (nCOS) method.

The authors in [239] first applied Volterra series-based frequency domain analysis to X-structured nonlinear energy harvesting systems shown in Figure 19a. By deriving the generalized frequency response functions (GFRFs) [240] up to the 7th order, the study established a direct analytical connection between structural parameters (assembly angle, rod length, layer number, spring stiffness) and the system's output frequency response. A key innovation was the construction of a power generation function (PGF), which can be expressed as

$$|P_{ave}(j\omega)| = \frac{\omega^2}{8} \left\{ \begin{aligned} & c_e \delta_0 \left[4|2Y^{\hat{}}(j\omega)|^2 + 16|2Y^{\hat{}}(j2\omega)|^2 \right] + \\ & \delta_3 \left[-4|Y^{\hat{}}(j0)||2Y^{\hat{}}(j\omega)|^2 - 16|Y^{\hat{}}(j0)||2Y^{\hat{}}(j2\omega)|^2 + 6|2Y^{\hat{}}(j\omega)|^2|2Y^{\hat{}}(j2\omega)| \right] + \\ & \delta_4 \left[4|Y^{\hat{}}(j0)|^2|2Y^{\hat{}}(j\omega)|^2 + |2Y^{\hat{}}(j\omega)|^4 - 12|Y^{\hat{}}(j0)||2Y^{\hat{}}(j\omega)|^2|2Y^{\hat{}}(j2\omega)| + \right. \\ & \left. 16|Y^{\hat{}}(j0)|^2|2Y^{\hat{}}(j2\omega)|^2 + 10|2Y^{\hat{}}(j\omega)|^2|2Y^{\hat{}}(j2\omega)|^2 + 4|2Y^{\hat{}}(j2\omega)|^4 \right] \end{aligned} \right\} \quad (16)$$

where $Y^{\hat{}}(j\omega)$, $Y^{\hat{}}(j0)$, and $Y^{\hat{}}(j2\omega)$ represent the first order harmonic output, the 0th order harmonic output (the DC component) and the second order harmonic output respectively. They are the functions of structural parameters via GFRFs. The PGF function for the first time linked the full chain of “structural parameters-frequency response-energy output” in an explicit analytical form. Using an X-structured electromagnetic harvester as a benchmark, the method revealed that the synergistic effect of stiffness and damping is critical for low-frequency performance. Figure 19b showed that the nonlinear system achieved a peak power of 1.474 W, compared to only 0.382 W for its linear equivalent. This work showed that frequency domain analysis could efficiently capture high-order nonlinear coupling effects that are difficult to quantify with traditional time-domain methods.

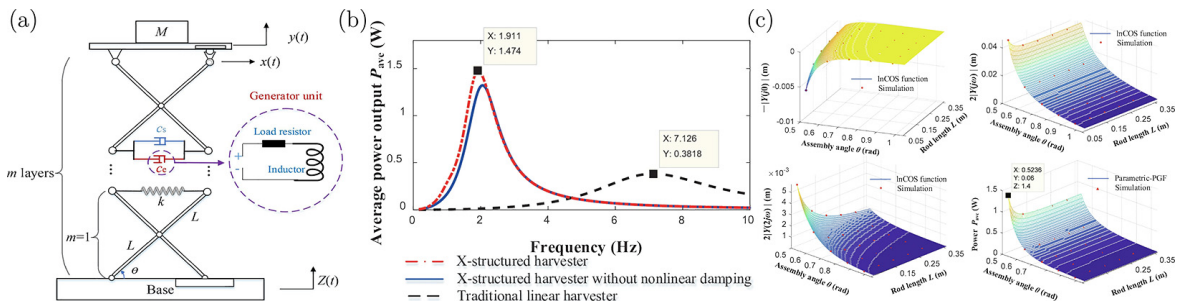


Figure 19. Advanced frequency domain modeling for X-structure energy harvesting: systems and performance; (a) X-structured nonlinear energy harvester for Volterra series analysis [239,241]; (b) power output comparison between nonlinear and linear X-structured harvesters [239], reprinted with permission from Copyright 2022, Springer Nature; (c) parameter-performance relationships of X-structured harvesters via InCOS method [241], reprinted with permission from Copyright 2022, Elsevier Ltd.

Building upon this, the authors in [241] introduced a parametric frequency domain approach, i.e., the linear-nonlinear characteristic output spectrum (InCOS) method. It is currently the most systematic and powerful

modeling framework for X-structure energy harvesting. The core innovation lies in the derivation of a parametric power generation function (parametric-PGF), which expresses the average output power as an explicit analytical function of design parameters, including assembly angle θ , rod length L , and equivalent electrical damping c_e .

The lnCOS method achieves this by decomposing the system's output spectrum into linear and nonlinear components using specially designed coefficient extractors, i.e., $CE_L(\cdot)$ and $CE_N(\cdot)$, and hence, the new lnCOS function could be given by

$$Y(j\omega) = \left(\bigoplus_{n=1}^N \left[CE_N \left(H_n(j\omega_1 j\omega_2, \dots, j\omega_n) \right) \otimes CE_L \left(H_n(j\omega_1 j\omega_2, \dots, j\omega_n) \right) \right] \right) \Psi(j\omega)^T. \quad (17)$$

This decomposition enables direct quantitative insights into parameter-performance relationships. As illustrated in Figure 19c, increasing θ from $\pi/6$ to $\pi/3$ shifts the resonance frequency from 2 Hz to 6 Hz while dramatically reducing power output from 1.4 W to below 0.1 W. Reducing L below 0.15 m strengthens nonlinear effects, increasing peak power to 1.5 W at $L = 0.06$ m. And increasing c_e from 50 to 200 Ns/m monotonically decreases output from 1.15 W to 0.65 W, revealing a trade-off between damping and vibration amplitude. Crucially, the method achieves high accuracy with relative errors mostly below 2%, while avoiding the computational complexity of harmonic balance or numerical integration.

The lnCOS/nCOS framework represents a fundamental theoretical tool that extends far beyond energy harvesting applications. By establishing explicit, analytical relationships between system parameters and output characteristics, it enables three key capabilities. First, it facilitates rapid parameter optimization without the need for iterative simulation. Second, it supports sensitivity analysis to identify dominant design variables, for instance, the system is highly sensitive to parameter changes when $L < 0.15$ m and $\theta < 0.6$ rad. Third, it enables direct performance prediction for any parameter combination within the design space.

Furthermore, the methodology is inherently applicable to any nonlinear system with linear-nonlinear coupling characteristics, and not limited to X-structures or energy harvesting. It provides a unified framework for analyzing how specific nonlinear parameters influence output spectra. This makes it invaluable for designing and optimizing a wide class of nonlinear dynamical systems.

The progress made from [239] to [241] demonstrates a systematic development of frequency domain analysis for X-structure energy harvesting. The early work confirmed the feasibility of Volterra series-based modeling, while the lnCOS method transformed this approach into a systematic, parametric, and highly efficient design tool. By making explicit the implicit relationships between structural parameters and energy output, this methodology not only speeds up the design of optimal X-structured harvesters but also provides deep understanding into the underlying nonlinear mechanisms. These insights are essential for the advanced multi-directional and multifunctional systems discussed in the following subsections. For a more comprehensive understanding of Volterra series-based frequency domain analysis theory and the nCOS method, readers can refer to [240,242].

5.1.3. Multi-Directional and Low-Frequency Harvesters

Based on the fundamental understanding of X-structure nonlinear dynamics, a major research focus has centered on extending energy harvesting capabilities to multiple directions and ultralow frequencies. By exploiting the inherent MDOF motion coupling and geometric nonlinearity of X-mechanisms, these designs overcome the unidirectional limitation of traditional harvesters while achieving broadband, low-frequency performance. This subsection reviews several systematic advances in this field, covering the development of multi-directional harvesters, dual-function integration, self-powered application validation, and a complementary dual-nonlinear integrated design.

As shown in Figure 20a, the authors of [243] introduced the first 3-DOF X-structured piezoelectric energy harvester, which enables simultaneous energy harvesting in vertical, horizontal, and rotational directions. By coupling an X-mechanism with piezoelectric cantilevers, the system utilized geometric nonlinearity and inertial coupling to achieve dual resonance peaks and broadband band-pass characteristics in all directions, as presented in Figure 20b. Figure 20c illustrated experimental validation results, with peak powers of 2.5 mW (vertical), 41.2 μ W (horizontal), and 86.2 μ W (rotational) at 9–13.5 Hz, and an effective harvesting frequency ranges over 20 Hz. The motion coupling effect, in which unidirectional excitation induces multi-directional structural responses, further broadened the operating bandwidth, particularly in the ultra-low frequency range below 10 Hz.

Besides, as shown in Figure 20d, the work presented in [244] provided systematic experimental validation of the dual-functional capability of the same 3-DOF X-structure, confirming both its energy harvesting and vibration isolation performance. With no structural innovations introduced, this study demonstrated the vibration isolation effectiveness of the structure for the first time. Excellent low-frequency isolation below 4 Hz is achieved in the vertical and horizontal directions, and the resonance frequency decreases as the excitation amplitude increases,

e.g., from 5.6 Hz at 1 mm amplitude to 4.9 Hz at 2 mm amplitude, as depicted in Figure 20e. This work confirmed that the HSLDS of the X-structure enables seamless integration of energy harvesting and vibration isolation, which serves as a critical validation for practical engineering applications.

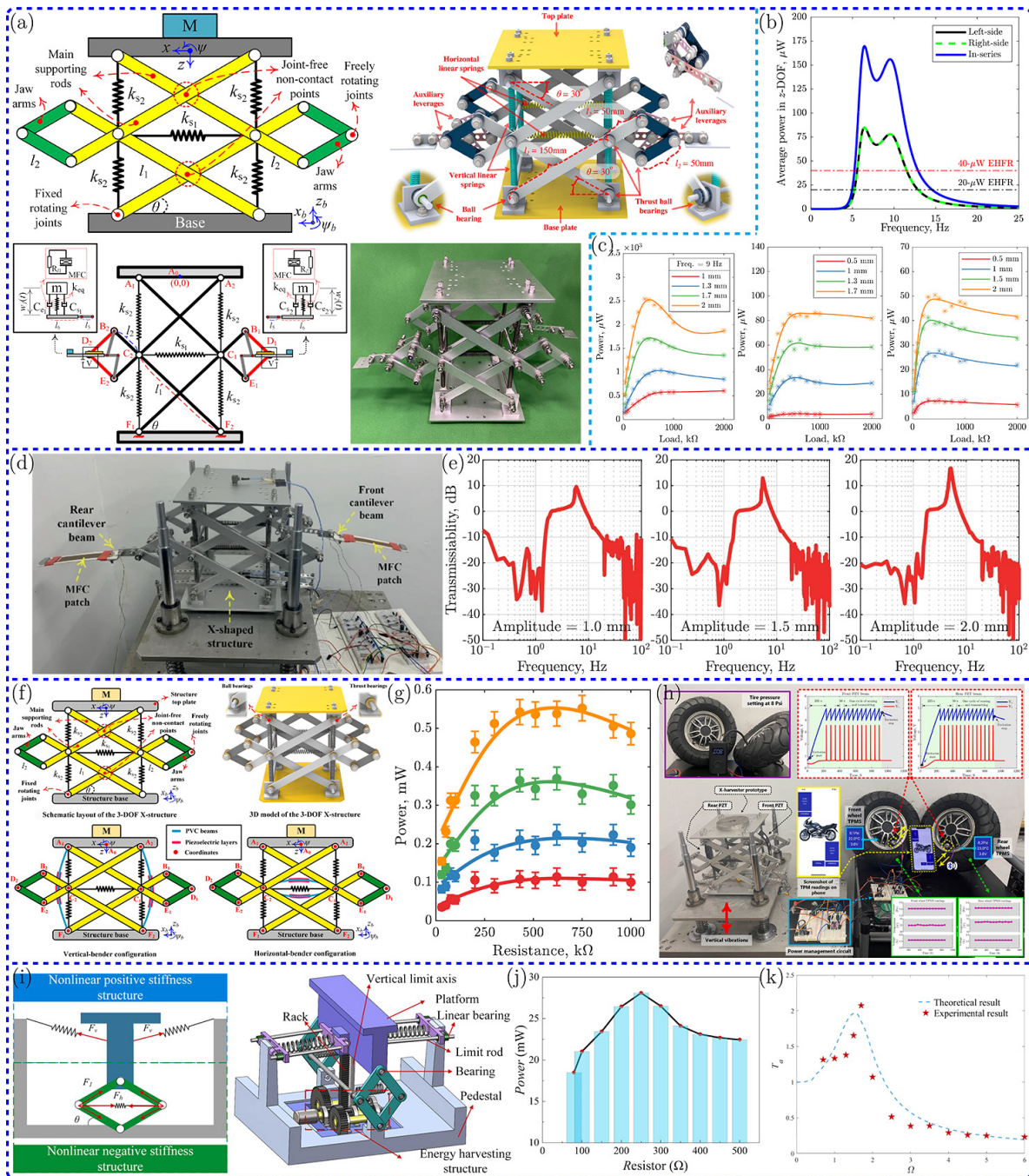


Figure 20. Key advances and performance of multi-directional low-frequency X-structured energy harvester: (a) 3-DOF X-structured piezoelectric energy harvester [243]; (b) vertical dual resonance and broadband bandpass response [243]; (c) experimental output power in three directions [243], reprinted with permission from Copyright 2023, Elsevier Ltd.; (d) 3-DOF X-structure for simultaneous energy harvesting and isolation [244]; (e) vibration transmissibility under various excitation amplitudes [244], reprinted with permission from Copyright 2025, Springer Nature; (f) vertical and horizontal bended piezoelectric beams [245]; (g) power–resistance curve of vertical-bender under 6 Hz [245]; (h) 3-DOF X-harvester powering an IoT sensor [245], reprinted with permission from Copyright 2025, Elsevier Ltd.; (i) DNSVI with gear-driven electromagnetic harvester (GEEH) [246]; (j) output power of DNSVI-GEEH at varied frequencies [246]; (k) vibration transmissibility of DNSVI-GEEH at 2.5 Hz [246], reprinted with permission from Copyright 2025, Elsevier Ltd.

In [245] the authors report a systematic advancement of the 3-DOF X-structured harvesting framework, with a focus on ultralow-frequency adaptability and real-world Internet of Things (IoT) application validation. As given

in Figure 20f, two piezoelectric beam mounting configurations, namely the vertical-bender and horizontal-bender designs, were proposed to enhance installation flexibility, and the vertical-bender configuration exhibits superior motion coupling effect and energy harvesting efficiency. This refined dynamic modeling that accounts for static equilibrium effects provided more precise guidance for structural design. As depicted in Figure 20g, the system achieved breakthrough ultralow-frequency performance, with a vertical peak power of 0.55 mW at 6 Hz and an effective harvesting range of 5–16 Hz. Critically, the system successfully powered commercial IoT sensors, including a tire pressure monitoring system shown in Figure 20h, heart rate/electrocardiogram (ECG) trackers, and temperature-humidity monitoring systems, under ultralow-frequency excitations for 2.5–6 Hz. This achievement bridges the gap between laboratory performance and practical engineering application. The system's normalized power density of 1.52 mW/g significantly outperforms comparable multi-directional harvesters.

Furthermore, as shown in Figure 20i, the authors in [246] proposed a conceptually distinct but complementary design, i.e., a double nonlinear stiffness vibration isolator (DNSVI) integrated with a gear-driven electromagnetic energy harvester (GEEH). By coupling an oblique spring structure that provides nonlinear positive stiffness and a rhomboid structure that offers nonlinear negative stiffness, the system achieves a wide QZS region. The gear-driven energy harvesting module converts vertical vibration into continuous unidirectional rotation, and reaches a peak output power of 27.5 mW at 4.5 Hz as shown in Figure 20j, which is significantly higher than that of typical piezoelectric or triboelectric integrated systems. The mechanical damping introduced by the harvester simultaneously reduces the system's resonance peak, demonstrating the synergistic integration of vibration isolation and energy harvesting, with an initial isolation frequency as low as 2.5 Hz, as given in Figure 20k.

The results above illustrate the progressive development of X-structure-based energy harvesting technologies. From the foundational 3-DOF multi-direction concept [243] and its dual-function experimental validation [244], to ultralow-frequency optimization and IoT application demonstration [245], and an alternative high-power electromagnetic integrated design [246], this body of work collectively demonstrates that the inherent nonlinearity of X-structures can be utilized to realize broadband, multi-directional energy harvesting with strong practical engineering viability. The successful power supply for commercial IoT sensors [245] and the high output power of 27.5 mW [246] mark important milestones toward the development of self-powered, multifunctional vibration control systems. Future research directions may include more operating bandwidth expansion, device miniaturization for embedded applications, and integration with energy storage and wireless transmission modules for autonomous monitoring and/or diverse smart systems.

5.1.4. Large Energy Sources

Moving beyond conventional vibration-based energy harvesting, the unique nonlinear characteristics of X-structures have been successfully adapted to ocean wave energy conversion. This field is characterized by ultralow frequencies, large amplitudes, and irregular excitations. This subsection reviews the pioneering application of X-mechanisms to wave energy harvesting, highlighting the progressive integration of nonlinear dynamics and mechanical innovations to address the challenges of marine environments.

As illustrated in Figure 21a, the authors in [170] developed the first bistable X-structured electromagnetic wave energy converter by integrating a bio-inspired X-mechanism with a unique mechanical-motion-rectifier (MMR) featuring an energy-storing-releasing mechanism. The specially designed MMR converts bidirectional linear wave motion into controllable unidirectional rotation. It stores mechanical energy in a spiral spring during compression. The rapid release of stored energy produces significantly enhanced instantaneous voltage and power output. This is different from conventional rectifiers that depend on velocity.

Unlike conventional bistable systems that use oblique springs or magnetic forces, this design uses the inherent negative stiffness of the X-structure together with an auxiliary horizontal spring. This achieves both wide range QZS and bistable characteristics across the full working stroke, as presented in Figure 21b. This enables efficient energy harvesting from ultralow frequency (4 s period) and large amplitude (0.15 m height) ocean waves.

As shown in Figure 21c, dynamic modeling and experimental validation confirmed that bistable inter-well oscillations are triggered under moderate wave heights, greatly improving energy capture. Under wave conditions of 0.15 m height and 4 s period, the system achieved a peak voltage of 15 V and peak power of 1 W. Field tests and self-powered demonstrations proved its capability to supply marine environmental sensors and wireless communication nodes. This marks an important advancement in applying X-mechanism nonlinearity to wave energy harvesting.

In [247] the authors further extended this foundational work by introducing an inerter into the X-structured wave energy harvesting system for the first time. This design aims to improve power capture under irregular wave conditions modeled by the JONSWAP spectrum. A double-layer X-structure mechanism was integrated with a linear

inertor placed either horizontally or vertically. The horizontal layout produces geometrically nonlinear inertance through X-structure deformation, while the vertical layout acts as a linear inertor counterpart.

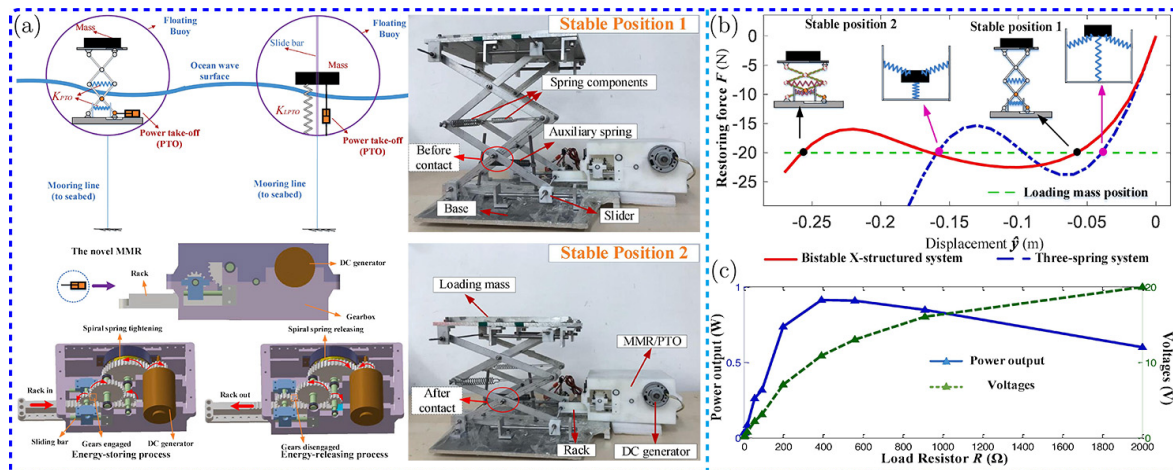


Figure 21. X-structure-based wave energy converters, including their configurations, mechanisms and performance; (a) bistable X-structured electromagnetic wave energy converter with MMR [170]; (b) QZS and bistable characteristics of the X-structured converter [170]; (c) energy harvesting performance of the bistable X-structured converter [170], reprinted with permission from Copyright 2021, Elsevier Ltd.

Ref. [247] reveals that the nonlinear inertance configuration, i.e., horizontal layout, achieves higher average power output than both linear inertance (vertical layout) and inerter-free systems, particularly under small initial installation parameters γ . As γ increases, the performance advantage of nonlinear inertance decreases. This work provides theoretical guidelines for enhancing X-structured wave energy converters through inertance-related nonlinear regulation. It demonstrates that purposefully introducing nonlinear inertial effects can further optimize wave energy capture.

The progress achieved in [170] and [247] illustrates the systematic evolution of X-structure-based wave energy harvesting. These results demonstrate that the X-structure's inherent nonlinearity can be strategically used to address the unique challenges of ocean wave energy harvesting, whether in stiffness, bistability, or inertance. This opens new ways for self-powered marine monitoring systems and coastal engineering applications.

5.2. Robotics and Adaptive Structures

The unique nonlinear characteristics of X-structures, particularly their tunable stiffness, bistability, and precision motion control, have recently been exploited in underwater robotics. They offer new paradigms for achieving both agility and energy efficiency. This subsection reviews a series of systematic studies focused on a common platform, i.e., a bioinspired robotic fish driven by an X-structure parallel mechanism that controls an elastic spine. These works collectively demonstrate the progressive refinement of X-structure-enabled robotic locomotion, from initial concept validation to controllable bistable propulsion and real-time stiffness modulation.

It can be observed from Figure 22a in [248] the authors developed the first untethered bimodal robotic fish (TBFiBot) by integrating an X-structure parallel mechanism as the core actuation module. By adjusting the pre-compression of an elastic spine, the robot achieves online switching between monostable and bistable modes. Monostable mode enables smooth, energy-efficient cruising, while bistable mode exploits snap-through instability for high-speed propulsion and sharp turns. Inverse kinematics of the X-structure and nonlinear modeling of the elastic spine allowed precise trajectory control and mode transition. As presented in Figure 22b, experimental results demonstrated that bistable mode nearly doubles swimming velocity and triples turning angular velocity, especially at low actuation frequencies. This confirms that X-structure-enabled bistable actuation can improve underwater robotic performance.

In [249] the authors further developed this concept by focusing on accurate controllability of the bistable propulsion mechanism. The double-layer X-structure parallel mechanism precisely regulates the deformation and snap-through behavior of the elastic spine. This enables adjustable energy barriers and controllable strain energy storage and release. Through optimized stiffness coordination between the elastic spine and caudal fin, the robotic swimmer achieved significant performance improvements. It reached a maximum swimming speed of 0.8 m/s, a cost of transport as low as 9 J/(m·kg), a peak turning rate of 107°/s, and a minimum turning radius of 0.31 body length, as shown in Figure 22c. This work is the first accurately controllable bistable elastic propulsion system

based on the X-mechanism. It overcomes the poor controllability and adjustability of conventional bistable swimming robots.

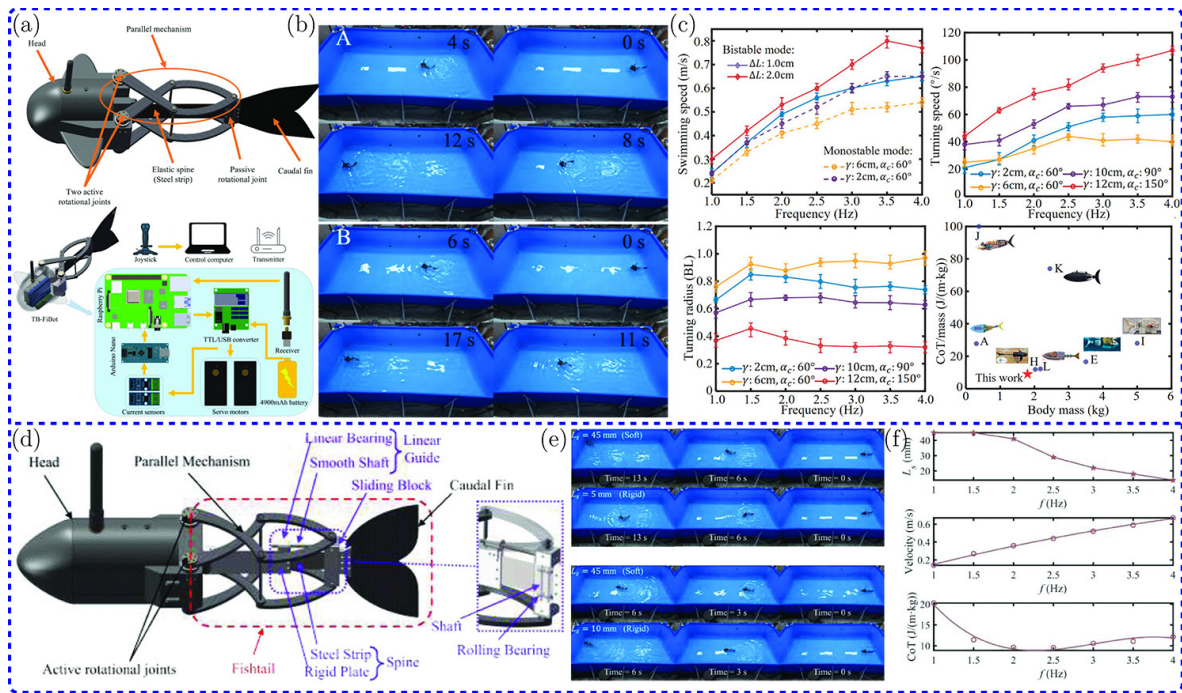


Figure 22. X-structure-enabled bioinspired robotic fish: configurations, mechanisms and locomotion performance: (a) untethered bimodal robotic fish (TBFiBot) with X-shaped parallel mechanism [248]; (b) locomotion performance comparison between bistable (top) and monostable (bottom) modes [248], reprinted with permission from Copyright 2024, IEEE; (c) propulsion and turning performance of the X-structured robotic swimmer [249], reprinted with permission from Copyright 2024, Mary Ann Liebert Inc.; (d) X-structure-based tail stiffness modulation mechanism [250]; (e) swimming speed performance of the stiffness-tunable robotic fish [250]; (f) cost of transport of the stiffness-tunable robotic fish [250], reprinted with permission from Copyright 2025, IEEE.

As depicted in Figure 22d, the work presented in [250] introduced a novel stiffness modulation mechanism that uses the X-structure to achieve online tail stiffness tuning without dedicated actuators. The X-mechanism drives a slider to change the effective cantilever length of an elastic steel strip, which enables real-time, continuous tail stiffness adaptation according to tail-beat frequency. A comprehensive Lagrangian dynamic model was established and validated. It reveals that low stiffness is beneficial for efficient cruising at low frequencies, while high stiffness increases speed at high frequencies. With an online stiffness optimization rule, the robot achieved efficient swimming over a wide frequency range of 0–4 Hz. It reached a maximum speed of 1.4 BL/s and a minimum transport cost of 9.5 J/(m·kg), see Figures 22e,f. This passive, lightweight, and high-bandwidth stiffness modulation method overcomes the drawbacks of additional actuators and high energy consumption in conventional variable-stiffness robotic fish.

Overall, this series of studies illustrates the powerful synergy between X-structure nonlinearity and bioinspired robotic design. The common platform, i.e., an X-structure parallel mechanism driving an elastic spine, serves as a versatile foundation for increasingly complex functions. These functions range from bimodal locomotion mode switching [248], to controllable bistable propulsion [249], and real-time stiffness adaptation [250]. Each advancement exploits a different aspect of X-structure mechanics to address specific challenges in underwater propulsion, such as geometric nonlinearity, tunable bistability, precision motion control. The results consistently show that X-structure-enabled designs achieve superior agility, efficiency, and environmental adaptability compared to conventional robotic fish. This establishes a new approach for high-performance bioinspired underwater vehicles. Future directions may include MDOF maneuvers, integration with energy harvesting for autonomous operation, and adaptation to diverse aquatic environments.

5.3. Spacecraft On-Orbit Capture and Post-Capture Applications

The unique challenges of on-orbit capture operations include extreme impact loads, post-capture multi-body coupling, and the need for ultralow frequency isolation in microgravity. These challenges have motivated a

dedicated research direction that applies X-structures to spacecraft protection. Unlike the general-purpose isolators discussed in previous sections, these studies focus on performance requirements for specific scenarios. The X-structure serves as a robust, passive core for developing mission-oriented systems. This subsection reviews advances in two interconnected phases of spacecraft capture missions. One phase is the capture event, where anti-impact capabilities are most important. The other is the post-capture phase, where vibration suppression and stability control become critical. The common theme across these works is the exploitation of X-structures nonlinearity to address the extreme dynamics of space operations while maintaining passive reliability.

5.3.1. On-Orbit Capture Scenarios

The capture of non-cooperative targets such as defunct satellites or large space debris imposes severe impact loads on the servicing spacecraft. These loads threaten both structural integrity and mission success. X-structures have been increasingly employed to address these challenges through their inherent nonlinear stiffness and energy dissipation characteristics. This subsection reviews several studies, which demonstrates the progressive refinement of X-structure-based anti-impact systems for on-orbit capture scenarios.

The authors in [251] established an accurate dynamic model of a bio-inspired isolation (BII) system based on the X-structure mechanism shown in Figure 23a. This model is specifically designed for vibration suppression during spacecraft capture. Unlike simplified models that neglect rod and joint masses, this work fully included the kinetic and gravitational potential energies of all components. This resulted in a set of implicit ordinary differential equations (IODEs). By solving these IODEs using the harmonic balance method and Matlab ODE15i solver, the study revealed that simplified models overestimate isolation performance by approximately 5% under simple responses, as presented in Figure 23b. They also produce completely different results under complex responses, demonstrating the critical importance of accurate modeling. The X-structured BII system was tested in both ground and zero-gravity environments. As shown in Figure 23c, it performed better than conventional SMD isolators, reducing peak acceleration by 7–10%. Parametric studies further clarified the influence of layer number, assembly angle, rod length, and damping, providing a theoretical foundation for applying X-mechanisms to on-orbit capture buffer systems.

From Figure 23d, it is evident that authors in [252] further developed this concept by creating a bio-inspired anti-impact manipulator (BAM). An X-structure mechanism is embedded directly inside the manipulator arm, which is a significant change from previous add-on isolation systems. This integrated design is compact and has adjustable preload forces to adapt to varying target masses and impact levels. A non-smooth dynamic model including the Karnopp friction model was derived to characterize preload friction. The Runge-Kutta-Henon (RKH) method was adopted to accurately solve the discontinuous equations. As depicted in Figure 23e, simulations and experiments showed that the BAM system performs better than traditional SMD isolators, with peak acceleration of the spacecraft platform reduced by 6.3%, and vibration decay time shortened by 14.2%. The study also identified a distinct stick-slip phenomenon caused by preload friction. It confirmed that optimal anti-impact performance is achieved by increasing layer count, reducing assembly angle, and tuning preload near a critical value.

Additionally, as given in Figure 23f, the authors in [48] introduced a new spider-inspired anti-impact hydraulic-based (SIAH) structure that replaces the linear damper in conventional X-structure designs with a hydraulic damper. This innovation converts geometric nonlinearity into component-level nonlinearity. It achieves weak coupling between stiffness and damping and enables both excellent stability and anti-impact performance. This solves the inherent balance problem in traditional designs. Figure 23g reveals that compared with previous X-structured systems, the SIAH structure achieved significant improvements, i.e., the amplitude of the capture mechanism was reduced by 78%, vibration decay time was shortened from 500 s to less than 5 s, peak acceleration of the satellite platform was decreased by 25%, and overall structure size was reduced by 66%. An air-bearing table experiment simulating microgravity conditions validated the design, with simulation errors controlled within 10%, see Figure 23h. This demonstrates its capability to capture large-mass space targets up to 2000–3000 kg.

These studies illustrate the systematic evolution of X-structure-based anti-impact systems for on-orbit capture. The progression starts with establishing the necessity of accurate dynamic modeling [251], through compact manipulator-integrated design with adjustable preload [252], to the hydraulic-based breakthrough achieving unprecedented impact mitigation and miniaturization [48]. Each result addresses critical issues in spacecraft capture scenarios. The development reflects a deepening exploitation of X-structure nonlinearity, i.e., from a geometric nonlinearity alone to a hybrid geometric-component nonlinearity. This enables increasingly compact, stable, and high-performance solutions for passive anti-impact protection in the extreme dynamics of on-orbit capture missions.

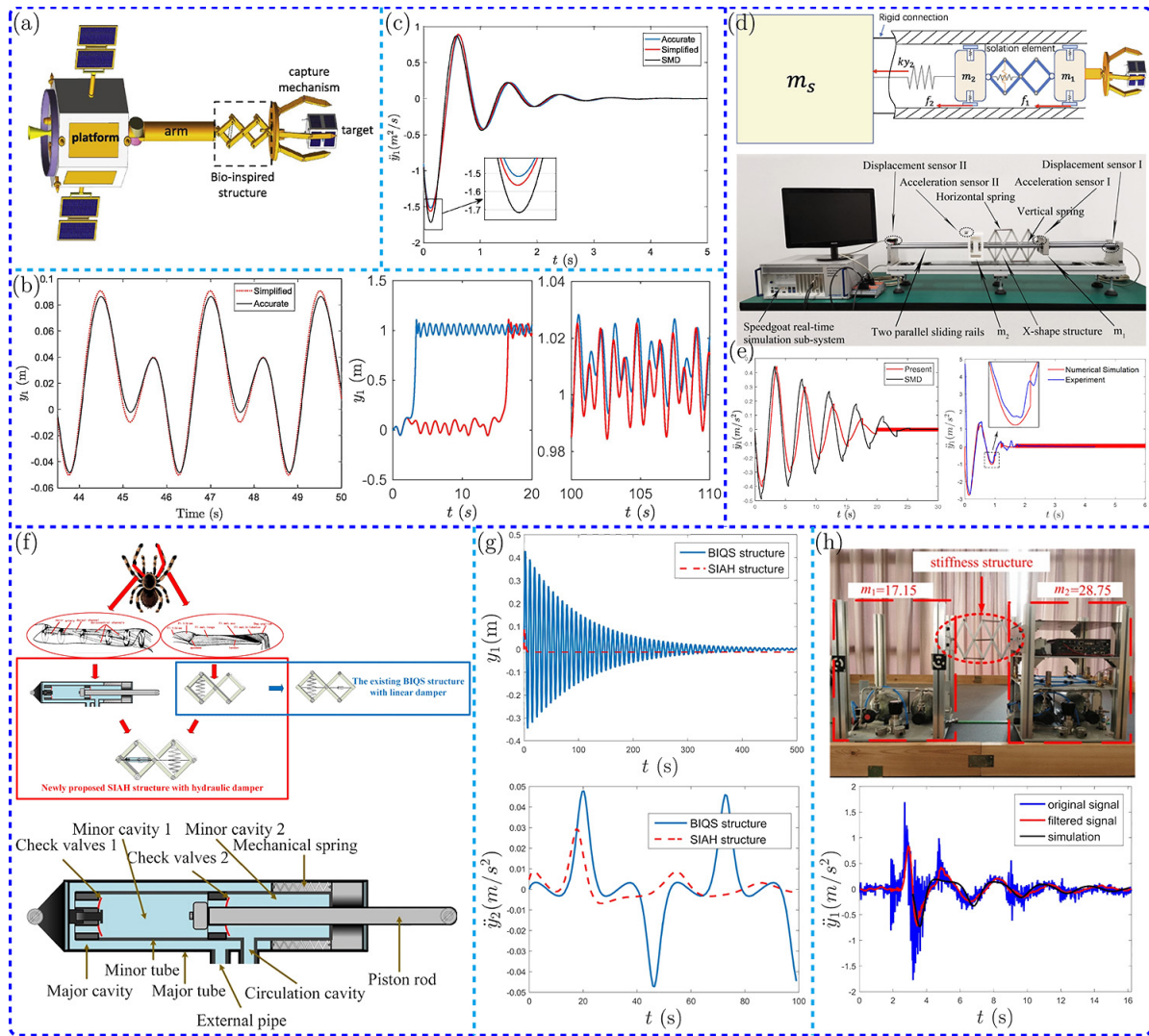


Figure 23. X-structure-based anti-impact systems for on-orbit capture: models, configurations and performance; (a) bio-inspired isolation (BII) system with X-shaped mechanism [251]; (b) vibration isolation performance comparison between accurate and simplified models [251]; (c) anti-impact performance of X-structured BII system [251], reprinted with permission from Copyright 2018, Elsevier Ltd.; (d) bio-inspired anti-impact manipulator (BAM) with embedded X-structure in Ref. [252]; (e) anti-impact and vibration decay performance of BAM system [252], reprinted with permission from Copyright 2020, Elsevier Ltd.; (f) spider-inspired anti-impact hydraulic-based (SIAH) structure in Ref. [48]; (g) performance improvements of SIAH structure compared with previous X-structured systems [48]; (h) microgravity experiment validation of SIAH structure for large mass capture [48], reprinted with permission from Copyright 2023, Springer Nature.

5.3.2. Post-Capture Vibration Suppression

Following the capture event, the combined spacecraft, comprising the servicer, target, and capture mechanism, enters a post-capture phase. This phase is characterized by complex coupled dynamics, including residual oscillations from impact, flexible vibrations of appendages such as solar arrays, and potential instability from the newly formed multibody system. X-structures have been systematically developed to address these challenges, evolving from foundational isolation concepts through MDOF platforms to fully coupled rigid-flexible models and multifunctional systems integrating energy harvesting.

Figure 24a shows the bio-inspired quadrilateral shape (BIQS) isolation system specifically for post-capture vibration suppression of free-floating spacecraft first proposed in [158], sharing the same structure with Figure 23a. A 2DOF dynamic model was established for the under-constrained floating spacecraft and a modified Lagrange equation was derived to handle impulsive impacts during capture. As shown in Figure 24b, numerical results showed that the BIQS isolator performs about 10% better than conventional SMD systems in reducing peak acceleration, and its inherent nonlinear damping provides better high-frequency attenuation. Parametric studies

confirmed that increasing layer count, reducing assembly angle, and shortening rod length effectively improve isolation performance.

It is clear from Figure 24c that the authors of [253] advanced the concept to full spatial vibration control by developing an X-shape structure-based Stewart isolation platform (XSSIP). Replacing the legs of a conventional Stewart platform with multilayer X-structure mechanisms, the system achieves 6-DOF passive vibration suppression for post-capture spacecraft. A complete 6-DOF dynamic model was derived for the under-constrained floating spacecraft. It is applicable to both periodic excitations such as those from rotating payloads and impulsive loads such as capture collisions. As presented in Figure 24d, Theoretical analyses and ADAMS multi-body simulations confirmed that the XSSIP achieves lower resonant frequencies, smaller displacement transmissibility, and faster vibration decay than traditional SMD-based Stewart platforms across all spatial directions.

Moreover, the work [254] established a key fully coupled dynamic model for a post-capture spacecraft equipped with an X-structure isolation system. This represents a major advancement over the simplified rigid-body models in previous studies. The model incorporates coupling among attitude motion, orbital drift, structural vibration, and the flexibility of solar arrays using the hybrid coordinate method. Under both periodic and impulsive excitations, the X-structure isolator performed better than conventional SMD isolators. It reduced maximum spacecraft acceleration by 28% and peak displacement transmissibility by 18%. An LQR-based cooperative controller was also designed to stabilize attitude drift, orbital motion, and flexible vibrations at the same time.

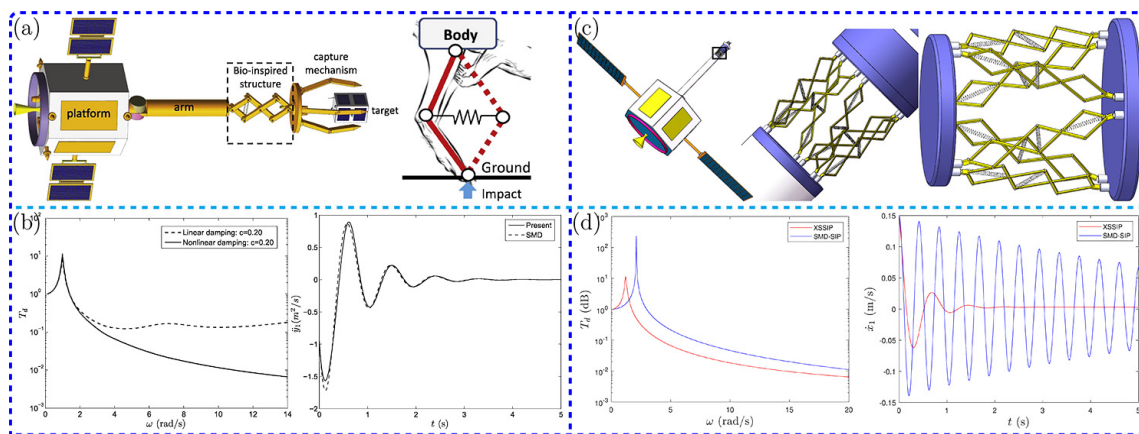


Figure 24. X-structure-based post-capture vibration suppression for spacecraft, including configurations, modeling and performance; (a) bio-inspired quadrilateral shape (BIQS) isolation system [158]; (b) vibration isolation performance of the BIQS system [158], reprinted with permission from Copyright 2018, Elsevier Ltd.; (c) X-structure-based Stewart isolation platform (XSSIP) [253]; (d) 6-DOF vibration suppression performance of XSSIP [253], reprinted with permission from Copyright 2020, Elsevier Ltd.

In addition, in [255] a new bio-inspired quadrilateral shape system integrated with an energy harvester (BQS-EH) is introduced. It achieves both broadband vibration isolation and energy harvesting in post-capture spacecraft. The design combines a nonlinear X-structure isolation structure with a monostable piezoelectric energy harvester, using geometric nonlinearity for both functions. A fully coupled dynamic model was developed, and the working mechanism was identified as targeted energy localization. This concentrates vibrational energy into the harvester, enhancing both vibration attenuation and power generation. Under both periodic and impulsive excitations, the BQS-EH system performs better than the pure isolation structure (BQS) with lower force transmissibility, broader effective isolation bandwidth, and simultaneous electrical output. Parametric studies revealed trade-offs and synergies between mass ratios, structural nonlinearity, damping, and electromechanical coupling. This provides optimal design guidelines for improving vibration isolation without sacrificing energy harvesting performance.

These studies show the systematic development of X-structure-based post-capture vibration suppression technology and confirm that X-mechanisms are a versatile platform for ensuring post-capture stability, protecting sensitive payloads, and even enabling self-powered monitoring in the demanding environment of space operations.

5.4. Nonlinear Energy Sink for Vibration Absorption and Energy Harvesting

Owing to its advantage in constructing nonlinear damping and stiffness characteristics, the X-structure has attracted wide attention in fields such as vibration control and energy harvesting. Nonlinear energy sinks and vibration absorption devices based on this structure have become research hotspots, and many scholars have conducted systematic studies and achieved fruitful results.

Inspired by kangaroos' body movements, the authors of [256] established a multi degrees-of-freedom vibration isolation platform model consisting of a protected object, a nonlinear energy sink, and an X-structure. They performed displacement transmissibility analysis and numerical calculations using the harmonic balance method and Runge–Kutta algorithm, revealing nonlinear behaviors and resonance phenomena in the system's time-frequency response. They also confirmed that quasi-periodic motion can serve as a predictor of periodic steady-state response or strong resonance, and verified the correctness of theoretical analysis through Adams simulations, demonstrating that the dynamic characteristics of structural parameters are adjustable and designable within a specific frequency band. Inspired by the walking, jumping, and running of quadrupeds, and in [257] a novel vibration isolation-absorption platform using a bistratal X-structure and a multi-vertebra structure was proposed. By establishing static and dynamic models and combining force/stiffness-displacement curves, they revealed the platform's load-bearing capacity and QZS characteristics, investigated the effects of different parameters on vibration suppression performance, and verified the results through numerical methods. The findings showed that the mutual promotion of vibration isolation and absorption significantly reduces resonance peaks, and the vibration suppression performance can be flexibly tuned by parameters such as initial installation angle and rod length ratio.

In [258], a nonlinear energy sink by combining the X-structure with viscoelastic damping was formulated. Using the complexification-averaging method and fast-slow analysis, they reduced the 5-dimensional real system to a 3-dimensional complex slow-varying equation, focusing on analyzing the system's dynamic characteristics and response mechanism. They found that the folding structure of the slow invariant manifold helps determine the threshold of the nonlinear damping ratio triggering saddle-node bifurcation, and the folding structure generates strongly modulated responses. Energy spectrum analysis confirmed that selecting appropriate nonlinear damping can enhance vibration reduction performance.

To improve ride comfort and meet lightweight design requirements of vehicles, the work presented in [259] proposed an inerter-based X-structure nonlinear energy sink and applied it to a half-vehicle system. Using Lagrange's method, they established a coupled dynamic model, obtained dynamic responses under road harmonic excitation via the harmonic balance method (HBM) and pseudo-arc-length method, and optimized the structural parameters of an inerter-based X-structure nonlinear energy sink using a genetic algorithm. The results showed that inerter-based X-structure nonlinear energy sink effectively reduces the resonance peaks and root mean square values of front and rear suspension deflections and dynamic tire loads, while slightly increasing those of vertical and pitching accelerations of the vehicle body; after optimization, the vertical and pitching accelerations of the vehicle body are reduced by 2.41% and 1.16% respectively. Inspired by human push-up movements, Yang et al. [260] designed a broadband energy harvester using a bio-inspired hexagonal skeleton structure combined with a nonlinear energy sink, as presented in Figure 25. They established dynamic equations and verified them through simulations and experiments. The harvester can cover a wide low-frequency range of 5–15 Hz, effectively converting hybrid vibration into electrical energy, providing a viable solution for low-frequency, broadband hybrid vibration energy harvesting.

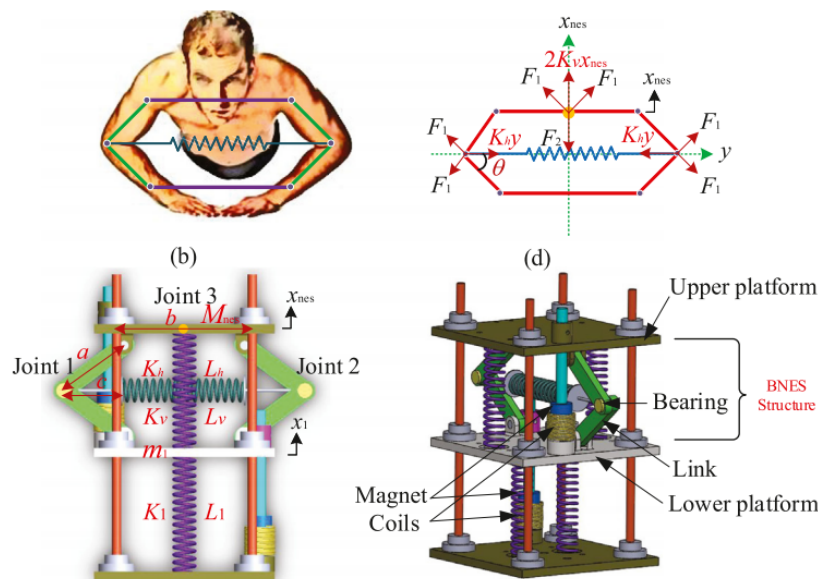


Figure 25. Nonlinear energy sink vibration absorption and energy harvesting with X-structure: broadband energy harvester combined with a nonlinear energy sink [260], reprinted with permission from Copyright 2022, Elsevier Ltd.

The X-structure exhibits significant advantages in vibration control and energy harvesting: it can flexibly achieve nonlinear stiffness and damping characteristics, has strong adjustability of structural parameters, adapts to various scenarios such as vibration isolation, absorption, and energy harvesting, effectively reduces resonance peaks, improves related performance, and provides important references for practical engineering applications. However, existing research still faces challenges: (i) the optimal matching mechanism of structural parameters in different scenarios is not fully clear; (ii) the stability and reliability under complex excitation environments need further verification; and (iii) its adaptability in miniaturized and integrated applications remains to be explored in depth.

6. Critical Discussion, Open Challenges, and Future Perspectives

The preceding sections have systematically outlined the development of X-structures/mechanisms research over the past 10 years. They cover progress from deepening the understanding of core nonlinear characteristics (Section 2), through innovative configurations for complex scenarios (Section 3) and integration with advanced technologies (Section 4), to the expansion into multifunctional applications (Section 5). These advances demonstrate that the X-structure/mechanism paradigm has become from a specialized vibration isolation concept into a versatile platform, capable of addressing diverse engineering challenges through geometric nonlinearity. However, this rapid progress also reveals ongoing issues or new questions that require critical examination. The following subsections summarize some key achievements, identify remaining challenges, and outline promising frontiers for future research. These directions include intelligent digitalization, cross-scale exploration, and sustainability-driven design etc. By critically reflecting on the current state of the field and its future potential, this review aims to provide a roadmap for further X-structure research.

6.1. Summary of Key Advances

In the past several years, the X-structure research has evolved from a focused exploration of geometric nonlinearity for vibration isolation into a broad and versatile tool or methodology covering multiple nonlinear characteristics, advanced configurations, and diverse application fields. The advances documented in Sections 2–5 are summarized here through two complementary perspectives. Table 1 provides a structured overview of how different nonlinear mechanisms have been exploited across application fields, while Table 2 brings together the representative performance characteristics of experimentally validated X-structure-based QZS isolators.

Table 1. Summary of key advances in X-structure research classified by nonlinear characteristics and application domains.

Characteristic/Mechanism	Vibration Isolation	Energy Harvesting	Robotics	Spacecraft	Advanced Control	Sensing
Nonlinear Stiffness	[8,14,16,109,115,153,154,161–169,171–175,177–179,186,188–191,195–205,222,223,225,226,229–232,234,243,244,246,255,261]	[170,229,235–239,241,243–246,255]	[248–250]	[48,158,251–255]	[206,207,209–220,222–224]	[225–227,229]
Nonlinear Damping	[9,109,155,180–182,192–194,200,202,204]	[236,237]	-	[48]	[208,214–216]	-
Nonlinear Inertia	[10,109,110,156,183–187,205,233,234]	[236,237,247]	-	-	-	-
Bistability	-	[170]	[248,249]	-	-	-
Configurational Innovation	[14,16,115,167,168,188–191,195–197,199–205,261]	-	-	-	-	-
Technology Integration	[230–234]	-	-		[206–220,222–224]	[225–227]
Multifunctional Integration	[229,243,246]	[229,242,245,246]	[248–250]	[48,158,251–255]		[229]

A central achievement has been the systematic deepening of understanding and controllability of X-structure nonlinearities. Nonlinear stiffness remains the most versatile and widely exploited characteristic. Research has progressed from fundamental QZS realization [8,153,154,161–163] to sophisticated compensation strategies. These strategies include contact mechanisms [164], magnetic nonlinearity [171–175], and active modulation [222,223], which achieve enlarged QZS ranges (up to 85% of total stroke [161]) and ultra-low resonant frequencies (as low as 0.75 Hz [191], 0.8 Hz [201], and 0.9 Hz [188]). Auxiliary absorbers integrated into multi-layer X-shaped structures effectively suppress high-frequency resonances and nonlinear dynamics without compromising low-frequency isolation [262,263]. Nonlinear damping has developed from theoretical characterization [9,155] to practical integration with viscoelastic materials [180,181], friction elements [182], and passive MR dampers [82]. This enables displacement-dependent dissipation that suppresses resonance without compromising high-frequency isolation.

Nonlinear inertia emerged as a distinct third dimension, with foundational studies [10,109,110,156,183–185] establishing the theoretical framework, hybrid inerter designs [186] enhancing performance, and novel S-type variants [187] expanding the tunable inertial landscape.

Table 2. Experimentally validated X-structure-based QZS isolators: representative performance characteristics.

QZS System	DoFs	QZS Range [%]	Load Capacity [kg]	Resonance Freq. [Hz]	Start Isol. Freq. [Hz]	Peak Trans. [dB]	10 Hz Trans. [dB]
LLS [8]	1	<20	0.55	1.32	2.64	~25	~20
vaX [153,154]	1	<20	~1	~1.6	~2.8	~30	<20
X-cube [161]	1	>85	~3	1	~1.6	~9	<20
NXSC [164]	1	54	~4	1.1	1.65	~10	~27
X-QZS [165]	1	~40	2.7	~2.2	2.8	~5	~10
CXSS [166]	1	~50	~8	2	3	~10	~20
X-combined structure [169]	1	~56	~4	~2.3	~3.2	~12	~5
NCM X-QZS [171]	1	50	5	1.8	4	~2	~16
RMLS [172]	1	~30	~1	2.03	~4	~2	~15
TMNS X-shape [173,174]	1	~40	~3.5	~1.1	~1.5	~20	~35
BQZSVI [175]	1	~50	2	~8	~9	~15	~7
HBIASV [109]	1	<20	~5	1.5	~2.5	~12	~18
HBIASV-NI [110]	1	<20	1.5	1.25	~2	~25	~40
NI X-structure [183–185]	1	~30	~2	1.3	1.9	~15	~18
S-type NI X-structure [187]	1	<20	2	~1.1	~2.2	~8	~10
MDVIS [188]	3	80 (T) ~30 (R)	~4	~0.9 (T) ~1.0 (R)	~1.4 (T) ~1.5 (R)	~10	<20
RXM [191]	2	N/A (T) ~100 (R)	~1	N/A (T) 0.75 (R)	N/A (T) ~2.0 (R)	~5	~15
Stewart platform [16]	6	~25 (T) ~25 (R)	10	~0.9 (T) ~3.8 (R)	~3 (T) ~7 (R)	~20	<30
Stewart platform [195]	6	~20 (T) ~20 (R)	3	~1 (T) ~1 (R)	~1.8 (T) ~3 (R)	~20	~25
X-Stewart mechanism [198]	1	<20	~1	1.77	2.32	~4	~12
Two-stage isolator [199]	2	N/A	~6	3.2	5	~22	<20
BKLS [201]	1	<20	1.2	~0.8	1.06	~2	<20
CIS-EMSD [204]	1	N/A	2	1.86	~3.5	~4	~14
SBI VI [205]	1	<20	1	~3.2	~5	~3	~18
EM X-mechanism [222]	2	~50 (T) ~90 (R)	1.2	~1.0 (T) ~1.0 (R)	~1.4 (T) ~1.4 (R)	~15	~20
X-VSVI [223]	1	N/A	2	~10	~13	~15	~15
BVI-TENG [229]	1	~70	~0.5	3	4.5	~9	~15
X-QZS Meta-ring [231]	1	~50	~1.5	1.83	~3	~10	~60
X-structure NEH [244]	3	N/A	N/A	~4.5 (T) ~N/A (R)	~5.5 (T) ~N/A (R)	~12	~20
DNSVI-GEEH [246]	1	~40	N/A	1.7	2.5	~6	~20

QZS Range = (QZS zone width / total stroke) × 100%; Freq.: Frequency; Isol.: Isolation; Trans.: Transmissibility. T: Translational motion; R: Rotational motion; N/A: data not available or not applicable due to design focus.

A defining trend is the architectural evolution from single-axis isolators to systems capable of addressing multi-directional and MODF excitations. Compact 3-DOF units [188–191] achieve simultaneous vertical, horizontal, and rotational isolation with resonant frequencies as low as 0.05 Hz theoretically. Stewart-platform integrations [16,195,196] extend this capability to full 6-DOF spatial vibration control. Hierarchical designs, including two-stage isolators [199] and bilayer/layered structures [197,200,261], demonstrate that functional differentiation through structural nesting can achieve better performance than single-layer configurations. Parallel developments in bio-inspired morphological extensions [201–205] illustrate how X-structure principles, independently explored by other research groups, have enriched the design space with diverse biological inspirations.

The empty spaces indicated that no representative work identified within the scope of this review due to the limitation of this literature review.

A transformative shift has been the combination with external technologies and the emergence of multifunctional systems. In active control, X-structure nonlinearity has been used both as a reference model for energy-saving control strategies [206–220] and as a physical element for active stiffness modulation via electromagnetic [222] and piezoelectric [223] actuation. Metamaterial integrations [230–234] have opened new frontiers in bandgap engineering and topological design, demonstrating programmable QZS characteristics with energy absorption ratios exceeding 98% [232]. Most notably, X-structures have become platforms for multifunctional integration, combining vibration isolation with energy harvesting [170,243–246], self-powered

sensing [229,245,246], robotic propulsion [248–250], and spacecraft protection [48,158,251–255]. These developments go beyond the traditional role of X-structures as simple isolators, positioning them as core components in complex, autonomous systems.

Table 2 summarizes the performance range achieved across representative QZS systems which has been experimentally validated by physical prototypes. Key milestones includes significant reductions in resonant frequency, e.g., 0.8 Hz in BKLS designs [201], and some rotational designs achieving full stroke QZS [191]. The QZS range has also expanded dramatically, from less than 20% of total stroke to more than 85% in the X-cube [161] and 90% in electromagnetically reshaped X-mechanism [222], which significantly improves the practical working stroke. Load capacity has been maintained or improved alongside QZS expansion, with some designs supporting up to 10 kg [16] while retaining ultra-low resonant frequencies. Additionally, the capability has evolved from single-axis to 3-DOF and 6-DOF systems, with consistent performance across translational and rotational direction [16,188–191,195,196].

The classification matrix in Table 1 reveals both areas of concentrated research activity and promising research gaps. Nonlinear stiffness and vibration isolation remain the most heavily researched intersections, reflecting the field's foundational strength. Energy harvesting has emerged as a dynamic application domain, leveraging stiffness, damping, and inertia mechanisms. Robotics and spacecraft applications are rapidly growing frontiers, with early demonstrations of bistable propulsion [248–250] and anti-impact capture [48,158,251–255] indicating significant future potential. On the other hand, certain intersections remain less explored, such as nonlinear inertia in robotics, bistability in sensing, and multifunctional integration in spacecraft, representing opportunities for future innovation.

In summary, the advances reviewed in this article show a clear development path, i.e., from characterizing geometric nonlinearity, through engineering it for specific performance targets, to integrating it with external technologies, and ultimately using it in multifunctional, application-driven systems. The thematic organization in Table 1 and the performance data in Table 2 together illustrate a field that has developed considerably while still having plenty of room for growth. The following sections critically examine the bottlenecks that have emerged alongside these advances and outline promising directions for future research.

6.2. Identified Practical Issues

Despite the remarkable advances described in the preceding sections, the X-structure/mechanism research field still has ongoing challenges and unresolved questions. Based on the field's development path, from foundational mechanism characterization to sophisticated multifunctional systems, this subsection critically examines both theoretical and practical bottlenecks that currently constrain the field.

There are many practical challenges in this field which have been gradually met and addressed. We have identified several ones, which in our views should be studied in the future research and development.

- (1) High-dimensional or high-DoF nonlinear dynamics. The inherent nonlinearity that makes X-structures so versatile also makes their dynamic behavior increasingly complex as configurations become more elaborate. MDOF systems, hierarchical designs, and coupled stiffness, damping, and inertia interactions lead to high-dimensional nonlinear dynamics that remain difficult to fully characterize comprehensively. While the nCOS method [239–241] represents a significant advance in establishing explicit parametric relationships for certain types of systems, its extension to strongly nonlinear regimes, multi-input excitations, and transient impact scenarios remains an unresolved challenge. Moreover, the interaction between geometric nonlinearity and material nonlinearity introduces more complexity, particularly in flexible or viscoelastic X-structure variants. Existing analytical frameworks may be limited to capture such complexity without using computationally intensive numerical simulations.
- (2) Fatigue life and long-term reliability. A notable issue in the literature may be the systematic investigation of fatigue behavior and long-term reliability of X-structures, although this is a general problem to all those literature methods. Most studies report performance under short-term excitation, but practical applications require decades of operational life, particularly in aerospace, automotive, and civil engineering. The cyclic loading of joints, springs, and flexible components will inevitably cause wear, preload relaxation, and material degradation. How these aging mechanisms affect the carefully tuned nonlinear characteristics, e.g., QZS range, nonlinear damping, bistability, remains largely unquantified. The lack of validated fatigue models and lifetime prediction tools represents a significant barrier to industrial application.
- (3) Uncertainty quantification and robustness. The performance of X-structures is determined by geometric parameters (rod lengths, assembly angles, layer numbers) and component properties (rod materials, spring stiffness, damping coefficients). Manufacturing tolerances, assembly variations, and environmental changes

(temperature, humidity) may therefore alter the intended nonlinear behavior. Limited studies have systematically measured such uncertainties or developed robust design methods that guarantee performance limits under parameter variations, although such issues have been examined for specific problems, e.g., [155]. The robustness demonstrated by certain control-integrated designs [217–220] suggests promising directions, but uncertainty propagation in passive X-structure systems need more systematic studies for in-depth understanding and quantification, targeting practical engineering applications.

- (4) High-level integration and validation. Real-world systems often require arrays, networks, or integrated subsystems. How X-structures interact when placed in series, parallel, or distributed configurations is still not well understood. The successful powering of IoT sensors [245] and the validated spacecraft capture experiments [255] are important steps, but scaling such demonstrations to full-system integration remains open problems.

Some other issues may include manufacturing complexity and precision, unmodelled dynamics due to assembly misalignments and so on. While X-structures are simple in concept, their practical production may still require some manufacturing precision to achieve the theoretically predicted nonlinear characteristics. Joint friction, clearances, and assembly misalignments can introduce unmodeled nonlinearities that differ from idealized behavior. Cost-efficient and reliable 3D printing and additive manufacturing are expected for rapid prototyping of flexible monolithic variants [230,231], and scalable manufacturing methods that preserve the designable nonlinearity while reducing cost and variability are needed.

Moreover, commercial viability, reliability and cost might be other practical issues to consider. No matter what materials are used, X-structures are relatively cost-effective, compared to other existing solutions for the same performance based on the literature studies. Developing design-for-manufacturing guidelines and using high-volume production techniques will be essential for commercialization. Many promising applications, such as spacecraft capture, deep-sea exploration, and high-speed robotics, require operation under extreme conditions. They include vacuum, radiation, extreme temperatures, corrosive environments, and high-impact loads. Some studies have demonstrated microgravity testing via air-bearing tables [48] or high-impact mitigation [252]. Manufacturing and fabrications of X-structures involve similar commonly-used mechanical components in those engineering practices, and thus in principle, the viability, reliability, cost-efficiency and callable applications of the X-structure method should not be problems, yet to be further examined in specific applications.

6.3. Several Future Research Frontiers

Looking beyond current limitations, several emerging directions show promise for expanding the capabilities, applicability, and impact of X-structure/mechanism technology. These frontiers build upon the foundational advances reviewed in this article while moving into interdisciplinary and forward-looking fields.

6.3.1. Towards Digital and Intelligent X-Structures

We foresee several areas where the main development of digital and intelligent X-structures can be focused on, namely:

- (1) AI-Aided design and optimization. The parameter space of X-structures includes geometry, materials, layering, and component properties. It is vast and highly nonlinear. Traditional parametric sweeps and gradient-based optimization are often insufficient to explore this space comprehensively. Machine learning approaches, particularly generative design, reinforcement learning, and surrogate modeling, offer great potential for change. Neural networks trained on finite element data or experimental measurements could rapidly predict nonlinear characteristics for novel configurations. Inverse design methods could automatically generate X-structure geometries and spring arrangement methods based on the basic X-unit, optimized for specific performance objectives, such as simplest structural design, maximized QZS range, targeted frequency band, minimum weight and/or highest reliability or stability. The parametric mapping demonstrated in meta-ring design [231] is a good example of the kind of structured design space that could benefit from AI-driven exploration.
- (2) Digital twins for health monitoring. The same nonlinear dynamics that enable superior vibration control also contain rich information about the system's structural health. Changes in stiffness, damping, or inertial characteristics that can be detected through vibration response may indicate early damage, joint wear, or preload loss. Developing digital twins, which are computational models that mirror physical X-structures in real time, would enable continuous health monitoring and predictive maintenance. The nCOS framework [239–241] provides a theoretical basis for establishing such digital representations. However, integrating them with sensor networks, real-time data assimilation, and decision-support systems remains largely unexplored.

- (3) Self-adaptive and reconfigurable systems. Most current X-structures are passive or require external actuation for stiffness modulation. Future designs could incorporate more self-adaptive mechanisms, such as using smart materials (shape memory alloys, MR elastomers) or passive feedback (e.g., centrifugal governors), to automatically tune nonlinear characteristics in response to changing loads or excitation spectra. Such self-adaptive X-structures could maintain optimal performance across a wide range of operating conditions without active control. They combine the reliability of passive systems with the flexibility of active ones.

6.3.2. Cross-Scale Applications

The applications of X-structures/mechanisms have been seen across the scales and in our view their future developments will be undertaken to address several aspects listed below:

- (1) Micro/nano-electromechanical systems (MEMS/NEMS). The principles of geometric nonlinearity that govern macroscopic X-structures are scale-invariant, suggesting their applicability at micro and nano scales. MEMS-scale X-structures are fabricated using surface micromachining or focused ion beam techniques. They could enable unprecedented vibration control and energy harvesting for micro-sensors, resonators, and implantable devices. However, scaling introduces new physics, such as surface forces, material size effects, and fabrication constraints. Preliminary explorations of flexible monolithic X-structures [230,231] show their potential, but systematic investigation of X-structure dynamics at the micro-scale remains an open frontier.
- (2) Metamaterial and architected material integration. The X-structure concept is naturally suitable for periodic assembly, forming metamaterials with engineered wave propagation characteristics [230–234]. Future work could explore hierarchical metamaterials where X-structures serve as both unit cells and building blocks for larger-scale architectures. This would achieve unprecedented combinations of stiffness, damping, and bandgap properties. Coupling X-structure metamaterials with other functional elements (e.g., piezoelectric patches and magnetic circuits) could yield active metamaterials with tunable wave control. The recent demonstration of meta-ring matrices with 98% energy absorption [231] illustrates the potential of such architected designs.
- (3) Bridging scales from unit cells to systems. An open problem lies in connecting the performance of individual X-structure units to that of large-scale assemblies. We need design methods that predict the collective behavior of hundreds or thousands of X-structure elements which can be constructed in a special form or architecture. These methods should account for interactions, tolerances, and statistical variations to enable applications such as vehicle seats, building foundations, or shipboard vibration isolation. Multiscale modeling frameworks, combining detailed finite element analysis of unit cells with homogenized models of assemblies, could help analysis and design in bridging this gap.

6.3.3. Sustainability-Driven Design

Sustainability is a vital design criterion of X-structures/mechanisms with the following aspects to be noticed:

- (1) Lifecycle analysis and environmental impact. As awareness of sustainability increases, the environmental impact of engineering solutions has become more important. X-structures can work in passive mode without extra energy input. They also feature long service life and high material efficiency, so they are naturally consistent with sustainability targets. However, quantitative lifecycle assessments (LCA) are still insufficient. Future research should evaluate the embodied energy, manufacturing emissions, and end-of-life recyclability of X-structure-based systems, particularly compared to active alternatives. Such assessments could help find optimization directions and provide strong support for sustainable engineering applications.
- (2) Biodegradable and renewable materials. Most X-structure prototypes utilize metals such as aluminum and steel or conventional polymers. Exploring biodegradable or bio-based materials including polylactic acid (PLA), cellulose composites, and mycelium-based structures can create new opportunities in temporary structures, biomedical implants, or environmentally sensitive deployments. The flexible monolithic designs [230,231] are particularly suitable for such material replacement, as they remove the need for independent metal joints and springs. However, the mechanical properties, long-term stability, and consistent nonlinear performance of bio-based materials must be systematically characterized.
- (3) Circular Economy Considerations. X-structures have a modular structure that includes rods, springs, and joints. This feature makes them easy to repair, reuse, and recycle at the component level. Designing for disassembly with standardized connections and material-compatible fastening methods can support circular economy models such as leasing instead of selling isolators. Although these topics are seldom discussed in academic research, they are becoming more important for industrial application.

6.3.4. Emerging Application Frontiers

Considering the very new applications of X-structures/mechanisms, several areas are emerging, namely:

- (1) Robotic and prosthetic systems. The successful application of X-structures to robotic fish [248–250] suggests broader opportunities in bioinspired robotics. X-structure joints can support limbs with adjustable stiffness, adaptive grippers, and energy-efficient movement for robots on land, in the air, and underwater. In prosthetics, X-structures can offer passive and adaptive flexibility that imitates biological joints. This may lower the complexity and energy use of powered prosthetic devices.
- (2) Automotive and aerospace systems. Although active suspension control has been researched [206–220], the practical use of X-structures in vehicle chassis, engine mounts, and landing gear has not been fully explored. The compact X-cube [161] and high-load Stewart platforms [16,195] show that such designs are technically feasible. However, full-vehicle testing, durability verification, and integration with existing vehicle structures still require further work. In aerospace, apart from capture scenarios, X-structures can be applied to payload isolation, launch vibration reduction, and pointing stability control for space telescopes.
- (3) Infrastructure and seismic protection. The high load capacity and ultra-low frequency isolation shown in several designs [14,189] make X-structures a good choice for civil engineering applications. These include building base isolation, bridge bearing replacement, and protection for precision equipment. However, scaling the design to support ton-level loads, long-term outdoor service, and seismic performance certification remain significant challenges. Hybrid designs that integrate X-structures with traditional rubber bearings or viscous dampers may provide practical ways for real-world implementation.
- (4) Smart functional structures and systems. The X-structures or X-mechanisms can be well applied in some “smart” systems as basic design motif or essential role to reshape desired stiffness, damping and inertial functions with special nonlinear dynamics, targeting a smart feature and/or a functional objective [10,224,230,231,264].

Reflecting on the 2022 outlook [12], several of the identified open problems have been seen with substantial progress. MDOF and compact isolation [16,188–191,195,196] have advanced considerably, with 3-DOF units and Stewart-platform integrations now achieving performance that was only conceptualized earlier. Combination with active control has been established into sophisticated frameworks that exploit beneficial nonlinearity for energy saving [206–220]. Micro/nano exploration remains in its infancy, though flexible monolithic designs [230,231] provide a pathway. Material and structural combinations, particularly those with viscoelastic materials [180,181], friction elements [182], and magnetic mechanisms [171–175], have developed rapidly.

Yet significant challenges persist. The systematic characterization of fatigue, reliability, and manufacturing variability identified in 2022 is yet to be addressed. Cross-scale design methodologies connecting unit cell to system performance are still open problems. Sustainability and circular economy considerations have received growing attention but have not yet been developed into quantitative frameworks or working prototypes.

Looking forward, the most transformative advances will likely emerge at the intersections, i.e., digital and physical systems including AI-aided design and digital twins, micro and macro structures such as hierarchical metamaterials, passive and active components like self-adaptive structures, and functional and sustainable designs such as biodegradable and circular systems. By addressing the bottlenecks identified above while pursuing these interdisciplinary frontiers, X-structure research can continue its trajectory from academic curiosity to engineering necessity. It will support safer, more efficient, and more sustainable systems across a wide range of engineering fields.

7. Closing Remarks

Building on the foundational framework established in the 2022 review [12], which systematically articulated the core principles of geometric nonlinearity in stiffness, damping, and inertia, the past several years have witnessed a further broadening and deepening of the research on X-structures.

The advances documented in this review collectively demonstrate that the X-structure paradigm has evolved from a specialized vibration isolation concept into a versatile tool or method, capable of addressing diverse engineering challenges. The deepening of core nonlinear mechanisms has pushed performance boundaries, i.e., QZS ranges now exceed 85% of total stroke, resonant frequencies have been reduced below 1 Hz, and novel nonlinear inertia effects (e.g., S-type characteristics and inertance-enhanced dynamics) have been discovered and systematically characterized. Architectural innovations have extended X-structures from single-axis isolators to compact 3-DOF units, full 6-DOF Stewart platforms, hierarchical multi-stage designs, and a variety of bio-inspired morphological variants. Integration with advanced technologies has opened new frontiers beyond passive vibration control. These technologies include active control frameworks that use beneficial nonlinearity to achieve energy savings exceeding 85%, metamaterial designs with 98% energy absorption, and self-powered sensing interfaces.

Application expansion into robotics, spacecraft on-orbit capture, wave energy harvesting, and multifunctional systems has confirmed the engineering feasibility of X-structure principles in challenging real-world scenarios.

Throughout this development, a consistent theme has emerged, i.e., working with nonlinearity in a deliberate and interactive way. This philosophy, explained in the 2022 review [12], has proven to be not just a theoretical ideal but a practical engineering strategy. The X-structure/mechanism approach now offers a comprehensive toolkit for designing beneficial nonlinearities that are controllable, adjustable, designable, and thus predictable and scalable across scales and applications.

Despite these remarkable advances, many challenges remain as discussed above. Future research directions hold promise for addressing these challenges and expanding the impact of the X-structure technology, including digital and intelligent X-structures such as AI-aided design and digital twins, cross-scale applications such as MEMS/NEMS and metamaterials, and sustainability-driven design such as lifecycle analysis and biodegradable materials. These are important to broaden frequency sensitivity and modelling of the structure and model frequency bands for energy transfer.

Actually, recent research confirms that X-structures are no longer limited to geometric nonlinearity alone, but are increasingly being combined with material nonlinearities and nonlinear interactions to unlock new performance regimes. Emerging studies have explored hyperelastic and architected polymer members, shape-memory alloys, and soft-hard hybrid composites within X-type topologies, enabling strain-dependent stiffness evolution, adaptive hysteresis, and temperature- or rate-sensitive responses. These material effects, when coupled with the inherent kinematic nonlinearity of X-structures, give rise to rich multi-stable and path-dependent behaviors that are difficult to obtain through geometry alone. However, this also introduces open challenges in constitutive modeling and parameter identification, as the interaction between material nonlinearity, contact, and large deformation leads to high-dimensional nonlinear dynamics that are not well captured by classical reduced-order models. Developing tractable yet accurate modeling frameworks—possibly based on energy landscapes, data-driven identification, or machine-learned surrogates—remains a key open problem for design optimization and reliability assessment.

Another promising but underexplored direction lies in nonlinear interaction and multi-physics-based X-structure designs. Recent work has begun to integrate electromagnetic, thermal, acoustic, and fluid-structure couplings into X-mechanisms, enabling tunable stiffness and damping through magnetic forces, electro-mechanical feedback, or thermally induced preloads. These multi-physics interactions offer powerful new design freedoms, but also raise issues of stability, control authority, and robustness under uncertain or extreme environmental conditions. Nonlinear coupling across physical domains can trigger internal resonances, bifurcations, or degradation mechanisms that are poorly understood at present. Open problems include how to systematically co-design geometry, material, and field-based actuation in a unified framework, how to ensure long-term reliability under fatigue, radiation, or extreme temperatures, and how to scale such systems from micro-/meta-structural levels to large civil or aerospace platforms. Addressing these challenges would position X-structures (and various X-like variants such as V, K, M, L or other shapes) as a general nonlinear systems technology, bridging mechanical design, materials science, and intelligent multi-physics engineering for next-generation adaptive and sustainable structures.

In the future, the X-structure/mechanism approach is well-positioned to make meaningful contributions to a greener, safer, more reliable, and more sustainable engineering future. By enabling passive, energy-efficient vibration control, supporting self-powered monitoring systems, and offering lightweight, compact solutions for extreme environments, X-structures meet the pressing demands of modern engineering. As research continues to deepen our understanding of geometric nonlinearity and its combination with materials, control, and manufacturing, the X-structure paradigm will undoubtedly inspire further innovations.

Author Contributions

Y.G.: conducted the literature review and prepared the initial draft; Z.W.: reviewed and revised the draft; T.Y.: contributed several different parts of this review; G.L. and M.W.: reviewed and revised thoroughly the draft. X.J.: supervised the review work, reviewed and revised the draft, coordinated the review study, and provided all funding support. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

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