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# Improved Stability Criteria for Delayed T-S Fuzzy Systems Using an Augmented Zero Equality Approach

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**Abstract:** This study attempts to develop improved stability criteria with less conservatism for Takagi-Sugeno fuzzy systems with a time-varying delay that is bounded and differentiable. An augmented Lyapunov-Krasovskii functional is constructed by incorporating extensive information about the time-varying delay and its derivative. An auxiliary function-based integral inequality is combined with an extended reciprocally convex matrix inequality to obtain a tighter bound for the functional derivative. Furthermore, Finsler's lemma and an augmented zero equality approach that takes advantage of the interrelationships between augmented vector elements are employed to develop a less conservative delay-dependent stability criterion for the addressed fuzzy systems. The proposed augmented zero equality approach is then integrated into extended Finsler's lemma to enhance the proposed stability criterion, with the dual goals of reducing computational complexity and attaining a larger delay bound. Two numerical examples demonstrate that the established stability criteria are more efficient and less conservative than those in recent literature.

**Keywords:** Takagi-Sugeno fuzzy systems; stability; time-varying delay; augmented Lyapunov-Krasovskii functional; augmented zero equality approach

## 1. Introduction

The Takagi-Sugeno (T-S) fuzzy model has been universally accepted as the most convenient and versatile method for dealing with complex nonlinear systems. It should be specified that the T-S fuzzy model uses membership functions to approximate smooth nonlinear systems as a set of local linear models of arbitrary accuracy [1,2]. This method enables more effective research and control of nonlinear systems by employing existing linear system theories. Consequently, significant research efforts have been committed to incorporating T-S fuzzy modeling for nonlinear systems (see for example [3–6]). It is vital to note that most practical systems experience time delays, which can have a significant impact on their stability and performance. Thus, considering the influence of time delays, it is significant to explore the stability of delayed T-S fuzzy systems. Despite the fact that numerous works have been reported on this topic (see for example [7–12]), there are still areas that demand further investigation.

Lyapunov stability theory is an effective tool for determining the stability criterion of a system in the presence of time-varying delays. In this context, there are two major ways to increase the stability region: building the appropriate Lyapunov-Krasovskii functional (LKF) and estimating a tighter constraint for its time derivative. The former approach commonly uses augmented LKFs, which include information about the system's state variables in various forms. Specifically, augmented LKFs contain information about the system's state variables, such as its derivative and integral components. A significant number of research on T-S fuzzy systems using this methodology have been reported in the literature [13–19]. Soon after, the development of delay-product-type functionals evolved into the augmented LKF to increase the delay bound [20,21]. The latter approach employs a variety of mathematical tools,

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including advanced integral inequalities and convex combination techniques (for example, refer to [22–27]). However, it is vital to stress that producing less conservative outcomes is critical both theoretically and practically, despite the fact that numerous approaches exist.

In [28], the authors presented zero equalities that add cross-terms to the time derivative of LKF terms, resulting in enhanced stability criteria. Soon after, several works have been published by applying the zero equality approach, which plays a critical role in establishing less conservative stability conditions (see for example [29–32]). It should be noted that while various zero equality methods help to extend the feasible regions, they raise the computing complexity of the stability criteria. In [33], the authors proposed a new approach by grafting zero equalities onto the extended Finsler's lemma, obtaining considerably improved stability conditions while decreasing computational complexity. Some intriguing papers for various dynamic systems have lately been published using this approach [34–37].

Motivated by the preceding discussions, this work presents improved stability criteria for delayed T-S fuzzy systems. To achieve a maximum time delay bound while maintaining system stability, an appropriate augmented LKF is built, which comprises comprehensive information about the system state, its derivative and integral components, and delay-product-type terms. The addressed stability problem is solved using a variety of mathematical strategies, including auxiliary function-based integral inequality, extended reciprocally convex matrix inequality, and an augmented zero equality approach. Two numerical examples exhibit that the proposed stability criteria yield less conservative outcomes than existing ones.

**Notation 1.** *The space of $m \times n$ real-valued matrices and the $n$-dimensional Euclidean space are represented by $\mathbb{R}^{m\times n}$ and $\mathbb{R}^n$, respectively; $\mathbb{S}^n$ denotes the space of $n \times n$ symmetric real-valued matrices and $\mathbb{S}^n_+$ is the space of $n \times n$ symmetric and positive definite real-valued matrices; the $n \times n$ identity matrix and the $m \times n$ zero matrix are denoted by $I_n$ and $0_{m\times n}$, respectively (simply, $0$ denotes a zero matrix of appropriate dimension); for any square matrix $Q$, $\mathtt{sym}\{Q\} = Q + Q^T$; $\mathtt{diag}\{\cdots\}$ and $\mathtt{col}\{\cdots\}$ mean the block diagonal and column matrices, respectively; $P^\perp \in \mathbb{R}^{n\times(n-r)}$ represents the right orthogonal complement of a matrix $P \in \mathbb{R}^{m\times n}$ with $\mathtt{rank}(P) = r < n$; the symbol '$*$' describes the symmetry-implied matrices' entries.*

This paper is briefly described as follows: The description of the T-S fuzzy system with a time-varying delay and the preliminaries are provided in Section 2. Section 3 establishes two set of stability criteria for the addressed system. The validation of the proposed stability criteria and the conclusion of this work are provided in Sections 4 and 5, respectively.

## 2. System Description and Preliminaries

Consider a nonlinear system with time-varying delay that can be described by the T-S fuzzy model as follows: *Rule i:* IF $\varpi_1(x(t))$ is $M_{i1}$, $\varpi_2(x(t))$ is $M_{i2}$, ... and $\varpi_p(x(t))$ is $M_{ip}$, THEN

$$\begin{aligned} &\dot{x}(t) = A_i x(t) + B_i x(t-\gamma(t)), \quad i = 1,2,\ldots,r, \\ &x(t) = \psi(t), \quad t \in [-\hat{\gamma}, 0], \end{aligned} \tag{1}$$

where $r$ represents the number of IF-THEN rules; $M_{ij}$ and $\varpi_j(x(t))$ $(j = 1,2,\ldots,p)$ are the fuzzy sets and premise variables, respectively; $x(t) \in \mathbb{R}^n$ and $\psi(t)$ denote the state vector and initial condition of the system, respectively; $\{A_i, B_i\} \in \mathbb{R}^{n\times n}$ are constant matrices and $\gamma(t)$ represents the differentiable time-varying delay that meets the following conditions:

$$0 \le \gamma(t) \le \hat{\gamma}, \quad \lambda_1 \le \dot{\gamma}(t) \le \lambda_2 < 1,$$

where $\hat{\gamma}$ and $\lambda_b$ $(b = 1,2)$ are given constants.

Using the singleton fuzzifier, product inference, and center average defuzzifier, one can obtain the following global fuzzy system model:

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{r} \Theta_i(x(t))[A_i x(t) + B_i x(t-\gamma(t))], \quad i = 1,2,\ldots,r, \\ &x(t) = \psi(t), \quad t \in [-\hat{\gamma}, 0], \end{aligned} \tag{2}$$

where $\Theta_i(x(t))$ denotes the normalized membership function satisfying

$$\Theta_i(x(t)) = \frac{\prod_{j=1}^{p} M_{ij}(\varpi_j(x(t)))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ij}(\varpi_j(x(t)))} \geq 0, \quad \sum_{i=1}^{r}\Theta_i(x(t)) = 1,$$

and $M_{ij}(\varpi_j(x(t)))$ represents the grade of membership of $\varpi_j(x(t))$ in $M_{ij}$ under rule $i$.

In the following section, two improved stability criteria for the T-S fuzzy system (2) are developed using the lemmas listed below.

**Lemma 1** ([23]). *For any matrix $Q \in \mathbb{S}_+^n$, the following inequality holds for all continuously differentiable function $x(t) : [a,b] \to \mathbb{R}^n$:*

$$(b-a)\int_a^b x^T(s)Qx(s)\mathrm{d}s \geq \sum_{k=1}^{3}(2k-1)\Omega_k^T Q \Omega_k,$$

*where*

$$\Omega_1 = \int_a^b x(s)\mathrm{d}s, \quad \Omega_2 = -\Omega_1 + \frac{2}{b-a}\int_a^b\int_v^b x(s)\mathrm{d}s\mathrm{d}v,$$
$$\Omega_3 = \Omega_1 - \frac{6}{b-a}\int_a^b\int_v^b x(s)\mathrm{d}s\mathrm{d}v + \frac{12}{(b-a)^2}\int_a^b\int_u^b\int_v^b x(s)\mathrm{d}s\mathrm{d}v\mathrm{d}u.$$

**Lemma 2** ([26]). *The following inequality holds for a scalar $\alpha \in (0,1)$ and matrices $\{Q,S\} \in \mathbb{S}_+^n$, $\{K_1,K_2\} \in \mathbb{R}^{n\times n}$:*

$$\begin{bmatrix} \frac{1}{\alpha}Q & 0 \\ 0 & \frac{1}{1-\alpha}S \end{bmatrix} \geq \begin{bmatrix} Q+(1-\alpha)L_1 & (1-\alpha)K_1+\alpha K_2 \\ * & S+\alpha L_2 \end{bmatrix},$$

*where $L_1 = Q - K_2 S^{-1} K_2^T$ and $L_2 = S - K_1^T Q^{-1} K_1$.*

**Lemma 3** ([38]). *Let $w \in \mathbb{R}^n$, $F \in \mathbb{S}^n$, and $H \in \mathbb{R}^{m\times n}$. Then the following results are equivalent: (i) $w^T F w < 0$, $\forall Hw = 0$, $w \neq 0$, (ii) $\exists K \in \mathbb{R}^{n\times m}$ such that $F + \mathtt{sym}\{KH\} < 0$, and (iii) $(H^\perp)^T F (H^\perp) < 0$.*

**Lemma 4** ([39]). *Let $w \in \mathbb{R}^n$, $S \subseteq \mathbb{R}^d$, $F : S \to \mathbb{S}^n$, and $H : S \to \mathbb{R}^{m\times n}$. Then the following statements are equivalent: (i) For each $s \in S$, $w^T F(s) w < 0$, $\forall H(s)w = 0$, $w \neq 0$, (ii) For each $s \in S$, $\exists K(s) \in \mathbb{R}^{n\times m}$ : $F(s) + \mathtt{sym}\{K(s)H(s)\} < 0$, and (iii) For each $s \in S$, $(H^\perp(s))^T F(s)(H^\perp(s)) < 0$.*

## 3. Main Results

This section presents two enhanced stability criteria for the addressed fuzzy system (2) using an augmented LKF and augmented zero equality method. For conciseness, the following vectors are defined:

$$\gamma_h(t) = \hat{\gamma} - \gamma(t), \quad \gamma_d(t) = 1 - \dot{\gamma}(t), \quad \vartheta_1(t) = \int_{t-\gamma(t)}^{t} x(s)\mathrm{d}s, \quad \vartheta_2(t) = \int_{t-\hat{\gamma}}^{t-\gamma(t)} x(s)\mathrm{d}s,$$
$$\vartheta_3(t) = \int_{t-\gamma(t)}^{t}\int_v^t x(s)\mathrm{d}s\mathrm{d}v, \quad \vartheta_4(t) = \int_{t-\hat{\gamma}}^{t-\gamma(t)}\int_v^{t-\gamma(t)} x(s)\mathrm{d}s\mathrm{d}v,$$
$$\vartheta_5(t) = \int_{t-\gamma(t)}^{t}\int_u^t\int_v^t x(s)\mathrm{d}s\mathrm{d}v\mathrm{d}u, \quad \vartheta_6(t) = \int_{t-\hat{\gamma}}^{t-\gamma(t)}\int_u^{t-\gamma(t)}\int_v^{t-\gamma(t)} x(s)\mathrm{d}s\mathrm{d}v\mathrm{d}u,$$
$$\xi_1(t) = \mathtt{col}\left\{x(t), x(t-\gamma(t)), x(t-\hat{\gamma}), \dot{x}(t), \dot{x}(t-\gamma(t)), \dot{x}(t-\hat{\gamma})\right\},$$
$$\xi_2(t) = \mathtt{col}\left\{\vartheta_1(t), \vartheta_2(t), \frac{1}{\gamma(t)}\vartheta_3(t), \frac{1}{\gamma_h(t)}\vartheta_4(t), \frac{1}{\gamma^2(t)}\vartheta_5(t), \frac{1}{\gamma_h^2(t)}\vartheta_6(t)\right\},$$
$$\xi_3(t) = \mathtt{col}\left\{\frac{1}{\gamma(t)}\vartheta_1(t), \frac{1}{\gamma_h(t)}\vartheta_2(t), \frac{1}{\gamma^2(t)}\vartheta_3(t), \frac{1}{\gamma_h^2(t)}\vartheta_4(t)\right\},$$
$$\xi(t) = \mathtt{col}\left\{\xi_1(t), \xi_2(t), \xi_3(t), \gamma(t)x(t), \gamma_h(t)x(t)\right\}, \quad e_g = [0_{n\times(g-1)n}, I_n, 0_{n\times(18-g)n}]^T, \quad g = 1,2,\ldots,18.$$

**Theorem 1.** *The fuzzy system (2) is asymptotically stable for given scalars $\hat{\gamma} > 0$ and $\lambda_1 < \lambda_2 < 1$ if there exist matrices $P_a \in \mathbb{S}_+^{7n}$, $Q_b \in \mathbb{S}_+^{6n}$, $R \in \mathbb{S}_+^{3n}$, $G_b \in \mathbb{S}^n$, $X_b \in \mathbb{R}^{9n\times 9n}$, and $N \in \mathbb{R}^{18n\times 6n}$ $(a = 1,2,3; b = 1,2)$, such that the following conditions hold for $\dot{\gamma}(t) \in \{\lambda_1, \lambda_2\}$ and $i = 1,2,\ldots,r$:*

$$\Pi_i^T \begin{bmatrix} \Phi(0,\dot{\gamma}(t)) & \Xi_1 X_2 \\ * & -\tilde{R}_{G_2} \end{bmatrix} \Pi_i < 0, \tag{3}$$

$$\Pi_i^T \begin{bmatrix} \Phi(\hat{\gamma},\dot{\gamma}(t)) & \Xi_2 X_1^T \\ * & -\tilde{R}_{G_1} \end{bmatrix} \Pi_i < 0, \tag{4}$$

*where* $\Phi(\gamma(t),\dot{\gamma}(t)) = \Phi_1(\gamma(t),\dot{\gamma}(t)) + \Phi_2(\gamma(t),\dot{\gamma}(t)) + \Phi_3(\gamma(t)) + \Phi_4 + \Phi_5(\gamma(t)) + \Phi_6(\gamma(t))$,

$$\begin{aligned}
\Phi_1(\gamma(t),\dot{\gamma}(t)) &= \mathtt{sym}\{[e_1,e_2,e_3,e_7,e_8,e_9,e_{10}](P_1+\gamma(t)P_2+\gamma_h(t)P_3) \\
&\quad \times [e_4,\gamma_d(t)e_5,e_6,e_1-\gamma_d(t)e_2,\gamma_d(t)e_2-e_3,e_1-\gamma_d(t)e_{13}-\dot{\gamma}(t)e_{15},\gamma_d(t)e_2-e_{14}+\dot{\gamma}(t)e_{16}]^T\} \\
&\quad + [e_1,e_2,e_3,e_7,e_8,e_9,e_{10}]\dot{\gamma}(t)(P_2-P_3)[e_1,e_2,e_3,e_7,e_8,e_9,e_{10}]^T, \\
\Phi_2(\gamma(t),\dot{\gamma}(t)) &= [e_1,e_2,e_3,e_4,0,e_7]Q_1[e_1,e_2,e_3,e_4,0,e_7]^T - [e_2,e_2,e_3,e_5,e_7,0]\gamma_d(t)Q_1[e_2,e_2,e_3,e_5,e_7,0]^T \\
&\quad + \mathtt{sym}\{[e_7,\gamma(t)e_2,\gamma(t)e_3,e_1-e_2,\gamma(t)e_9,\gamma(t)(e_7-e_9)]Q_1[0,\gamma_d(t)e_5,e_6,0,e_1,-\gamma_d(t)e_2]^T\} \\
&\quad + [e_2,e_2,e_3,e_5,0,e_8]\gamma_d(t)Q_2[e_2,e_2,e_3,e_5,0,e_8]^T - [e_3,e_2,e_3,e_6,e_8,0]Q_2[e_3,e_2,e_3,e_6,e_8,0]^T \\
&\quad + \mathtt{sym}\{[e_8,\gamma_h(t)e_2,\gamma_h(t)e_3,e_2-e_3,\gamma_h(t)e_{10},\gamma_h(t)(e_8-e_{10})]Q_2[0,\gamma_d(t)e_5,e_6,0,\gamma_d(t)e_2,-e_3]^T\}, \\
\Phi_3(\gamma(t)) &= [e_1,e_4,0]\hat{\gamma}^2 R[e_1,e_4,0]^T \\
&\quad + \mathtt{sym}\{[\gamma(t)e_9+\gamma_h(t)(e_7+e_{10}),\hat{\gamma}e_1-e_7-e_8,\frac{\hat{\gamma}^2}{2}e_1-\gamma(t)e_9-\gamma_h(t)(e_7+e_{10})]\hat{\gamma}R[0,0,e_4]^T\}, \\
\Phi_4 &= \hat{\gamma}(e_1G_1e_1^T + e_2(G_2-G_1)e_2^T - e_3G_2e_3^T), \\
\Phi_5(\gamma(t)) &= -\frac{2\hat{\gamma}-\gamma(t)}{\hat{\gamma}}\Xi_1\tilde{R}_{G_1}\Xi_1^T - \frac{\hat{\gamma}+\gamma(t)}{\hat{\gamma}}\Xi_2\tilde{R}_{G_2}\Xi_2^T - \frac{\gamma_h(t)}{\hat{\gamma}}\mathtt{sym}\{\Xi_1X_1\Xi_2^T\} - \frac{\gamma(t)}{\hat{\gamma}}\mathtt{sym}\{\Xi_1X_2\Xi_2^T\}, \\
\Phi_6(\gamma(t)) &= \mathtt{sym}\left\{N\begin{bmatrix} e_7^T-\gamma(t)e_{13}^T \\ e_8^T-\gamma_h(t)e_{14}^T \\ e_9^T-\gamma(t)e_{15}^T \\ e_{10}^T-\gamma_h(t)e_{16}^T \\ e_{17}^T-\gamma(t)e_1^T \\ e_{18}^T-\gamma_h(t)e_1^T \end{bmatrix}\right\}, \quad R_{G_b} = R + \begin{bmatrix} 0 & G_b & 0 \\ G_b & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\tilde{R}_{G_b} &= \mathtt{diag}\{R_{G_b},3R_{G_b},5R_{G_b}\}, \quad \Upsilon_i = A_ie_1^T + B_ie_2^T - e_4^T, \quad \Pi_i = \mathtt{diag}\{\Upsilon_i^{\perp},I\}, \\
\Xi_b &= [e_{b+6},e_b-e_{b+1},e_{b+16}-e_{b+6},-e_{b+6}+2e_{b+8},e_b+e_{b+1}-2e_{b+12},e_{b+6}-2e_{b+8}, \\
&\quad e_{b+6}-6e_{b+8}+12e_{b+10},e_b-e_{b+1}+6e_{b+12}-12e_{b+14},-e_{b+6}+6e_{b+8}-12e_{b+10}], \quad b=1,2.
\end{aligned}$$

**Proof.** Consider the following augmented LKF for the addressed fuzzy system (2):

$$V(x(t)) = \sum_{a=1}^{3} V_a(x(t)), \tag{5}$$

where

$$\begin{aligned}
V_1(x(t)) &= \zeta_1^T(t)(P_1+\gamma(t)P_2+\gamma_h(t)P_3)\zeta_1(t), \\
V_2(x(t)) &= \int_{t-\gamma(t)}^{t}\zeta_2^T(t,s,t-\gamma(t))Q_1\zeta_2(t,s,t-\gamma(t))\mathrm{d}s + \int_{t-\hat{\gamma}}^{t-\gamma(t)}\zeta_2^T(t-\gamma(t),s,t-\hat{\gamma})Q_2\zeta_2(t-\gamma(t),s,t-\hat{\gamma})\mathrm{d}s, \\
V_3(x(t)) &= \hat{\gamma}\int_{t-\hat{\gamma}}^{t}\int_{v}^{t}\zeta_3^T(t,s)R\zeta_3(t,s)\mathrm{d}s\mathrm{d}v,
\end{aligned}$$

and

$$\begin{aligned}
\zeta_1(t) &= \mathtt{col}\left\{x(t),x(t-\gamma(t)),x(t-\hat{\gamma}),\vartheta_1(t),\vartheta_2(t),\frac{1}{\gamma(t)}\vartheta_3(t),\frac{1}{\gamma_h(t)}\vartheta_4(t)\right\}, \\
\zeta_2(j,s,k) &= \mathtt{col}\left\{x(s),x(t-\gamma(t)),x(t-\hat{\gamma}),\dot{x}(s),\int_s^j x(v)\mathrm{d}v,\int_k^s x(v)\mathrm{d}v\right\}, \\
\zeta_3(t,s) &= \mathtt{col}\left\{x(s),\dot{x}(s),(x(t)-x(s))\right\}.
\end{aligned}$$

Time derivatives of $V_a(x(t))$ $(a=1,2,3)$ along the solution trajectory of (2) are calculated as follows:

$$\dot{V}_1(x(t)) = 2\zeta_1^T(t)(P_1+\gamma(t)P_2+\gamma_h(t)P_3)\begin{bmatrix} \dot{x}(t) \\ \gamma_d(t)\dot{x}(t-\gamma(t)) \\ \dot{x}(t-\hat{\gamma}) \\ x(t)-\gamma_d(t)x(t-\gamma(t)) \\ \gamma_d(t)x(t-\gamma(t))-x(t-\hat{\gamma}) \\ x(t)-\frac{\gamma_d(t)}{\gamma(t)}\vartheta_1(t)-\frac{\dot{\gamma}(t)}{\gamma^2(t)}\vartheta_3(t) \\ \gamma_d(t)x(t-\gamma(t))-\frac{1}{\gamma_h(t)}\vartheta_2(t)+\frac{\dot{\gamma}(t)}{\gamma_h^2(t)}\vartheta_4(t) \end{bmatrix} + \zeta_1^T(t)\dot{\gamma}(t)(P_2-P_3)\zeta_1(t)$$
$$= \xi^T(t)\Phi_1(\gamma(t),\dot{\gamma}(t))\xi(t), \quad (6)$$

$$\begin{aligned}\dot{V}_2(x(t)) &= \zeta_2^T(t,t,t-\gamma(t))Q_1\zeta_2(t,s,t-\gamma(t)) - \zeta_2^T(t,t-\gamma(t),t-\gamma(t))\gamma_d(t)Q_1\zeta_2(t,t-\gamma(t),t-\gamma(t)) \\ &\quad + 2\int_{t-\gamma(t)}^{t}\zeta_2^T(t,s,t-\gamma(t))Q_1\frac{\partial}{\partial t}[\zeta_2(t,s,t-\gamma(t))]\mathrm{d}s \\ &\quad + \zeta_2^T(t-\gamma(t),t-\gamma(t),t-\hat{\gamma})\gamma_d(t)Q_2\zeta_2(t-\gamma(t),t-\gamma(t),t-\hat{\gamma}) \\ &\quad - \zeta_2^T(t-\gamma(t),t-\hat{\gamma},t-\hat{\gamma})Q_2\zeta_2(t-\gamma(t),t-\hat{\gamma},t-\hat{\gamma}) \\ &\quad + 2\int_{t-\hat{\gamma}}^{t-\gamma(t)}\zeta_2^T(t-\gamma(t),s,t-\hat{\gamma})Q_2\frac{\partial}{\partial t}[\zeta_2(t-\gamma(t),s,t-\hat{\gamma})]\mathrm{d}s \\ &= \xi^T(t)\Phi_2(\gamma(t),\dot{\gamma}(t))\xi(t), \end{aligned} \quad (7)$$

$$\begin{aligned}\dot{V}_3(x(t)) &= \zeta_3^T(t,t)\hat{\gamma}^2R\zeta_3(t,t) + 2\hat{\gamma}\int_{t-\hat{\gamma}}^{t}\int_{v}^{t}\zeta_3^T(t,s)R\frac{\partial}{\partial t}[\zeta_3(t,s)]\mathrm{d}s - \hat{\gamma}\int_{t-\hat{\gamma}}^{t}\zeta_3^T(t,s)R\zeta_3(t,s)\mathrm{d}s \\ &= \xi^T(t)\Phi_3(\gamma(t))\xi(t) - \hat{\gamma}\int_{t-\gamma(t)}^{t}\zeta_3^T(t,s)R\zeta_3(t,s)\mathrm{d}s - \hat{\gamma}\int_{t-\hat{\gamma}}^{t-\gamma(t)}\zeta_3^T(t,s)R\zeta_3(t,s)\mathrm{d}s. \end{aligned} \quad (8)$$

Consider the following zero-value term for matrices $G_b \in \mathbb{S}^n$ $(b = 1, 2)$ based on the work in [28]:

$$\begin{aligned} 0 = \hat{\gamma}\Bigg(& x^T(t)G_1x(t) - x^T(t-\gamma(t))G_1x(t-\gamma(t)) - 2\int_{t-\gamma(t)}^{t}x^T(s)G_1\dot{x}(s)\mathrm{d}s \\ & + x^T(t-\gamma(t))G_2x(t-\gamma(t)) - x^T(t-\hat{\gamma})G_2x(t-\hat{\gamma}) - 2\int_{t-\hat{\gamma}}^{t-\gamma(t)}x^T(s)G_2\dot{x}(s)\mathrm{d}s\Bigg). \end{aligned} \quad (9)$$

Adding (9) to (8) yields the following result:

$$\dot{V}_3(x(t)) = \xi^T(t)\left(\Phi_3(\gamma(t))+\Phi_4\right)\xi(t) - \hat{\gamma}\int_{t-\gamma(t)}^{t}\zeta_3^T(t,s)R_{G_1}\zeta_3(t,s)\mathrm{d}s - \hat{\gamma}\int_{t-\hat{\gamma}}^{t-\gamma(t)}\zeta_3^T(t,s)R_{G_2}\zeta_3(t,s)\mathrm{d}s. \quad (10)$$

Using Lemma 1, the two integral terms in (10) are bounded as follows:

$$-\hat{\gamma}\int_{t-\gamma(t)}^{t}\zeta_3^T(t,s)R_{G_1}\zeta_3(t,s)\mathrm{d}s \le -\frac{\hat{\gamma}}{\gamma(t)}\xi^T(t)\Xi_1\tilde{R}_{G_1}\Xi_1^T\xi(t), \quad (11)$$
$$-\hat{\gamma}\int_{t-\hat{\gamma}}^{t-\gamma(t)}\zeta_3^T(t,s)R_{G_2}\zeta_3(t,s)\mathrm{d}s \le -\frac{\hat{\gamma}}{\gamma_h(t)}\xi^T(t)\Xi_2\tilde{R}_{G_2}\Xi_2^T\xi(t). \quad (12)$$

The quadratic terms in (11) and (12) are bounded with aid of Lemma 2 and matrices $X_b \in \mathbb{R}^{9n\times 9n}$ $(b = 1, 2)$ as follows:

$$\begin{aligned} &\xi^T(t)\left\{-\frac{\hat{\gamma}}{\gamma(t)}\Xi_1\tilde{R}_{G_1}\Xi_1^T - \frac{\hat{\gamma}}{\gamma_h(t)}\Xi_2\tilde{R}_{G_2}\Xi_2^T\right\}\xi(t) \\ &= -\xi^T(t)\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix}^T\begin{bmatrix}\frac{\hat{\gamma}}{\gamma(t)}\tilde{R}_{G_1} & 0 \\ 0 & \frac{\hat{\gamma}}{\gamma_h(t)}\tilde{R}_{G_2}\end{bmatrix}\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix}\xi(t) \\ &\le \xi^T(t)\Bigg\{-\Xi_1\tilde{R}_{G_1}\Xi_1^T - \Xi_2\tilde{R}_{G_2}\Xi_2^T - \frac{\gamma_h(t)}{\hat{\gamma}}\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix}^T\begin{bmatrix}\tilde{R}_{G_1}-X_2\tilde{R}_{G_2}^{-1}X_2^T & X_1 \\ * & 0\end{bmatrix}\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix} \\ &\qquad -\frac{\gamma(t)}{\hat{\gamma}}\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix}^T\begin{bmatrix}0 & X_2 \\ * & \tilde{R}_{G_2}-X_1^T\tilde{R}_{G_1}^{-1}X_1\end{bmatrix}\begin{bmatrix}\Xi_1^T \\ \Xi_2^T\end{bmatrix}\Bigg\}\xi(t) \\ &= \xi^T(t)\left\{\Phi_5(\gamma(t)) + \frac{\gamma_h(t)}{\hat{\gamma}}\Xi_1X_2\tilde{R}_{G_2}^{-1}X_2^T\Xi_1^T + \frac{\gamma(t)}{\hat{\gamma}}\Xi_2X_1^T\tilde{R}_{G_1}^{-1}X_1\Xi_2^T\right\}\xi(t). \end{aligned} \quad (13)$$

Based on the work [33], the following augmented zero equality is constructed from the relationships between the augmented vector $\xi(t)$ and for a matrix $N \in \mathbb{R}^{18n\times 6n}$:

$$0 = \xi^T(t)\mathtt{sym}\left\{ N \begin{bmatrix} e_7^T - \gamma(t)e_{13}^T \\ e_8^T - \gamma_h(t)e_{14}^T \\ e_9^T - \gamma(t)e_{15}^T \\ e_{10}^T - \gamma_h(t)e_{16}^T \\ e_{17}^T - \gamma(t)e_1^T \\ e_{18}^T - \gamma_h(t)e_1^T \end{bmatrix} \right\} \xi(t) = \xi^T(t)\Phi_6(\gamma(t))\xi(t). \tag{14}$$

Combining (6)–(14), the following upper bound for $\dot{V}(x(t))$ is obtained:

$$\dot{V}(x(t)) \le \xi^T(t)\Phi^*(\gamma(t), \dot{\gamma}(t))\xi(t), \tag{15}$$

where $\Phi^*(\gamma(t), \dot{\gamma}(t)) = \Phi(\gamma(t), \dot{\gamma}(t)) + \frac{\gamma_h(t)}{\hat{\gamma}}\Xi_1 X_2 \tilde{R}_{G_2}^{-1} X_2^T \Xi_1^T + \frac{\gamma(t)}{\hat{\gamma}}\Xi_2 X_1^T \tilde{R}_{G_1}^{-1} X_1 \Xi_2^T$.

Thus, $\xi^T(t)\Phi^*(\gamma(t), \dot{\gamma}(t))\xi(t) < 0$ subject to $\Upsilon_i \xi(t) = 0\ \forall i = 1, 2, \ldots, r$ is a stability criteria of the fuzzy system (2). It can be further expressed using Lemma 3 as $(\Upsilon_i^{\perp})^T \Phi^*(\gamma(t), \dot{\gamma}(t))(\Upsilon_i^{\perp}) < 0$, which is affinely dependent on $\gamma(t)$ and $\dot{\gamma}(t)$. The Schur complement can then be used to equivalently describe it as (3) and (4). As a result, the fuzzy system (2) is asymptotically stable if the conditions (3) and (4) are met. □

**Remark 1.** *It should be noted in Theorem 1 that the inclusion of the augmented zero equality* (14) *derived using the relationships between the terms considered in the augmented vector* $\xi(t)$ *requires an additional free variable* $N$*, whose dimension can be defined by the dimension of the augmented vector. As a result, the addition of this augmented zero equality will surely increase the overall number of decision variables, but it also opens up the possibility of obtaining a broader delay bound.*

The following corollary, based on Theorem 1, improves the stability criterion of the T-S fuzzy system (2) while reducing the number of decision variables.

**Corollary 1.** *The fuzzy system* (2) *is asymptotically stable for given scalars* $\hat{\gamma} > 0$ *and* $\lambda_1 < \lambda_2 < 1$ *if there exist matrices* $P_a \in \mathbb{S}_+^{7n}$, $Q_b \in \mathbb{S}_+^{6n}$, $R \in \mathbb{S}_+^{3n}$, $G_b \in \mathbb{S}^n$, *and* $X_b \in \mathbb{R}^{9n \times 9n}$ $(a = 1, 2, 3; b = 1, 2)$, *such that the following conditions hold for* $\dot{\gamma}(t) \in \{\lambda_1, \lambda_2\}$ *and* $i = 1, 2, \ldots, r$*:*

$$\tilde{\Pi}_i^T(0) \begin{bmatrix} \tilde{\Phi}(0, \dot{\gamma}(t)) & \Xi_1 X_2 \\ * & -\tilde{R}_{G_2} \end{bmatrix} \tilde{\Pi}_i(0) < 0, \tag{16}$$

$$\tilde{\Pi}_i^T(\hat{\gamma}) \begin{bmatrix} \tilde{\Phi}(\hat{\gamma}, \dot{\gamma}(t)) & \Xi_2 X_1^T \\ * & -\tilde{R}_{G_1} \end{bmatrix} \tilde{\Pi}_i(\hat{\gamma}) < 0, \tag{17}$$

*where*

$$\begin{aligned} \tilde{\Phi}(\gamma(t), \dot{\gamma}(t)) &= \Phi_1(\gamma(t), \dot{\gamma}(t)) + \Phi_2(\gamma(t), \dot{\gamma}(t)) + \Phi_3(\gamma(t)) + \Phi_4 + \Phi_5(\gamma(t)), \\ \tilde{\Upsilon}_i(\gamma(t)) &= \mathtt{col}\{\Upsilon_i, e_7^T - \gamma(t)e_{13}^T, e_8^T - \gamma_h(t)e_{14}^T, e_9^T - \gamma(t)e_{15}^T, e_{10}^T - \gamma_h(t)e_{16}^T, e_{17}^T - \gamma(t)e_1^T, e_{18}^T - \gamma_h(t)e_1^T\}, \\ \tilde{\Pi}_i(\gamma(t)) &= \mathtt{diag}\{(\tilde{\Upsilon}_i(\gamma(t)))^{\perp}, I\}. \end{aligned}$$

**Proof.** The proof of this corollary is almost the same as that of Theorem 1. By eliminating (14) and using Lemma 4, one can easily obtain the following result:

$$\xi^T(t)\tilde{\Phi}(\gamma(t), \dot{\gamma}(t))\xi(t) < 0 \quad \text{subject to} \quad \tilde{\Upsilon}_i(\gamma(t))\xi(t) = 0, \quad \forall i = 1, 2, \ldots, r.$$

The remaining steps are similar to those in Theorem 1, thus they are removed. □

**Remark 2.** *Although Corollary 1 is derived using a similar procedure to Theorem 1, it offers a less conservative stability criterion for the T-S fuzzy system* (1)*. To be precise, Corollary 1 applies zero equalities in* (14) *to the extended Finsler's lemma, eliminating the free variable* $N \in \mathbb{R}^{18n \times 6n}$*. As a result, the stability criterion proposed in Corollary 1 provides a larger delay bound and has fewer decision variables than Theorem 1. It will be clearly illustrated in the following section, with two numerical examples containing detailed data.*

## 4. Numerical Examples

This section provides two well-known examples to demonstrate the improvements and validity of the proposed stability criteria for the fuzzy system (2).

**Example 1.** *Consider the T-S fuzzy system* (2) *with the following parameter values:*

$$A_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0 & -2.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.$$

*The corresponding membership functions are selected as follows:*

$$\Theta_1(x(t)) = \frac{1}{1 + \exp(-2x_1(t))}, \quad \Theta_2(x(t)) = 1 - \Theta_1(x(t)).$$

*Table 1 shows the maximum allowable upper bound (MAUB) $\hat{\gamma}$ for various $\lambda_1$ and $\lambda_2$ values derived using the proposed methods as well as other existing works. It is clearly visible from Table 1 that the MAUB of the delay achieved by the methods in Theorem 1 and Corollary 1 is substantially bigger than that obtained in previously published ones [13, 14, 16, 19]. From a detailed standpoint, the improvement from Theorem 1 validates the effectiveness of the augmented LKF proposed in this paper, as well as the augmented zero equality approach, whereas the improvement from Corollary 1 demonstrates the effectiveness of incorporating the augmented zero equality approach into the extended Finsler's lemma (Lemma 4).*

*Although both approaches in Theorem 1 and Corollary 1 far better than the existing results, their computational complexity is demonstrated in Table 2. It is concluded from Table 2 that Corollary 1 outperforms Theorem 1 in terms of both increasing the delay bound and reducing the computational complexity.*

**Table 1.** MAUB $\hat{\gamma}$ under various values for $\lambda = -\lambda_1 = \lambda_2$.

| Methods | $\lambda = 0.03$ | $\lambda = 0.10$ | $\lambda = 0.50$ | $\lambda = 0.90$ |
|---|---|---|---|---|
| Ref. [13] | 5.0366 | 2.1270 | 1.4882 | 1.4330 |
| Ref. [14] | 5.6506 | 2.4674 | 1.6450 | 1.5103 |
| Ref. [16] | 6.4162 | 2.8295 | 1.7398 | 1.5946 |
| Ref. [19] | 6.099 | 3.823 | 2.373 | 1.954 |
| Theorem 1 | 8.2048 | 4.7907 | 2.5152 | 1.9825 |
| Corollary 1 | 10.5675 | 9.2588 | 5.5016 | 3.8940 |

**Table 2.** Number of decision variables (DVs).

| Methods | DVs |
|---|---|
| Theorem 1 | $385n^2$+$19n$ |
| Corollary 1 | $277n^2$+$19n$ |

*On the other hand, under the initial state $\psi(t) = [-1, 1]^T$, Figure 1 depicts the state responses of the T-S fuzzy system* (2) *with time-varying delay $\gamma(t) = (10.5675/2) + (10.5675/2)\sin(0.06t/10.5675)$. The curves show that the delayed T-S fuzzy system eventually remains stable for the MAUB obtained in this paper.*

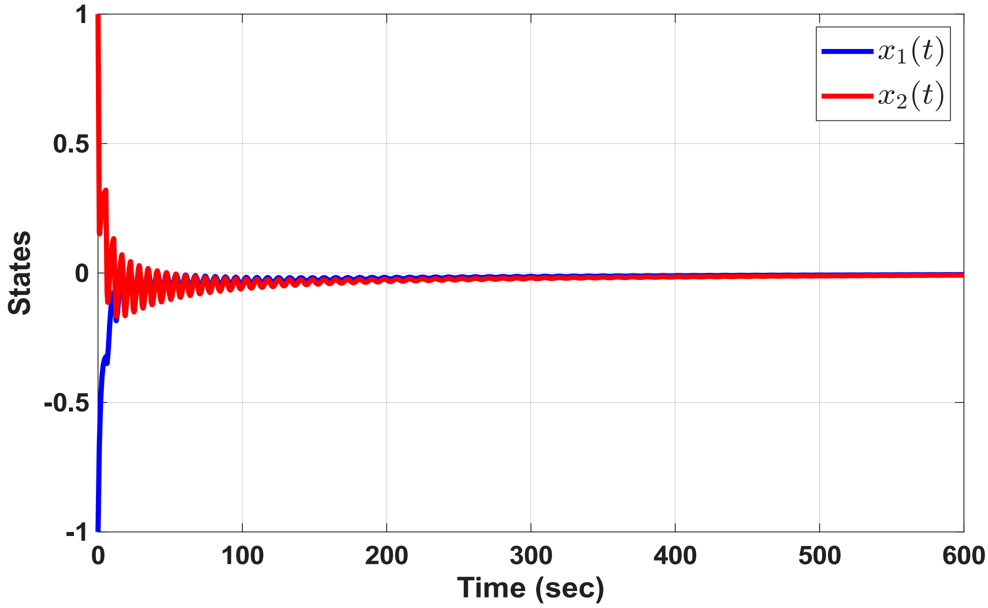


**Figure 1.** Evolution of the system state $x(t)$.

**Example 2.** *Consider the T-S fuzzy system* (2) *with the following matrices:*

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.5 & 1 \\ 0 & -0.75 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 0 \\ 1 & -0.85 \end{bmatrix}.$$

*The membership functions are chosen as described in Example 1. The MAUB $\hat{\gamma}$ is calculated for various $\lambda_1$ and $\lambda_2$ values using the stability criteria in Theorem 1 and Corollary 1 and compared with previous works, which are presented in Table 3. It is observed from Table 3 that the upper bound of delay obtained by Corollary 1 is much larger than that obtained by [15–19] and Theorem 1. This indicates that the delay range of asymptotic stability of the T-S fuzzy system* (1) *derived by Corollary 1 is wider. Thus, the stability criterion developed in Corollary 1 is both more efficient and less conservative.*

**Table 3.** MAUB $\hat{\gamma}$ under various values for $\lambda = -\lambda_1 = \lambda_2$.

| Methods | $\boldsymbol{\lambda = 0.20}$ | $\boldsymbol{\lambda = 0.40}$ | $\boldsymbol{\lambda = 0.60}$ |
|---|---|---|---|
| Ref. [15] | 1.8728 | 1.6517 | 1.5205 |
| Ref. [16] | 1.9216 | 1.7317 | 1.6121 |
| Ref. [17] | 2.0101 | 1.8487 | 1.7349 |
| Ref. [18] | 2.0813 | 1.8954 | 1.7521 |
| Ref. [19] | 2.089 | 1.975 | 1.871 |
| Theorem 1 | 1.9479 | 1.7711 | 1.6553 |
| Corollary 1 | 2.2516 | 2.2472 | 2.2395 |

## 5. Conclusions

This paper has presented enhanced delay-dependent stability criteria for T-S fuzzy systems subject to time-varying delays using a refined analytical framework that reduces conservatism. The improved stability conditions have been developed using a variety of mathematical techniques, including the augmented LKF method and the augmented zero equality approach. Specifically, Corollary 1 has significantly increased the allowable delay bound while reducing computational complexity compared to Theorem 1. The effectiveness and superiority of the proposed stability criteria have been validated using numerical examples that provide better outcomes than those by existing methods. It would be an interesting problem to apply the proposed method to reachable set analysis of T-S fuzzy systems with time-varying delay, which is our future research topic.

## Author Contributions

B.K.: Conceptualization, methodology, writing—original draft, software, writing—reviewing and editing; R.S.: Writing—original draft, software, writing—reviewing and editing; A.K.: Writing—original draft, software, writing—reviewing and editing; A.P.: Writing—original draft, software, writing—reviewing and editing. All authors have read and agreed to the published version of the manuscript.

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The authors declare no conflict of interest.

## Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

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