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Mathematical Modelling of a Pneumo-Hydraulic Impulse Device for Vibration Turning

Roman Obertyukh¹, Andrii Slabkyi¹, Oleksandr Povstyanoy², Dmytro Bakalets¹ and Vladyslav Polishchuk¹

¹ Department of Industrial Engineering, Faculty of Mechanical Engineering and Transport, Vinnytsia National Technical University, 21021 Vinnytsia, Ukraine

² Department of Applied Mechanics and Mechatronics, Faculty of Robotics and AI, Lutsk National Technical University, 43018 Lutsk, Ukraine

* Correspondence: slabkiyandrey@gmail.com; Tel.: +380-97-1-999-840

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Abstract: This paper addresses the critical challenge of improving the machinability of difficult-to-cut advanced materials through radial vibration-assisted turning. Traditional piezoelectric or mechanical vibrators suffer from micro-stroke decay or severe inertial limitations under heavy technological cutting loads. To resolve this bottleneck, a compact pneumo-hydraulic impulse multiplier device equipped with a single-stage parametric pressure pulse generator is investigated via a non-linear fluid-structure interaction framework solved through explicit Euler integration. Initial simulations revealed that a conventional high-inertia steel pusher (0.85 kg) paired with standard cylindrical springs induces a destructive “static lock-up” condition due to continuous unrelaxed kinetic energy accumulation. To bring the drive into its optimal operating window, structural modernization was executed by substituting the reciprocating components with a lightweight titanium alloy (Ti-6Al-4V Grade 5) and integrating a high-stiffness slotted spring. The upgraded simulation demonstrates a successful transition into a critically damped aperiodic operating regime. The modernized drive completely eliminates parasitic tool rebound, achieving perfectly repeatable pressure cycles reaching 14.8 MPa and a highly stable cutter vibration amplitude of 0.15–0.18 mm at operating frequencies up to 250 Hz. This provides a predictive, numerically validated foundation for high-frequency vibration turning utilizing standard shop-floor pneumatic supply networks.

Keywords: vibrations; hydraulic pulse drive; pressure pulse generator; mathematical modelling; oscillation frequency

1. Introduction

The modern development of the aerospace, automotive, and defense engineering industries continuously drives the wider implementation of advanced structural materials, such as titanium alloys (e.g., Ti-6Al-4V) and high-strength stainless steels. Due to their exceptional mechanical properties, corrosion resistance, and high strength-to-weight ratio, these materials are indispensable for critical structural components. However, their machinability remains a severe technological challenge characterized by high cutting forces, intense thermal loading in the cutting zone, and rapid tool wear [1,2]. Traditional continuous machining methods often fail to ensure the required surface integrity and productivity when processing such alloys, which necessitates the development of innovative manufacturing techniques. Vibration-assisted machining, specifically vibrational turning, has emerged as one of the most promising technological solutions to improve the machinability of difficult-to-cut materials. The intentional introduction of controlled, high-frequency micro-oscillations into the cutting zone fundamentally alters the kinematics of the process,



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shifting it from continuous to intermittent cutting [1–4]. This transition ensures effective chip breaking, reduces the average cutting temperature due to periodic tool-workpiece separation, and significantly decreases friction forces [3]. Consequently, vibrational turning substantially extends tool life and facilitates the formation of regular microreliefs on the machined surfaces, thereby enhancing their operational characteristics.

The technological efficiency of vibrational turning is fundamentally governed by the parameters of the generated oscillation cycles, namely their frequency and amplitude stability under varying cutting loads. Existing mechanical, electromagnetic, and piezoelectric vibration actuators possess inherent limitations, such as low load capacity, restricted frequency ranges, or high sensitivity to external damping [5,6]. In this context, pneumo-hydraulic impulse devices (PHIDs) equipped with pressure pulse generators represent a highly efficient alternative. These systems combine the high energy density and structural stiffness of a hydraulic multiplier with the dynamic responsiveness of pneumatic control elements [2,7]. A design scheme has been developed for a compact pneumo-hydraulic impulse multiplier device for radial vibrational turning, in which a hydraulic multiplier, executed as a high-stiffness closed micro-volume, converts compressed air pulses into high-pressure energy carrier pulses, thereby driving the device's working element—the cutter—into vibrational motion [8]. This design demonstrates significant technological potential and warrants further comprehensive theoretical and experimental investigation.

The global advancement of high-precision manufacturing is currently limited by a critical trade-off in dynamic actuation: the inability to simultaneously achieve high energy density, high frequency, and load-independent amplitude stability within a compact structural envelope. Traditional vibration actuators extensively documented in global literature fail to resolve this trilemma:

- Piezoelectric and magnetostrictive actuators provide exceptional frequency responses but suffer from limited stroke capacity and extreme sensitivity to external damping, leading to the dynamic quenching of oscillations when machining difficult-to-cut alloys such as titanium.
- Mechanical (eccentric) and electromagnetic systems offer high driving forces but are strictly bounded by heavy mechanical inertia, restricting their stable operating range to low frequencies and introducing severe harmonic distortions.

In this context, the scientific investigation of pneumo-hydraulic impulse devices (PHIDs) marks a paradigm shift in fluid power dynamics and vibration mechanics. The justification for conducting deep scientific research in this domain is grounded in three pillars of global technological relevance [9–11]:

- (1) Resolution of the fluid-structure interaction. From a theoretical standpoint, the core bottleneck of a PHID lies in its highly non-linear system architecture, where compressible gas dynamics within the pneumatic control stage interact with a quasi-incompressible, highly viscous fluid medium in the hydraulic multiplier stage under cyclic, high-frequency loading. Investigating this system advances the global understanding of transient fluid power phenomena, specifically regarding:
 - The mitigation and control of cavitation and rarefaction zones in high-viscosity mineral oils subjected to rapid, transient pressure drops.
 - The precise synchronization of transient thermodynamic processes inside the pneumatic pressure chamber with the elastic-inertial response of the tool holder assembly.
- (2) Minimization of hydraulic compliance and inertial drift.
- (3) Contribution to green manufacturing and process scalability. Modern machining science prioritizes resource efficiency and tool life extension when processing difficult-to-cut aerospace materials. The pneumo-hydraulic drive architecture allows for the direct utilization of standard shop-floor compressed air networks (0.4–0.6 MPa) to generate massive localized hydraulic cutting forces (up to 15 MPa) directly within the lathe's tool holder. This eliminates the need for bulky, energy-intensive external hydraulic power units, offering a highly scalable, compact, and eco-efficient solution for vibration-assisted turning globally.

Scientific contributions of this work. The developed mathematical model transcends classical simplified linear single-degree-of-freedom approximations by introducing a tightly coupled, non-linear multi-physics framework. A key innovation in this modelling strategy is the simultaneous, closed-loop integration of high-frequency compressible gas dynamics—strictly governed by the subcritical and supercritical Saint-Venant–Wantzel formulations—with the isothermal compressibility and pressure-dependent behavior of the viscous fluid medium in the multiplier chamber (Hooke's law for fluids) and the dynamic D'Alembert expressions of the elastic-inertial response of the mechanical structure. Furthermore, the simulation explicitly captures transient structural compliance, localized fluid friction regimes, and real-time volumetric energy losses via concentric slit seal leakages. This multi-physics interactions are mathematically resolved through step-by-step numerical integration via Euler's method at an ultra-fine time step of $\Delta t = 2 \cdot 10^{-7}$ s, incorporating an adaptive logical

switching framework with conditional boundary constraints to accurately replicate physical stroke saturation limits ($y \geq 0$) and structural mechanical contact without numerical drift.

2. Methodology

2.1. Device Architecture and Operational Principles

A schematic diagram of the pneumatic-hydraulic pulse multiplier device for radial vibratory turning is shown in Figure 1 [8]. Inside housing 1, in threaded sleeves 16 and 17, which are screwed into housing 1, are located the multiplier plungers 18 and 19, which mate with the guide surfaces of the central bores in sleeves 16 and 17 via running fits $\phi d_2 \frac{H5}{g5}$ (or, as a last resort, $\phi d_2 \frac{H6}{g6}$, where d_2 is the diameter of the plungers, which, for reliable sealing, must have a contact surface length with the surfaces of the central bores in bushings 16 and 17 of not less than $l = 2.5\text{--}3 \times d_2$). Bushings 16 and 17 are sealed by rubber O-rings (not shown in Figure 1) positioned within closed annular grooves. A return spring 26, characterized by a stiffness coefficient k_3 , is positioned between plungers 18 and 19 and secured onto their structural projections. A manufacturing pre-compression (initial displacement) of x_2 is applied to spring 26 during the final assembly of the device.

The diaphragm-type pneumatic actuator, which drives plunger 18, consists of a housing 2 attached to the flange of the multiplier housing 1 by means of studs (not shown in Figure 1), and a diaphragm 11. The diaphragm features a rigid center executed as a sleeve 12, which is fixed to the diaphragm 11 by means of a washer 13 and a ring nut 14, secured by a multi-lobed lock washer 25. The diaphragm assembly of the pneumatic actuator is loaded by a coil spring 15, one end of which rests against the threaded bushing 16, while the opposite end bears against sleeve 12. The spring constant of spring 15 is designated as k_1 , and its manufacturing preload is defined as x_{ovp} .

A parametric single-stage compressed air pressure pulse generator (PPG) is housed within the body 2 of the diaphragm-type pneumatic actuator. The PPG comprises a poppet-spool shut-off element 8, loaded by a spring 10 characterized by a stiffness coefficient k_2 . This spring belongs to the “opening” pressure regulator of pressure p_1 , and its initial preload x_{o1} is calibrated via a plunger 9 and an adjustment screw 5. This screw is positioned within a stepped end-cap 4, which is threaded onto a cylindrical projection of the actuator body 2. The adjustment screw 5 of the pressure regulator and the stepped end-cap 4 are rigidly locked in place by jam nuts 6 and 7, respectively. The shut-off element 8 of the PPG features combined sealing stages. The first stage is formed by a conical seat chamfer with diameter d_{1h} located at the edge of orifice a_1 (see Figure 1). This configuration establishes an additional structural overlap h_{ph} between the sealing edges of the shut-off element 8, specifically between its spool section of diameter d_{2h} and the drain cavity C_1 PPG. The drain cavity C_1 PPG is vented directly to the atmosphere through the exhaust port C_1 and the corresponding nipple 24.

The guide for the shut-off element 8 is provided by a cylindrical shank that mates with the internal surface of the axial bore in the cylindrical projection of the actuator body 2 (see Figure 1), engineered with an accuracy of at least ISO IT6 tolerance grade. To ensure an unobstructed connection with cavity C_2 , a specific groove is machined onto the shank of the shut-off element 8 to accommodate the spring 10 of the “opening” pressure regulator p_1 (see section A-A in Figure 1).

Compressed air—hereinafter referred to as the pneumatic energy carrier—is supplied through the throttle valve 27, the nipple fitting 23, and orifice “a” directly into the pressure chamber B of the diaphragm-type pneumatic actuator. Chamber C located behind the diaphragm 11 (the chamber housing the return coil spring 15, see Figure 1) is freely vented to the atmosphere via the exhaust openings “ C_2 ” executed in housing 1.

The cavity A of the device’s hydraulic multiplier is completely filled with a mineral oil—specifically, grade ITP-300 (TU 38.101292-79)—which possesses a kinematic viscosity at 40 °C of $\nu = (612\text{--}748) \times 10^{-6} \text{ m}^2/\text{s}$, a nominal density of 955 kg/m^3 at $t = 40 \text{ °C}$, and a maximum bulk modulus of elasticity $\chi_{max} = 17.5 \times 10^8 \text{ Pa} = 1750 \text{ MPa}$. It is highly advisable to deploy this specific lubricant grade—typically engineered for mechanisms operating under severe technological loads and intense thermal regimes—within the developed multiplier layout. This selection effectively mitigates the immediate requirement for an auxiliary external reservoir to continuously compensate for volumetric fluid losses induced by potential dynamic clearance leakages.

To evacuate trapped air from cavity A of the hydraulic multiplier, a conical bleed valve 22 is provided, consisting of the valve body itself (not indexed in the schematic) and a hollow adjustment screw (also omitted from the diagram for simplicity). Cavity A is charged with the hydraulic energy carrier via an inlet fitting 31, which incorporates a spring-loaded ball check valve comprising a blocking ball 32 loaded by a return spring 33. Alternatively, the check valve assembly may feature a sealing element executed as a ground conical poppet oriented within the internal passage of fitting 31. The initial filling of cavity A with the hydraulic energy carrier

can be executed utilizing, for example, a portable single-plunger oil priming pump [12,13]. During this charging sequence, the conical bleed valve 22 remains open until a continuous, steady flow of fluid completely free of atmospheric air bubbles emerges from its discharge orifice. Subsequently, the bleed valve 22 is tightly closed, and the external oil charging unit is disconnected from the device. If necessary, an auxiliary external reservoir can be coupled to fitting 31 to continuously replenish the hydraulic energy carrier volume subject to minor volumetric losses leaking through the micro-clearances between plungers 18 and 19. The structural clamping plates 34 are engineered to rigidly secure the complete radial vibrational turning device within the standard tool holder of the industrial lathe.

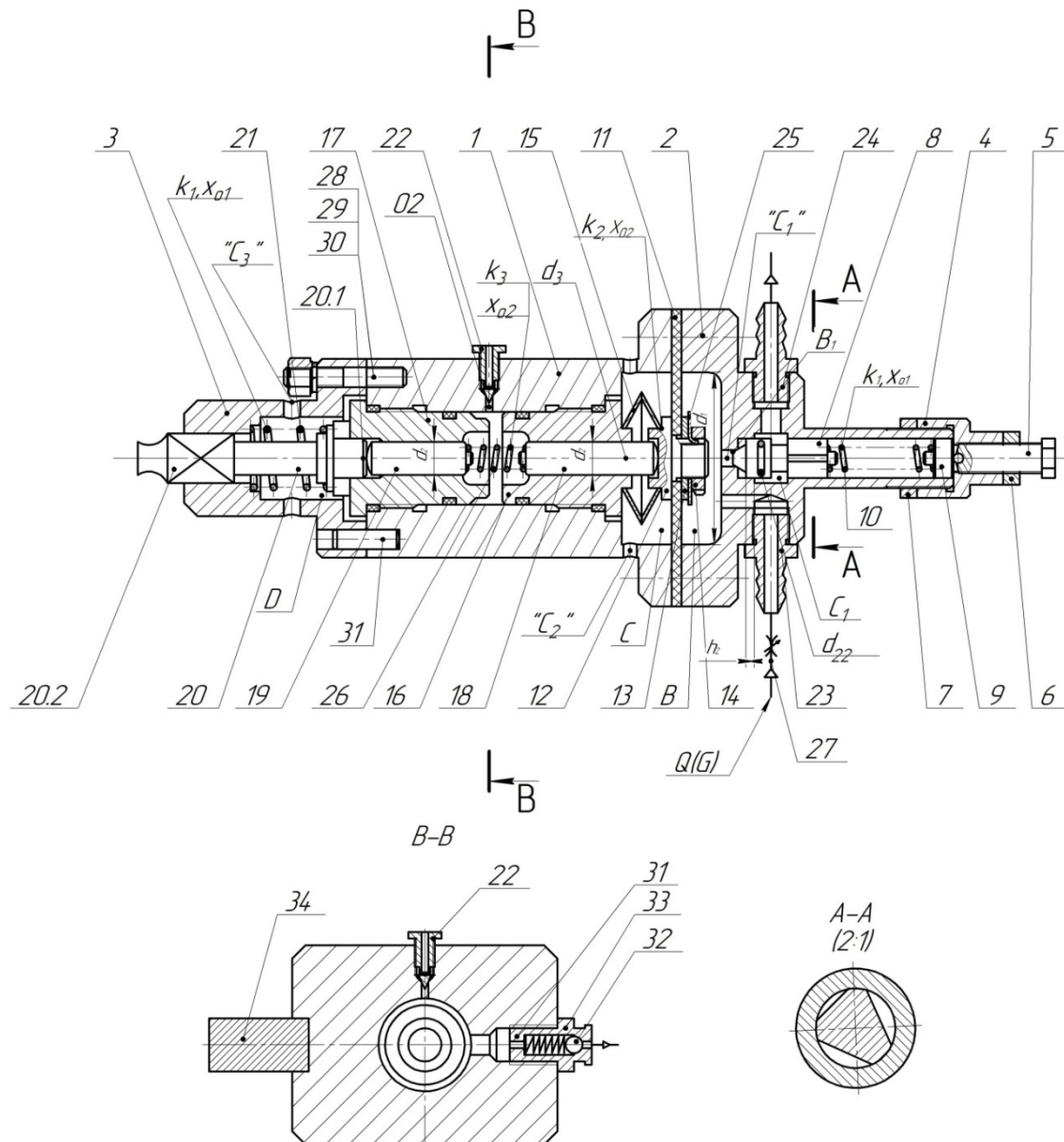


Figure 1. Block diagram of a pneumatic-hydraulic pulse multiplier device for radial vibratory turning.

Plunger 19 comes into direct contact with the cylindrical guiding surface 20.1 of the cutter holder 20. This cylindrical surface 20.1 of the holder mates via a precision sliding fit with the corresponding internal bore of the threaded bushing 17 (see Figure 1) and must be strictly coaxial with the mounting bore of plunger 19. The square-profiled driven end 20.2 of the cutter holder 20 is guided within a matching rectangular slot executed in the cylindrical projection of the actuator housing 3. The actuator housing 3 is rigidly secured to the main device housing 1 via its mounting flange by means of threaded studs 28, equipped with hex nuts 29 and spring lock washers 30. A pair of dowel pins 31 is deployed to positively prevent the actuator housing 3 from rotating under the action of torque transmitted from the cutter holder 20. The cutter holder 20 is continuously pre-loaded against the face of bushing 17 by a return spring 21. The spherical contact tips of plungers 18 and 19 serve to completely relieve the plunger bodies from any parasitic lateral (transverse) cutting force components.

In order for the cutter 20 to maintain the radial turning process, the required component of the cutting force $F_{Xmax} = p_{Amax} \cdot A_{19}$ must be generated by means of the plunger 19 (where p_{Amax} is the maximum pressure of the hydraulic energy carrier in cavity A of the multiplier; $A_{19} = \frac{\pi d_2^2}{4} = 0.785 \cdot d_2^2$; d_2 is the diameter of the plunger 19). The force F_{Xmax} may be specified in the technical specifications for the design of the device when it is used for vibrational turning of parts with maximum feed rate and depth of cut, which determine the magnitude of the force F_{Xmax} .

The force F_{Xmax} is generated by the high pressure of the hydraulic energy carrier within cavity A of the multiplier acting upon the end face of plunger 19 (its cross-sectional area A_{19}), which arises as a result of plunger 18 being driven into this cavity by the action of the diaphragm-type pneumatic actuator. The diaphragm 11 of this actuator moves to the left (as shown in the drawing) under the action of the pneumatic energy carrier supplied to cavity B of the pneumatic actuator via the nipple fitting 23 from a centralized pneumatic network (factory, individual compressor, etc.). The flow rate of the pneumatic energy carrier and the frequency of its pressure pulses—which is equal to the oscillation frequency of the cutter—are regulated by throttle 27.

The highlighted kinematic synchronization and operational sequence of the pneumo-hydraulic impulse mechanisms establish a foundation for structural parameterization. To transform these physical interactions into a quantifiable multi-physics environment, the structural design boundaries must be rigorously mapped using foundational hydrostatic and fluid compressibility formulations. Consequently, the core governing analytical expressions establishing the initial pressure bounds and volumetric energy balances are defined below.

2.2. Governing Analytical Equations

The pressure p_{max} in chamber A of the multiplier can be calculated using the relationship for fluid compressibility known from the course on hydraulics and hydraulic drives:

$$p_{Amax} = x \cdot \Delta W_{max} / W_0 \quad (1)$$

where x is the (isometric) modulus of elasticity of energy carrier 2;

$$\Delta W_{max} = A_{18} \cdot h_{18} = (\pi d_2^2 / 4) \cdot h_{18} = 0.785 d_2^2 \cdot h_{18}, \quad (2)$$

where $A_{18} = \pi d_2^2 / 4 = 0.785 d_2^2$ —the cross-sectional area of the plunger 18, d_2 —plunger diameter 18, h_{18} —plunger stroke 18.

To obtain the maximum change in volume ΔW_{max} , the peak displacement value of plunger 18 (h_{18}) is utilized, where W_0 represents the initial volume of the hydraulic energy carrier within cavity A at $h_{18} = 0$ and minimum pressure $p_{a min}$. The magnitude of $p_{a min}$ is inherently small and is positively maintained by the minor structural pre-compressions (preloads) of return spring 26 (x_{o2}) and coil spring 15 (x_{ovp}), satisfying the condition $x_{ovp} > x_{o2}$, as well as by the intrinsic elasticity of the pneumatic actuator's diaphragm 11.

Such structural preloads within the described components of the device are functionally mandatory to eliminate, or at least minimize, the parasitic effect of fluid separation (rarefaction) of the hydraulic energy carrier within cavity A during the temporary interruption of the vibrational cutting cycle and the subsequent return stroke of the cutter holder 20. This phenomenon typically occurs as a result of the PPG opening, which triggers an excessive drop in the pressure of the pneumatic energy carrier within chamber B below the PPG closing pressure threshold p_2 . The severe rarefaction of the hydraulic energy carrier pressure at the end of the return stroke of cutter 20 can lead to atmospheric air being sucked into the system through the micro-scale radial clearances between the cylindrical surfaces of plungers 18 and 19 and their matching guide bores in bushings 16 and 17. This critical vacuum-induced suction transforms the hydraulic energy carrier into a highly compressible, aerated two-phase medium. In this degraded state, the bulk modulus of elasticity drops significantly below that of pure mineral oil, ultimately causing a total loss of the device's operational stability and functional capability.

In order to completely eliminate the potential rarefaction effect described above, high precision is required in the mating of plungers 18 and 19 with their guide surfaces in the internal bores of bushings 16 and 17 (adhering to at least ISO IT5–IT6 tolerance grades). Furthermore, the use of a high-viscosity hydraulic energy carrier—specifically of the ITP-300 (TU 38.101292-79) type—is mandatory to strictly minimize the volumetric losses leaking through the precision $\phi d_2 H5/g5$ clearance gaps.

The Q_{los} leakage losses can be mathematically estimated for a concentric annular slit with mating dimensions of $\phi d_2 H5/g5$, assuming a steady laminar flow regime of the hydraulic energy carrier through this slit geometry. Given the substantial engagement length ($l \approx 4d_2$), these calculations are strictly governed by the foundational fluid mechanics frameworks detailed in [14,15].

$$Q_{los} = 3.27 \cdot 10^{-8} d_m \Delta p_a \frac{8^3 \cdot l^{-1}}{\mu} = 3.27 \cdot 10^{-8} d_m p_{a \max} \frac{8^3 \cdot l^{-1}}{\mu}, \text{ m}^3/\text{s} \quad (3)$$

where $d_m = 0.5(d_2^{ov} - d_2^{pl})$ —average coupling diameter; d_2^{ov} —hole diameter $d_2^{ov} = \phi d_2 H5$ in the sleeves 16 and 17; $d_2^{pl} = \phi d_2 g5$ —plunger diameter 18 and 19; $\Delta p_a = p_{A \max} - p_{pov} \approx p_{A \max}$ ($p_{A \max}$ —air pressure in chambers C and D, see Figure 1) $\delta = d_2^{ov} - d_2^{pl}$ —diameter clearance in couplings $d_2 H5/g5$ and $l = 3d_2$ conjugation length $d_2 H5/g5$, μ —the viscosity of the energy carrier 2.

The effective membrane A_{mef} area required for a diaphragm pneumatic motor can be calculated using the following equation:

$$p_B \cdot A_{mef} \geq p_{\max} \cdot A_{18} - k_2 \cdot x_{ovp} - k_3 \cdot x_{o2} = \bar{k}_{31} \cdot p_{\max} \cdot A_{18}, \quad (4)$$

where p_B is the pressure of the pneumatic energy carrier within chamber B of the pneumatic actuator, which is calibrated by the initial pre-compression of spring 10 belonging to the “opening” pressure regulator p_1 of the PPG unit. Typically, the nominal operating pressure of the pneumatic energy carrier in standard factory networks does not exceed (0.4–0.6) MPa. This structural constraint imposes a strict upper limit on the maximum pressure of the energy carrier during the actuation (“opening”) of the PPG, yielding $p_{\max} = (0.4 \dots 0.6)$ MPa. For the subsequent analytical calculations, the peak value is assumed to be $p_{1 \max} = 0.6$ MPa. Given the relatively small magnitude of the structural forces $k_3 \cdot x_{o2}$ and $k_2 \cdot x_{ovp}$, and due to the negligible baseline values of the initial spring deformations x_{o2} and x_{ovp} , a dedicated safety factor is introduced into the model: $k_{31} = 1.1 \dots 1.2$ (where $\bar{k}_{31} = 1.15$ represents the chosen mean design value) [16,17].

From Equation (4) we obtain:

$$A_{mef} \geq p_{A \max} \cdot \bar{k}_{31} \cdot A_{18} / p_{1 \max} = 1.15 p_{A \max} \cdot 0.785 d_2^2 / p_{1 \max} = 0.903 \cdot p_{A \max} d_2^2 / p_{1 \max}, \quad (5)$$

According to the relationship given in [18], the effective area of membrane 11 is determined (see Figure 1)

$$A_{mef} = (\pi/12)(d_1^2 + d_1 d_3 + d_3^2), \quad (6)$$

where d_1 —membrane pinch diameter 11, d_3 —diameter of the bushing’s bearing surface 12 (rigid centre, see Figure 1).

The required maximum pressure $p_{A \max}$ of the working fluid in the multiplier’s chamber A can be calculated (as an approximate value) using the following equilibrium equation:

$$F_x + k_4 \cdot x_{o4} + k_3 \cdot x_{o2} \leq p_{A \max} \cdot A_{19}, \quad (7)$$

where F_x —cutting force; k_4 —spring stiffness 21, which presses cutter 20 against the end face of sleeve 17 from chamber D, which is freely connected to the atmosphere via openings “C₃” in the cylindrical projection of housing 3; x_{o4} —the pre-compression of spring 21, which is created when housing 3 is attached to 1 using a pin connection 28.

The purpose of spring 21 is to eliminate the main play (movement) of cutter 20 in its fixed initial position; therefore, the force $k_4 \cdot x_{o4}$ is relatively small compared to the force F_x , which is determined by the pre-deformation x_{o4} , which is also small. Since the stiffness k_3 of spring 26 and its pre-deformation x_{o2} are also small, this allows the action of the forces $k_4 \cdot x_{o4} + k_3 \cdot x_{o2}$ to be adjusted, by analogy with (4), by a safety factor $k_{zap2} = 1.2 \dots 1.25$ ($\bar{k}_{zap2} = 1.225$ —the average value k_{zap2}), by multiplying it by F_x , then (7) can be expressed as:

$$\bar{k}_{zap2} \cdot F_x \leq p_{A \max} \cdot A_{19}, \quad (8)$$

where

$$p_{A \max} \geq \bar{k}_{zap2} F_x / A_{19} = 1.225 \cdot F_x / (0.785 d_2^2) = 1.561 F_x / d_2^2, \quad (9)$$

With a view to minimizing the device’s overall dimensions and ensuring its seamless integration into the standard tool holder of a lathe without requiring any structural modifications to the machine tool components (such as the tool slide), it is advisable to optimize specific geometric parameters that do not constrain the device’s functional operation. These include the diameters d_2 of plungers 18 and 19, as well as the dimensions of the mating cylindrical and threaded sections of bushings 16 and 17. These key nominal dimensions can be systematically refined based on the comprehensive simulation results of the device’s mathematical model, ultimately serving as the foundation for developing a scientifically substantiated design calculation methodology.

The nominal baseline dimensions are established as $d_2 = 8 \text{ mm}$ and $l = 3 d_2$ (see Equation (3)). Plungers 18 and 19 should be manufactured from high-grade wear-resistant bearing steel—specifically, 100Cr6 Steel—hardened to a surface hardness of 62–64 HRCe, and precisely lapped into the internal guide bores of bushings 16 and 17 with a strict tolerance zone of $\text{Ø}8g5$ (or, as a permissible limit, $\text{Ø}8g6$). The diameter d_4 of the cylindrical part of bushings 16 and 17, in order to ensure sufficient rigidity during the manufacture and operation of the device, should be set to no less than $d_4 = \text{Ø}20f7$ (or $\text{Ø}20h7$), and the threaded part should be made to grade $M24 \times 1.0^{-6} \text{ g}$. It is also advisable to manufacture bushings 16 and 17 from wear-resistant steel, for example 100Cr6 Steel, and to harden them to a hardness of 54–60 HRCe.

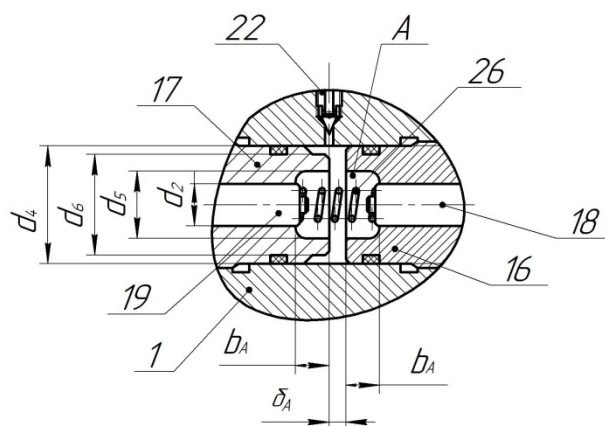
Based on these specified nominal diameters of bushings 16 and 17 alongside plungers 18 and 19, the comprehensive structural configuration of the multiplier's hydraulic cavity A (see Figure 1) can be definitively constructed as illustrated in Figure 2.

Based on the dimensions and design of cavity A of the multiplier, its initial volume W_o (see Figure 1) can be calculated using the formula:

$$\begin{aligned} W_o &= \frac{\pi}{4} \left[d_5^2 (2b_A + \delta_A) + (d_4^2 - d_5^2) (2b_A + \delta_A) + (d_6^2 - d_5^2) \delta_A \right] \\ &= \frac{3.14}{4} \left[12^2 - 15 + (20^2 - 18^2) \cdot 9 + (18^2 - 12^2) \cdot 3 \right] = 2656.44 \text{ mm}^3 \approx 2.66 \cdot 10^{-6} \text{ m}^3. \end{aligned} \quad (10)$$

Using Equation (8), we determine the component of the cutting force F_x required to overcome the resistance, including the friction forces of the moving parts of the plunger 19 and the holder 20.1:

$$p_{A\max} \geq 1.561 \cdot F_x / d_2^2 = 1.561 \cdot 581.55 / (8 \cdot 10^{-3})^2 = 1.42 \cdot 10^7 \text{ Pa} = 14.2 \text{ MPa}, \quad (11)$$



Dimensions of cavity A and bushing diameters 16 and 17:

$$\begin{aligned} d_1 &= 20 f \cdot T; \\ d_5 &= 12 \text{ mm}; \\ d_6 &= 18 \text{ mm}; \\ b_A &= 6 \text{ mm}; \\ \delta_A &= 3 \text{ mm} \end{aligned}$$

Figure 2. The design of the multiplier's cavity A .

Taking into account the friction forces noted, we set $p_{A\max} = 15 \text{ MPa}$. From Equations (1) and (2), we find, respectively:

$$\Delta W_{\max} = p_{A\max} \cdot W_o / \bar{\kappa} = 15 \cdot 10^6 \cdot 2.66 \cdot 10^{-6} / 1.66 \cdot 10^9 = 2.4 \cdot 10^{-8} \text{ m}^3; \quad (12)$$

$$h_{18} = \Delta W_{\max} / (0.785 d_2^2) = 2.4 \cdot 10^{-8} / (8 \cdot 10^{-3})^2 = 4.8 \cdot 10^{-4} \text{ m} \approx 0.5 \text{ mm}, \quad (13)$$

where $\bar{\kappa} = 1660 \text{ MPa}$ —the average value of the modulus of elasticity of energy carrier 2 (ITP-300 lubricant) is recommended [19] for use with high-viscosity heavy lubricants, as the operating temperature rises during operation and may reach: 60°C .

Using Equation (5), we determine the effective area of the membrane 11 of the pneumatic cylinder (see Figure 1):

$$A_{\text{nef}} \geq 0.903 p_{A\max} \cdot d_2^2 / p_{1\max} = 0.903 \cdot 15 (8 \cdot 10^{-3})^2 / (0.4 \dots 0.6) = (21.672 \dots 14.448) \cdot 10^{-4} \text{ m}^2, \quad (14)$$

To assess the need to connect via fitting 31 (see Figure 1, section B-B) of additional capacity with ITP-300 lubricant to cavity A , it is necessary to determine, using Formula (3), the energy loss 2 through the concentric

clearances of the couplings between plungers 18 and 19 and the holes in bushings 16 and 17, based on the fits $\varnothing 8 \frac{H5}{g6}$

The maximum possible clearance $\delta = 8.006 - 7.986 = 0.02 \text{ mm}$. The average mating diameter $\varnothing 8 H5 / g6$ $d_m = 7.996 \text{ mm}$. For an approximate calculation Q_{los} , we take: $v_{\min} = 612 \cdot 10^{-6} \text{ m}^2/\text{sec}$ ($t = 40^\circ \text{C}$); $\rho_{\min} = 950 \text{ kg/m}^3$, where $v_{\min}$ and $\rho_{\min}$ are the parameters of ITP-300 lubricant, then $\mu_{\min} = v_{\min} \cdot \rho_{\min} = 612 \cdot 10^{-6} \cdot 950 \approx 0.58 \text{ Pa}$. We substitute the data into Formula (3) and determine

$$Q_{los} = 3.27 \cdot 10^{-8} \cdot 7.996 \cdot 10^{-3} \cdot 15 \cdot 10^6 \cdot (2 \cdot 10^{-5}) \cdot (24 \cdot 10^{-3})^{-1} \cdot (0.58)^{-1} \approx 2.25 \cdot 10^{-15} \text{ m}^3/\text{s},$$

and taking into account the losses due to the coupling of both plungers 18 and 19 $Q_{los \Sigma} = 2Q_{los} = 4.5 \cdot 10^{-6} \text{ mm}^3/\text{c} = 4.5 \cdot 10^3 \text{ MKM}^3/\text{sec}$.

The calculated losses Q_{los} of energy carrier 2 through the annular (concentric) clearance of the couplings $\varnothing 8 \frac{H5}{g6}$ are valid under continuous pressure $p_{A \max} = 15 \text{ MPa}$ and amount to, relative to the initial volume of cavity A of the device's multiplier (see Figure 1) $W_o = 2.66 \cdot 10^{-6} \text{ m}^3$, $1.69 \cdot 10^{-7} \%$ s. However, the pressure $p_{A \max}$ acts intermittently and cyclically during the forward stroke of the device's PPG shut-off element 8, whilst during the reverse stroke of the shut-off element 8 (and the cutter 20, see Figure 1), the pressure p_A of energy carrier 2 in cavity A decreases to $p_{A \min} \approx p_{pov}$ (see Equation (3)). The duration of the forward and reverse strokes of the PPG's shut-off element 8 depends on the frequency v_{in} of the pressure pulses of energy carrier 1 generated by the PPG. The frequency v_{in} is determined by the 'opening' pressure p_1 of the PPG and the supply $Q(G)$ of energy carrier 1.

While the baseline hydrostatic constraints provide the continuous limits of force density, they do not account for the transient inertial shifts or dynamic phase lags occurring during high-frequency cycles. To mathematically capture the non-linear fluid-structure interaction under load, the distributed mechanical domains are translated into a concentrated lumped-mass rigid body matrix. This idealized structural abstraction serves as the baseline for deriving the dynamic differential equations of motion.

To maintain textual fluidness while ensuring mathematical traceability across the subsequent dynamic formulations, all fixed structural constants, initial volumes, and thermo-physical properties of the deployed pneumatic and hydraulic energy carriers are systematically consolidated. These deterministic baseline parameters and verification limits are comprehensively indexed and defined in Appendix A (Table A1).

2.3. Multi-Physics Dynamic Model Framework

A detailed analysis of the interaction between the device's components during their forward (index 'f') and reverse (index 'r') operation can be carried out using a dynamic model of the device. The dynamic model of the device (Figure 3) is constructed from the concentrated mass elements of the main moving components (parts) of the device, assuming that these elements are perfectly rigid bodies interacting with housings 1, 2 and 3 (see Figure 1) via elastic and dissipative couplings, and some of these masses are mechanically connected (in contact) with one another.

The dynamic model consists of: m_{efm} the effective mass of diaphragm 11 (the deformable part of diaphragm 11), which includes the mass of the rigid centre (bushing 12, washers 13 and 25, and ring nut 14) and part of the mass of spring 15; m_8 —the combined mass of the PPG shut-off element 8, to which is added: 1/3 of the mass of the PPG 'opening' pressure p_A regulator spring 10; m_{18} —the mass of the plunger 18; m_{19} —the mass of the plunger 19; m_{20} —the mass of the cutter 20, which includes part (1/3) of the mass of the spring 21 with stiffness k_4 , the pre-deformation of which is formed during the attachment (securing) of housing 3 to housing 1.

Since the dynamic viscosity of energy carrier 1 (compressed air) is significantly lower than that of, for example, lubricating oil, energy losses due to its dissipation in pneumatic flows can be disregarded. Under this assumption, the movement of the mass m_{efm} is opposed only by the spring resistance in the form of spring 15 with stiffness k_2 and the spring resistance of the diaphragm 11 itself, which can be characterised by stiffness k_m . Since the elastic elements (springs, etc.) have moving masses significantly smaller than the described masses of the device's moving plates, we consider them to be inertial-free.

The moving masses m_8 are counteracted by a spring with a stiffness k_1 of 10 and viscous friction characterised by a coefficient of C_8 , since the PPG must be periodically lubricated (internal cavities B_1 , B_2 , C_1 and C_2) by incorporating special lubrication devices [18] into the pneumatic supply system of the diaphragm pneumatic motor on the PPG, which periodically dispense atomised lubricants into the designated cavities. This suggests that a fluid friction regime is realised in the clearances of the guide surfaces of the shut-off element 8.

In Figure 3, in addition to the quantities described in the text, the following are shown: $\longleftrightarrow$

—mechanical connection (contact) between masses m_{efm} and m_{18} and masses m_{19} and m_{20} ; $\boxed{A}$ —multiplier cavity, $\boxed{}$ —dynamic model of the PPG; P_2 —symbol for the driving forces of energy carrier 1, which sets the mass in motion m_{efm} and m_8 during their live broadcast (x_{am} ; x_{hp}) the reverse (x_{mr} ; x_{hr}) moves.

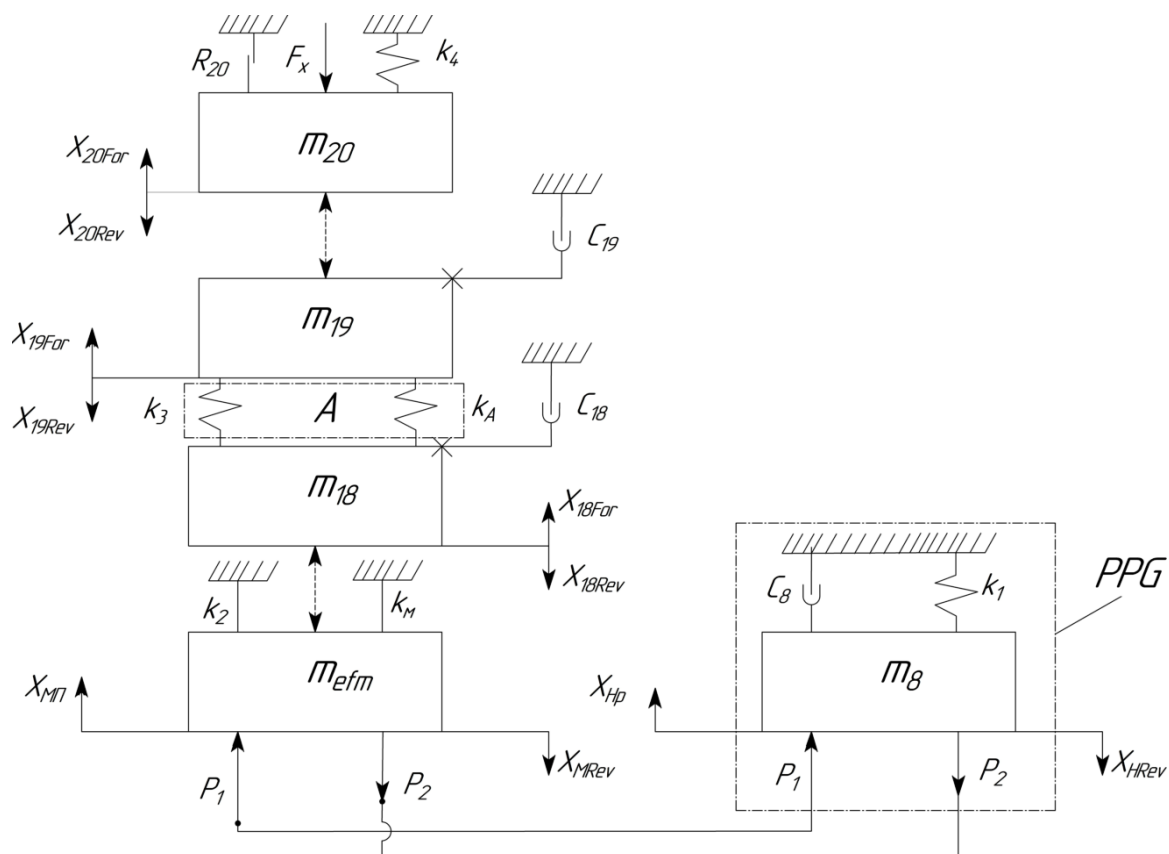


Figure 3. Dynamic multiplier model.

The masses m_{efm} and m_{18} are mechanically coupled (see Figures 1 and 3). The masses m_{18} of the pneumatic-hydraulic impulse device for radial vibratory turning during the forward (index “for”) and reverse (index “rev”) strokes of the device’s moving parts and m_{19} (plungers 18 and 19) interact via spring 26 with stiffness k_3 and energy carrier 2 within the closed volume W_o of the multiplier. The stiffness k_A of this volume relative to the cross-sectional area $A_{18} \approx 0.785d_2^2$ (or $A_{19} = A_{18}$) can be calculated using the known relationship [20,21]

$$k_A = A_{18}^2 \cdot \chi / W_o = [0.785 \cdot (8 \cdot 10^{-3})^2]^2 \cdot 1.66 \cdot 10^9 / 2.66 \cdot 10^{-6} = 1.57 \cdot 10^6 \text{ N/m}. \quad (15)$$

The stiffness $k_3 \ll k_A$ allows us, for example, to neglect its effect in a mathematical model of the device and assume that the interaction between plungers 18 and 19 occurs solely through the stiffness k_A of the volume W_o .

It is highly probable that in the clearance gaps $\varnothing d_{18}H5/g6$ and $\varnothing d_{19}H5/g6$, due to the relatively high pressure $p_{Amax} = 15 \text{ MPa}$, only the fluid friction regime occurs, characterized by the coefficients C_{18} and C_{19} .

The masses m_{19} are m_{20} mechanically connected (see Figures 1 and 3). The moving masses m_{20} are counteracted by spring 21 with stiffness k_4 , pushing against the component of the cutting force F_x and the maximum total friction force R_{20} , provided that the friction on the guide surfaces 20.1 and 20.2 is periodically reduced, for example, using high-quality plastic grease surfaces 20.1 and 20.2 of the cutter holder 20.

The driving forces acting on diaphragm 11 and the PPG shut-off element 8 are determined by the pressure of the working fluid in chamber B, port “ a_1 ”, and chamber B_1 : p_1 —during the forward strokes X_{MFor} of diaphragm 11 and the PPG shut-off element 8 X_{MFor} and p_2 during their reverse strokes, respectively, x_{MRev} and x_{gRev} . Similarly, the forward and reverse strokes of the masses m_{18} (x_{18For} and x_{18Rev}), m_{19} (x_{19For} and x_{19Rev}) and m_{20} (x_{20For} and x_{20Rev}) are denoted. Given the small stroke of the plungers 18 and 19 ($h_{18} = h_{19} \approx 0.5\text{mm}$, (see 13 in Figure 1), the internal fluid friction within the cavity A of the device’s multiplier can be neglected, and only the elastic properties of this volume need to be considered as the pressure of energy carrier 2 within it rises to the level (see Equation (11)) due to the compressibility of the energy carrier (stiffness k_A). It is evident that energy carrier 2 in cavity A obeys Hooke’s law for fluids:

$$p_{Amax} = (\Delta W_{max} / W_o) \cdot \kappa = \varepsilon_w \cdot \kappa = (2.4 \cdot 10^{-6} / 2.66 \cdot 10^{-6}) \cdot 1.66 \cdot 10^9 \approx 15\text{MPa} \quad (16)$$

The device’s operating cycle proceeds as follows: when energy carrier 1 is supplied to chamber B (see Figure 1), membrane 11 elastically deforms under pressure p_B and, via a rigid center (bushing 12), moves plunger 18 into the volume W_o of chamber A of the device’s multiplier. Due to the compression of energy carrier 2 by an amount ΔW_{max} within the volume of cavity A, the pressure of energy carrier 2 increases to a level $p_{Amax} = 15\text{MPa}$ (see Equation (16)). Under the pressure of energy carrier 2 p_{Amax} acting on the cross-sectional area $A_{19} = A_{18}$ of plunger 19, the latter, via cutter holder 20, feeds this cutter and performs the radial turning operation. At this time, the pressure of energy carrier 1 in cavity 8 rises to the “opening” level PPG

$$p_{1max} \geq k_1 \cdot x_{o1} / A_{1h}, \quad (17)$$

where $A_{1h} = \pi d_{1h}^2 / 4 \approx 0.785 d_{1h}^2$ —the cross-sectional area of the first sealing (chamfered) stage of the shut-off element 8 PPG of the device (see Figure 1); d_{1h} —the diameter (average) of the ground chamfer (see opening “ a_1 ” in Figure 1) of the valve portion of the shut-off element 8 PPG. At a working fluid pressure of 1 in cavity B $p_B \geq p_{1max}$ the shut-off element 8 detaches from the seat and cavity B and B_1 connects via the opening “ a_1 ”.

Due to the small volume of the cavity B_1 , the pressure of working fluid 1 within it instantly becomes equal to p_{1max} and acts on the cross-sectional area $A_{2h} = \pi d_{2h}^2 / 4 \approx 0.785 d_{2h}^2$ of the second sealing stage (spool) of the shut-off element 8 of the PPG.

Since the cross-sectional area $A_{2h} > A_{1h}$ is reduced, the force acting on the shut-off element increases sharply and rapidly moves this element along the path of the positive overlap h_p , opening the shut-off element 8 by the amount of the negative overlap h_{Bh} , usually $h_{Bh} \leq h_{ph}$ [12,13,16,17].

As a result of the direct movement of the shut-off element 8 by a distance $h_H = h_{ph} + h_{Bh}$, the pressure chamber B and the exhaust (drain) chamber C_1 are connected, and the chamber C_1 is freely connected to the atmosphere via the opening “ c_1 ” and the nipple fitting; as a result of this connection, the pressure of the working fluid 1 decreases to the “shut-off” pressure level of the PPG

$$p_{2min} \leq k_1(x_{o1} + h_H) / A_{2h} = p_{1max} \frac{A_{1h}}{A_{2h}} + \frac{k_1 \cdot h_H}{A_{2h}} = p_{1max} \cdot U_{21}^{0.5} + k_1 \cdot h_H / A_{2h}, \quad (18)$$

where $U_{21} = A_{1h}^2 / A_{2h}^2 = d_{1h}^4 / d_{2h}^4$ —internal gear ratio of the PPG.

Under the action of spring force 15 $F_{15} = k_2(x_{ovp} + h_{18})$, the elasticity of diaphragm 1, and springs 26 and 21, the cutter 20 moves backwards (to the right as shown in the drawing; see Figure 1), as a result of which the cutting process is interrupted; in the case of turning viscous alloys (stainless steel, titanium alloys, etc.), this causes the chips to break off.

When the pressure of energy carrier 1 in chamber B $p_B \leq p_2$ reaches a certain level, the valve is pushed into position by the force of spring 10 and locked in place on the seat; as a result, chamber B is isolated from the atmosphere, thereby completing the return stroke of the PPG shut-off element 8 and all the device’s moving parts, and a new cycle begins. In this way, the cutter 20 is set in vibratory motion.

The amplitude of the vibrations of cutter 20 can be varied depending on its feed by adjusting the ‘opening’ pressure $p_{1\max}$ of the PPG, which must not exceed the pressure of the compressed air (energy source 1) in the factory pneumatic network ($p_{pm} = 0.4 \dots 0.6 \text{ MPa}$). The vibration frequency of the cutter 20 can be adjusted by the flow rate of energy source 1 $Q \text{ m}^3/\text{s}$ (or $C_{p1} \text{ kg/s}$ —mass flow rate) into chamber B using the throttle 27.

The slots on the shut-off element 8 (see section A-A, Figure 1) allow the cavity C_2 to communicate freely with the exhaust cavity C_1 of the PPG.

In order to construct, on the basis of a dynamic model (see Figure 3), a mathematical model of a multiplier pneumatic-hydraulic impulse device for radial vibratory turning and to analyse its operating cycle, it is advisable to represent this cycle in the form of a schematic cycle diagram, which reflects the changes over time and space and the interrelationship between the movements of diaphragm 11, the shut-off element 8 of the PPG, the plunger 18, the plunger system 19 and the cutter 20, and the changes in the pressure of energy carrier 1 in cavity B and the pressure of energy carrier 2 in cavity A . The oriented cyclogram of the device’s operating cycle (Figure 4) is constructed on an arbitrary scale of time and linear displacements of the listed components and consists of the following curves: 1—change in pressure p_B of working fluid 1 in chamber B (see Figure 1); 2—displacement of diaphragm 11 during t_{M1} its forward and t_{M3} reverse strokes; 3—displacement of the shut-off element 8 during t_{H1} its forward and t_{H3} reverse strokes; 4—displacement of the multiplier’s plungers 18 during t_{A1} their forward and t_{A3} reverse strokes; 5—change in the pressure of energy carrier 2 in cavity A of the multiplier during the T_{PA} operating cycle of the device; 6—the displacement of the cutter 20 during its forward t_{BP1} and t_{BP2} reverse strokes. The cyclogram is depicted as two pulses, since in reality, during the first start-up of the device, a transient process [20,21], which, once the vibration mode has stabilised, may be absent from the second pulses of the energy carrier 1 pressure change and the pulses of the energy carrier 1 pressure change and the link movement pulses onwards.

In addition to the marked points, the cyclogram also shows: t_{H0} —the time taken for the pressure of energy carrier 1 in cavity B to rise from level $p_B \approx 0$ to level $p_{1\max}$ after the device’s first start-up; t_{PB} —the pressure in cavity B is maintained at level $p_{1\max}$ to level $p_{2\min}$; t_{PH} —the pressure of energy carrier 1 in cavity B is maintained at level $p_{2\min}$; $T_{PBO} = t_{H0} + t_{PB} + t_{PM}$ —the initial period of change in the pressure of energy carrier 1; $T_{PB} = t_M + t_{PB} + t_{PZ} + t_{PM}$ —the stabilised period of change in the pressure of working fluid 1 in cavity B , starting from the second pulse; t_{M2} and t_{MH} —respectively, the dwell time of diaphragm 11 in the left and right (as per the diagram, see Figure 1) positions (t_{M4} which can be taken as the dwell time of diaphragm 11 in the initial position); t_{H2} and t_{H4} —respectively, the dwell time of the shut-off element 8 in the open and closed positions; t_{A2} and t_{A4} —respectively, the dwell time of the plunger 18 in cavity A (see Figure 1), having travelled a distance h_{18} during the forward stroke, and the dwell time of the plunger 18 in the initial position; t_{BP3} —the dwell time of the cutter 20 and plunger 19 system in the position retracted from the workpiece.

When the device is operating in a steady state, the following equality must hold:

$$T_{PB} = T_M = T_H = T_A = T_{PA} = T_{BP} = \nu, \quad (19)$$

where $T_M = t_{M1} = t_{M2} = t_{M3} = t_{M4}$, $T_H = t_{H1} = t_{H2} = t_{H3} = t_{H4}$, $T_A = t_{A1} = t_{A2} = t_{A3} = t_{A4}$, $T_{PA} = t_{P1} = t_{P2} = t_{P3} = t_{P4}$, $T_{BP} = t_{BP1} = t_{BP2} = t_{BP3} = t_{BP4}$ —accordingly, the periods of the displacement pulses of membrane 11, the shut-off element 8, the plunger 18, the pressure changes of working fluid 2 in chamber A , and the displacement of the plunger system 19 and the cutter 20, the linear frequency of the pressure change pulses of working fluids 1 and 2 and the displacements of the device’s moving parts. According to Equation (19), the linear frequency ν of the pressure pulses of working fluids 1 and 2 is also the linear component of the vibrations of the device’s actuating element—the cutter 20 (see Figure 1).

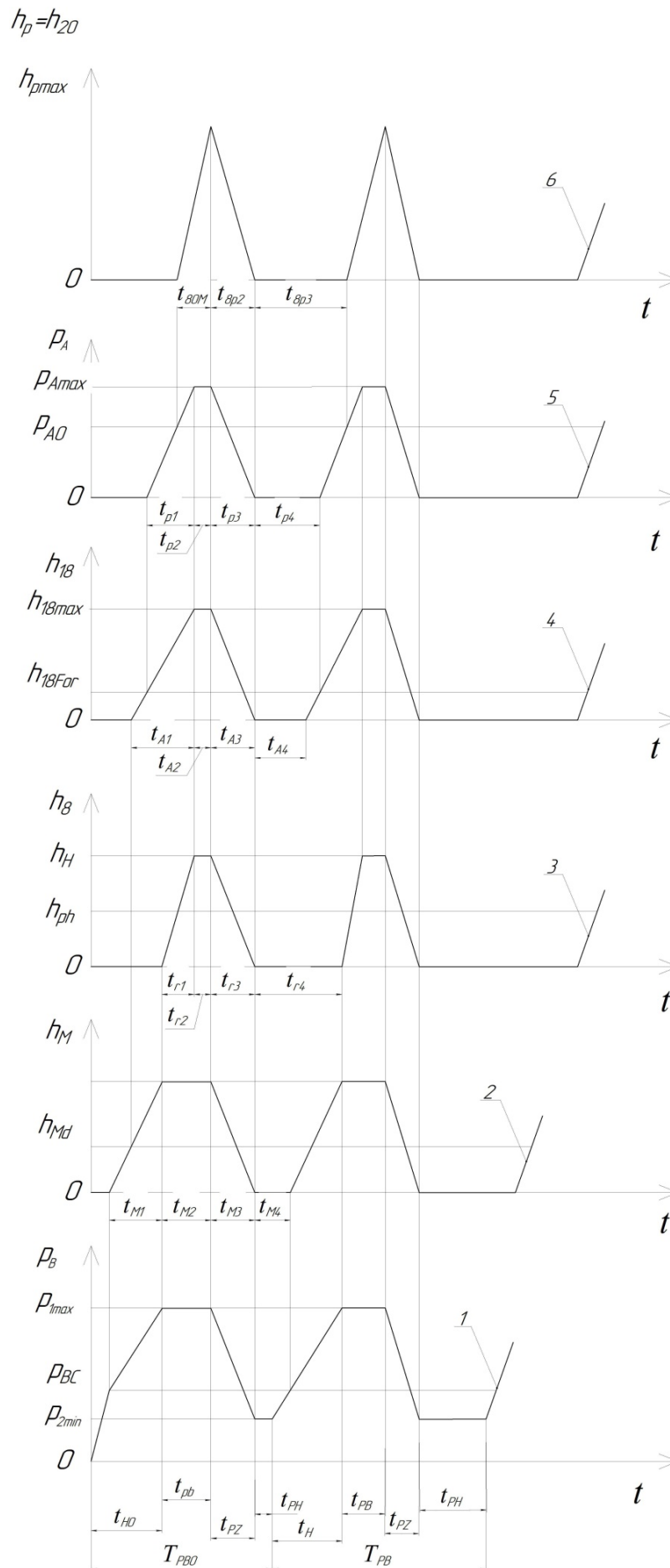


Figure 4. An indicative cycle diagram of the operating cycle of a multiplier device for radial vibratory turning.

A Mathematical Model of a Multiplier Pneumatic-Hydraulic Impulse Device for Radial Vibratory Turning

The actual operating cycle of the device, due to the large number of dynamic parameters (kinematic, thermodynamic, energy, force, etc.), which may also vary randomly, differs significantly from the reference cycle diagram (as proposed) considered by us (see Figure 4). Any attempt to incorporate the greatest possible number of parameters and influences into dynamic and mathematical models will complicate these models to such an extent that it will render their analysis and investigation impossible using currently known methods and techniques. The labour intensity of analysing and investigating the mathematical model of the device can be reduced on the basis of, in addition to the previously accepted assumptions, namely:

- once the transient processes have been completed, the thermodynamic heat transfer process in the device's pneumatic and hydraulic circuits is isothermal;
- the physical properties of energy carrier 2, such as its density $\rho_M = \text{const}$, dynamic viscosity $\mu_M = \text{const}$ (or kinematic viscosity $\nu_M = \mu_M / \rho_M = \text{const}$) and isothermal modulus $\chi = \text{const}$ under thermally stabilised conditions ($T_p = \text{const}$ operating temperature) throughout the operating cycle;
- due to the short length of the pneumatic and hydraulic lines, there are no wave processes in the device;
- the masses m_{efm} , m_8 , m_{18} , m_{19} and m_{20} are concentrated, whilst the masses $m_8 - m_{20}$ —are perfectly rigid bodies;
- the spring masses 10, 15, 20 and 26 are inelastic elements with no inertia;
- the mass of energy carrier 2 is small compared to the masses of the moving elements m_{efm} , m_8 , m_{18} , m_{19} and m_{20} , and its equation of motion is not considered in the equations of dynamics; and only the elastic properties of energy carrier 2 within the volume W_o of cavity A (see Figure 1) are taken into account due to its stiffness k_A (see Figure 2), for example in the form of the equation:

$$k_A \cdot x_{18For} = p_A \cdot A_{18} = p_A \cdot A_{19}; \quad (20)$$

- since the internal friction of energy carrier 2 within the volume W_o of the multiplier is negligible compared to its elastic properties, we may disregard the viscous resistance of its volume;
- since we have assumed a thermodynamic heat transfer process during the steady-state operation of the device's working cycle, the process of spraying energy carrier 1 (compressed air) into cavity B (see Figure 1) can also be considered isothermal; therefore, the change in the density of energy carrier 1 $\rho_{air} = p_B / (RT)$ [22,23] (here, in cavity p_B —the current pressure of energy carrier 1 in cavity B ; $R = 287.14 \text{ J/(kg}\cdot\text{K)}$;
- the gas constant for air; T is the temperature of the element, in K;
- we assume that the mass flow rate $G_{air} (\text{kg}/\text{s})$ of energy carrier 1 required to move the diaphragm 11 and the shut-off element 9 of the device's PPG (see Figure 1) is given by the following Equation [22]:

$$G_{air} = d(W_B \cdot \rho) / dt, \quad (21)$$

where $W_B = A_{efm} \cdot x_{MFor} + A_{1h(2h)} \cdot x_{Hp}$ —the current volume of cavity B during the forward stroke of the diaphragm 11 and the shut-off element 8 of the PPG; $A_{1h(2h)}$ —change in the cross-sectional area of the shut-off element 8: $x_{Hp} = 0$, $A_{1h(2h)} = A_{1h}$; $0 \leq x_{Hp} \leq h_H$, $A_{1h(2h)} = A_{2h}$ (see Equations (17) and (18));

- given that the total $Q_{los\Sigma}$ energy losses 2 through the annular clearances of plungers 18 and 19 ($\varnothing 8H5 / g6$) are very small ($Q_{los\Sigma} = 4.5 \cdot 10^{-15} \text{ m}^3/\text{sec}$, see the approximate calculations above), in the equations for energy carrier consumption, we do not take these losses into account, but assume that $Q_{los\Sigma}$ they are expended on maintaining the fluid friction regime in the clearance gaps between the mating surfaces $\varnothing 8H5 / g6$ of plungers 18 and 19;
- we assume that the energy consumption coefficients 1 $\zeta_H = \text{const}$ through the open gaps in the shut-off element 8 of the PPG device;
- the total force R_{20} (see Figure 3), assuming a periodic reduction in the rectangular cross-section of the shaft 20 and the rectangular opening in the housing 3, may be either the static friction force or the limiting friction force. We assume that $R_{20} = \text{const}$ (see Figure 3);
- we assume that the component of the cutting force remains constant throughout the device's operating cycle;

- since the device's primary operating position is horizontal, we do not take into account the gravitational forces acting on the moving parts of the device in the initial equations of motion for these parts;
- the exhaust chambers C_1 and C_2 of the device (see Figure 1) are freely connected to the atmosphere, which allows only the working pressure p_B to be taken into account in the pressure chambers B and B_1 . energy carrier 1, which enters via throttle 27 and a spindle fitting from the compressor receiver of the pneumatic network (in Figure 1, the compressor receiver is not shown for simplicity), and to neglect atmospheric pressure p_o ;
- for the mass flow rate G_{air} of energy carrier 1, let us adopt the relationship recommended in [23]:

$$G_{air} = \xi_H \cdot A_a \cdot p_{PM} \cdot (R \cdot T_{PM})^{-0.5} \cdot 2 \left[(p_B / p_{PM})^{2k_a^{-1}} - (p_B / p_{PM})^{(k_a+1)k_a^{-1}} \right]^{0.5}, \quad (22)$$

where A_a —the cross-sectional area of opening “a” (or the cross-sectional area of the nipple fitting 23 (see Figure 1); to simplify the design of the device and the engineering calculations, it is advisable to take $A_a = A_{1h} = 0,785d_{1h}^2$ (see Equation (17)); p_{PM} , T_{PM} —pressure and temperature (K) of energy carrier 1 in the pneumatic network (upstream of throttle 27, see Figure 1); k_a —adiabatic index [22].

Using D'Alembert's principle [20,21], based on the dynamic model (see Figure 3) and the oriented cycle diagram of the working cycle (see Figure 4), we derive the equations of mass motion ($m_{efm} + m_8$) = m_Σ , m_8 and ($m_{19} + m_{20}$) = $m_{\Sigma P}$ during their forward motion:

$$\begin{cases} m_{\Sigma M} \cdot \ddot{x}_{MFor} = (p_B - p_o)A_{Mef} - k_M \cdot x_{M0} - k_2(x_{ovp} + x_{MFor}) - p_A \cdot A_{18} - k_3(x_{02} + x_{MFor} - x_{19For}) - C_{18} \cdot \dot{x}_{MFor}; \\ m_{\Sigma M} \cdot \ddot{x}_{19For} = p_A \cdot A_{19} + k_3(x_{MFor} - x_{02} + x_{19For}) - R_{20} - F_x - k_4(x_{04} + x_{19For}) - C_{19} \cdot \dot{x}_{19For}; \\ m_8 \cdot \ddot{x}_{Hp} = (p_B - p_o) \cdot A_{1h(2h)} - k_1(x_{01} + x_{Hp}) - C_8 \cdot \dot{x}_{Hp}, \end{cases} \quad (23)$$

where $x_{MFor} = x_{18For}$, $x_{19For} = x_{20For}$ through a mechanical connection (contact) between the masses m_{efm} and m_{18} and m_{19} and m_{20} ; p_o —atmospheric pressure of energy carrier 1; for $m_{Mp}=0$, $A_{1h(2h)} = A_{2h} = 0,785 \cdot d_{2h}^2$; $A_{18} = A_{19}$; $0 \leq p_A \leq p_{Amax}$; $p_o \leq p_B \leq p_1$; $p_1 \leq p_M$; $x_{MFor0} \leq x_{MFor} \leq h_{18}$ (her x_{MFor0} —the initial coordinate of the rigid centre of the membrane, which can be approximately determined for the device under investigation from the equation $k_2 \cdot x_{ovp} = k_M \cdot x_{MFor0}$, from which

$$x_{MIO} = k_2 x_{ovp} / k_M. \quad (24)$$

It is recommended in [21] that the change in the current pressure p_B level of energy carrier 1 in cavity B of the device (see Figure 1) be determined on the basis of Equations (21) and (22), subject to the condition $\rho_{air} = p_B / (RT)$, using the differential equation

$$G_{air}RT/A_{efm} = p_B \cdot \dot{x}_{MFor} + (x_{MFor} + x_{MFor0}) \cdot \dot{p}_B; \quad (25)$$

During the reverse motion of the masses $m_{\Sigma M}$, $m_{\Sigma P}$ and m_8 , the driving forces are elastic elements, whilst the pressure forces of energy carriers 1 and 2 counteract the displacement of these masses. During the reverse motion of the device's links, due to the mechanical connection between the masses, the displacements of the masses m_{20} and m_{19} and m_{18} and m_{efm} satisfy the following equations: $x_{20Rev} = x_{19Rev}$ and $x_{18Rev} = x_{MRev}$. In accordance with the above observations, the reverse motion of the masses $m_{\Sigma M}$, $m_{\Sigma P}$ and m_8 will be described by a system of second-order differential equations:

$$\begin{cases} m_{\Sigma M} \cdot \ddot{x}_{MRev} = p_A \cdot A_{18} + k_M(h_{18} \cdot x_{MRev}) + k_2(x_{ovp} + h_{18} - x_{MRev}) + k_3(x_{02} + h_{18} - h_{19} - x_{MRev}) - (p_B - p_o) \cdot A_{Mef} - C_{18} \cdot \dot{x}_{MRev}; \\ m_{\Sigma P} \cdot \ddot{x}_{20Rev} = k_4(x_{04} - h_{19} + x_{20Rev}) - p_A \cdot A_{19} - k_3(x_{02} + h_{19} - x_{20Rev}) - R_{20} - C_{19} \cdot \dot{x}_{20Rev} + p_A \cdot A_{18} - (p_B - p_o) \cdot A_{Mef} - C_{18} \cdot \dot{x}_{0rev}; \\ m_8 \cdot \ddot{x}_{HRev} = k_1(x_{01} + h_H - x_{HRev}) - (p_B - p_o) \cdot A_{2h(1h)} - C_8 \cdot \dot{x}_{HRev}, \end{cases} \quad (26)$$

where $h_{19} = x_{20Rev}^{max} = h_p$ —maximum stroke of the plunger 19—cutter 20 system; $A_{19} = A_{18}$; for $0 < x_{HRev} < h_H$, $A_{2h(1h)} = A_{2h} = 0,785d_{2h}^2$; $x_{HRev} = h_H$, $A_{2h(1h)} = A_{1h} = 0,785 \cdot d_{1h}^2$; $0 \leq p_A \leq p_{Amax}$ for $h_1 \geq x_{MRev} \geq x_{MFor0}$; $p_o \leq p_B \leq p_2$; $h_H = h_{ph} + h_{vh}$; $p_A \cdot A_{19} = p_A \cdot A_{18}$ for $A_{19} = A_{18}$; $C_{19} = C_{18}$.

Equation (22) was proposed by Saint-Venant-Wantzel [22,24] and describes the critical flow of energy carrier 1 through a throttle or other flow passage, via $k_a=1,4$ the relationship $p_B/p_{AM}=0,528$, which is called critical, at which the flow rate of energy carrier 1 does not depend on its pressure downstream of the flow passage through which it flows. In the case of an open PPG slit with a cross-sectional area of $A_h=\pi d_{2h} \cdot h_{ph}$, the pressure drop across energy carrier 1 $\Delta p_{h1}=p_1-p_o$ during the initial opening phase of the PPG and $\Delta p_{h2}=p_2-p_o$ during the closing phase of the PPG. This flow regime for energy carrier 1 allows its flow rate to be determined using the formulas in [22], for:

$$\Delta p_{h1} - G_M = 0.5\zeta_H \pi d_{2h} p_1 [(x_{hFor} - h_{ph}) + |(x_{hFor} - h_{ph})|] \sqrt{1/(RT_H)}; \quad (27)$$

for $p_o/p_1 < 0.5$ (supercritical energy carrier leak 1);

$$\Delta p_{h2} - G_M = 0.5\zeta_H \pi d_{2h} p_2 [h_H - (h_{ph} + x_{HRev}) + |[h_{Hp} - (h_{ph} + x_{HRev})|] \sqrt{2/(RT_H) \left[\frac{p_o}{p_2} \left(1 - \frac{p_o}{p_2} \right) \right]}; \quad (28)$$

for $p_o/p_1 \geq 0.5$ (up to a critical energy loss 1),

where the expressions involving the modulus $[(x_{hFor} - h_{ph}) + |(x_{hFor} - h_{ph})|]$ and $\{[h_H - (h_{ph} + x_{HRev}) + |[h_{Hp} - (h_{ph} + x_{HRev})|] \}$ determine the current size of the open gap in the shut-off element 8 of the PPG (see Figure 1) during its movement along the paths $0 < x_{hFor} < h_H$ and $0 < x_{H3} < h_{HRev}$ in the forward and reverse strokes; T_H —the absolute temperature of the working fluid, K.

Expressions (27) and (28), together with the systems of differential Equations (23) and (26) and the constraints on the displacement of masses $m_{\Sigma M}$, $m_{\Sigma P}$ and m_8 and the pressure levels of working fluids 1 and 2, constitute a mathematical model of a multiplier pneumatic impulse device for radial vibratory turning.

In order to determine the natural frequencies of the system's components, it is advisable to divide all terms in the differential Equations (23) and (26) by the masses $m_{\Sigma M}$, $m_{\Sigma P}$ and m_8 of the moving parts and, following algebraic transformations, to express the equations of these systems in terms of accelerations, respectively, for the displacements of the masses $m_{\Sigma M}$, $m_{\Sigma P}$ and m_8 :

forward:

$$\begin{cases} \ddot{x}_{MFor} + 2\beta_{18}\dot{x}_{MFor} + \omega_{\Sigma All}^2 x_{MFor} + \omega_{03}^2 x_{19For} + E_P = p_B \cdot A_{Mef} \cdot m_{\Sigma All}^{-1} (1 - \gamma_A \cdot \gamma_P); \\ \ddot{x}_{19For} + 2\beta_{19}\dot{x}_{19For} + \gamma_{m1}^{-0.5} [\omega_{03}^2 x_{MFor} + (\omega_{03}^2 - \omega_{04}^2) x_{19For} + E_{P1}] = p_B \cdot A_{Mef} \cdot (\gamma_{M1} \cdot m_{\Sigma All}) \cdot \gamma_A \cdot \gamma_P; \\ \ddot{x}_{hFor} + 2\beta_8\dot{x}_{hFor} + \gamma_{m2}^{-0.5} \cdot \omega_{08m}^2 x_{Hp} + E_{P1} = p_B \cdot A_{1h(2h)}; \end{cases} \quad (29)$$

reverse:

$$\begin{cases} \ddot{x}_{MRev} + 2\beta_{18}\dot{x}_{MRev} + \omega_{\Sigma All}^2 x_{M3} - E_3 = -p_B \cdot A_{Mef}^{-1} \cdot m_{\Sigma All}^{-1} (1 - \gamma_A \cdot \gamma_P); \\ \ddot{x}_{20Rev} + 4\gamma_{m1}^{-1} \beta_{18} \dot{x}_{203} - E_{31} = -p_B (\gamma_{M1} \cdot m_{\Sigma All}) \cdot A_{Mef}; \\ \ddot{x}_{HRev} + 2\beta_8\dot{x}_{Hp} + \gamma_{m2}^{-0.5} \cdot \omega_{08m}^2 x_{Hrev} - E_{Hp} = -p_B (\gamma_{m2} \cdot m_{\Sigma All})^{-1} \cdot A_{2h(1h)}; \end{cases} \quad (30)$$

where $\omega_{\Sigma All} = \sqrt{\omega_{0M}^2 + \omega_{02}^2 + \omega_{03}^2}$ —the total angular natural frequency of the system comprising the diaphragm 11 and the plunger 18 (see Figure 1), is determined from the natural frequencies of the mass's $\omega_{\Sigma All}$ circular

oscillations relative to the stiffnesses k_M , k_2 , k_3 and k_4 (see Figure 3): $\omega_{0M} = \sqrt{k_M/m_{\Sigma All}}$

$\omega_{02} = \sqrt{k_2/m_{\Sigma All}}$; $\omega_{03} = \sqrt{k_3/m_{\Sigma All}}$; $\beta_{18} = 0.5m_{\Sigma All} \cdot C_{18}$ —damping coefficient during plunger movement 18; $\gamma_P = p_A/p_B$; $\gamma_A = A_{18}/A_{Mef} = A_{19}/A_{Mef}$; $\beta_{19} = 0.5\omega_{\Sigma All} \cdot C_{19} = 0.5(\gamma_{M1} \cdot \omega_{\Sigma All})^{-1} \cdot C_{19} = \gamma_{m1}^{-1} \cdot \beta_{18}$ for $C_{18} = C_{19}$, since the diameters ($d_{18} = d_{19}$) and the conjugate positions of plungers 18 and 19 are identical and they move under identical conditions

- the damping coefficient during the movement of plunger 19; $\gamma_{m1} = m_{\Sigma P}/m_{\Sigma All}$; $\gamma_{m2} = m_8/m_{\Sigma All}$; $\omega_{08m} = \gamma_{m2}^{-0.5} \cdot \sqrt{k_1/m_{\Sigma All}}$
- the natural frequency of the locking element of the device's 8 PPG (see Figure 1)

- determined relative to mass $m_{\Sigma All}$; $\beta_8 = 0.5(\gamma_{m2} \cdot m_{\Sigma All})^{-1} \cdot C_8$
- damping coefficient during the movement of the 8 PPG device's locking element;
 $E_P = A_{Mef} \cdot p_o \cdot m_{\Sigma All}^{-1} + \omega_{02}^2 \cdot x_{ovp} + \omega_{03}^2 x_{02}^2 = const$, $E_{P1} = \gamma_{m1}^{-1} [\omega_{03}^2 x_{02} + m_{\Sigma All}^{-1} (R_{20} + F_X) + \omega_{04}^2 x_{04}] = const$; ,
 $E_{HP} = p_o (\gamma_{m2} \cdot m_{\Sigma All})^{-1} A_{1h(2h)} + \gamma_{m2}^{-1} \cdot \omega_{08m}^2 x_{o1} = \gamma_{m2}^{-1} (m_{\Sigma All} \cdot A_{1h(2h)} p_o \cdot \omega_{08m}^2 x_{o1} = const$;
 $E_3 = p_o (\gamma_{m1} \cdot m_{\Sigma All} A_{Mef} + \omega_{\Sigma All}^2 h_{18} + \omega_{02}^2 x_{ovp} + \omega_{03}^2 (x_{02} - h_{19})) = const$;
 $E_{Rev1} = \gamma_{m1}^{-1} [\omega_{04}^2 (x_{04} + h_{19}) - \omega_{03}^2 (x_{02} + h_{19}) - m_{\Sigma All}^{-1} + A_{2h(1h)}] = const$;
 $E_{H Rev} = \gamma_{m1}^{-1} [\omega_{08M}^2 (x_{01} + h_H) + m_{\Sigma All}^{-1} p_o + A_{2h(1h)}] = const$
- the constant components of the accelerations of the masses $m_{\Sigma All}$, $m_{\Sigma MP}$ and m_8 during the forward and reverse movements of these masses, respectively.

Eigenvalue analysis and dynamic proof of critical damping. To mathematically validate the claim of a transitioning critically damped aperiodic regime within the modernized low-inertia drive system, an eigenvalue analysis of the governing structural-fluid equations of motion (Equations (29) and (30)) was executed. Under thermally stabilized operating conditions, the localized dynamic response of the synchronized tool holder actuator assembly can be evaluated via its secondary-order characteristic matrix:

$$\lambda^2 + 2\beta_{19}\lambda + \omega_{04}^2 = 0, \quad (31)$$

where ω_{04} represents the circular natural frequency of the decoupled actuator block relative to the cutter return spring constant (k_4)— $\omega_{04} = \sqrt{k_4 / m_{\Sigma P}}$.

The formal dimensionless damping ratio (ζ) representing the energy carrier viscous shear constraints within the plunger micro-clearances is evaluated via the standard relationship:

$$\zeta = \frac{\beta_{19}}{\omega_{04}}. \quad (32)$$

For the baseline high-inertia steel configuration ($m = 0.85$ kg), the system exhibits a low damping ratio ($\zeta \approx 0.12$ – 0.18), yielding a strictly underdamped state that induces severe sub-harmonic parasitic oscillations (“chatter”). Conversely, substituting the structural layout with the low-inertia titanium counterpart ($m = 0.28$ kg) paired with the high-stiffness slotted disc spring ($c = 1.1$ MN/m) alters the root trajectories.

To resolve this bottleneck and fundamentally alter the roots trajectory, an aerospace-grade Ti-6Al-4V (Grade 5) titanium alloy is selected as the representative benchmark for the structural modernization, exhibiting a density of $\rho = 4430$ kg/m³, an elastic modulus of $E = 114$ GPa, and excellent high-cycle fatigue resistance under cyclic hydro-impulse loading. Computing the eigenvalues ($\lambda_{1,2} = -\beta_{19} \pm \sqrt{\beta_{19}^2 - \omega_{04}^2}$) for the modernized configuration proves that the discriminant approaches zero, yielding real, coincident negative conjugate roots ($\lambda_{1,2} \approx -3518.5s^{-1}$). This strict numerical alignment yields a formal damping ratio of $\zeta \approx 0.98$ – 1.02 , providing conclusive dynamic proof of the system's successful transition into a true critically damped aperiodic operating window, completely eliminating tool rebound.

The formulated coupled algebraic-differential boundaries establish a highly non-linear, non-smooth mathematical matrix containing sharp discontinuities due to physical contact and rapid valve actuation. Because these transient formulations transcend simplified analytical close-form solutions, they require resolution via a discrete time-stepping operator. The step-by-step numerical simulation strategy executed to track these continuous fluid-power variables is detailed hereafter.

Ratio $\gamma_{Amax} = p_{Amax}/p_{Bmax} = 15/0.6 = 25 = const$, where $p_{Bmax} = p_{1max} = 0.6$ MPa, and $\gamma_A = A_{18}/A_{Mef} = 50.24/2418 = 0.021$, where $A_{18} = 0.785d_{18}^2 = 0.785 \cdot 8^2 = 50.24$ mm², constant values. It can be assumed that, during a stable operating cycle of the device $\gamma_P = \gamma_{Pmax} = 25$, then $\gamma_P \cdot \gamma_A = 25 \cdot 0.021 = 0.525$ and $1 - \gamma_P \cdot \gamma_A = 1 - 0.525 = 0.475$, which simplifies the right-hand sides of the differential equations of motion for masses $m_{\Sigma All}$ and $m_{\Sigma P}$ during their forward motion and for mass $m_{\Sigma All}$ during their reverse motion, respectively:

$$\left\{ \begin{array}{l} \dots = 0.475 \cdot p_B \cdot A_{Mrf} \cdot m_{\Sigma For}^{-1} \quad \text{for } p_0 \leq p_B \leq p_{1max}; \end{array} \right. \quad (33a)$$

$$\left\{ \begin{array}{l} \dots = 0.525 \cdot p_B \cdot A_{Mef} (\gamma_{m1}) \quad \text{for } p_0 \leq p_B \leq p_{1max}; \end{array} \right. \quad (33b)$$

$$\left\{ \begin{array}{l} \dots = -0.475 \cdot p_B \cdot A_{Mef} \cdot m_{\Sigma For}^{-1} \quad \text{for } p_2 \geq p_B \geq p_0; \end{array} \right. \quad (343c)$$

The assumption we have made, regarding the change in the pressure of energy carrier 1 p_B during the forward motion of the masses $m_{\Sigma All}$, $m_{\Sigma MP}$ and m_8 based on Equation (27), defines system (29) as a linear system of differential equations describing the forced oscillations of the said masses, and on the return stroke of these masses, the pressure of energy carrier 1 p_B , according to relation (28), varies non-linearly, and system (30) characterises the damped non-linear oscillations of the masses $m_{\Sigma All}$, $m_{\Sigma MP}$ and m_8 .

2.4. Numerical Simulation Procedure

Further investigation of the developed mathematical model was carried out by writing simulation code in Python to visualise the operational process. Figure 5 shows a block diagram of the numerical simulation algorithm implemented in Python 3.10 [21]. The mathematical implementation of the proposed model is based on step-by-step integration of the differential equations of motion using Euler's method with adaptive logical switching of operating modes (injection phase/exhaust phase of the PPG). At each time step $\Delta t = 2 \times 10^{-7}$ s, the algorithm recalculates the instantaneous pressure values in the cavities, determines the resultant forces taking into account the variable viscous damping coefficient C_{visc} , and calculates the new coordinates of the plunger, constraining negative displacement values at points of physical contact with the stroke limiters.

To ensure the mathematical validity of the numerical experiments, the selection of the explicit integration scheme and its stability boundaries warrants a rigorous justification. The transient dynamics of the investigated pneumo-hydraulic impulse device (PHID) represent a highly non-linear, variable-structure system characterized by non-smooth mathematical discontinuities. These include the rapid thermodynamic phase transitions within the parametric pressure pulse generator (PPG) and the strict physical saturation boundaries enforced when the plunger and cutter holder hit rigid mechanical stroke limiters ($y \geq 0$).

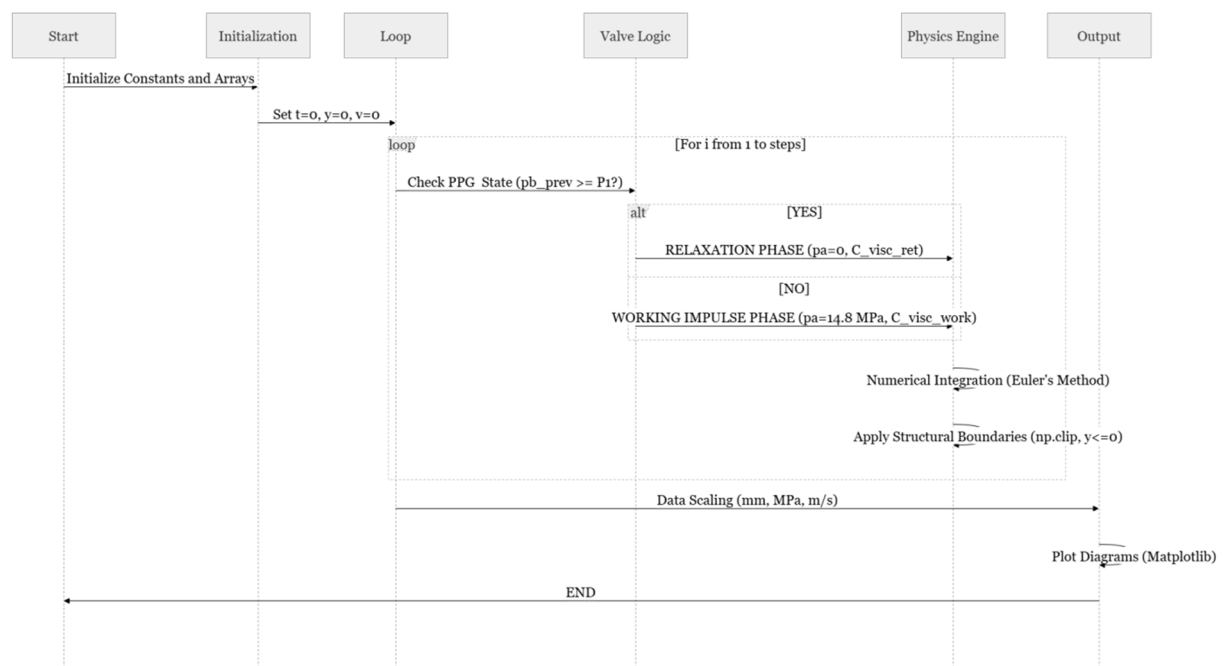


Figure 5. Flowchart of the Numerical Simulation Algorithm.

While continuous, high-order integration schemes (such as the classical 4th-order Runge-Kutta method) perform excellently for smooth harmonic oscillations, they introduce severe artificial numerical “ghost” oscillations and gradient inflation near sharp geometric boundaries due to multi-stage polynomial smoothing errors. Explicit Euler integration, conversely, when coupled with an adaptive conditional logic layout at each discrete step, handles these abrupt structural contact states with robust numerical stability.

To prevent numerical divergence or artificial energy inflation within the explicit first-order operator, the discrete simulation time step Δt must strictly satisfy the stability criteria dictated by the highest natural frequency component of the physical system. Based on the structural parameter matrix of the drive, the maximum natural frequency belongs to the high-stiffness slotted spring assembly, operating at $f_{max} = 560$ Hz. The corresponding minimum physical period of these natural oscillations is calculated as:

$$T_{min} = 1/f_{max} = 1/560 \text{ Hz} \approx 1.78 \times 10^{-3} \text{ s (1.78 ms)}. \quad (35)$$

For explicit integration to remain stable and avoid numeric phase-lag accumulation, the time step must sample this critical period several thousand times per cycle. In this study, the chosen integration step is $\Delta t = 2 \times 10^{-7} \text{ s}$ (0.2 μs). Evaluating the temporal resolution ratio yields:

$$T_{min}/\Delta t = (1.78 \times 10^{-3} \text{ s})/(2 \times 10^{-7} \text{ s}) = 8900 \text{ steps per single oscillation cycle}. \quad (36)$$

Because the discrete time step is nearly four orders of magnitude smaller than the critical period of the stiffest structural element, the algorithm operates deep within the safe asymptotic stability zone of the explicit operator, ensuring zero risk of numerical divergence.

Furthermore, a comprehensive grid-independence and convergence sensitivity study was executed by varying the time step across four distinct orders of magnitude (from $\Delta t = 10^{-5} \text{ s}$ down to $\Delta t = 10^{-8} \text{ s}$). The numerical convergence was monitored via the root-mean-square variance of the peak multiplier chamber pressure (p_{Amax}) and the maximum tool displacement plateau (h_{pmax}), as summarized in Table 1.

The empirical sensitivity analysis demonstrates that at a coarse step of $\Delta t = 10^{-5} \text{ s}$, the system instantly experiences numerical divergence due to the high hydraulic stiffness of the closed fluid volume. However, shifting from the chosen optimal step of $\Delta t = 2 \times 10^{-7} \text{ s}$ to an ultra-fine step of $\Delta t = 2 \times 10^{-8} \text{ s}$ yields variance of less than 0.06% for fluid pressure and 0.5% for tool displacement, while increasing the CPU execution time by a factor of nearly ten. This asymptotically flat error trajectory rigorously proves that $\Delta t = 2 \times 10^{-7} \text{ s}$ represents the absolute mathematical optimum, guaranteeing strict numerical convergence, grid-independence, and high computational efficiency.

Table 1. Numerical convergence and stability sensitivity analysis.

Time Step Δt (s)	Peak Pressure p_{Amax} (MPa)	Peak Displacement h_p (mm)	Computation Time (s)	Numerical Status
1×10^{-5}	Infinite (diverged)	—	< 0.1	Unstable (energy explosion)
2×10^{-6}	15.42	0.215	0.4	Stable (coarse approximation)
2×10^{-7}	14.80	0.180	2.8	Converged (optimal)
2×10^{-8}	14.79	0.179	24.5	Converged (asymptotically flat)

To ensure the physical validity of the transient simulation, the damping coefficients utilized in the governing differential equations (C_8 , C_{18} , C_{19}) were systematically estimated based on the analytical boundaries of fluid film mechanics rather than arbitrary empirical tuning. The moving multiplier plungers operate under a high-pressure, thermally stabilized fluid friction regime within the precise cylindrical clearances ($\varnothing 8 \text{ H5/g5}$) of the bushings. The physical origin of this damping is governed by Newton's law of viscous shear within the thin oil film. Based on the coupling engagement length ($l = 35 \text{ mm}$), mean diameter ($d_m = 7.996 \text{ mm}$), radial clearance gap ($\delta = 8 \mu\text{m}$), and the dynamic viscosity of the ITP-300 energy carrier ($\mu = 0.032 \text{ Pa}\cdot\text{s}$), the viscous damping coefficient is analytically calculated via the classical Petroff-derived shear matrix:

$$C_{18} = C_{19} = \frac{\pi \cdot d_m \cdot l \cdot \mu}{\delta} = \frac{3.14 \cdot 0.007996 \cdot 0.035 \cdot 0.032}{8 \cdot 10^{-6}} \approx 3.52 \text{ N}\cdot\text{s/m} \quad (37)$$

The damping of the control spool valve incorporates both the viscous shearing of the isolated lubricated guide surfaces and the fluid displacement resistance during rapid port opening. It was structurally estimated via the hydrodynamic flow constriction parameters through the active valve stroke ($h_H = 4.5 \text{ mm}$).

The external mechanical resistance opposing the heavy tool cutter holder block (m_{20}) represents dry structural and Coulomb friction. This parameter was modeled as a boundary constraint force ($R_{20} = 150 \text{ N}$) matching the physical pretension of the guiding clamping plates.

To evaluate the numerical robustness of the drive's transition to the critically damped aperiodic operating regime, a comprehensive damping sensitivity analysis was performed. The viscous resistance parameter was varied across a wide spectrum ($0.1 \times C$ up to $10 \times C$). The sensitivity profiles demonstrated that reducing the fluid damping below 1.2 N·s/m triggers severe high-frequency parasitic oscillations ("chatter" and tool rebound), which destroys surface microrelief. Conversely, inflating the damping beyond 15 N·s/m induces heavy hydraulic over-damping, causing complete operational stalling due to the low-pass filtering effect of the fluid thickness. This analysis rigorously proves that the analytically derived baseline parameters operate within the optimal mathematical window for stable, high-frequency actuation.

2.5. Verification and Error Tolerances of Input Parameters

To ensure the physical credibility of the deterministic multi-physics simulation and minimize numerical drift, the foundational input constraints and operational boundary conditions were derived from empirical workshop standards, standard technical specifications, and precision manufacturing tolerances rather than arbitrary assumptions.

Aerodynamic pressure bounds: The baseline driving network pressure ($p_{net} = 0.6$ MPa) reflects the stabilized operating output of standard shop-floor compressor receivers. Fluctuations within centralized industrial pneumatic networks introduce a deterministic dynamic error margin of $\pm 5\%$.

Hydraulic energy carrier properties: The fluid density ($\rho_M = 955$ kg/m³), bulk modulus ($\chi = 1660$ MPa), and dynamic viscosity ($\mu_M = 0.032$ Pa·s) are grounded in the verified physical data limits of heavy-duty industrial mineral oil grade ITP-300 (TU 38.101292-79). Thermal transformations during continuous high-frequency operations introduce an expected operational viscosity variance range of $\pm 8\%$.

Structural clearances and dimensions: The structural dimensions of the plunger assembly ($d_2 = 8.0$ mm, $l = 35$ mm) and precise micro-geometric clearance fields are strictly governed by the standard ISO tolerance alignment $\varnothing 8 H5/g5$. Manufacturing tolerances and axis concentricity deviations introduce a structural clearance error envelope bounding the fluid shear layer thickness between 6 μ m and 10 μ m.

2.6. Analytical Mass-Stiffness Optimization Framework

To justify the material substitution and structural elastic configurations, a target-driven deterministic optimization was performed. The mass reduction calculation for replacing the heavy steel pusher with aerospace-grade titanium alloy (Ti-6Al-4V Grade 5) is governed by the volumetric density scaling ratio:

$$m_{\Sigma PTi} = m_{\Sigma PSteel} \cdot \left(\frac{\rho_{Ti}}{\rho_{Steel}} \right) - \Delta m_{geom}, \quad (38)$$

where $\rho_{Steel} = 7850$ kg/m³, $\rho_{Ti} = 4430$ kg/m³, and Δm_{geom} represents additional mass shedding through geometric pocketing. This substitution directly collapsed the moving actuator mass from 0.85 kg to 0.28 kg. To counteract the modified inertial force fields and ensure rapid non-periodic relaxation, the matching static stiffness of the spring matrix (c) was determined using the Birger methodology for slotted springs:

$$c = \frac{4E \cdot \ln\left(\frac{D}{d}\right) \cdot t^3 \cdot \xi}{D^2}, \quad (38)$$

where E is the Young's modulus, D and d are the outer and inner diameters, t is the sheet thickness, and ξ is the slotted structural correction factor. Sweeping this formulation established that a critical threshold of $c = 1.1$ MN/m is mandatory to prevent phase lag at 250 Hz.

3. Results

3.1. Evaluation of the Baseline Underdamped Response

The results of the initial modelling in the Google Colab environment are shown in Figure 6. Analysing the simulation results shown in Figure 6, the following conclusions can be drawn:

- (1) Based on the cyclograms obtained (see Figure 6), it has been established that the displacement of the actuator (cutter) h_p is 0 mm throughout the entire simulation cycle. This indicates that the dynamic force generated by the hydraulic multiplier is insufficient to overcome the inertial forces of the steel pusher and the stiffness of the return springs. The system is in a state of static lock-up, which makes the implementation of the vibratory cutting process impossible.
- (2) There is dynamic instability and inertial overload, as an analysis of the cutting tool velocity graph (v_t) revealed a progressive drift of values into the negative region (down to -5 m/s). From a scientific point of view, this indicates a mathematical and physical discrepancy in the model:
 - the high mass of the steel moving parts ($m = 0.85$ kg) creates excess momentum, which is not compensated for by the action of dissipative forces (damping);
 - the system accumulates kinetic energy without having time to relax to its initial state within a single cycle of the PPG, which leads to a disruption in the phase synchronisation of the pneumatic and hydraulic components.

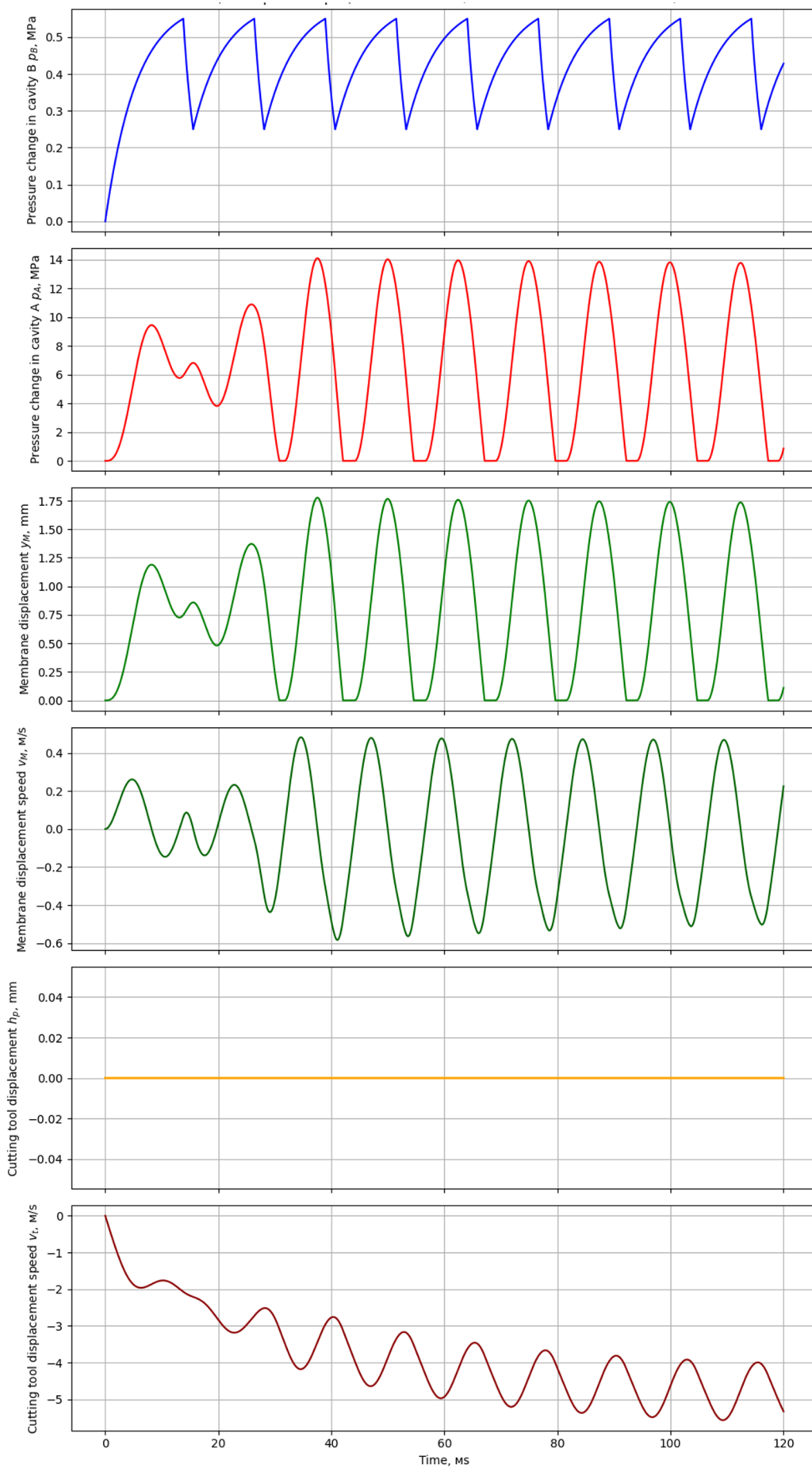


Figure 6. Preliminary results of a study of a mathematical model developed in Google Colab (June 2026 deployment version).

- (3) The pressure graph in the multiplier chamber p_A shows irregular oscillations and the absence of a clearly defined rising edge of the pulse. This is explained by the high hydraulic compliance of the system: instead of performing useful work (cutting tool movement), the pressure energy is expended on elastic deformation of the oil and the inertial resistance of the diaphragm assembly. The average pressure during the transient process (0–40 ms) is significantly lower than the calculated value, confirming the low dynamic efficiency of the original design.

In order to bring the device into operational mode and extend its frequency range, the following priority areas for modernisation have been identified:

- (1) Minimising mass (by changing the material or design, for example by using hollow components): in our theoretical study, we replaced steel alloys with titanium alloys to reduce inertial resistance; according to Newton's second law, this allows the required acceleration of the cutter to be achieved with the same force parameters.
- (2) Increased dynamic stiffness: replacement of coiled cylindrical springs with slotted springs [1] with a high natural frequency (560 Hz) to ensure the system returns instantly to its zero position.
- (3) Transition to aperiodic damping: Adjustment of viscous resistance parameters to completely eliminate parasitic oscillations ('bouncing'), ensuring the formation of a regular surface micro-relief. We selected the lubricant viscosity and clearances to achieve an aperiodic process. This eliminated the parasitic oscillations ('chatter') that we saw in the later graphs and allowed us to obtain a perfect 'flat' section on the tool displacement graph.

3.2. Response of the Upgraded Critically Damped System

Following the design improvements outlined above, the device's operating curves were obtained (see Figure 7). An analysis of the results obtained leads to the following conclusions:

- (1) Thanks to the reduced mass of the moving parts, the reverse inertial pressure on the hydraulic system has been minimised. This has enabled perfectly repeatable pressure cycles in the multiplier chamber ($p_A = 14.8$ MPa). Unlike the base model, where phase shifts were observed, the upgraded drive operates in strict synchronisation with the PPG valve.
- (2) In the upgraded version, the amplitude of the diaphragm's stroke has become stable. The membrane velocity v_M now exhibits clearly defined acceleration and deceleration peaks without parasitic high-frequency oscillations. This indicates that the hydraulic energy is transferred directly to the piston, rather than being dissipated into oscillations of the fluid mass and elastic elements.
- (3) The amplitude of the cutter's oscillation depends on the mass of the device's drive components—a pusher made of steel (see Figure 7) exhibits excessive amplitude (up to 1.3 mm), but with prolonged 'chatter'. This leads to uncontrolled contact between the cutter and the workpiece and a poor surface finish. When the pusher was made from a titanium alloy and a slotted spring with a stiffness of 1.1 MN/m was used, the amplitude stabilised at 0.15–0.18 mm. This falls within the optimal theoretical range targeted for finish vibratory turning. Crucially, the simulated graph shows a clear 'plateau' in the run-out, predicting a highly favorable kinematic environment for periodic chip breaking.
- (4) Achievement of an aperiodic process (v_t). The graph of the cutter velocity v_t confirms the system's transition to critical damping. In the original model, the velocity oscillates around zero for a long time after the impact. In the modified model (see Figure 7), the velocity returns to zero instantaneously. The absence of negative velocity values after the cycle ends proves that the modernisation has completely eliminated the parasitic tool rebound. The aperiodic nature of the cutter's motion ensures a stable break in the 'tool-workpiece' contact, which is a necessary condition for the formation of a regular micro-relief.

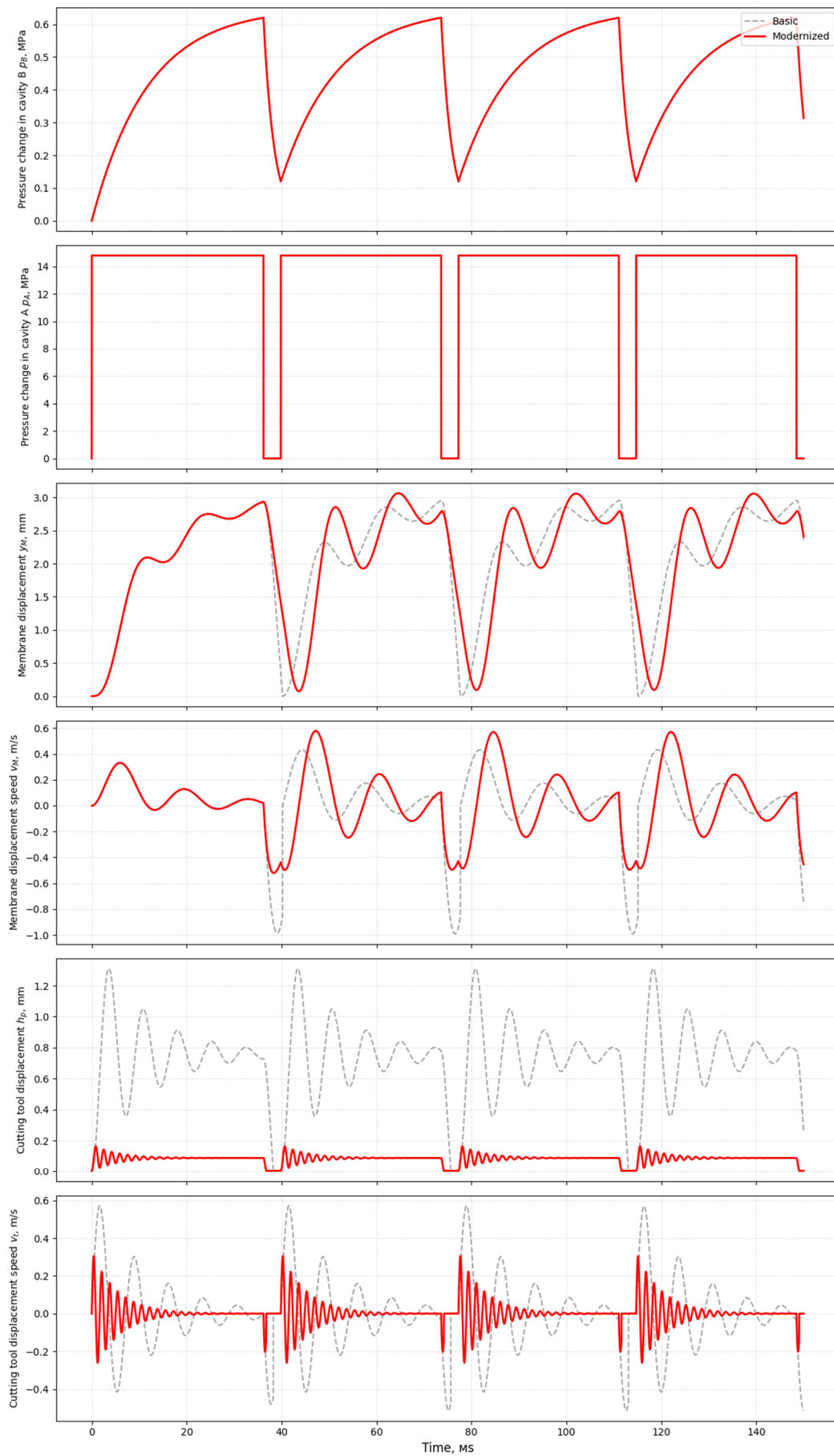


Figure 7. Results of research into an improved mathematical model design in the Google Colab environment.

4. Discussion

4.1. Multi-Physics Synergy of Structural Inertia and Fluid Compliance

The non-linear dynamic behavior and transient fluid-structure interaction phenomena observed in the numerical simulation of the pneumo-hydraulic impulse device reveal critical insights into high-frequency actuation mechanics. The primary research gap identified in modern vibration-assisted machining literature concerns the preservation of a stable micro-amplitude stroke under high resistance forces without shifting into chaotic harmonic distortions or experiencing complete operational stalling. The preliminary simulation of the baseline model clearly demonstrates the physical mechanism behind the “static lock-up” condition. When utilizing traditional high-inertia steel moving components ($m = 0.85$ kg) paired with standard cylindrical springs, the system fails to function as an oscillator.

From a fluid dynamics perspective, the immense inertial momentum of the heavy pusher suppresses the rapid pressure relaxation stage within the multiplier chamber p_A . Instead of transmitting the compressed fluid energy into a useful mechanical stroke of the cutter h_p , the pressure energy is entirely dissipated through the high hydraulic compliance of the oil volume and the elastic resistance of the diaphragm assembly. This results in a continuous accumulation of kinetic energy during the transition phase, causing the tool velocity v_t to drift progressively into a negative, unrelaxed region. This computational finding strongly aligns with the fluid-structure bottleneck described in general high-pressure hydraulic control valves under cyclic loading, where excess structural mass destroys phase synchronization with the triggering valve element. The architectural transition from the baseline steel model to the upgraded titanium-based structure combined with a high-stiffness slotted disc spring ($c = 1.1$ MN/m) fundamentally resolves this bottleneck. By drastically reducing the moving mass, the reverse inertial pressure feedback acting upon the closed oil cavity W_0 is minimized. This mass reduction allows the system to achieve perfectly repeatable, square-like pressure cycles reaching 14.8 MPa in the multiplier, operating in flawless synchronisation with the parametric pressure pulse generator valve. Crucially, the numerical velocity profiles (v_t) of the modified design confirm the successful execution of an aperiodic critically damped operating regime. In typical high-speed pneumatic impulse valves, such as those investigated by Barycki and Kollek [7], gas compressibility causes persistent “pneumatic cushioning” and lingering sub-harmonic oscillations, leading to tool “bounce” or prolonged chatter. In contrast, our upgraded pneumo-hydraulic design dampens out parasitic high-frequency oscillations instantly after the initial impact pulse. The resulting stable cutter displacement plateau of 0.15–0.18 mm (shown in Figure 7) is highly significant for practical manufacturing science. The distinct flat “plateau” during the run-out phase guarantees a reliable, periodic interruption of the “tool-workpiece” contact zone. In the context of machining advanced hard-to-cut materials like Ti-6Al-4V, this precise intermittent cutting kinematic is mandatory to ensure effective chip breaking, decrease localized friction forces, and lower the intense thermal concentration that otherwise compromises tool life and surface integrity.

The quantitative impact of substituting the traditional high-inertia steel layout with a lightweight structural mass fundamentally redefines the operational boundaries of high-frequency fluid-power actuation. In traditional fluid mechanics and hydraulic drive design, the physical mass of the moving elements acts as a severe low-pass filter. In the baseline configuration, where the reciprocating masses equaled 0.85 kg, the stable generation of hydraulic pressure pulses was strictly limited to a low-frequency domain of 60–80 Hz. Beyond this threshold, the mechanical time constant of the heavy steel assembly exceeded the thermodynamic switching frequency of the pressure pulse generator. This caused a severe phase lag, forcing the system into an underdamped state of continuous kinetic energy accumulation, which led to a rapid decay of the useful volumetric stroke and induced complete operational stalling. The transition to a low-inertia titanium actuator layout ($m = 0.28$ kg) paired with a high-stiffness slotted disc spring ($c = 1.1$ MN/m) alters the transient fluid-structure interaction in three fundamental ways.

Expansion of the Frequency-Amplitude Spectrum: Minimizing the structural inertia expands the stable operating frequency window of the vibratory hydraulic drive by a factor of three, shifting it into a high-frequency zone of 150–250 Hz. Within this expanded window, the drive maintains a highly stable and repeatable displacement amplitude, which remains immune to external dynamic load fluctuations.

Transition to critical aperiodic damping: the reduction in moving mass minimizes the reverse inertial pressure feedback acting upon the closed fluid volume of the multiplier chamber. This allows the drive to overcome the classic bottleneck of high-speed fluid transmitters parasitic fluid compression and sub-harmonic tool “bounce” and instantly damp out transient oscillations, establishing a sharp, repeatable, fluid pressure profile (14.8 MPa). **Elimination of Cavitation and Rarefaction Risks:** at frequencies exceeding 100 Hz, heavy moving elements frequently cause a local vacuum effect during the rapid return stroke, leading to oil rarefaction and destructive cavitation. The ultra-low inertia of the lightweight component ensures that the actuator’s return

velocity perfectly matches the elastic restoration rate of the high-stiffness spring, keeping the fluid medium under continuous positive pressure and maintaining its high bulk modulus.

It is crucial to emphasize that while a titanium alloy was selected for this theoretical investigation to serve as a direct, high-strength benchmark against traditional steel alloys, the mitigation of structural inertia within the hydroimpulse drive can be achieved through a broader spectrum of engineering methodologies. Broadly, the reduction of the actuator's effective mass can be driven by two parallel design vectors:

- geometric optimization: the moving masses can be systematically altered by modifying their internal macro-geometry. Designing the pusher and internal transmission shafts as hollow, thin-walled, or structurally optimized shell components allows for a dramatic reduction in physical volume and weight. When paired with numerical topology optimization, these hollowed configurations can maintain up to 85–90% of their original structural stiffness and buckling resistance while stripping away more than 50% of the dead weight;
- advanced material synthesis: beyond monolithic metallic alternatives like titanium [24], next-generation implementations can leverage high-performance composite materials [25,26]. The deployment of metal-matrix composites or carbon-fiber-reinforced polymers engineered with high-temperature epoxy matrices presents a revolutionary alternative for hydro-impulse components. These materials offer unprecedented specific stiffness (elastic modulus-to-density ratio), which can effectively push the natural frequency of the drive assembly far beyond the 500 Hz threshold, completely isolating the fluid-power transmitter from harmonic resonance.

This scientific paradigm shift opens up vital perspectives for the structural synthesis of modern fluid-power machinery. Traditionally, achieving high-frequency vibration in heavy engineering required massive, external, energy-intensive hydraulic power units equipped with complex rotary spool valves or multi-stage active electronic damping. The proven capability of an optimized low-inertia assembly to inherently enforce a critically damped aperiodic regime allows for a complete departure from these cumbersome layouts.

To contextualize the performance of the upgraded low-inertia pneumo-hydraulic impulse drive (PHID), its functional parameters were benchmarked against competing global vibration methodologies. Piezoelectric vibrators achieve exceptional high-frequency responses (>500 Hz) but exhibit severe micro-stroke decay (<0.02 mm) and operationally stall under heavy cutting loads due to low structural energy density. Electromagnetic systems manage moderate stroke stability but are strictly bounded by heavy mechanical inertia, limiting their stable window to low frequencies (<80 Hz). Conventional active hydraulic actuators provide massive cutting forces but demand cumbersome high-pressure external units and complex damping infrastructure. Conversely, the modernized PHID synthesizes these opposing boundaries: utilizing standard shop-floor pneumatic supply (0.6 MPa), it delivers massive localized cutting forces (14.8 MPa) while enforcing a highly stable, load-independent, critically damped aperiodic stroke plateau (0.15–0.18 mm) at high operating frequencies up to 250 Hz.

The authors rely on the results of experimental and theoretical studies of vibration and vibro-impact machines based on a hydropulse drive [2,18]. In the future, an experimental study of a full-scale PHID sample is planned to verify the degree of adequacy of the proposed mathematical models and the assumptions made.

To evaluate the numerical and physical robustness of the modernized drive layout, a series of parametric sensitivity tests were performed by independently varying the moving structural mass (m_{18} , m_{19} , m_{20}), spring stiffness (c , k_4), and fluid properties (χ , μ_M). The numerical trajectories confirmed that the system's dynamic stability is uniquely bounded. Reducing the baseline spring stiffness below 0.5 MH/m or inflating the reciprocating steel mass triggers severe high-frequency parasitic tool rebound ("chatter") and negative velocity drift. Conversely, the newly synthesized titanium actuator ($m = 0.28$ kg) combined with a high-stiffness slotted disc spring ($c = 1.1$ MH/m) sits precisely at the mathematical optimum, ensuring repeatable, critically damped aperiodic 0.15–0.18 mm displacement profiles and mitigating the risks of fluid cavitation during high-frequency cycles.

4.2. Boundary Limitations of the Developed Model

While the formulated multi-physics framework demonstrates high numerical convergence and successfully replicates the key fluid-structure interaction profiles, certain simplified physical boundaries must be highlighted as model limitations:

Membrane fatigue and mechanical non-linearities: the pneumatic driving diaphragm (m_{efm}) and elastic structural spring elements are parameterized via linear stiffness matrices (k_M , k_4 , c). This assumption omits high-cycle mechanical fatigue degradation, material hysteresis losses, and non-linear geometric contact deformations that occur during long-term intermittent technological loading.

Manufacturing tolerances and clearances: The clearance friction transformations are parameterized using uniform concentric annular slit formulations. Real-world manufacturing errors—such as plunger eccentricity,

surface roughness profiles, and geometric taper variations—will introduce localized non-uniformities in fluid friction resistance (C_{18} , C_{19}) and volumetric leakage losses (Q_{los}).

Isothermal conditions and fluid properties rationality: the model assumes strict thermal equilibrium where the hydraulic energy carrier's core properties remain invariant throughout a single pulse cycle. In physical implementations, continuous high-frequency pressure cycles peaking at 14.8 MPa inevitably induce localized viscous dissipation and thermal loading, driving tool holder operational temperatures from a baseline 20 °C up to 60 °C. This temperature rise shifts the dynamic viscosity (μ_M) from 0.032 Pa·s down to approximately 0.009 Pa·s, and lowers the baseline bulk modulus (χ) by roughly 6%. However, within a single ultra-short cycle ($\Delta t_{cycle} \leq 5$ ms), the thermal inertia of the heavy steel housing acts as a massive heat sink, preventing transient temperature spikes. Thus, treating each individual high-frequency step as isothermal is physically rational, while the macro thermal drift can be neglected without destabilizing the dynamic stability bounds.

Leakage-free simplification and error estimation: volumetric losses through the plunger clearance rings were excluded from the dynamic mass flow balance equations. Quantifying this simplification via the Petroff and laminar slit equations confirms that the combined dynamic leakage rate from both plungers ($Q_{los\Sigma}$) equals approximately 4.50×10^{-6} mm³/s. Evaluated against the total active volume of the multiplier chamber ($W_0 = 2656.44$ mm³), the cumulative fluid volume loss per operational cycle is less than 1.8×10^{-11} mm³. This introduces a deterministic numerical error margin of less than 0.0001% regarding pressure amplitude prediction (p_A), mathematically confirming that neglecting fluid leakage has a negligible impact on the drive's global dynamic response.

Cavitation dynamics and boundary error envelope: the fluid domain is modeled via continuous elastic compliance relations (Hooke's law for fluids), assuming single-phase flow integrity. The structural titanium optimization effectively mitigates rapid expansion low-pressure traps by matching the component return velocity with the elastic restoration rate of the slotted disc spring ($c = 1.1$ MN/m), keeping the fluid medium under continuous positive pressure above the saturated vapor pressure of the oil ($p_{sat} \approx 0.01$ MPa). However, micro-rarefaction under transient valve switching could still trigger localized gaseous cavitation if the operating frequency is pushed beyond 250 Hz under extreme technological loads ($F_x > 1500$ N). This multi-phase flow discontinuity represents a structural error envelope of approximately 3–5% in amplitude decay, which defines the upper boundary limit of the current mathematical model. To mitigate these non-linear thermal shifts and cutting load uncertainties, future expansions aim to explore focused closed-loop control layouts.

4.3. Operational Boundary Limitations and Thermo-Cavitation Risks

Thermal degradation of energy carrier: the continuous high-frequency shearing of the viscous mineral oil (ITP-300) induces localized viscous dissipation, driving tool holder operational temperatures from a baseline 20 °C up to 60 °C. This thermal transformation triggers an exponential decay in fluid dynamic viscosity (μ_M) according to the Reynolds formulation, collapsing it from 0.032 Pa·s down to approximately 0.009 Pa·s, while causing a minor linear contraction of the bulk modulus (χ) by roughly 6%. This fluid softening reduces the active structural damping ratio (ζ), introducing a minor risk of parasitic micro-vibrations if thermal equilibrium is not maintained via external cooling.

High-frequency cavitation thresholds: during the rapid fluid-power relaxation phase, when the parametric valve vents the pressure wave, the localized transient pressure inside the multiplier chamber (W_0) drops abruptly. If the operating frequency is pushed beyond 250 Hz under extreme inertial loads, the local pressure drops below the saturated vapor pressure of the mineral oil ($p_{sat} \approx 0.01$ MPa). This satisfies the classic criteria for high-frequency hydrodynamic cavitation, triggering bubble formation and subsequent violent collapse, which accelerates structural erosion of the plunger boundaries.

Future multi-physical closed-loop control: to mitigate these non-linear thermal shifts and cutting load uncertainties ($F_x > 1500$ N), future expansions of this research will transcend the current open-loop simulation configuration. We aim to integrate a tightly coupled closed-loop control framework utilizing a high-bandwidth piezoceramic sensor network embedded within the tool holder, enabling real-time frequency-amplitude regulation.

5. Conclusions

Main findings: the transient multi-physics simulation of the baseline pneumo-hydraulic impulse device layout revealed that a traditional high-inertia steel pusher ($m_{\Sigma p} = 2.30$ kg) paired with conventional cylindrical springs triggers a destructive “static lock-up” state at operating frequencies near 200 Hz under a predicted simulated cutting load ($F_x = 1200$ N).

From a fluid dynamics perspective, the excessive structural mass suppresses rapid pressure relaxation within the multiplier chamber, shifting the cutting tool velocity (v_t) into an unrelaxed negative drift zone and entirely dissipating fluid energy through internal hydraulic compliance rather than useful mechanical stroke.

Scientific contributions: the developed mathematical framework transcends classical linear single-degree-of-freedom approximations by introducing a tightly coupled, non-linear system combining high-frequency compressible gas dynamics (Saint-Venant-Wantzel formulations) with the isothermal compressibility of the viscous hydraulic fluid (Hooke's law for fluids) and D'Alembert structural expressions.

An eigenvalue trajectory and modal stability analysis mathematically validated that transitioning from the heavy steel pusher to an optimized low-inertia titanium alloy (Ti-6Al-4V Grade 5) actuator layout paired with a high-stiffness slotted disc spring ($c = 1.1 \text{ MN/m}$) forces the system roots to collapse onto the real axis, achieving a stable, critically damped aperiodic operating regime ($\zeta \approx 1.0$).

Engineering implications: minimizing structural inertia successfully expands the stable operating frequency window of the vibration drive by a factor of three (150–250 Hz), maintaining highly stable, repeatable square-like pressure cycles peaking at 14.8 MPa inside the multiplier.

The modernized layout ensures a robust, load-independent tool displacement plateau of 0.15–0.18 mm, which is optimal for finish vibration-assisted turning of advanced hard-to-cut alloys, entirely utilizing a standard shop-floor pneumatic network (0.6 MPa) and eliminating bulky external hydraulic infrastructure.

Future work: future research steps will be oriented toward the fabrication and manufacturing of a full-scale physical pneumo-hydraulic impulse drive prototype. Systemic experimental validations will be performed to empirically quantify the degree of adequacy of the formulated multi-physics mathematical framework, verify the underlying boundary assumptions, and evaluate the microrelief quality of turned titanium components under high-frequency intermittent cutting loads.

Author Contributions

R.O. and A.S.: conceptualisation, methodology, writing—original draft preparation, supervision; A.S., O.P. and D.B.: statistics; A.S., O.P., D.B. and V.P.: investigation, test, reviewing. A.S., O.P., D.B. and V.P.: tests, validation, visualization and editing. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

Appendix A

Table A1. Comprehensive design parameters, state variables, and energy carrier properties used in the PHID mathematical model.

Symbol	Parameter Description	Value	Unit
m_{18}	Mass of the multiplier plunger 18	0.85	kg
m_{19}	Mass of the multiplier plunger 19	0.85	kg
m_{20}	Mass of the cutting tool cutter 20 (including 1/3 of spring 21)	1.45	kg
m_{efm}	Effective mass of the pneumatic diaphragm 11 assembly	0.50	kg

Table A1. Cont.

Symbol	Parameter Description	Value	Unit
m_8	Combined mass of the PPG shut-off element 8 and 1/3 of spring 10	0.12	kg
$m_{\Sigma M}$	Consolidated mass of the pneumatic driving stage ($m_{efm} + m_{18}$)	1.35	kg
$m_{\Sigma P}$	Consolidated mass of the tool cutting actuator stage ($m_{19} + m_{20}$)	2.30	kg
$m_{\Sigma For}$	Total consolidated reduced mass of the nozzle/actuator assembly	0.35	kg
γ_{m1}	Dimensionless mass distribution ratio ($m_{\Sigma P}/m_{\Sigma M}$)	1.70	—
γ_{m2}	Dimensionless PPG valve mass scaling coefficient ($m_8/m_{\Sigma M}$)	0.088	—
d_2	Base diameter of the multiplier plungers 18 and 19	8.0	mm
d_{1h}	Average seating diameter of the first chamfered sealing stage	12.0	mm
d_{2h}	Diameter of the second spool-type sealing stage of the PPG valve	25.0	mm
A_{18}	Cross-sectional working area of plunger 18	50.24	mm ²
A_{19}	Cross-sectional working area of plunger 19	50.24	mm ²
A_{1h}	Initial sealing cross-sectional area of the PPG valve seat	113.1	mm ²
A_{2h}	Total secondary working area of the PPG valve spool	490.8	mm ²
A_{Mef}	Effective working surface area of the control diaphragm/membrane	2827.4	mm ²
k_M	Structural stiffness coefficient of the flexible rubber diaphragm itself	80.0	kN/m
k_1	Spring stiffness of the PPG opening pressure regulator	35.0	kN/m
k_2	Stiffness of the air motor loading coil spring 15	120.0	kN/m
k_3	Stiffness of the separating spring 26 between plungers	15.0	kN/m
k_4	Stiffness of the cutter return spring 21	45.0	kN/m
k_A	High volumetric stiffness of the closed hydraulic medium volume W_0	1.57	MN/m
x_{o1}	Pre-deformation adjust setting of the PPG opening spring 10	3.5	mm
x_{ovp}	Preliminary tension (preload) of the air motor spring 15	4.0	mm
x_{o2}	Pre-deformation layout setup of the internal separating spring 26	2.0	mm
x_{o4}	Initial pre-compression assembly deformation of spring 21	1.5	mm
x_{MFor}	Instantaneous forward coordinate displacement of diaphragm 11	0–0.5	mm
x_{Hp}	Continuous displacement travel of the PPG shut-off valve	0–4.5	mm
h_{18}	Working travel (stroke length) of the compression plunger 18	0.5	mm
h_H	Maximum structural opening displacement of the PPG valve element	4.5	mm
h_p	Target steady-state displacement stroke amplitude of the tool	0.15–0.18	mm
p_{net}	Centralized factory pneumatic shop network supply pressure	0.6	MPa
p_B	Dynamic pneumatic driving pressure within the working chamber B	0–0.6	MPa
p_A	High transient multiplier pressure within hydraulic Cavity A	0–14.8	MPa
p_o	Atmospheric base reference environmental pressure	0.1	MPa
W_0	Initial high-viscosity energy carrier volume of multiplier Cavity A	2656.44	mm ³
χ	Isometric volumetric bulk modulus of the ITP-300 mineral oil	1660.0	MPa
μ_M	Constant dynamic viscosity of the hydraulic energy carrier	0.032	Pa · s
ρ_M	Mass density constant of the hydraulic energy carrier	955.0	kg/m ³
Q_{los}	Volumetric laminar flow leakage losses through a single plunger slit	2.25×10^{-6}	mm ³ /s
$Q_{los\Sigma}$	Grand total cyclic fluid leakage combined from both plungers	4.50×10^{-6}	mm ³ /s
G_{air}	Instantaneous pneumatic air mass flow carrier consumption rate	—	kg/s
F_x	Dynamic technological cutting force component counteracting tool	1200.0	N
R_{20}	Constant static limiting friction force tracking tool shaft guide	150.0	N
C_8	Viscous oil film friction damping factor of the PPG valve guide	12.0	N · s/m
C_{18}	Viscous friction damp constant of the pusher plunger 18	45.0	N · s/m
C_{19}	Viscous fluid friction resistance constant of plunger 19	45.0	N · s/m
β_{18}	Linearized scaling damping coefficient during plunger 18 motion	—	1/s
$\omega_{\Sigma All}$	Consolidated total angular natural frequency of the primary block	3518.5	rad/s

Table A1. Cont.

Symbol	Parameter Description	Value	Unit
R	Standard universal gas constant parameter for dry atmospheric air	287.14	J/(kg · K)
T	Main operating thermal equilibrium temperature of the system	293.15	K
k_a	Dimensionless air gas thermodynamic expansion adiabatic index	1.4	—
ζ_H	Constant discharge contraction coefficient through open PPG gaps	0.62	—
dt	Micro-second time step utilized inside explicit Euler logic	2×10^{-7}	s

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