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Protocol-Based State Estimation for a Type of Delayed Neural Networks Subject to Probability Constraint

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Abstract: This article discusses the state estimation issue for a type of delayed neural networks (NNs). The investigated NN systems are assumed to face adversarial threats which posed on the data propagation process. Also, the information transmission delays between the sensor and estimator are taken into consideration. Moreover, with hope to better characterize the real-world situations, a constraint is posed on the measurement output by using a saturation function. The purpose of this article is to provide a framework for estimation of the state of NNs, ensuring that the estimation error is enforced not to escape a specific range in probability not less than a predetermined value. With the hope to coordinate the information propagation, the so-called Round-Robin protocol is used in the signal transmission channel. The main results are obtained by virtue of convex optimization algorithms, where the requested estimator parameters can be computed by solving the provided inequalities. On basis of the acquired main results, we further proceed to seek the locally optimal parameters according to different engineering demands. At last, the main theoretical results as well as the design method are demonstrated via an example.

Keywords: delayed neural networks; sensor saturations; probability constraint; deception attacks; Round-Robin protocol

1. Introduction

Originated in the middle of last century, the research on neural networks (NNs) has recently gained renewed interest ranging from academia (e.g., universities, institutes) to industries (e.g., Google, Baidu, DeepMind). The NNs nowadays has shown a strong power in many fields such as prediction and estimation, automatic control, especially in those relevant to artificial intelligence like pattern recognition, natural language recognition, intelligent city, to name but a few. This gives the rise of renewed research enthusiasm on this topic, and leads to a large amount of research fruits reported, see, e.g., [1–5].

Among all the research areas of neural networks, the state estimation issue possesses a paramount role, due to that the accurate state of each neuron is key to the subsequent applications such as optimization and approximation. Thus the estimation of NNs state has stirred a vast quantity of attention. It is, nevertheless, always the case that in practice, the collected information for state estimation is not perfect but with certain unanticipated constraints, such as data missing, dropout and outlier. Moreover, time delays exist in real-world implementations especially the systems in the context of network, which is recognized as one of the dominating sources for undesired dynamics oscillation, divergence, and even instability [6–8]. On the other hand, it is widely recognized that due to physical constraint or device protection, sensors used for the measurements collection usually have capacity limitations. That is, the outputs of the sensors are confined by certain maximum values, which, if not properly tackled, will cause significant degradation on the measurement information and the subsequent state estimation [9, 10]. As a consequence, many results have been published on estimation of NNs in the occurrence of sensor saturation as well as time delays, see, e.g., [11–13] for some recent advances, among which, however, more complicated network-induced engineering complexities have not been taken into account.

The adoption of network within the system control framework has been more and more popular, owing to the merits that network possesses. However, the extensive use of network to communicate among system modules will inevitably face cyber threats which make the security issue a research hot spot within the societies of systems, communication and control engineering [14–18]. The basic purpose of the research on cyber threats is to maintain the original systems' function and performance even if there exists malicious attacks. Usually, the cyber attack would performed against the data sharing channel by which the different modules propagate information within a system. Take the remote estimation for instance. The most perforable part (always the only possible way) for the adversary to launch attacks to a system is the communication channel during the data transmission procedure. By firstly capturing the information, the attackers could manipulate (erase, replace, or even destroy) the original signals as they want. Under such attacks, both function and performance of the systems under threat would be degraded or even collapsed. To date, there have been many sorts of cyber attack forms studied in literature, among which some extensively examined forms contain the so-called denial of service (DoS) attacks [19,20], deception attacks [21–23] as well as replay attacks [24,25]. Recently, due to the impact on system performance, the security property of a system has gained growing attention [26,27]; however, the relevant results on delay neural networks subject to network-induced complexities have been much scattered.

Another issue that should not be ignored resulting from the use of network is the network congestion in practical data transmission. This is mainly because the real-world communication channel is always subject to its physical constraint, namely, the bandwidth [28,29]. Fortunately, many protocols have been designed to conquer such a difficulty in a way that the nodes cannot send the information until permitted, according to certain predetermined rules [30, 31]. Some popularly used protocols, for instance, Round-Robin protocol (RRP) [9, 32], stochastic communication protocol (SCP) [1, 5, 19] and try-once-discard protocol (TODP) [23, 33] have been extensively studied by academia and used by industry. Nevertheless, as far as we know, under the regulation of the RRP, the state estimation for NNs with time delays while taking into account both sensor saturations and malicious attacks has been not adequately investigated.

In most control theory, it is usually requested to achieve the predetermined performances with a hard bound. That is to say, the performances are set to be met in 100% accuracy. It is worth emphasizing that, in the real-world applications there are many random factors that make such a hard constraint impossible to achieve. This gives the rise of so-called chance-constrained (also known as probability-guaranteed) design whose objective could be met as long as the probability of the performance can be confined within an allowable range is not less than a pre-specified value. In other words, the principle of the chance-constrained design aims to achieve the performance in probability less than 1. Such a design method has its merits especially that it could avoid unnecessary and sometimes impossible hard performance constraints. Up to now, though some progress on the problem of chance-constrained design has been discussed and certain initial results have been published [34–37], there are still many issues left for researchers to further examine. It is, therefore, the purpose of us to address the chance-constrained state estimation problem for delayed NNs subject to both sensor saturation and malicious attacks.

As a response to above discussions, this paper endeavors to propose an state estimation approach for a sort of delayed NNs, under the constraint of predetermined probability, and simultaneously takes into consideration of the sensor saturations and deception attacks. The novelties of our work are highlighted as follows: (i) the model of NNs considered in this paper takes into account simultaneously several frequently seen network-induced complexities (i.e., time delays, saturations and attacks), which is much closer to applications compared to the existing models; (ii) we utilize RRP to mitigate the effect from limited communication resource posed on the state estimation performance; and (iii) the chance-constrained design principle is used in our paper, which avoid unnecessary stringent design constraint and is thus much more welcome in practical engineering.

2. Problem Formulation

The neural networks under consideration is as follows:

$$\begin{cases} x(s+1) = A(s)x(s) + D(s)f(x(s)) + W(s)g(x(s-\tau)) \\ \qquad\qquad + L(s)\mu(s) \\ y(s) = \sigma(C(s)x(s)) + E(s)v(s) \\ x(l) = \phi(l),\ l \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(s) \in \mathbb{R}^{n_x}$ and $y(s) \in \mathbb{R}^{n_y}$, respectively, stand for the state and the measurement output; $A(s) = \mathrm{diag}\{a_1(s), a_2(s), \ldots, a_{n_x}(s)\}$ is the coefficient matrix with $|a_i(s)| < 1$; $D(s) = [d_{ij}(s)]_{n_x \times n_x}$ is the connection weighting matrix, and $W(s) = [w_{ij}(s)]_{n_x \times n_x}$ represents the delayed connection weighting matrix; $\mu(s) \in \mathbb{R}^{\mu}$ and $v(s) \in \mathbb{R}^{v}$ denote the process and measurement disturbances; $\tau \in \mathbb{Z}^{+}$ denotes the discrete delay; $f(x(s)) \in \mathbb{R}^{n_x}$ and $g(x(s-\tau)) \in \mathbb{R}^{n_x}$ are activation functions of the neuron; $L(s), C(s)$ and $E(s)$ are known constant matrices; $\phi(l)$ is the initial condition.

The saturation function $\sigma(\cdot)$ is given by

$$\sigma(u) = [\sigma(u_1)\ \sigma(u_2)\ \cdots\ \sigma(u_{n_y})]^{\mathrm{T}} \tag{2}$$

with $\sigma(u_i) = \text{sign}(u_i)\min\{\bar{u}, |u_i|\}, i = 1, 2, \cdots, n_y$, where $\bar{u} > 0$ is the saturation threshold.

According to [38], we can always find a scalar $0 < b_i < 1$ satisfying the following inequality:

$$(\sigma(u_i) - b_i u_i)(\sigma(u_i) - u_i) \leq 0. \tag{3}$$

Assumption 1. *The noises $\mu(s)$ and $v(s)$ are assumed to be unknown-but-bounded (UBB) satisfying the following inequality constraints:*

$$\begin{cases} \mu(s) \in \mathcal{P}(s) \triangleq \{\mu(s) : \mu^{\mathrm{T}}(s)P^{-1}(s)\mu(s) \leq 1\} \\ v(s) \in \mathcal{R}(s) \triangleq \{v(s) : v^{\mathrm{T}}(s)R^{-1}(s)v(s) \leq 1\} \end{cases} \tag{4}$$

with two known matrices $P(s) > 0$ and $R(s) > 0$.

Assumption 2. *The neural activation function $f(\cdot)$ and $g(\cdot)$ ($f(0) = g(0) = 0$) satisfy*

$$[f(x) - f(y) - U_1(x - y)]^{\mathrm{T}}[f(x) - f(y) - U_2(x - y)] \leq 0 \tag{5}$$

$$[g(x) - g(y) - V_1(x - y)]^{\mathrm{T}}[g(x) - g(y) - V_2(x - y)] \leq 0 \tag{6}$$

where U_j, V_j ($j = 1, 2$) are known matrices.

Let $y(s) = [y_1^{\mathrm{T}}(s)\ y_2^{\mathrm{T}}(s)\ \cdots\ y_{n_y}^{\mathrm{T}}(s)]^{\mathrm{T}}$, where $y_i(s)(i = 1, 2, \cdots, n_y)$ is the measurement output collecting by the i-th sensor node. Then, the measurement data the estimator received $\tilde{y}(s)$ can be expressed as $\tilde{y}(s) = [\tilde{y}_1^{\mathrm{T}}(s)\ \tilde{y}_2^{\mathrm{T}}(s)\ \cdots\ \tilde{y}_{n_y}^{\mathrm{T}}(s)]^{\mathrm{T}}$.

In this paper, as shown in Figure 1, the RRP is adopted to regulate the transmission of measurement data.

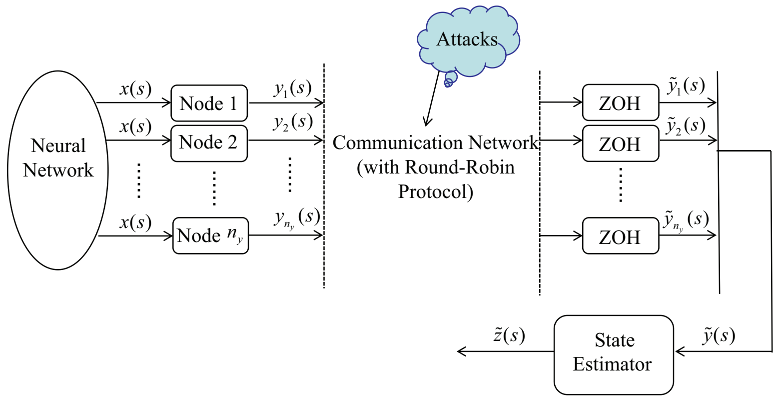

Figure 1. The structure of state estimation with RRP under deception attacks.

Assume that $\xi(s) \in \{1, 2, \cdots, n_y\}$ is the chosen node which gets the permission to use the transmission channel at step s. By regulation of the RRP, we can obtain that $\xi(s + n_y) = \xi(s)$ for all $s \in [0, N]$. Denote $\mathcal{V} = \{\xi(0), \xi(1), \cdots, \xi(n_y - 1)\}$. It is easy to see that $\mathcal{V}$ is a permutation of $\{1, 2, \cdots, n_y\}$ characterizing the sequence of transmission. Therefore, we can acquire $\xi(s)$ by

$$\xi(s) = \mathcal{V}(\text{mod}(s, n_y) + 1). \tag{7}$$

By virtue of zero-order holds, the updating rule of $\tilde{y}_i(s)(s > 0, i = 1, \cdots, n_y)$ is set to be

$$\tilde{y}_i(s) = \begin{cases} y_i(s), & \text{if } i = \xi(s) \\ \tilde{y}_i(s-1), & \text{otherwise} \end{cases} \tag{8}$$

with the initial condition $\tilde{y}_i(l) = 0(-\tau \leq l \leq 0)$.

Then, $\tilde{y}(s)$ can be written as

$$\tilde{y}(s) = \Phi_{\xi(s)}y(s) + (I - \Phi_{\xi(s)})\tilde{y}(s - 1) \tag{9}$$

where $\Phi_{\xi(s)} = \text{diag}\{\delta(\xi(s) - 1), \delta(\xi(s) - 2), \cdots, \delta(\xi(s) - n_y)\}$ is a matrix representing the update, and $\delta(\cdot)$ is the Kronecker delta operator.

In this paper, we also take the deception attack into account. We consider the following deception attack scenario. During the data transmission process, the attackers first capture the propagated data, on basis of which new signals will be generated before injecting to the captured data as deception signals. In such a way, the attackers have the capability to deteriorate the estimation performance through manipulating the transmitted information. Here, it should be emphasized that although the attackers have the ability to obtain/manipulate the shared information, they have also, on the other hand, certain constraints on the capability on launching such attacks. To be specific, the attackers are assumed to perform the attacks successfully at some reasonable rate. As such, the measurement information actually received by estimator, $\tilde{y}(s)$, is described by

$$\begin{aligned}\tilde{y}(s)=&(1-\beta(s))\Phi_{\xi(s)}y(s)+\beta(s)\tilde{\Phi}_{\xi(s)}\zeta(s)\\&+(I-\Phi_{\xi(s)})\tilde{y}(s-1)\end{aligned}\tag{10}$$

where $\tilde{\Phi}_{\xi(s)}=\Phi_{\xi(s)}\mathcal{I}_{n_y}$ with $\mathcal{I}_{n_y}^{\mathrm{T}}=[\underbrace{1,1,\cdots,1}_{n_y}]$; $\zeta(s)$ is a Bernoulli distributed sequence denoting the deception attack with $||\zeta(s)||\leq\theta$ where $\theta>0$ is given. Moreover, we also assume that

$$\mathrm{Prob}\left\{\beta(s)=1\right\}=\bar{\beta},\quad \mathrm{Prob}\left\{\beta(s)=0\right\}=1-\bar{\beta}\tag{11}$$

where $\bar{\beta}$ is known.

Define

$$\tilde{\beta}(s)\triangleq\beta(s)-\bar{\beta},\tag{12}$$

and it is not difficult to have that

$$\mathbb{E}\left\{\tilde{\beta}(s)\right\}=0,\tag{13}$$

and

$$\begin{aligned}\mathbb{E}\left\{\tilde{\beta}^{\mathrm{T}}(s)\tilde{\beta}(s)\right\}&=\mathbb{E}\left\{(\beta(s)-\bar{\beta})^{\mathrm{T}}(\beta(s)-\bar{\beta})\right\}\\&=\bar{\beta}(1-\bar{\beta})\triangleq\alpha^2.\end{aligned}\tag{14}$$

Then, we have

$$\begin{aligned}\tilde{y}(s)=&(1-\bar{\beta})\Phi_{\xi(s)}y(s)+\bar{\beta}\tilde{\Phi}_{\xi(s)}\zeta(s)\\&-\tilde{\beta}(s)\Phi_{\xi(s)}y(s)+\tilde{\beta}(s)\tilde{\Phi}_{\xi(s)}\zeta(s)\\&+(I-\Phi_{\xi(s)})\tilde{y}(s-1).\end{aligned}\tag{15}$$

Denote $\tilde{x}(s)\triangleq[x^{\mathrm{T}}(s)\ \tilde{y}^{\mathrm{T}}(s-1)]^{\mathrm{T}}$, and $\omega(s)\triangleq[\mu^{\mathrm{T}}(s)\ v^{\mathrm{T}}(s)]^{\mathrm{T}}$, $\bar{\Phi}_{\xi(s)}\triangleq I-\Phi_{\xi(s)}$, we obtain the following augmented system

$$\begin{cases}\begin{aligned}\tilde{x}(s+1)=&\mathcal{A}_1(s)\tilde{x}(s)+\mathcal{D}(s)f(H\tilde{x}(s))+\mathcal{W}(s)\\&\times g(H\tilde{x}(s-\tau))+(\mathcal{L}_1(s)+\tilde{\mathcal{L}}_1(s))\omega(s)\\&+(\mathcal{G}_1(s)+\tilde{\mathcal{G}}_1(s))\zeta(s)\\&+(\mathcal{H}_1(s)+\tilde{\mathcal{H}}_1(s))\phi(C(s)H\tilde{x}(s))\\ \tilde{y}(s)=&\mathcal{A}_2(s)\tilde{x}(s)+(\mathcal{L}_2(s)+\tilde{\mathcal{L}}_2(s))\omega(s)\\&+(\mathcal{G}_2(s)+\tilde{\mathcal{G}}_2(s))\zeta(s)\\&+(\mathcal{H}_2(s)+\tilde{\mathcal{H}}_2(s))\phi(C(s)H\tilde{x}(s))\end{aligned}\end{cases}\tag{16}$$

where

$$
\begin{aligned}
&\mathcal{A}_1(s) \triangleq \begin{bmatrix} A(s) & 0 \\ 0 & \bar{\Phi}_{\xi(s)} \end{bmatrix}, \ \mathcal{D}(s) \triangleq \begin{bmatrix} D(s) \\ 0 \end{bmatrix}, \\
&\mathcal{W}(s) \triangleq \begin{bmatrix} W(s) \\ 0 \end{bmatrix}, \ \mathcal{L}_1(s) \triangleq \begin{bmatrix} L(s) & 0 \\ 0 & (1-\bar{\beta})\Phi_{\xi(s)}E(s) \end{bmatrix}, \\
&\tilde{\mathcal{L}}_1(s) \triangleq \begin{bmatrix} 0 & 0 \\ 0 & -\tilde{\beta}(s)\Phi_{\xi(s)}E(s) \end{bmatrix}, \ \mathcal{G}_1(s) \triangleq \begin{bmatrix} 0 \\ \bar{\beta}\tilde{\Phi}_{\xi(s)} \end{bmatrix}, \\
&\tilde{\mathcal{G}}_1(s) \triangleq \begin{bmatrix} 0 \\ \tilde{\beta}(s)\tilde{\Phi}_{\xi(s)} \end{bmatrix}, \ \mathcal{H}_1(s) \triangleq \begin{bmatrix} 0 \\ (1-\bar{\beta})\Phi_{\xi(s)} \end{bmatrix}, \\
&\tilde{\mathcal{H}}_1(s) \triangleq \begin{bmatrix} 0 \\ -\tilde{\beta}(s)\Phi_{\xi(s)} \end{bmatrix}, \ H \triangleq [I \ 0], \\
&\mathcal{A}_2(s) \triangleq \begin{bmatrix} 0 & \bar{\Phi}_{\xi(s)} \end{bmatrix}, \ \mathcal{L}_2(s) \triangleq \begin{bmatrix} 0 & (1-\bar{\beta})\Phi_{\xi(s)}E(s) \end{bmatrix}, \\
&\tilde{\mathcal{L}}_2(s) \triangleq \begin{bmatrix} 0 & -\tilde{\beta}(s)\Phi_{\xi(s)}E(s) \end{bmatrix}, \ \mathcal{G}_2(s) \triangleq \bar{\beta}\tilde{\Phi}_{\xi(s)}, \\
&\tilde{\mathcal{G}}_2(s) \triangleq \tilde{\beta}(s)\tilde{\Phi}_{\xi(s)}, \ \mathcal{H}_2(s) \triangleq (1-\bar{\beta})\Phi_{\xi(s)}, \\
&\tilde{\mathcal{H}}_2(s) \triangleq -\tilde{\beta}(s)\Phi_{\xi(s)}.
\end{aligned}
$$

For system (1), we construct the following state estimator:

$$\hat{x}(s+1) = K(s)\hat{x}(s) + G(s)\tilde{y}(s) \tag{17}$$

where $K(s)$ and $G(s)$ are parameters we need to determine.

Define $e(s) \triangleq \tilde{x}(s) - \hat{x}(s)$. Then, from (16) and (17), we can easily get the following estimation error dynamics:

$$
\begin{aligned}
e(s+1) =& (\mathcal{A}_1(s) - G(s)\mathcal{A}_2(s))\tilde{x}(s) - K(s)\hat{x}(s) \\
&+ \mathcal{D}(s)f(H\tilde{x}(s)) + \mathcal{W}(s)g(H\tilde{x}(s-\tau)) \\
&+ (\mathcal{L}_1(s) - G(s)\mathcal{L}_2(s))\omega(s) \\
&+ (\tilde{\mathcal{L}}_1(s) - G(s)\tilde{\mathcal{L}}_2(s))\omega(s) \\
&+ (\mathcal{G}_1(s) - G(s)\mathcal{G}_2(s))\zeta(s) \\
&+ (\tilde{\mathcal{G}}_1(s) - G(s)\tilde{\mathcal{G}}_2(s))\zeta(s) \\
&+ (\mathcal{H}_1(s) - G(s)\mathcal{H}_2(s))\phi(C(s)H\tilde{x}(s)) \\
&+ (\tilde{\mathcal{H}}_1(s) - G(s)\tilde{\mathcal{H}}_2(s))\phi(C(s)H\tilde{x}(s)).
\end{aligned} \tag{18}
$$

Assumption 3. *The initial condition and its estimate are assumed to satisfy:*

$$\mathbb{E}\left\{e^{\mathrm{T}}(l)Q^{-1}(l)e(l)\right\} \leq 1 \tag{19}$$

where $Q(l)(-\tau \leq l \leq 0)$ is known and positive definite.

In this paper, we shall determine parameters $K(s)$ and $G(s)$ in (17) ensuring that the following constraint is satisfied:

$$\mathbb{P}\left\{e^{\mathrm{T}}(s)\mathfrak{Q}^{-1}(s)e(s)\right\} \geq \mathbf{p} \tag{20}$$

where $\mathfrak{Q}(s) > 0$ is a given matrix characterizing the estimation precision, and $\mathbf{p}$ is a predetermined constant with $0 < \mathbf{p} < 1$.

3. Main Results

The following existing results would be useful in our further development.

Lemma 1. (S-procedure [39]) *Let $\psi_0(\cdot), \psi_1(\cdot), \ldots, \psi_m(\cdot)$ be quadratic functions of the variable $\varsigma \in \mathbb{R}^{n_x}$: $\psi_j(\varsigma) \triangleq \varsigma^{\mathrm{T}} S_j \varsigma (j = 0, 1, \ldots, m)$, where $S_j^{\mathrm{T}} = S_j$. If there exist $\epsilon_1 \geq 0, \epsilon_2 \geq 0, \ldots, \epsilon_m \geq 0$ such that $S_0 - \sum_{j=1}^{m} \epsilon_j S_j \leq 0$, then the following is true:*

$$\psi_1(\varsigma) \leq 0, \ldots, \psi_m(\varsigma) \leq 0 \rightarrow \psi_0(\varsigma) \leq 0.$$

Lemma 2. (Schur Complement Equivalence) *Given constant matrices* $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$*, where* $\mathcal{N}_1 = \mathcal{N}_1^{\mathrm{T}}$ *and* $0 < \mathcal{N}_2 = \mathcal{N}_2^{\mathrm{T}}$*, then* $\mathcal{N}_1 + \mathcal{N}_3^{\mathrm{T}}\mathcal{N}_2^{-1}\mathcal{N}_3 < 0$ *if and only if*

$$\begin{bmatrix} \mathcal{N}_1 & \mathcal{N}_3^{\mathrm{T}} \\ \mathcal{N}_3 & -\mathcal{N}_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\mathcal{N}_2 & \mathcal{N}_3 \\ \mathcal{N}_3^{\mathrm{T}} & \mathcal{N}_1 \end{bmatrix} < 0.$$

Lemma 3. [40] *Given a matrix* $\mathcal{M} > 0$ *and a vector* b *of compatible dimensions, an ellipsoid* $\mathcal{S}$ *is defined by*

$$\mathcal{S} \triangleq \left\{ z | (z-b)^{\mathrm{T}} \mathcal{M} (z-b) \le 1 \right\}$$

with z *a stochastic variable. If*

$$\mathbb{E} \left\{ z | (z-b)^{\mathrm{T}} \mathcal{M} (z-b) \right\} \le 1 - \boldsymbol{p}$$

holds for any given $0 < \boldsymbol{p} < 1$*, then we have*

$$\mathbb{P} \left\{ z \in \mathcal{S} \right\} \ge \boldsymbol{p}.$$

3.1. State Estimator Design

First, define

$$\begin{aligned}
\hat{\mathcal{L}}_1(s) &\triangleq \begin{bmatrix} 0 & 0 \\ 0 & -\alpha \Phi_{\xi(s)} E(s) \end{bmatrix}, \ \hat{\mathcal{L}}_2(s) \triangleq \begin{bmatrix} 0 & -\alpha \Phi_{\xi(s)} E(s) \end{bmatrix}, \\
\hat{\mathcal{G}}_1(s) &\triangleq \begin{bmatrix} 0 \\ \alpha \tilde{\Phi}_{\xi(s)} \end{bmatrix}, \ \hat{\mathcal{G}}_2(s) \triangleq \alpha \tilde{\Phi}_{\xi(s)}, \\
\hat{\mathcal{H}}_1(s) &\triangleq \begin{bmatrix} 0 \\ -\alpha \Phi_{\xi(s)} \end{bmatrix}, \ \hat{\mathcal{H}}_2(s) \triangleq -\alpha \Phi_{\xi(s)}.
\end{aligned}$$

Then, the following lemmas are necessary to be recalled.

Lemma 4. *Let* $K(s)$ *and* $G(s)$ *be given. If there exists a series of matrices* $\{Q(s) > 0\}_{0 \le s \le N+1}$*, series of positive constants* $\{\varepsilon_1(s), \varepsilon_2(s), \varepsilon_3(s), \varepsilon_4(s), \varepsilon_5(s), \varepsilon_6(s), \varepsilon_7(s)\}_{0 \le s \le N}$ *such that*

$$\begin{bmatrix} \Xi(s) & \Gamma^{\mathrm{T}}(s) & \hat{\Gamma}^{\mathrm{T}}(s) \\ \star & -Q(s+1) & 0 \\ \star & \star & -Q(s+1) \end{bmatrix} \le 0 \tag{21}$$

where

$$\begin{aligned}
\Xi(s) &\triangleq -\operatorname{diag}\{1,0,0,0,0,0,0,0\} - \sum_{i=1}^{7} \varepsilon_i(s) \Lambda_i(s), \\
\Gamma(s) &\triangleq [\Gamma_1(s) \ \mathcal{D}(s) \ \mathcal{W}(s) \ \Gamma_2(s) \ \Gamma_3(s) \ \Gamma_4(s) \ \Gamma_5(s) \ 0], \\
\hat{\Gamma}(s) &\triangleq [0 \ \ 0 \ \ 0 \ \ \hat{\Gamma}_1(s) \ \hat{\Gamma}_2(s) \ \hat{\Gamma}_3(s) \ \ 0 \ \ 0], \\
\Lambda_1(s) &\triangleq \operatorname{diag}\{-1,0,0,0,0,0,I,0\}, \\
\Lambda_2(s) &\triangleq \operatorname{diag}\{-1,0,0,0,0,0,0,I\}, \\
\Lambda_3(s) &\triangleq \operatorname{diag}\{0,0,0,0,0,I,0,0\} - \mathcal{I}_1^{\mathrm{T}} M C(s) H N(s) \\
&\quad - N^{\mathrm{T}}(s) H^{\mathrm{T}} C^{\mathrm{T}}(s) M^{\mathrm{T}} \mathcal{I}_1 \\
&\quad + N^{\mathrm{T}}(s) H^{\mathrm{T}} C^{\mathrm{T}}(s) B C(s) H N(s), \\
\Lambda_4(s) &\triangleq \operatorname{diag}\{0,I,0,0,0,0,0,0\} - \mathcal{I}_2^{\mathrm{T}} \bar{U}_1 H N(s) \\
&\quad - N^{\mathrm{T}}(s) H^{\mathrm{T}} \bar{U}_1^{\mathrm{T}} \mathcal{I}_2 + N^{\mathrm{T}}(s) H^{\mathrm{T}} \bar{U}_2 H N(s), \\
\Lambda_5(s) &\triangleq \operatorname{diag}\{0,0,I,0,0,0,0,0\} - \mathcal{I}_3^{\mathrm{T}} \bar{V}_1 H Z(s) \\
&\quad - Z^{\mathrm{T}}(s) H^{\mathrm{T}} \bar{V}_1^{\mathrm{T}} \mathcal{I}_3 + Z^{\mathrm{T}}(s) H^{\mathrm{T}} \bar{V}_2 H Z(s), \\
\Lambda_6(s) &\triangleq \operatorname{diag}\left\{-2,0,0,\operatorname{diag}\left\{P^{-1}(s), R^{-1}(s)\right\},0,0,0,0\right\}, \\
\Lambda_7(s) &\triangleq \operatorname{diag}\left\{-\theta^2,0,0,0,I,0,0,0\right\}
\end{aligned}$$

with

$$\begin{aligned}
\Gamma_1(s) &\triangleq (\mathcal{A}_1(s) - G(s)\mathcal{A}_2(s) - K(s))\hat{x}(s), \\
\Gamma_2(s) &\triangleq \mathcal{L}_1(s) - G(s)\mathcal{L}_2(s), \\
\Gamma_3(s) &\triangleq \mathcal{G}_1(s) - G(s)\mathcal{G}_2(s), \\
\Gamma_4(s) &\triangleq \mathcal{H}_1(s) - G(s)\mathcal{H}_2(s), \\
\Gamma_5(s) &\triangleq (\mathcal{A}_1(s) - G(s)\mathcal{A}_2(s))T(s), \\
\hat{\Gamma}_1(s) &\triangleq \hat{\mathcal{L}}_1(s) - G(s)\hat{\mathcal{L}}_2(s), \\
\hat{\Gamma}_2(s) &\triangleq \hat{\mathcal{G}}_1(s) - G(s)\hat{\mathcal{G}}_2(s), \\
\hat{\Gamma}_3(s) &\triangleq \hat{\mathcal{H}}_1(s) - G(s)\hat{\mathcal{H}}_2(s), \\
\mathcal{I}_1 &\triangleq [0\ 0\ 0\ 0\ 0\ I\ 0\ 0], \\
\mathcal{I}_2 &\triangleq [0\ I\ 0\ 0\ 0\ 0\ 0\ 0], \\
\mathcal{I}_3 &\triangleq [0\ 0\ I\ 0\ 0\ 0\ 0\ 0], \\
N(s) &\triangleq [\hat{x}(s)\ 0\ 0\ 0\ 0\ 0\ T(s)\ 0], \\
Z(s) &\triangleq [\hat{x}(s-\tau)\ 0\ 0\ 0\ 0\ 0\ 0\ T(s-\tau)], \\
M &\triangleq \frac{1}{2}(B+I),\ \bar{U}_1 \triangleq \frac{1}{2}(U_1+U_2),\ \ \bar{V}_1 \triangleq \frac{1}{2}(V_1+V_2), \\
\bar{U}_2 &\triangleq \frac{1}{2}(U_1^{\mathrm{T}}U_2 + U_2^{\mathrm{T}}U_1),\ \ \bar{V}_2 \triangleq \frac{1}{2}(V_1^{\mathrm{T}}V_2 + V_2^{\mathrm{T}}V_1),
\end{aligned}$$

then the following is true:

$$\mathbb{E}\left\{e^{\mathrm{T}}(s)Q^{-1}(s)e(s)\right\} \le 1. \tag{22}$$

Proof. The proof is performed by following the mathematical induction principle. At the first step, we know from Assumption 3 that

$$\mathbb{E}\left\{e^{\mathrm{T}}(0)Q^{-1}(0)e(0)\right\} \le 1. \tag{23}$$

Then, at the deductive step, for any $s > 0$, we assume that the following is true:

$$\mathbb{E}\left\{e^{\mathrm{T}}(s)Q^{-1}(s)e(s)\right\} \le 1. \tag{24}$$

Consequently, we only need to show that inequality (19) holds at $s+1$ by using the the given condition. As a matter of fact, from (24), it is easy, according to [41] to see that we can always find a vector $q(s) \in \mathbb{R}^{n_q}$ with $\mathbb{E}\left\{q^{\mathrm{T}}(s)q(s)\right\} \le 1$ satisfying

$$\tilde{x}(s) = \hat{x}(s) + T(s)q(s) \tag{25}$$

where $T(s)$ is such that $Q(s) = T(s)T^{\mathrm{T}}(s)$.

By substituting (25) into (18), (18) can be rewritten by:

$$\begin{aligned}
e(s+1) =& (\mathcal{A}_1(s) - G(s)\mathcal{A}_2(s))T(s)q(s) \\
&+ (\mathcal{A}_1(s) - G(s)\mathcal{A}_2(s))\hat{x}(s) - K(s)\hat{x}(s) \\
&+ \mathcal{D}(s)f(H\tilde{x}(s)) + \mathcal{W}(s)g(H\tilde{x}(s-\tau)) \\
&+ (\mathcal{L}_1(s) - G(s)\mathcal{L}_2(s))\omega(s) \\
&+ (\tilde{\mathcal{L}}_1(s) - G(s)\tilde{\mathcal{L}}_2(s))\omega(s) \\
&+ (\mathcal{G}_1(s) - G(s)\mathcal{G}_2(s))\zeta(s) \\
&+ (\tilde{\mathcal{G}}_1(s) - G(s)\tilde{\mathcal{G}}_2(s))\zeta(s) \\
&+ (\mathcal{H}_1(s) - G(s)\mathcal{H}_2(s))\phi(C(s)H\tilde{x}(s)) \\
&+ (\tilde{\mathcal{H}}_1(s) - G(s)\tilde{\mathcal{H}}_2(s))\phi(C(s)H\tilde{x}(s)).
\end{aligned} \tag{26}$$

Denote

$$\eta(s) = [1\ f^{\mathrm{T}}(H\tilde{x}(s))\ g^{\mathrm{T}}(H\tilde{x}(s-\tau))\ \omega^{\mathrm{T}}(s)\ \zeta^{\mathrm{T}}(s)\ \phi^{\mathrm{T}}(C(s)H\tilde{x}(s))\ q^{\mathrm{T}}(s)\ q^{\mathrm{T}}(s-\tau)]^{\mathrm{T}},$$

then, we have

$$e(s+1) = (\Gamma(s) + \tilde{\Gamma}(s))\eta(s) \tag{27}$$

where

$$\begin{aligned}\Gamma(s) &= [\Gamma_1(s)\ \mathcal{D}(s)\ \mathcal{W}(s)\ \Gamma_2(s)\ \Gamma_3(s)\ \Gamma_4(s)\ \Gamma_5(s)\ 0],\\ \tilde{\Gamma}(s) &= [0\ \ 0\ \ 0\ \ \tilde{\Gamma}_1(s)\ \tilde{\Gamma}_2(s)\ \tilde{\Gamma}_3(s)\ \ 0\ \ 0]\end{aligned}$$

with

$$\begin{aligned}\tilde{\Gamma}_1(s) &= \tilde{\mathcal{L}}_1(s) - G(s)\tilde{\mathcal{L}}_2(s),\\ \tilde{\Gamma}_2(s) &= \tilde{\mathcal{G}}_1(s) - G(s)\tilde{\mathcal{G}}_2(s),\\ \tilde{\Gamma}_3(s) &= \tilde{\mathcal{H}}_1(s) - G(s)\tilde{\mathcal{H}}_2(s).\end{aligned}$$

Subsequently, by taking (13) and (14), one can deduce that

$$\begin{aligned}&\mathbb{E}\left\{e^{\mathrm{T}}(s+1)Q^{-1}(s+1)e(s+1)\right\}\\ =&\mathbb{E}\left\{\eta^{\mathrm{T}}(s)(\Gamma(s)+\tilde{\Gamma}(s))^{\mathrm{T}}Q^{-1}(s+1)(\Gamma(s)+\tilde{\Gamma}(s))\eta(s)\right\}\\ =&\eta^{\mathrm{T}}(s)\Gamma^{\mathrm{T}}(s)Q^{-1}(s+1)\Gamma(s)\eta(s)\\ &+\eta^{\mathrm{T}}(s)\hat{\Gamma}^{\mathrm{T}}(s)Q^{-1}(s+1)\hat{\Gamma}(s)\eta(s)\\ \triangleq&\eta^{\mathrm{T}}(s)\Lambda(s)\eta(s).\end{aligned} \tag{28}$$

On the other hand, it can be obtained from $\mathbb{E}\left\{q^{\mathrm{T}}(s)q(s)\right\} \leq 1$ that

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_1(s)\eta(s)\right\} \leq 0. \tag{29}$$

Similarly, it is inferred from $\mathbb{E}\left\{q^{\mathrm{T}}(s-\tau)q(s-\tau)\right\} \leq 1$

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_2(s)\eta(s)\right\} \leq 0. \tag{30}$$

Next, for the saturation function $\phi(\cdot)$, we are readily to acquire from (3) that

$$\begin{aligned}&(\phi(C(s)H\tilde{x}(s)) - BC(s)H\tilde{x}(s))^{\mathrm{T}}\\ &\qquad\times(\phi(C(s)H\tilde{x}(s))C(s)H\tilde{x}(s)) \leq 0\end{aligned} \tag{31}$$

where $B \triangleq \mathrm{diag}\{b_1, b_2, \cdots, b_{ny}\}$, and then by denoting $M \triangleq \frac{1}{2}(B+I)$, (31) can be further expressed as

$$\begin{aligned}&\phi^{\mathrm{T}}(C(s)H\tilde{x}(s))\phi(C(s)H\tilde{x}(s)) - \phi^{\mathrm{T}}(C(s)H\tilde{x}(s))M\\ &\times C(s)H\tilde{x}(s) - \tilde{x}^{\mathrm{T}}(s)H^{\mathrm{T}}C^{\mathrm{T}}(s)M^{\mathrm{T}}\phi(C(s)H\tilde{x}(s))\\ &-\tilde{x}^{\mathrm{T}}(s)H^{\mathrm{T}}C^{\mathrm{T}}(s)BC(s)H\tilde{x}(s)) \leq 0.\end{aligned} \tag{32}$$

Consequently, by substituting (25) into (32), (32) can be described by

$$\begin{aligned}&\eta^{\mathrm{T}}(s)\mathrm{diag}\left\{0,0,0,0,0,I,0,0\right\}\eta(s) - \eta^{\mathrm{T}}(s)\mathcal{I}_1^{\mathrm{T}}MC(s)\\ &\times HN(s)\eta(s) - \eta^{\mathrm{T}}(s)N^{\mathrm{T}}(s)H^{\mathrm{T}}C^{\mathrm{T}}(s)M^{\mathrm{T}}\mathcal{I}_1\eta(s)\\ &+\eta^{\mathrm{T}}(s)N^{\mathrm{T}}(s)H^{\mathrm{T}}C^{\mathrm{T}}(s)BC(s)HN(s)\eta(s) \leq 0\end{aligned} \tag{33}$$

or,

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_3(s)\eta(s)\right\} \leq 0. \tag{34}$$

By the same line, for the nonlinear function $f(\cdot)$, we can obtain from (5) that

$$(f(H\tilde{x}(s)) - U_1H\tilde{x}(s))^{\mathrm{T}}(f(H\tilde{x}(s)) - U_2H\tilde{x}(s)) \leq 0, \tag{35}$$

by denoting $\bar{U}_1 \triangleq \frac{1}{2}(U_1 + U_2)$, $\bar{U}_2 \triangleq \frac{1}{2}(U_1^{\mathrm{T}}U_2 + U_2^{\mathrm{T}}U_1)$, can be equivalently described by

$$\begin{aligned}&\eta^{\mathrm{T}}(s)\mathrm{diag}\left\{0, I, 0, 0, 0, 0, 0, 0\right\}\eta(s) - \eta^{\mathrm{T}}(s)\mathcal{I}_2^{\mathrm{T}}\bar{U}_1 \\ &\times HN(s)\eta(s) - \eta^{\mathrm{T}}(s)N^{\mathrm{T}}(s)H^{\mathrm{T}}\bar{U}_1^{\mathrm{T}}\mathcal{I}_2\eta(s) \\ &+ \eta^{\mathrm{T}}(s)N^{\mathrm{T}}(s)H^{\mathrm{T}}\bar{U}_2HN(s)\eta(s) \le 0\end{aligned} \tag{36}$$

or,

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_4(s)\eta(s)\right\} \le 0. \tag{37}$$

Similarly, one has

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_5(s)\eta(s)\right\} \le 0. \tag{38}$$

From (4), we can obtain the following inequalities:

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_6(s)\eta(s)\right\} \le 0. \tag{39}$$

For the deception attack signal $\zeta(s)$, we have from $||\zeta(s)|| \le \theta$ that

$$\mathbb{E}\left\{\eta^{\mathrm{T}}(s)\Lambda_7(s)\eta(s)\right\} \le 0. \tag{40}$$

According to Lemma 2, we know from (21) that

$$\Lambda(s) - \mathrm{diag}\left\{1, 0, 0, 0, 0, 0, 0, 0\right\} - \sum_{i=1}^{7}\varepsilon_i(s)\Lambda_i(s) \le 0 \tag{41}$$

which, taking Lemma 1 into account, leads to

$$\eta^{\mathrm{T}}(s)(\Lambda(s) - \mathrm{diag}\left\{1, 0, 0, 0, 0, 0, 0, 0\right\})\eta(s) \le 0. \tag{42}$$

Therefore, the inequality (22) holds under the given condition. This proof is now complete. □

Theorem 1. *Given a real-valued constant* $0 < \mathbf{p} < 1$ *and a series of matrices* $\{\mathfrak{Q}(s) > 0\}_{0 \le s \le N+1}$. *The inequality (20) is achieved if there are sets of positive constants* $\{\varepsilon_1(s), \varepsilon_2(s), \varepsilon_3(s), \varepsilon_4(s), \varepsilon_5(s), \varepsilon_6(s), \varepsilon_7(s)\}_{0 \le s \le N}$ *such that*

$$\begin{bmatrix}\Xi(s) & \Gamma^{\mathrm{T}}(s) & \hat{\Gamma}^{\mathrm{T}}(s) \\ \star & -\frac{1}{1-\mathbf{p}}\mathfrak{Q}(s+1) & 0 \\ \star & \star & -\frac{1}{1-\mathbf{p}}\mathfrak{Q}(s+1)\end{bmatrix} \le 0. \tag{43}$$

Moreover, $K(s)$ *and* $G(s)$ *can be obtained by resorting to solving the provided matrix inequalities step by step.*

Proof. By noting that $Q(s+1) = \frac{1}{1-\mathbf{p}}\mathfrak{Q}(s+1)$, we can easily prove Theorem 1 with help of Lemma 4. Thus, the detailed proof is omitted to save space. □

Remark 1. *Theorem 1 gives the sufficient condition under which the investigated problem is solvable, while guaranteeing that estimation errors* $e(s)$ *are confined within certain predetermined ellipsoidal range in a probability not less than the given one. The requested gains can then be found by dealing with the presented set of inequalities.*

3.2. Optimization Problem

In the following, we shall consider the optimization problems for the designed state estimator by making trade-off between two different performance indices.

Problem 1: With the purpose of seeking the locally optimal estimation accuracy under the satisfactory probability constraint, we shall calculate the estimator gains by minimizing the matrix trace of $\mathfrak{Q}(s)$.

Corollary 1. *Given* $\mathbf{p}$. *On basis of Theorem 1, a series of minimized-trace matrices* $\{\mathfrak{Q}(s)\}_{0 \le s \le N+1}$ *are guaranteed if the following is solvable:*

$$\min_{\substack{\mathfrak{Q}(s+1),K(s),G(s),\varepsilon_1(s),\varepsilon_2(s),\\ \varepsilon_3(s),\varepsilon_4(s),\varepsilon_5(s),\varepsilon_6(s),\varepsilon_7(s)}} \quad \mathrm{tr}[\mathfrak{Q}(s+1)] \tag{44}$$

$$\text{subject to} \quad (43)$$

In the following, assume $\mathbf{p}$ is time-varying and let $\mathbf{p}(s)$ be the probability constraint at specific step s. Denote $\mathbf{r}(s) \triangleq \frac{1}{1-\mathbf{p}(s)}$.

Problem 2: Under the satisfactory estimation accuracy described by matrix $\mathfrak{Q}(s)$, we shall minimize the variable $\mathbf{r}(s)$ with the aim of guaranteeing the minimum value of the probabilistic constraint at each time instant.

Corollary 2. *Given the series of $\{\mathfrak{Q}(s)\}_{0\le s\le N+1}$. On basis of Theorem 1, the minimum value of $\mathbf{p}(s)$ can be achieved provided that the following is feasible:*

$$\min_{\substack{\mathbf{r}(s),K(s),G(s),\varepsilon_1(s),\varepsilon_2(s),\\ \varepsilon_3(s),\varepsilon_4(s),\varepsilon_5(s),\varepsilon_6(s),\varepsilon_7(s)}} \quad \mathbf{r}(s) \tag{45}$$

$$\text{subject to} \quad \begin{cases} 1<\mathbf{r}(s)<+\infty \\ \begin{bmatrix} \Xi(s) & \Gamma^{\mathrm{T}}(s) & \hat{\Gamma}^{\mathrm{T}}(s) \\ \star & -\mathbf{r}(s)\mathfrak{Q}(s+1) & 0 \\ \star & \star & -\mathbf{r}(s)\mathfrak{Q}(s+1) \end{bmatrix} \le 0 \end{cases}$$

Notice that both Corollaries 1 and 2 can be easily proved according to Theorem 1; thus, the detailed proofs are omitted to avoid unnecessary denotation.

Remark 2. *Up to now, the addressed problem has been studied for delayed NNs subject malicious attack and sensor saturations. The feasibility of the investigated problem has been cast into the solvability of certain inequalities. We can obtain the desired estimator gains by finding the solutions to those provided matrix inequalities at each step. Within the established framework, we also have focused on the locally optimal estimator design issue via presenting two sub-optimal design algorithms.*

4. A Numerical Example

We now use an example to demonstrate the developed algorithm.

The NNs parameters are given as follows:

$$A(s) = \begin{bmatrix} -0.2+0.5\sin(0.2s) & 0 \\ 0 & 0.6\cos(0.5s) \end{bmatrix},$$

$$D(s) = \begin{bmatrix} 0.6-0.8\sin(0.5s) & -0.5 \\ 0.75 & 0.3\cos(0.3s) \end{bmatrix},$$

$$W(s) = \begin{bmatrix} 0.2\sin(0.5s) & 0.2 \\ 0 & 0.3\cos(0.3s) \end{bmatrix},$$

$$L(s) = \begin{bmatrix} 0.6\cos(s) \\ 0.5\sin(s) \end{bmatrix}, \quad E(s) = \begin{bmatrix} 0.3\sin(s) \\ 0.2\cos(s) \end{bmatrix},$$

$$C(s) = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}, \quad \tau = 2.$$

Select $\mu(s) = 0.4\cos(0.5s)$ and $v(s) = 0.5\sin(0.8s)$. Obviously, based on the Assumption 1, we can set $P(s) = 0.16$ and $R(s) = 0.25$.

Choose the following nonlinear activation functions

$$f(x(s)) = \begin{bmatrix} 0.4x_1(s) - \tanh(0.1x_1(s)) + 0.2x_2(s) \\ 0.8x_2(s) - \tanh(0.6x_2(s)) \end{bmatrix},$$

$$g(x(s)) = \begin{bmatrix} 0.5x_1(s) - \tanh(0.2x_1(s)) + 0.1x_2(s) \\ 0.7x_2(s) - \tanh(0.1x_1(s)) + 0.2x_1(s) \end{bmatrix},$$

to make the Assumption 2 is satisfied, we set

$$U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix},\ U_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0 & 0.8 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.7 \end{bmatrix},\ V_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.7 \end{bmatrix}.$$

Assume:

$$x(l) = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ \hat{x}(l) = \begin{bmatrix} 2.3 \\ 1.5 \\ 0.5 \\ 0.4 \end{bmatrix},\ \tilde{y}(l) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$Q(l) = \begin{bmatrix} 1.0396 & 0.1448 & 0.1448 & 0.1159 \\ 0.1448 & 1.1940 & 0.2414 & 0.1931 \\ 0.1448 & 0.2414 & 1.1940 & 0.1931 \\ 0.1159 & 0.1931 & 0.1931 & 1.1071 \end{bmatrix},\ -\tau \le l \le 0.$$

Choose

$$\mathbf{p} = 0.9,\ b_1 = b_2 = 0.1,\ \bar{\beta} = 0.6,$$
$$V = \{2, 1\},\ \theta = 0.25.$$

Figures 2–7 show the simulation results. Specifically, Figures 2 and 3 illustrate system state $x(s)$ (i.e., $x_1(s)$ and $x_2(s)$) and the corresponding estimates. Estimation errors $e_1(s)$ and $e_2(s)$ are shown in Figure 4, while Figures 5 and 6 depict the ideal measurement output $y(s)$ (i.e., $y_1(s)$ and $y_2(s)$) and the corresponding actual received measurement output. Figure 7 indicates the time step at which the system is attacked by adversaries.

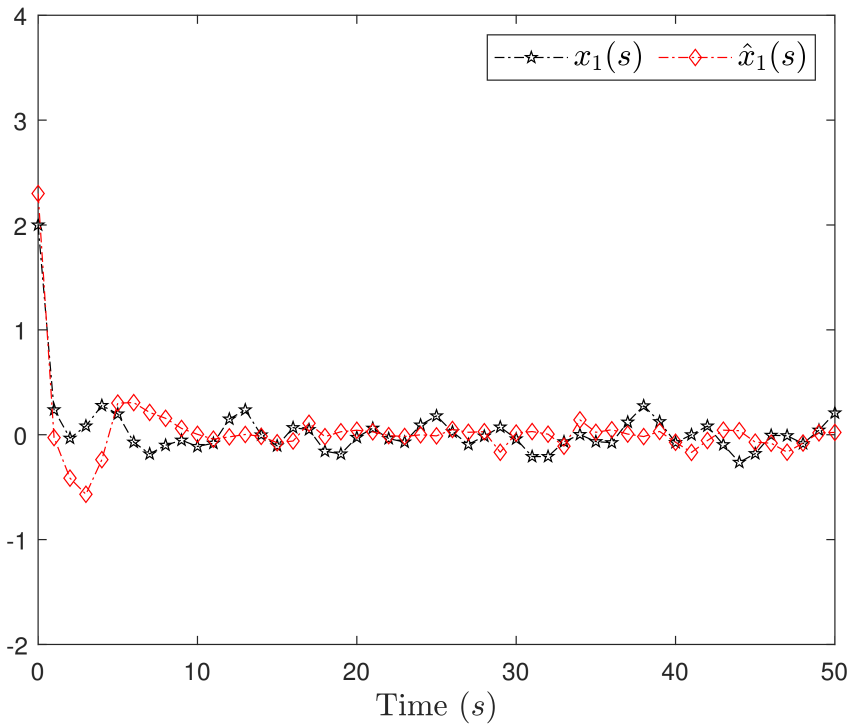

Figure 2. Trajectories of $x_1(s)$ and $\hat{x}_1(s)$.

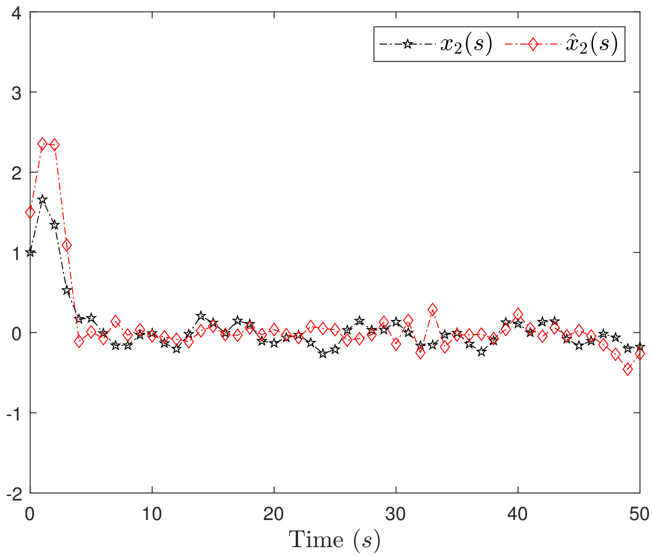

Figure 3. Trajectories of $x_2(s)$ and $\hat{x}_2(s)$.

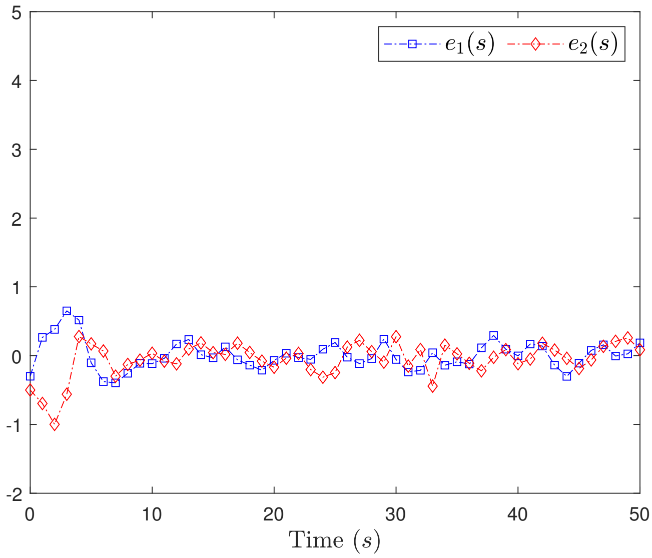

Figure 4. Trajectories of $e_1(s)$ and $e_2(s)$.

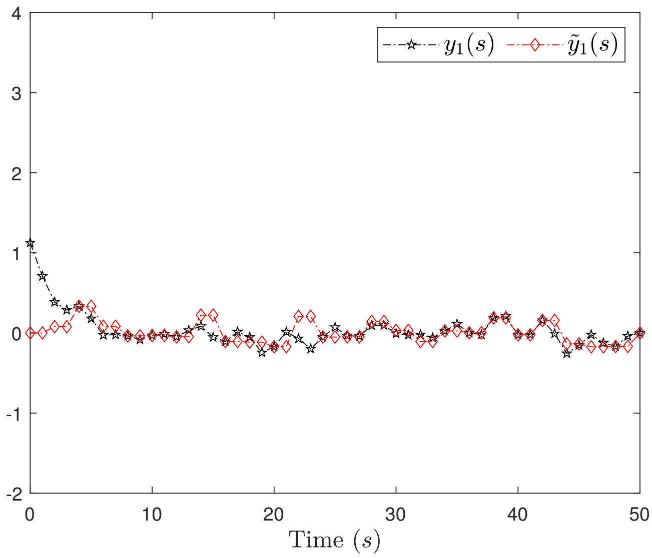

Figure 5. Trajectories of $y_1(s)$ and $\tilde{y}_1(s)$.

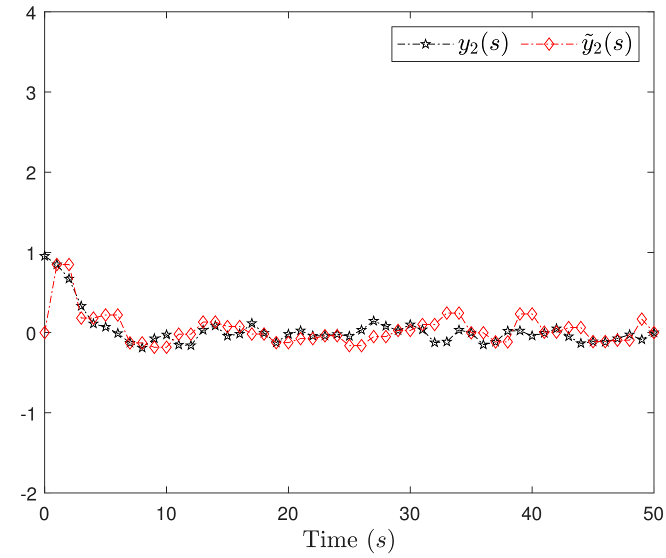

Figure 6. Trajectories of $y_2(s)$ and $\tilde{y}_2(s)$.

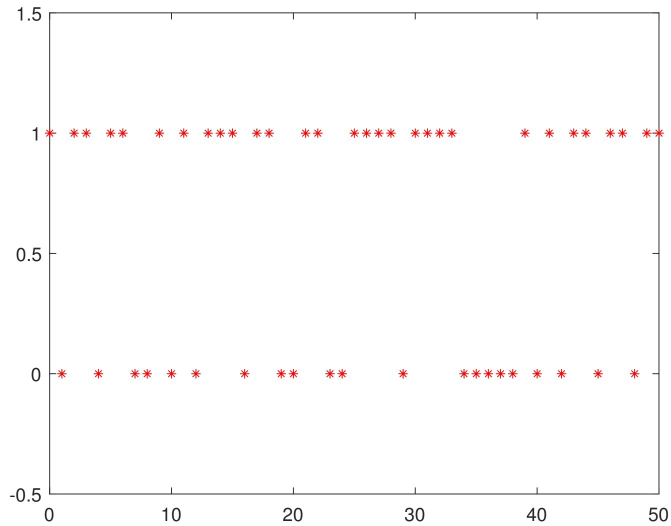

Figure 7. Whether the system suffers the deception attacks or not.

5. Conclusions

This article has proposed an algorithm to estimate the state information for a type of delayed neural networks under the constraints of both sensor saturations and deception attacks. The RRP has been utilized to regulate data sharing to make better use of the restricted communication resources. We have presented the sufficient conditions of the feasibility of the addressed state estimation problem, guaranteeing the estimation errors are enforced to stay within an predetermined range with an allowable probability not less than a predetermined one. According to the proposed algorithm, the requested estimator gains can be found by solving s series of matrix inequalities. Two sub-optimization problems have been put forward to seek estimators of locally optimal performance. Finally, we have used an example to demonstrate the proposed state estimation approach. One of our future research topics is to extend the main results to more general systems with more performance requirements or more network-induced phenomena [42,43].

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