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# Observer-Based Leader-Follower Coordination in Fractional-Order Multi-Manipulator Systems

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**Abstract:** This study focuses on the leader-follower control problem of fractional-order multi-manipulator systems in which certain state components of the follower manipulators are unmeasurable. To address this issue, a nonlinear state observer is developed to estimate the unknown states from system outputs. The design parameters of both the observer and the leader-follower control law are then determined using the linear matrix inequality (LMI) framework. Furthermore, by applying Lyapunov stability theory, sufficient conditions ensuring leader-follower consensus are established. The effectiveness and practicality of the proposed approach are subsequently demonstrated through numerical simulations.

**Keywords:** consensus analysis; fractional-order; manipulator system; leader-following control; state observer

## 1. Introduction

In recent years, manipulators have been widely applied in various fields owing to their distinctive advantages, which makes the tracking control of manipulator systems a central topic of research worldwide. For instance, in [1], to address the limited degrees of freedom of a single mobile manipulator, the stability of multiple mobile manipulators is investigated based on a Lagrangian equation model. The work in [2] focuses on solving the tracking control problem of manipulators with asymmetric, time-dependent output bounds by employing an event-triggered adaptive scheme. Ref. [3] develops a consensus control algorithm for flexible joint manipulator based on regional potential function constraints to address the tracking control under regional constraints. In multi-manipulator systems, each manipulator is capable of exchanging information and interacting with others, which enables the collaborative execution of complex tasks beyond the capability of a single manipulator and significantly enhances the overall efficiency and flexibility of the system [4–6].

In the work on tracking control of multi-manipulator systems, control strategies are generally classified into three categories according to the system's dynamic control methods: decentralized control, centralized control, and distributed control [7,8]. Among these, the decentralized control approach must address the coupling relationships among subsystems, making it more suitable for manipulator system reconfiguration, and it is primarily adopted in traditional manipulator systems. The centralized control method is similar to the traditional servo system control. Besides, the distributed control method can be further classified as distributed consensus control [9,10] and distributed formation control [11,12]. With low energy consumption, low communication burden, fast response speed and high reliability, it is more suitable for multi-manipulator system control and practical engineering applications. For example, in [13], a distributed regional optimal coverage controller is designed to achieve coverage control of manipulators with strong nonlinear characteristics. Ref. [14] proposes a distributed predictive control algorithm based on iterative linear quadratic regulator to ensure formation stability and cooperative control optimization in multi-manipulator cooperative handling. The distributed consensus control of multi-manipulator system with elastic joints is studied in [15]. According to whether the system has leaders or



not, it can be classified into tracking control with leaders and consensus control without leaders [16,17]. Among them, coordinated control with leaders can be further divided into tracking control (single leader) and inclusive control (multiple leaders) [18–20]. In their study on leader-following and leaderless consensus in the task space, a distributed control strategy applicable to multi-robot networks comprising kinematically and dynamically heterogeneous agents is introduced in [21].

Multi-manipulator systems, especially those involving flexible joints, elastic links, or complex interaction dynamics, often exhibit inherent memory and hereditary properties due to material elasticity, frictional effects, and unmodeled dynamics. These characteristics cannot be accurately captured by conventional integer-order models. Fractional-order models provide an effective way to describe such behaviors by incorporating nonlocal dynamics and memory effects into the system representation. Compared with integer-order formulations, they offer greater modeling flexibility and can more accurately reflect the dynamic response of systems with viscoelastic or hysteresis-like properties. Moreover, from a control perspective, the use of fractional-order dynamics allows for additional degrees of freedom in system design, which can be exploited to achieve improved robustness and smoother transient performance. In the context of observer design, this facilitates better handling of uncertainties and disturbances that exhibit temporal correlations. In recent years, a lot of researchers have studied the consensus problem of fractional-order multi-agent systems [22,23]. In [22], a bounded estimation approach was applied to address the containment control issue of fractional-order multi-agent systems. Meanwhile, [23] investigated the consensus problem of such systems by employing an observer-based method. There has been extensive research on modeling robotic manipulator systems by using fractional-order equations [24]. However, in practical scenarios, multi-agent systems are often subject to various uncertainties and adverse factors, such as Denial-of-Service (DoS) attacks, actuator faults, and time-varying communication topologies, which significantly challenge the achievement of reliable consensus. To address these issues, a variety of control strategies have been developed. For instance, boundary-based control approaches have been proposed to reduce dependence on full-state information, thereby lowering implementation cost. Buffer mechanisms have been introduced to mitigate the impact of DoS attacks by maintaining control continuity during communication interruptions [25]. In addition, adaptive pinning control methods have been applied to fractional-order singular multi-agent systems with switching topologies, effectively handling memory-dependent dynamics and network variations [26]. Moreover, distributed control frameworks based on fractional Lyapunov theory and LMI techniques have been widely adopted to ensure robust consensus in the presence of uncertainties and disturbances [27]. However, existing works mainly focus on integer-order multi-agent/manipulator systems. Fractional-order works seldom consider observer-based consensus under directed topologies. The integration of state estimation and consensus control in fractional-order settings remains limited.

Following the previous discussion, this work is devoted to addressing the tracking control issue in fractional-order multi-manipulator systems characterized by directed communication topologies. Unlike conventional integer-order observer-based consensus frameworks, the proposed method explicitly incorporates fractional-order dynamics with memory effects, leading to fundamentally different system behavior and stability characteristics. A nonlinear observer design tailored for fractional-order multi-manipulator systems is developed, where stability is established using fractional Lyapunov theory rather than classical approaches. The proposed framework simultaneously addresses fractional dynamics, directed topology, and leader-follower coordination, which are typically studied separately in existing works. Ultimately, simulation studies are performed to confirm both the effectiveness and applicability of the developed approach.

## 2. Preliminaries

### 2.1. Algebraic Graph Theory

The system communication topology composed of  $n$  manipulators can be described by a directed graph  $\Theta = (V, E, A)$ , here  $V = (V_1, V_2, \dots, V_n)$  is the set of nodes and  $E \subseteq V \times V$  is an edge set. The undirected edge  $E_{ij} = (V_j, V_i) \in E$  indicates that the manipulator  $i$  transmits the manipulator  $j$ .  $A = [a_{ij}]_{n \times n}$  is the adjacency matrix. In this paper, assuming that the main diagonal element satisfies  $a_{ii} = 0, i = 1, 2, \dots, n$ . When  $(V_i, V_j) \in E$ , we have  $a_{ij} = 1$ . Otherwise,  $a_{ij} = 0$ . The degree matrix  $D \in R^{n \times n}$  is constructed as a diagonal matrix that captures the degree of each node in the graph, namely,  $D = \text{diag}\{d_i\}$ ,  $d_i = \sum_{j=1}^n |a_{ij}|$ ,  $i = 1, 2, \dots, n$ .



Define the Laplace matrix  $L = [l_{ij}] \in R^{n \times n}$  is  $L = D - A$ , where  $l_{ii} = \sum_{j=1}^n |a_{ij}|$ ,  $\forall i = j$  and  $l_{ij} = -a_{ij}$ ,  $\forall i \neq j$ .

$$l_{ij} = \begin{cases} \sum_{j=1, j \neq i}^n |a_{ij}|, & \forall i = j \\ -a_{ij}, & \forall i \neq j \end{cases}$$

It is expressed as

In the communication topology, the communication matrix  $M \in R^{n \times n}$  between the leader and the follower manipulator is defined as  $M = \text{diag}\{m_i\}$ ,  $i = 1, 2, \dots, n$ , where  $m_i$  is the weighted gain. If there is a follower manipulator that can obtain the state information from the leader, the weighted gain  $m_i = 1$ . Otherwise,  $m_i = 0$ . In addition, the matrix  $H \in R^{n \times n}$  is defined as  $H = L + M$ , with  $H \geq 0$ . The presence of a directed spanning tree in a digraph  $\Theta$  is ensured if there exists a root node with directed paths leading to all remaining nodes. Concerning the Laplace matrix corresponding to the directed graph has the following conclusion.

**Lemma 1** ([28]). *If a directed graph  $\Theta$  contains a directed spanning tree, its corresponding Laplace matrix  $L$  has a zero eigenvalue and the other  $n-1$  eigenvalues all have positive real parts.*

## 2.2. Fractional Calculus Theory

**Definition 1** ([29]). *(Caputo integral) The Caputo fractional-order integral of the continuous derivable function  $f(t) \in R^n$  is*

$${}_0^C I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(s)}{(t-s)^{1-\alpha}} ds \quad (1)$$

of which  $\Gamma(\cdot)$  is the Gamma function.

**Definition 2.** *(Caputo differential) The Caputo fractional-order differential of the continuous derivable function  $f(t) \in R^n$  is*

$${}_0^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}} ds \quad (2)$$

where  $n = [\alpha] + 1$ , and  $f^{(n)}(\cdot)$  is the  $n$ -order derivative of  $f(\cdot)$ .

The common properties of Caputo fractional-order differential [30] are as follows

(1) *Linear property*

$${}_0^C D_t^\alpha [af(t) \pm bh(t)] = a[{}_0^C D_t^\alpha f(t)] \pm b[{}_0^C D_t^\alpha h(t)]$$

(2) *Superposition*

$${}_0^C D_t^p {}_0^C D_t^q [g(t)] = {}_0^C D_t^{p+q} [g(t)]$$

where  $a, b \in R$ , and  $g(t), h(t)$  are continuous function,  $n-1 < p < n$ ,  $p \in R$ ,  $0 < q < 1$ .

**Definition 3** ([31]). *(Kronecker product) Let matrices  $A = [a_{ij}] \in R^{m \times n}$ ,  $B = [b_{ij}] \in R^{p \times q}$ , then the Kronecker product of matrices  $A, B$  is*

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix} \in R^{mp \times nq} \quad (3)$$

The common properties of Kronecker product of the above matrices are as follows

$$(1) \quad a(A \otimes B) = (aA) \otimes B = A \otimes (aB), \quad a \in R;$$

- (2)  $(A+B) \otimes C = A \otimes C + B \otimes C$ ,  $A, B$  are of the same order matrices;  
 (3)  $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$ ,  $A, B$  are nonsingular matrices;  
 (4)  $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ ,  $C, D$  are matrices with appropriate dimension.

**Lemma 2** ([32]). Let  $x(t) \in R^n$  be a continuously derivable vector function, and for  $\forall t \geq t_0$ , the following three conditions are equivalent

- (1)  ${}_0^C D_t^\alpha [x^T(t)Px(t)] \leq 2x^T(t)P[{}_0^C D_t^\alpha x(t)]$ ;  
 (2)  ${}_0^C D_t^\alpha [x^T(t)Px(t)] \leq 2{}_0^C D_t^\alpha [x^T(t)]Px(t)$ ;  
 (3)  ${}_0^C D_t^\alpha [x^T(t)Px(t)] \leq {}_0^C D_t^\alpha [x^T(t)]Px(t) + x^T(t)P[{}_0^C D_t^\alpha x(t)]$ ,

where the matrix  $P \in R^{n \times n}$  is a symmetric positive definite matrix.

**Lemma 3** ([33]). For given symmetric matrix  $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ , where  $S_{11} \in R^{r \times r}$ , the conditions (1), (2) and (3) are equivalent

- (1)  $S < 0$ ;  
 (2)  $S_{11} < 0$ ,  $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$ ;  
 (3)  $S_{22} < 0$ ,  $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$ .

**Lemma 4** ([34]). If there is matrix  $Q > 0$  and matrices  $S, T$  with appropriate dimension, then for any  $x(t), y(t) \in R^n$ , the condition (4) holds

$$2x^T(t)STy(t) \leq x^T(t)SQS^T x(t) + y^T(t)T^T Q^{-1}Ty(t) \quad (4)$$

### 3. Problem Description

The fractional-order multi-manipulator system under study comprises a single leader and  $n$  followers. The dynamic model of the leader, as well as that of each follower, is represented by the fractional-order equation:

$$\begin{cases} {}_0^C D_t^\alpha x_0(t) = Ax_0(t) + f(t, x_0(t)) \\ y_0(t) = Cx_0(t) \end{cases} \quad (5)$$

$$\begin{cases} {}_0^C D_t^\alpha x_i(t) = Ax_i(t) + f(t, x_i(t)) + Bu_i(t) \\ y_i(t) = Cx_i(t), \quad i = 1, 2, \dots, n \end{cases} \quad (6)$$

where  $A \in R^{4 \times 4}$ ,  $B \in R^4$ ,  $C \in R^{2 \times 4}$  are state matrix, input matrix and output matrix of the system, respectively.  $x_0(t)$ ,  $x_i(t) \in R^4$  denote the state of the leader and the  $i$ th follower manipulator, respectively.  $y_0(t)$ ,  $y_i(t) \in R^2$  denote the output of the leader and the  $i$ th follower, respectively.  $f(\cdot) \in R^4$  is nonlinear smooth function satisfying the Lipschitz condition,  $u_i(t) \in R$  denotes the control input of the system. Equations (5) and (6) are obtained from the modeling of the actual single-link manipulator system.

$$x_i(t) = [x_i^1(t), x_i^2(t), x_i^3(t), x_i^4(t)]^T,$$

where  $x_i^1(t)$ ,  $x_i^2(t)$ ,  $x_i^3(t)$ ,  $x_i^4(t)$  denote the motor angular displacement, the motor angular velocity, connecting rod angular displacement and connecting rod angular velocity, respectively. Reference [35] derived a state-space model from the Euler-Lagrange system of the original manipulator model, whereas the model in this paper replaces the integer-order model in Reference [35] with a more accurate fractional-order model.

The following assumptions are imposed on the fractional-order multi-manipulator systems (5) and (6).

**Assumption 1.** The directed graph  $\Theta$  of communication topology contains a directed spanning tree, and the leader can send information to at least one follower manipulator, so that at least one value of the weighted gain  $m_i$  is nonzero.

**Assumption 2.** At any time  $t$ , for nonlinear smooth function  $f(\cdot) \in R^n$ , there exists a constant  $\delta > 0$  that makes any  $x(t), y(t) \in R^n$  satisfy the inequality (7)

$$\|f(t, x(t)) - f(t, y(t))\| \leq \delta \|x(t) - y(t)\| \quad (7)$$

where  $\delta$  is the Lipschitz constant.

**Lemma 5** ([36]). Let  $x(t) = 0$  be the equilibrium point of the non-autonomous fractional-order system, then (8) holds

$${}_0^C D_t^\alpha x(t) = f(t, x(t), x_0(t)) \in R^n \quad (8)$$

If there is a Lyapunov function and a nonnegative number  $\psi$ , such that

$${}_0^C D_t^\alpha V(t, x(t)) \leq -\psi V(t, x(t)) \leq 0 \quad (9)$$

the system is said to be asymptotically stable.

**Definition 4.** If the state of any nonlinear leader and follower satisfies the following equality

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\| = 0, \quad i = 1, 2, \dots, n \quad (10)$$

the fractional-order multi-manipulator system (5) and (6) can realize coordinated tracking control.

In this paper, to estimate the unmeasured states, a nonlinear state observer is developed to overcome the difficulty in measuring the state components of the follower manipulator, so that the variation curve of the state components of the follower under the observer can be used to accurately and effectively approximate that in practical measurement.

**Remark 1.** Compared with integer-order systems, fractional-order systems have the following three advantages. (1) *Modeling Capability:* Fractional-order models inherently incorporate memory and hereditary properties, which enable them to capture complex dynamic behaviors more accurately than integer-order models. (2) *Robustness to Disturbances:* Due to their nonlocal nature, fractional-order dynamics provide an additional degree of freedom in controller design. This allows for more flexible shaping of system responses, leading to improved robustness against external disturbances and parameter uncertainties compared to traditional integer-order approaches. (3) *Control Performance in Practice:* In practical implementations, fractional-order controllers can achieve smoother transient responses and better disturbance attenuation. However, this advantage comes at the cost of increased computational complexity and implementation challenges.

**Remark 2.** When the communication topology does not contain a directed spanning tree, the information flow among agents may become insufficient to guarantee global consensus or accurate state estimation, which can lead to performance degradation or even instability of the proposed method. The Lipschitz condition is mainly employed to facilitate the Lyapunov-based stability analysis and to derive sufficient conditions for the convergence of the observer. In practice, however, many nonlinear systems may only satisfy local Lipschitz conditions or may exhibit mild nonlinearities that deviate from strict Lipschitz continuity. When the Lipschitz condition is partially violated, the theoretical guarantees on convergence and boundedness may no longer strictly hold. Some problems caused by nonlinearity and uncertainty can be addressed by introducing static neural networks. For a detailed analysis, please refer to reference [37].

#### 4. Design of State Observer

To control the fractional-order multi-manipulator systems (5) and (6), the algorithm from [35] is employed, as expressed in (11).

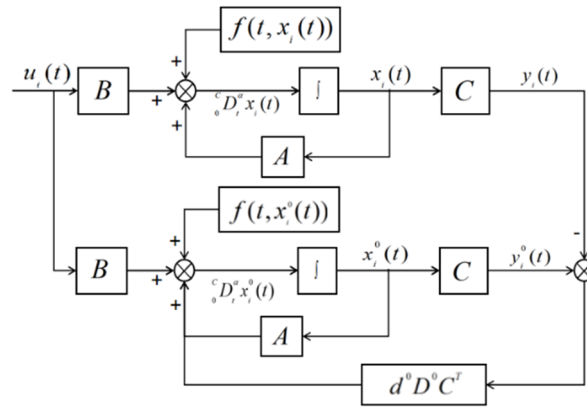
$$u_i(t) = -K \left[ \sum_{j=1}^n a_{ij} (x_i^0(t) - x_j^0(t)) + m_i (x_i^0(t) - x_0(t)) \right] \quad (11)$$

where  $i = 1, 2, \dots, n$ ,  $K \in R^{1 \times 4}$  is the feedback gain matrix of the system to be designed.

The following nonlinear state observer is designed, and the structural diagram is shown in Figure 1.

$${}^C_0 D_t^\alpha x_i^0(t) = Ax_i^0(t) + Bu_i(t) + d^0 D^0 C^T [y_i^0(t) - y_i(t)] + f(t, x_i^0(t)) \quad (12)$$

where  $i = 1, 2, \dots, n$ , and  $d^0$  is gain coefficient of the observer,  $D^0 \in R^{4 \times 4}$  is the gain matrix of the observer to be designed, and  $x_i^0(t)$ ,  $y_i^0(t)$  are the estimated values of  $x_i(t)$ ,  $y_i(t)$ , respectively.



**Figure 1.** The structure diagram of state observer.

Combining (5), (6) and (11), we can get

$${}^C_0 D_t^\alpha x_i^0(t) = Ax_i^0(t) + Bu_i(t) + d^0 D^0 C^T [y_i^0(t) - y_i(t)] + f(t, x_i^0(t)) \quad (13)$$

According to Definition 3, Equation (13) can be represented as

$${}^C_0 D_t^\alpha e(t) = (A \otimes I_n) e(t) - (BK \otimes H) [e(t) + e^0(t)] + F(t, x(t), x_0(t)) \quad (14)$$

where  $F(t, x(t), x_0(t)) = [F(t, x_1(t), x_0(t))^T, F(t, x_2(t), x_0(t))^T, \dots, F(t, x_n(t), x_0(t))^T]^T$ .  $e(t)$ ,  $e^0(t)$  are error vectors, which are respectively defined as

$$e(t) = [e_1(t)^T, e_2(t)^T, \dots, e_n(t)^T]^T, \quad e^0(t) = [e_1^0(t)^T, e_2^0(t)^T, \dots, e_n^0(t)^T]^T, \quad (15)$$

where  $e_i(t) = x_i(t) - x_0(t)$ ,  $e_i^0(t) = x_i^0(t) - x_i(t)$ ,  $F_i(t, x_i(t), x_0(t)) = f(t, x_i(t)) - f(t, x_0(t))$ .

Similarly, combining (6) and (12), one can obtain

$${}^C_0 D_t^\alpha e_i^0(t) = (A + d^0 D^0 C^T C) e_i^0(t) + F_i^0(t, x_i(t), x_0(t)) \quad (16)$$

where  $F_i^0(t, x_i(t), x_0(t)) = f(t, x_i^0(t)) - f(t, x_i(t))$ .

The matrix vector of (16) can be stated as

$${}^C_0 D_t^\alpha e^0(t) = [(A + d^0 D^0 C^T C) \otimes I_n] e^0(t) + F^0(t, x_i(t), x_0(t)) \quad (17)$$

where  $F^0(t, x_i(t), x_0(t)) = [F^0(t, x_1(t), x_0(t))^T, F^0(t, x_2(t), x_0(t))^T, \dots, F^0(t, x_n(t), x_0(t))^T]^T$ .

**Remark 3.** In this study, we design a fractional-order state observer, as illustrated in Figure 1, where  $x_i^0(t)$ ,  $y_i^0(t)$  denote the estimates of the original system states  $x_i(t)$ ,  $y_i(t)$ , respectively. The fractional-order state observer represents a significant extension of traditional state estimation theory into the fractional domain. Unlike conventional integer-order observers, which can only provide approximate descriptions, the fractional-order observer is capable of accurately modeling physical processes characterized by long-memory and hereditary effects. Such properties are intrinsic to many real-world systems, including electrochemical reactions and heat conduction dynamics, which are inherently fractional in nature. In addition, the memory-dependent

structure enables better attenuation of persistent disturbances and uncertainties, leading to enhanced robustness. From a dynamic perspective, fractional-order observers exhibit Mittag-Leffler-type convergence rather than purely exponential behavior, providing more flexible transient responses and potentially improved stability margins in highly coupled systems. Furthermore, in distributed multi-manipulator scenarios with communication constraints, the use of historical information can partially compensate for information loss or delays. These features make fractional-order observers a more expressive and robust framework for observer design in mechanical systems with complex dynamics.

**Theorem 1.** Under Assumptions 1 and 2, and given scalars  $\alpha_1, \alpha_2, \delta_1, \delta_2, d^0$ , if there are symmetric positive definite matrices  $P_1, P_2$  that satisfy conditions (18) and (19) respectively, then the state of each follower and leader in the original fractional-order multi-manipulator system can converge to the expected target value

$$\text{Sym}\{AP_1\} + \delta_1(I_n + P_1^2) + B\Psi + \alpha_1 P_1 < 0 \quad (18)$$

$$\text{Sym}\{AP_2\} + \delta_2 I_n + P_2 \Pi P_2 + 2d^0 D^0 C^T C P_2 + \alpha_2 P_2 < 0 \quad (19)$$

where  $\text{Sym}\{AP_1\} = AP_1 + P_1 A^T$ ,  $\Psi = 2\text{Re}_{\min}\{\lambda(H)\}KP_1$ ,  $\text{Re}_{\min}\{\lambda(H)\}$  is defined as the minimum value among the real parts of the eigenvalues of  $H$ ,  $H = L + M$ .  $\text{Sym}\{AP_2\} = AP_2 + P_2 A^T$ ,  $\Pi = \delta_2 I_n - \frac{1}{\mu} H^T H$ .

**Proof.** Construct the Lyapunov function, for  $\forall t \geq t_0$

$$V(t) = e^T(t)(P_1^{-1} \otimes I_n)e(t) + e^0(t)^T(P_2^{-1} \otimes I_n)e^0(t) \quad (20)$$

By taking the fractional-order differential of (20), according to Lemma 2, we can get

$${}_0^C D_t^\alpha V(t) \leq 2e^T(t)(P_1^{-1} \otimes I_n)[{}_0^C D_t^\alpha e(t)] + 2e^0(t)^T(P_2^{-1} \otimes I_n)[{}_0^C D_t^\alpha e^0(t)] \quad (21)$$

Substituting (14) and (17) into (21), one can obtain

$$\begin{aligned} {}_0^C D_t^\alpha V(t) &\leq 2e^T(t)(P_1^{-1} A \otimes I_n)e(t) + 2e^T(t)(P_1^{-1} BK \otimes H)(e(t) + e^0(t)) + 2e^T(t)(P_1^{-1} \otimes I_n)F(t, x_i(t), x_0(t)) \\ &\quad + 2e^0(t)^T(P_2^{-1} A \otimes I_n)e^0(t) + 2e^0(t)^T[P_2^{-1}(d^0 D^0 C^T C) \otimes I_n]e^0(t) + 2e^0(t)^T(P_2^{-1} \otimes I_n)F^0(t, x_i(t), x_0(t)) \end{aligned} \quad (22)$$

If  $q = 2e^T(t)(P_1^{-1} K \otimes H)e^0(t)$ , according to Lemma 4, we can derive

$$q \leq e^T(t)(\mu P_1^{-1} K B K^T B^T P_1^{-1} \otimes I_n)e(t) + e^0(t)^T(\frac{1}{\mu} H^T H \otimes I_n)e^0(t) \quad (23)$$

where  $\mu > 0$ .

By substituting (23) into (22), it can be deduced

$$\begin{aligned} {}_0^C D_t^\alpha V(t) &\leq 2e^T(t)(P_1^{-1} A \otimes I_n)e(t) + 2e^T(t)(P_1^{-1} BK \otimes H)e(t) + e^T(t)(\mu P_1^{-1} B K K^T B^T P_1^{-1} \otimes I_n)e(t) \\ &\quad + 2e^T(t)(P_1^{-1} \otimes I_n)F(t, x_i(t), x_0(t)) + 2e^0(t)^T(P_2^{-1} A \otimes I_n)e^0(t) + 2e^0(t)^T[P_2^{-1}(d^0 D^0 C^T C) \otimes I_n]e^0(t) \\ &\quad + e^0(t)^T(\frac{1}{\mu} H^T H \otimes I_n)e^0(t) + 2e^0(t)^T(P_2^{-1} \otimes I_n)F^0(t, x_i(t), x_0(t)) \end{aligned} \quad (24)$$

Suppose  $F = F(t, x_i(t), x_0(t))$ ,  $F^0 = F^0(t, x_i(t), x_0(t))$ , contacted Assumption 2 and Young inequality, yields

$$2e^T(t)(P_1^{-1} \otimes I_n)F \leq \delta_1 e^T(t)[(P_1^{-2} + I_n) \otimes I_n]e(t) \quad (25)$$

$$2e^0(t)^T(P_2^{-1} \otimes I_n)F^0 \leq \delta_2 e^0(t)^T[(P_2^{-2} + I_n) \otimes I_n]e^0(t) \quad (26)$$

where  $\delta_1 > 0$ ,  $\delta_2 > 0$ .

Substituting (25) and (26) into (24), we can get

$$\begin{aligned}
{}_0^C D_t^\alpha V(t) &\leq 2e^T(t)(P_1^{-1}A \otimes I_n)e(t) + 2e^T(t)(P_1^{-1}BK \otimes H)e(t) + e^T(t)(\mu P_1^{-1}BKK^T B^T P_1^{-1} \otimes I_n)e(t) \\
&\quad + e^T(t)[\delta_1(P_1^{-2} + I_n) \otimes I_n]e(t) + 2e^0(t)^T(P_2^{-1}A \otimes I_n)e^0(t) + 2e^0(t)^T[P_2^{-1}(d^0 D^0 C^T C) \otimes I_n]e^0(t) \\
&\quad + e^0(t)^T\left(\frac{1}{\mu}H^T H \otimes I_n\right)e^0(t) + e^0(t)^T[\delta_2(P_2^{-2} + I_n) \otimes I_n]e^0(t)
\end{aligned} \quad (27)$$

Assuming that

$$\begin{aligned}
W_1 &= [2P_1^{-1}A + \delta_1(P_1^{-2} + I_n) + \mu P_1^{-1}K \times K^T P_1^{-1}] \otimes I_n + 2P_1^{-1}K \otimes H, \quad W_2 = [2P_2^{-1}A + \delta_2(P_2^{-2} + I_n) \\
&\quad + 2P_2^{-1}(d^0 D^0 C^T C)\frac{1}{\mu}H^T H] \otimes I_n
\end{aligned}$$

we have

$${}_0^C D_t^\alpha V(t) \leq e^T(t)W_1 e(t) + e^0(t)^T W_2 e^0(t) \quad (28)$$

Supposing  $Q_1 = (P_1 \otimes I_n)W_1(P_1 \otimes I_n)$ ,  $Q_2 = (P_2 \otimes I_n)W_2(P_2 \otimes I_n)$ , it can be obtained

$$Q_1 \leq [\text{Sym}\{AP_1\} + \delta_1(I_n + P_1^2) + 2\text{Re}_{\min}\{\lambda(H)\}BKP_1] \otimes I_n \quad (29)$$

$$Q_2 \leq [\text{Sym}\{AP_2\} + \delta_2 I_n + P_2(\delta_2 I_n + \frac{1}{\mu}H^T H)P_2 + 2d^0 D^0 C^T CP_2] \otimes I_n \quad (30)$$

Combining (18) and (19), the following holds

$$Q_1 \leq (-\alpha_1 P_1) \otimes I_n \quad (31)$$

$$Q_2 \leq (-\alpha_2 P_2) \otimes I_n \quad (32)$$

According to Equation (31), it can be obtained that

$$W_1 \leq -\alpha_1 (P_1^{-1} \otimes I_n) \quad (33)$$

Similar to the derivation process of (33), (32) can be arranged as

$$W_2 \leq -\alpha_2 (P_2^{-1} \otimes I_n) \quad (34)$$

Combining (33) and (34), (28) can be processed as

$${}_0^C D_t^\alpha V(t) \leq -\alpha V(t) \leq 0 \quad (35)$$

where  $\alpha = \max\{\alpha_1, \alpha_2\}$ .

According to Lemma 5, it can be concluded that (5) and (6) are stable under the control algorithm (11) and observer (12), and that the state of each follower manipulator and the leader in the original fractional-order multi-manipulator system can converge to the expected target value.  $\square$

**Remark 4.** Theorem 1 establishes sufficient conditions for ensuring that the leader-follower manipulator system reaches the desired objective, expressed in the form of a matrix inequality. Nevertheless, the derived inequality is not in a tractable linear matrix inequality (LMI) form, which hinders direct computation. Therefore, an additional reformulation step is required to transform it into an equivalent LMI, thereby enabling efficient numerical solution.

**Remark 5.** The use of the Caputo fractional derivative enables the proposed observer to capture the memory and hereditary characteristics of dynamical systems, which are not represented in classical integer-order models. In terms of convergence behavior, fractional-order observers typically exhibit a Mittag-Leffler type convergence rather than the conventional exponential convergence observed in integer-order systems. Such convergence provides greater flexibility in shaping transient responses and can contribute to improved robustness under certain conditions. Compared with conventional observer designs, such as high-gain and sliding mode observers, the proposed method achieves a balanced trade-off between convergence performance and robustness. While some existing approaches may improve robustness at the cost of increased sensitivity to noise or more restrictive assumptions, the proposed observer maintains stable estimation performance with a relatively simple structure.

Under the conclusion of Theorem 1, assuming that  $\Psi = B^T$  in the condition (18), the following Theorem can be obtained.

**Theorem 2.** Under Assumptions 1 and 2, and given scalars  $\alpha_1, \alpha_2, \delta_1, \delta_2, d^0$ , if there are symmetric positive definite matrices  $P_1, P_2$  that satisfy conditions (36) and (37) respectively, then the state of each follower and leader in the original fractional-order multi-manipulator system can realize leader-following consensus

$$\begin{pmatrix} \text{Sym}\{AP_1\} + \delta_1 I_n + BB^T + \alpha_1 P_1 & P_1 \\ P_1 & -\frac{1}{\delta_1} I_n \end{pmatrix} < 0 \quad (36)$$

$$\begin{pmatrix} \text{Sym}\{AP_2\} + \delta_2 I_n + 2d^0 D^0 C^T C P_2 + \alpha_2 P_2 & P_2 \\ P_2 & -\Pi^{-1} \end{pmatrix} < 0 \quad (37)$$

where  $D^0 = (C^T C + I_n)^{-1} P_1$  is the gain matrix of the state observer. Therefore, the feedback gain matrix of the system is  $K = \xi B^T P_1^{-1}$ , where  $\xi = \frac{1}{2\text{Re}_{\min}\{\lambda(H)\}}$  is the coupling gain.

**Proof.** According to Lemma 3, (36) can be arranged as

$$\text{Sym}\{AP_1\} + \delta_1 (I_n + P_1^2) + BB^T + \alpha_1 P_1 < 0 \quad (38)$$

By substituting (36) into (27), it can be derived

$$Q_1 \leq [BB^T + 2\text{Re}_{\min}\{\lambda(H)\} B K P_1 - \alpha_1 P_1] \otimes I_n \quad (39)$$

Since  $\Psi = B^T$ , we can get

$$B\Psi = BB^T \quad (40)$$

where  $\Psi = 2\text{Re}_{\min}\{\lambda(H)\} K P_1$ .

By taking (40) into (39), it can be obtained

$$Q_1 \leq (-\alpha_1 P_1) \otimes I_n \quad (41)$$

Combining (32) and (41), similar to the derivation process of (35), we can get that

$${}_0^C D_t^\alpha V(t) \leq -\alpha V(t) \leq 0 \quad (42)$$

where  $\alpha = \max\{\alpha_1, \alpha_2\}$ .

According to Lemma 5, it can be concluded that (5) and (6) under the control algorithm (11) and observer (12) can achieve leader-following consensus.

As  $P_1 = P_1^T > 0, H > 0$ , one has

$$K = \xi B^T P_1^{-1} \quad (43)$$

The gain matrix  $D^0$  in the nonlinear state observer (12) can be designed by

$$(C^T C + I_n) D^0 = P_1 \quad (44)$$

Due to  $C = [c_i]_{2 \times 4}, c_i \geq 0$ , we can get  $(C^T C + I_n) > 0$ . Thus, we can further obtain  $D^0 = (C^T C + I_n)^{-1} P_1$ .  $\square$

**Remark 6.** Theorem 2 establishes sufficient conditions for guaranteeing leader-follower consensus in multi-manipulator systems, formulated within the framework of linear matrix inequalities (LMIs). This formulation not only provides a rigorous criterion for consensus but also offers a computationally feasible approach for controller synthesis.

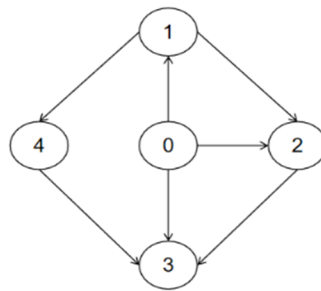
**Remark 7.** The proposed observer-controller framework involves mainly algebraic operations and avoids iterative optimization or high-dimensional matrix computations, resulting in a moderate computational burden



suitable for embedded implementation. The fractional-order derivative can be realized via standard numerical approximations. In practical implementations, performance is influenced by factors such as sampling rate, approximation accuracy, and processor capability. A higher sampling rate and higher-order approximation generally improve estimation accuracy but increase computational cost, while lower-order implementations reduce complexity at the expense of precision. Therefore, a trade-off between accuracy and efficiency is required, depending on the available hardware resources.

## 5. Numerical Simulation

In this section, a fractional-order multi-manipulator system consisting of one leader and four linearized follower manipulators is considered. Figure 2 presents the communication topology of the system, where node 0 denotes the leader and nodes 1–4 represent the follower manipulators.



**Figure 2.** Communication topology.

According to Figure 2, one can obtain

$$H = L + M = \begin{bmatrix} 3 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Combining Assumption 1 and Lemma 1, we get  $L \geq 0, M \geq 0$ . In order to ensure  $H > 0$ , a random diagonal matrix  $C_0$  is added. The eigenvalues of  $H_0$  can be obtained, which are

$$\lambda_1 = 3.8014, \lambda_2 = 1.3465, \lambda_3 = 1.5111, \lambda_4 = 1.0833.$$

Therefore  $\text{Re}_{\min}\{\lambda(H_0)\} = \lambda_4 = 1.0833$ ,  $H_0 = H + C_0$ ,  $C_0 = \text{diag}\{c_i\}$ ,  $c_i \in (0,1)$ ,  $i = 1, 2, \dots, n$ .

By substituting  $\text{Re}_{\min}\{\lambda(H_0)\} = \lambda_4 = 1.0833$  into the calculation formula of coupling gain  $\xi = \frac{1}{2\text{Re}_{\min}\{\lambda(H_0)\}}$ , we can get  $\xi = 0.4616$ .

Given the parameters in the Theorem 2 as:  $\alpha_1 = 20$ ,  $\delta_1 = 2.5$ ,  $\alpha_2 = 20$ ,  $\delta_2 = 3.5$ ,  $d^0 = 0.01$ . And the initial states of leader and follower manipulators are

$$[X_0, X_1, X_2, X_3, X_4] = [-0.25, 0.15, -0.2, -0.08, -0.15].$$

The parameter  $\alpha$  of the fractional-order system is set to 0.9. The initial values of the leader are selected as

$$X_0 = [0.8944, 0.1375, 0.3900, 0.9274]^T.$$

The initial conditions for the follower manipulators are given by

$$X_1 = [0.9175, 0.7136, 0.6183, 0.3433]^T, X_2 = [0.9360, 0.1248, 0.7306, 0.6465]^T,$$

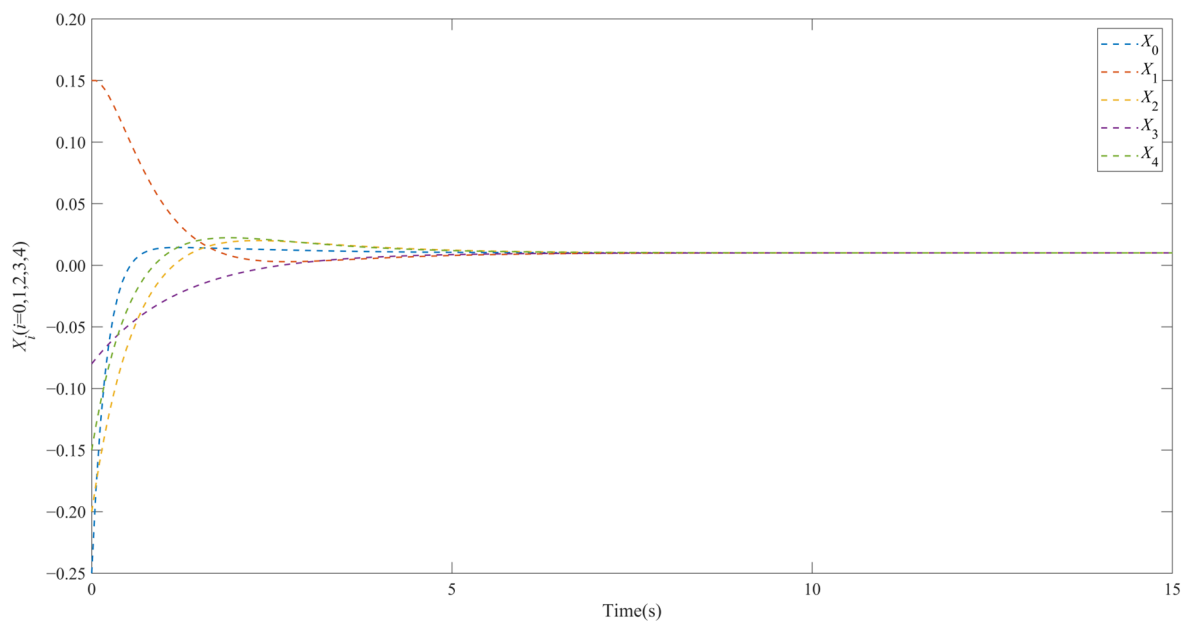
$$X_3 = [0.8332, 0.3983, 0.7498, 0.8352]^T, X_4 = [0.3225, 0.5523, 0.9791, 0.5493]^T.$$

According to the conditions (36) and (37),  $P_1, K, D^0, P_2$  can be solved as

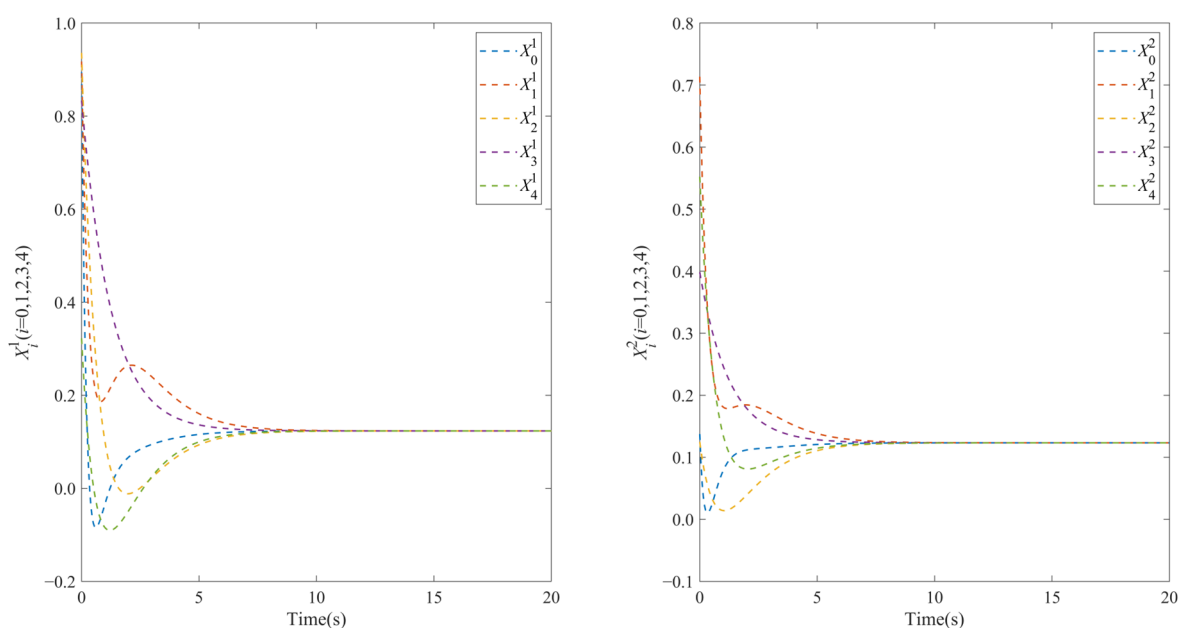
$$P_1 = \begin{bmatrix} 4.6468 & 1.1085 & 3.4316 & 0.5034 \\ 1.1085 & 14.3437 & -0.4508 & -2.4743 \\ 3.4316 & -0.4508 & 4.0842 & -1.1473 \\ 0.5034 & -2.4743 & -1.1473 & 10.9126 \end{bmatrix}, K = [-1.0297, 0.8695, 1.0612, 0.3562],$$

$$D^0 = \begin{bmatrix} 2.3234 & 0.5543 & 3.4316 & 0.5034 \\ 0.5543 & 7.1719 & -0.4508 & -2.4743 \\ 3.4316 & -0.4508 & 4.0842 & -1.1473 \\ 0.5034 & -2.4743 & -1.1473 & 10.9126 \end{bmatrix}, P_2 = \begin{bmatrix} 0.0786 & 0.0256 & 0.0546 & -0.0007 \\ 0.0256 & 0.3374 & -0.0302 & -0.0974 \\ 0.0546 & -0.0302 & 0.0744 & 0.0219 \\ -0.0007 & -0.0974 & 0.0219 & 0.1774 \end{bmatrix}.$$

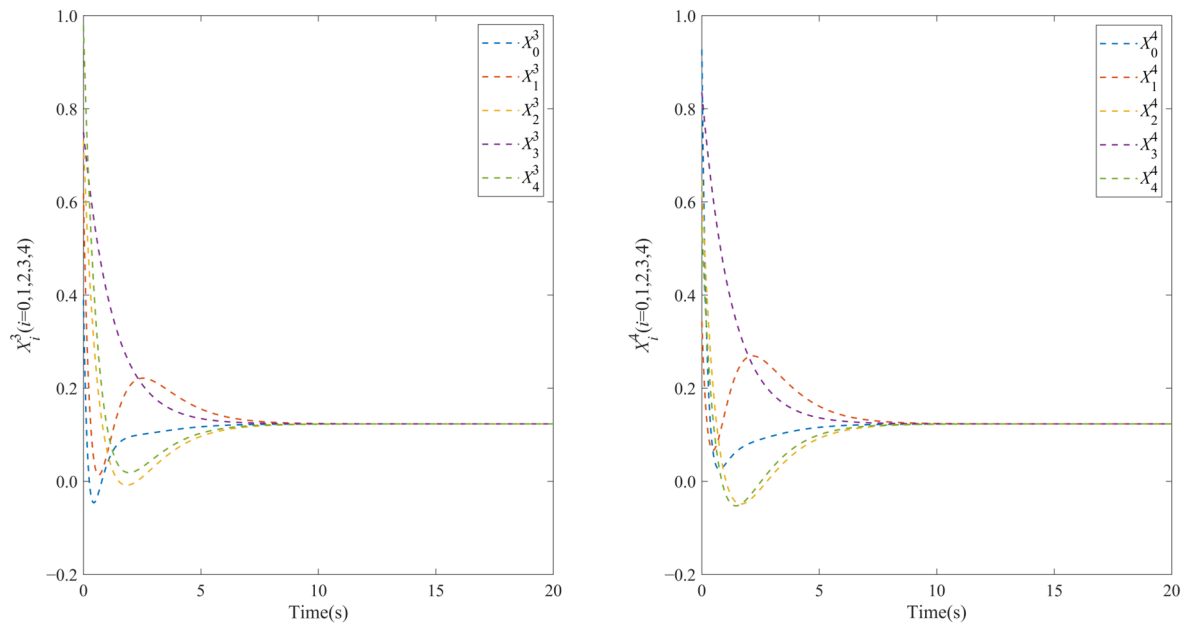
Under the control algorithm (11), the state curves of the leader and each follower manipulator, motor angular displacement and velocity, connecting rod angular displacement and velocity in (5) and (6), are shown in Figures 3–5, respectively.  $X_i = [X_i^1, X_i^2, X_i^3, X_i^4]^T$  ( $i = 0, 1, 2, 3, 4$ ) represent the state components of the leader and the  $i$ th following manipulator, respectively.



**Figure 3.** The state curves of the leader and each follower manipulator.



**Figure 4.** The state curves of motor angular displacement and velocity.

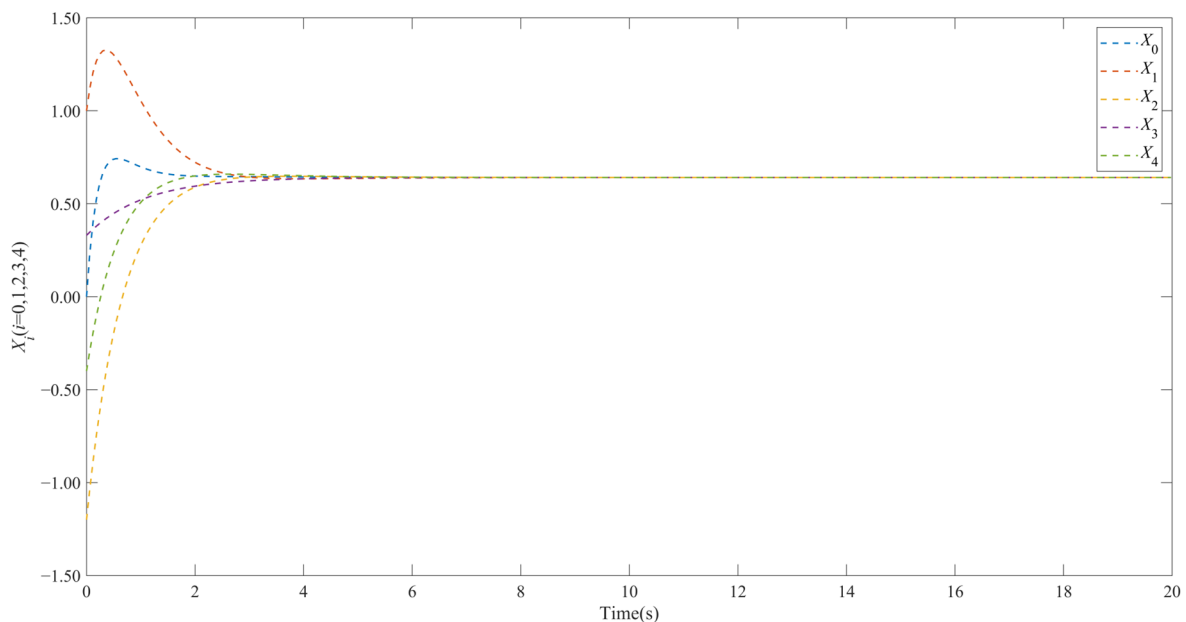


**Figure 5.** The state curves of connecting rod angular displacement and velocity.

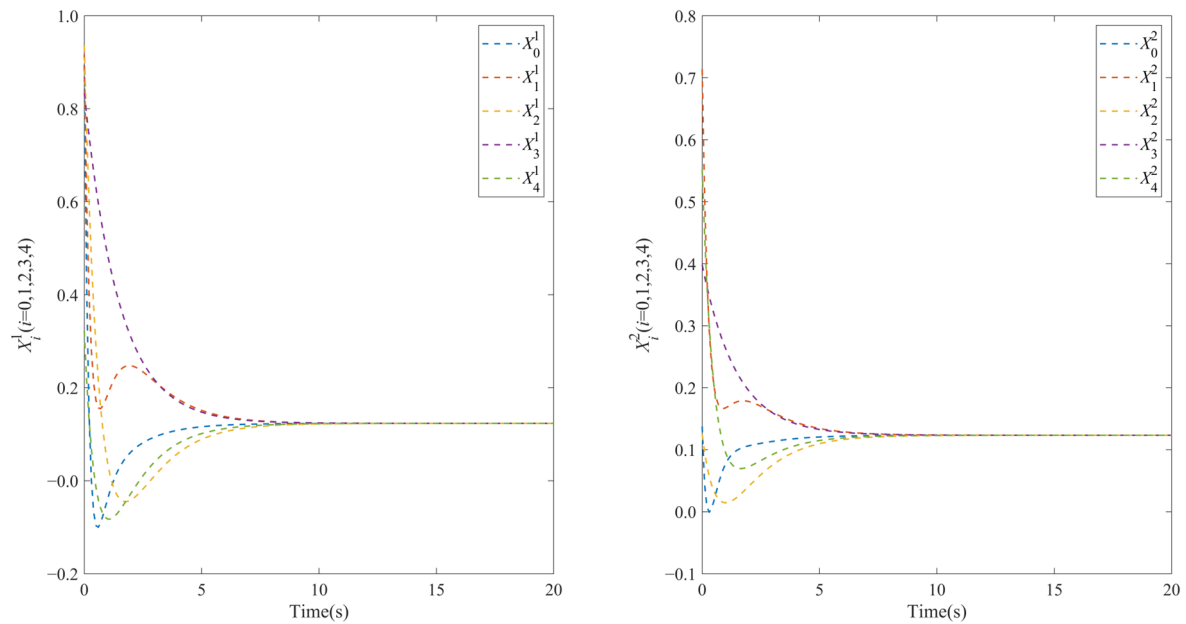
Under the observer (12), the state curves of leader and each follower manipulator, motor angular displacement and velocity, connecting rod angular displacement and velocity, are shown in Figures 6–8, respectively. Similarly,  $X_i = [X_i^1, X_i^2, X_i^3, X_i^4]^T$  ( $i = 0, 1, 2, 3, 4$ ) represent the state components of the leader and the  $i$ th follower manipulator, respectively.

Therefore,  $K$  is replaced by  $K_1$ , and  $K_1 = d^0 K^0$ . And  $d^0$  is the correction coefficient,  $K^0$  is determined by  $P_2$ .

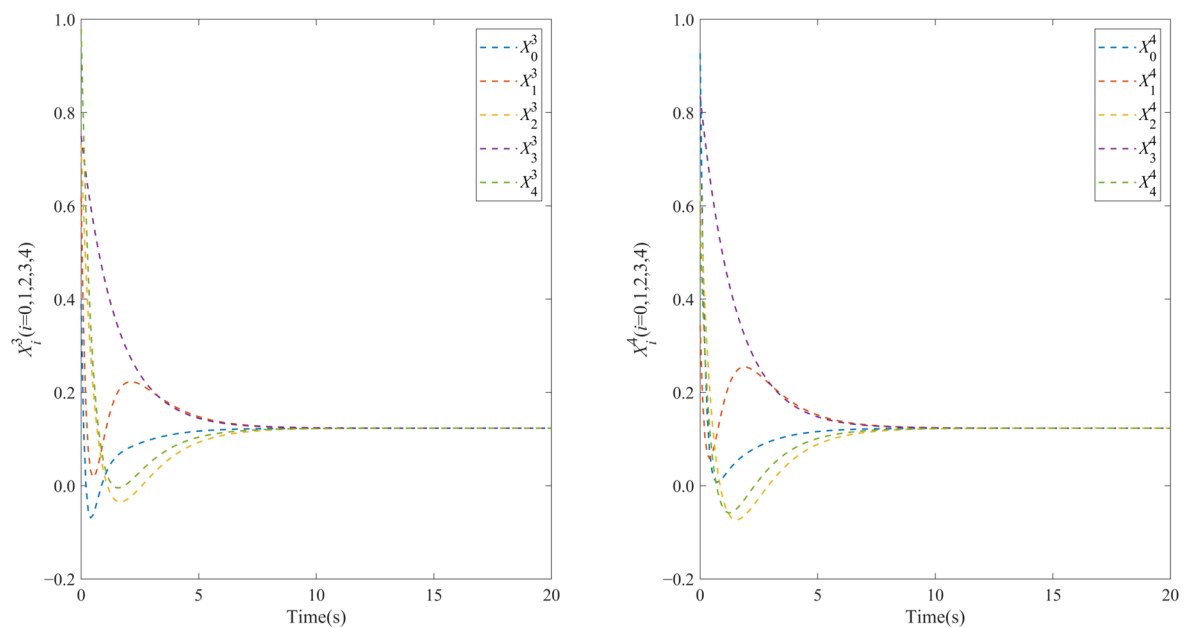
It can be seen from Figures 6–8 that for (5) and (6) under the control algorithm (11) and observer (12), the state of the leader and the follower manipulators can finally converge to the expected target value, which can realize leader-following consensus. Therefore, it is further proved that observer (12) is reasonable and effective.



**Figure 6.** The state curves of the leader and each follower manipulator.



**Figure 7.** The state curves of motor angular displacement and velocity.



**Figure 8.** The state curves of connecting rod angular displacement and velocity.

**Remark 8.** When the parameter  $\xi$  is designed as  $\xi \geq \frac{1}{2\text{Re}_{\min}\{\lambda(H_0)\}}$ , the calculation results need to be selected accordingly. Within its critical range, when  $\xi$  takes different values, it can still be obtained through numerical simulation that the state of each follower manipulator and leader in the fractional-order multi-manipulator system can converge to the expected target value, which will change with the value of  $\xi$ .

**Remark 9.** The gain matrix  $D^0$  of the designed state observer is obtained on the basis of matrices  $C$  and  $P_1$ . During numerical simulation, it can control the error between the state components of the follower manipulators and the original state components of the follower manipulators within a small range, and then an appropriate observer gain coefficient  $d^0$  is selected for accurate control.

**Remark 10.** In this paper, the communication topology  $\Theta$  designed is assumed to have a directed spanning tree, so its corresponding  $L$  is semi-positive. To ensure the positive de-finiteness of  $H$ , during numerical simulation,

a random constant diagonal matrix  $C_0$  is added without affecting the stability of the original fractional-order system and the consensus of the achieved goal, so we can substitute  $Re_{\min}\{\lambda(H_0)\}$  for  $Re_{\min}\{\lambda(H)\}$ .

**Remark 11.** In practical scenarios, the presence of bounded disturbances and noisy measurements may affect the estimation accuracy. According to the derived stability conditions, the estimation error is guaranteed to remain bounded provided that the uncertainties and disturbances are bounded. This implies that the observer exhibits inherent robustness against such perturbations. Specifically, measurement noise may lead to a non-zero steady-state estimation error, whose magnitude is related to the noise level and system parameters. However, the convergence properties of the observer are preserved under the proposed design, as ensured by the fractional-order Lyapunov-based analysis. Moreover, the design parameters in the observer can be appropriately tuned to balance convergence speed and noise sensitivity, which provides additional flexibility in practical implementations. A more comprehensive quantitative analysis of observer performance under different noise intensities is an interesting topic and will be considered in future work.

**Remark 12.** Although the numerical results illustrate the convergence behavior of the proposed method, its performance can also be characterized from a theoretical perspective. Based on the derived stability conditions, the estimation/consensus error is guaranteed to remain bounded and converge asymptotically, which implicitly provides an upper bound on the error dynamics. In particular, the convergence behavior follows a Mittag-Leffler type decay, which can be used to characterize the convergence rate of the system. Although a detailed statistical analysis (e.g., Monte Carlo evaluation) is not explicitly included, the theoretical results offer sufficient insight into the error bounds and convergence characteristics of the proposed approach.

## 6. Conclusions

This paper has investigated the coordinated tracking control problem of a fractional-order multi-manipulator system under a directed communication topology. A nonlinear state observer has been developed to estimate the unmeasurable state components of the follower manipulators. A distributed coordinated tracking control algorithm has been formulated, with the observer and controller parameters obtained via linear matrix inequalities. Sufficient conditions for leader-follower consensus have been established using Lyapunov stability theory. Numerical simulations have confirmed the validity and effectiveness of the proposed approach.

## Author Contributions

W.W.: visualization, investigation; S.H.: writing—original draft preparation, software; H.Z.: supervision, writing—reviewing and editing. All authors have read and agreed to the published version of the manuscript.

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## Institutional Review Board Statement

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## Informed Consent Statement

Not applicable.

## Data Availability Statement

Data will be made available on request.

## Conflicts of Interest

The authors declare no conflicts of interest.

## Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

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