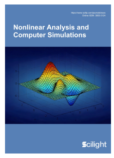
Scilight

Nonlinear Analysis and Computer Simulations
https://www.sciltp.com/journals/nacs



Check for updates

Article

Modeling the Chaos and Bifurcation in Solow's Business Trade Cycle Model by Using Delay Differential Equations

Dipesh [1,*] and Pankaj Kumar [2]

[1] Department of Mathematics, SR University, Warangal-506371, Telangana, India
[2] Department of Mathematics, Lovely Professional university, Phagwara-144401, India
* Correspondence: dipeshdalal2411@gmail.com

How To Cite: Dipesh; Kumar, P. Modeling the Chaos and Bifurcation in Solow's Business Trade Cycle Model by Using Delay Differential Equations. *Nonlinear Analysis and Computer Simulations* **2026**, *1*(2), 8. https://doi.org/10.53941/nacs.2026.100008

Received: 7 February 2026
Revised: 15 April 2026
Accepted: 5 May 2026
Published: 11 May 2026

Abstract: This research extends the traditional Solow model by introducing a delay parameter via a delay differential equation to examine the dynamics of business cycles. Alongside the Solow farmwork, the Harrod-Domar model and a modified Solow model are considered to provide a broader perspective on growth dynamics and stability analysis. This study reveals that the inclusion of delays causes fluctuations in stability, leading to Hopf bifurcation, limit cycles, and chaotic behavior, thereby capturing the complex evolution of the trade cycle. The dynamics highlight how minor changes in the system parameters can reshape long-term economic trajectories. The analysis, conducted using MATLAB, underscores the significance of the Solow and Harrod-Domar models, as well as their variants, for understanding economic growth and industrial development. This approach helps stakeholders predict and mitigate economic changes by identifying key thresholds and dynamic patterns, thereby promoting resilience, stability, and long-term growth.

Keywords: business-trade cycle; equilibrium; stability; Hopf-bifurcation

2020 MSC: 34Kxx

1. Introduction

During the Second World War, mathematical modeling became an important part of economic theory. Paul Anthony Samuelson, an important contributor in this process, formalized the issue of dynamic analysis in his 1947 book, "Foundations of Economic Analysis." These early efforts have evolved into modern dynamic economic modeling, especially through the use of delay differential equations. These equations are useful for describing economic growth, capital accumulation, and the business cycle. The physical sciences make extensive use of ordinary differential equations, which are essential for comprehending intricate engineering systems [1]. In economics, for instance, they stand for GDP, investment, income, consumption, and economic growth. Time is an implicit variable in autonomous differential equations, also called difference equations, which make up the bulk of ordinary differential equations in economics. On the other hand, considerably less attention has been paid to the more challenging delay-differential equations. Given that important economic variables are tracked at discrete time intervals, delay differential equations are a more logical choice for modeling economic processes. However, because of the inherent complexity in their asymptotic behavior, delay differential equations can be more challenging to analyze. For a more comprehensive analysis of the economic applications of differential equations, two books are recommended: the introductory work by Gandolfo [2] and the more advanced work by Brock and Malliaris [3]. For information on ordinary differential equations in economic dynamics, both volumes are great resources. Acemoglu [4] has a more recent book that calls for a solid grounding in mathematics.

Economists have produced numerous significant differential equations over the span of more than 60 years, the earliest of which being Solow's growth model, which was partially inspired by Harrod and Domar's writings in Neoclassical growth theory. Three factors drive economic growth: labor, capital, and technology. This is the central tenet of neoclassical growth theory. Because of its fundamental simplicity and significance, Acemoglu [5] argues that Solow's growth model is the first and the foremost requirement. The GDP and national debt under the delayed external investment were examined by Chen et al. [6,7]. The relationship between GDP and national debt was examined by Dmitriev and Kurkina [8]. Financial planning should concentrate on lowering or eliminating debt by attaining fiscal surpluses, according to Fourie and Blom, rather than depending on loans [9]. Strachinariu looked on the relationship between public debt and important macroeconomic indices [10]. Using simulations to investigate the impact of debt and equity financing on business success, Li and Wan used system dynamics to create a model for business finance structure [11]. Numerous additional models have also been added to the literature in an effort to employ mathematical models to describe the underlying features of these issues [12–16]. Shananin et al. [17] looked at how consumer financing affected family economics in Russia during the COVID-19 pandemic. Gimaltdinov investigated Ramsey-type models as optimal control issues [18]. Mazoumfard and Glantz [19] looked at bank earnings under monopolistic competition and the effect of tax pressure. Tadmon and Njike [20] looked at Okun's law and the method for calculating the minimal reservation pay based on model parameters. The reduction order of derivatives from units, which results in delays in the fluctuation of financial assets, was studied by Arabob et al. [21]. Using the competition model, Wang et al. investigated the dynamics of bank data and produced the findings [22]. Investment types include holding and retainage. Assuming a one-way financial exchange between the parent and subsidiary banks, comes studied the three-level Lotka-Volterra model [23]. Savings and capital, as well as loans and deposits, are important categories of banking services, according to Selyutin and Rudenko [24].

To evaluate the stability of the banking industry, Marasco et al. looked at the Fokker-Planck-Kolmogorov stochastic equation, which is employed for n-level in the banking industry and in the three-level Lotka-Volterra model [25]. Ruan conducted a thorough analysis of the exponential characteristic equation's zeros [26]. Segura et al. studied the dependency parameter in solow model for economic growth [27]. Kulikov et al. examined the existence of periodic solution in economics growth using delay [28]. Borges et al. and Tian & Huang studied the dynamics of solow model with respect to different parameters [29,30]. Ferrara et al. modified the neoclassical growth model using delay and provide some open problems for future [31].

The paper's primary contribution is the expansion of Solow's economic model by including delay factors, which demonstrate how temporal delays in labour and capital accumulation cause transitions from stability to limit cycles and, finally, chaotic behaviour via Hopf bifurcation. By integrating analytical and numerical simulations, the study identifies crucial thresholds influencing business trade cycle stability and offers practical insights for economic forecasting, policy design, risk management, and investment strategies.

2. The Harrod-Domar Model

Originally created by Harrod [32] and Domar [33], the Harrod–Domar framework examines economic growth from the perspectives of savings and capital productivity. It was initially proposed to study business cycle dynamics. The basic differential equation form of the model is:

$$\frac{dQ}{dt} = abQ - \zeta Q \tag{1}$$

where:

- $Q(t)$ denotes the total economic output at time t,
- b is the marginal productivity of capital (a constant),
- a is the proportion of income saved (savings rate),
- ζ is the capital depreciation coefficient.

This relation clearly illustrates that increases in either savings or productivity can drive economic expansion, though it notably excludes labor force and demographic factors.

Paul Samuelson [34] later applied differential equations in 1941 to examine equilibrium stability under various demand–supply settings.

3. Modified Solow Model

To complement the Harrod–Domar model, Robert Solow introduced an enhanced growth model incorporating labor expansion and capital accumulation [35]. His production function is expressed as:

$$Q = G(K, N) \tag{2}$$

where:

- K denotes the capital stock,
- N is the labor force size,
- G is a neoclassical production function.

The Inada conditions [36] imposed on G are:

$$\frac{\partial G}{\partial K} > 0, \quad \frac{\partial G}{\partial N} > 0, \quad \frac{\partial^2 G}{\partial K^2} < 0, \quad \frac{\partial^2 G}{\partial N^2} < 0$$

These ensure G is strictly concave and diminishing in marginal productivity. Furthermore, if G is homogeneous of degree one:

$$G(\xi K, \xi N) = \xi G(K, N), \quad \forall \xi > 0 \tag{3}$$

Choosing $\xi = 1/N$, we define:

- $q = Q/N$: output per worker,
- $k = K/N$: capital per worker.

This gives a reduced form:

$$q = g(k) \tag{4}$$

Investment $J(t)$ increases capital stock but is offset by depreciation. So

$$J(t) = \frac{dK}{dt} + \zeta K \tag{5}$$

Considering a closed economy in which every output is either invested in or consumed:

$$Q(t) = R(t) + J(t) \tag{6}$$

Letting consumption and investment per capita be:

- $r(t) = R/N$,
- $j(t) = J/N$,

we derive:

$$q(t) = r(t) + \frac{dk}{dt} + \left(\zeta + \frac{1}{N}\frac{dN}{dt}\right)k \tag{7}$$

Assuming full employment and exponential labor growth $N = N_0 e^{\nu t}$, and assuming a fixed savings fraction a, the key Solow differential equation becomes:

$$\frac{dk}{dt} = ag(k) - (\zeta + \nu)k \tag{8}$$

Assuming linear production $g(k) = \rho k$, we obtain:

$$\frac{dk}{dt} = a\rho k - (\zeta + \nu)k \tag{9}$$

In practical economies, delays are common due to lagged savings, capital formation, or labor availability. Introducing a delay τ, the model evolves into:

$$\begin{gathered} \frac{dk}{dt} = a\rho k(t-\tau) - (\zeta + \nu)k(t-\tau) \\ \Rightarrow \frac{dk}{dt} = -\theta k(t-\tau) \end{gathered} \tag{10}$$

where $\theta = (\zeta + \nu) - a\rho$.

Letting $B(\xi) = k(t)$, with $\xi = \eta t$, and choosing $\eta = \frac{1}{\tau}$, we get:

$$\frac{dB}{d\xi} = -\theta\tau B(\xi - 1) = -\alpha B(\xi - 1), \quad \text{where } \alpha = \theta\tau \tag{11}$$

The linear delay operator becomes:

$$\mathcal{L}(B) = \frac{dB}{d\xi} + \alpha B(\xi - 1) \tag{12}$$

Assuming a solution of the form $B(\xi) = e^{\lambda\xi}$, substitution yields the characteristic equation:

$$\lambda + \alpha e^{-\lambda} = 0 \tag{13}$$

With $\lambda = a + ib$, the real and imaginary components satisfy:

$$a = -\alpha e^{-a}\cos b, \quad b = \alpha e^{-a}\sin b \tag{14}$$

Lemma 1. *The functions $\xi^j e^{\lambda\xi}$, for $j = 0,1,\ldots,m$, are solutions to (11) if and only if λ is a root of multiplicity $\geq m+1$.*

Proof: Let:

$$A(\mu) = \mu^j e^{\lambda\mu}$$

We differentiate using the Leibniz rule for the operator:

$$\mathcal{L}(A) = \frac{d}{d\mu}A(\mu) + \alpha A(\mu - 1)$$

Start with $A(\mu) = e^{\lambda\mu}$:

$$\mathcal{L}(e^{\lambda\mu}) = \lambda e^{\lambda\mu} + \alpha e^{\lambda(\mu-1)} = e^{\lambda\mu}(\lambda + \alpha e^{-\lambda}) = e^{\lambda\mu}g(\lambda)$$

Now for general j, apply:

$$\mathcal{L}(\mu^j e^{\lambda\mu}) = \left(\frac{d}{d\mu} + \alpha T_1\right)(\mu^j e^{\lambda\mu})$$

where T_1 is the translation operator $T_1 A(\mu) = A(\mu - 1)$.

We can express this in terms of derivatives of g. By the chain rule:

$$\frac{\partial^m}{\partial\lambda^m}\left[e^{\lambda\mu}g(\lambda)\right] = e^{\lambda\mu}\sum_{k=0}^{m}\binom{m}{k}g^{(k)}(\lambda)\mu^{m-k}$$

Thus:

$$\mathcal{L}(\mu^m e^{\lambda\mu}) = e^{\lambda\mu}\sum_{k=0}^{m}\binom{m}{k}g^{(k)}(\lambda)\mu^{m-k}$$

This vanishes iff all $g^{(k)}(\lambda) = 0$ for $k = 0,1,\ldots,m$, i.e., the root λ has multiplicity $\geq m+1$. □

Lemma 2. *Real root behavior of the transcendental equation:*

- *If $\alpha < 0$: exactly one positive real root.*
- $0 < \alpha < \frac{1}{e}$*: two negative real roots.*
- $\alpha = \frac{1}{e}$*: one double root at $\lambda = -1$.*
- $\alpha > \frac{1}{e}$*: no real roots.*

Proof: Define:

$$g(\lambda) = \lambda + \alpha e^{-\lambda}, \quad g'^{(\lambda)} = 1 - \alpha e^{-\lambda}$$

We study the real-valued function $g: \mathbb{R} \to \mathbb{R}$. Let's consider the function:

$$h(\lambda) = \lambda e^{\lambda} = -\alpha$$

This is the Lambert W-function equation:

$$\lambda e^{\lambda} = -\alpha$$

Case 1. $\alpha < 0$, *then* $-\alpha > 0$, *and the function* $h(\lambda) = \lambda e^{\lambda}$ *has exactly one positive solution. So* $g(\lambda) = 0$ *has one positive real root.*

Case 2. $0 < \alpha < 1/e$, *then* $-\alpha \in (-1/e, 0)$. *The function* $h(\lambda) = \lambda e^{\lambda}$ *has two negative real roots in this domain because the inverse* $W(z)$ *is real and double-valued for* $z \in (-1/e, 0)$. *Hence,* $g(\lambda) = 0$ *has two negative roots.*

Case 3. $\alpha = 1/e$

Then:

$$\lambda e^{\lambda} = -\alpha = -1/e \Rightarrow \lambda = -1$$

Check multiplicity:

$g(\lambda) = \lambda + \alpha e^{-\lambda} = -1 + \frac{1}{e}e = 0$ ✓$g'(\lambda) = 1 - \alpha e^{-\lambda} = 1 - \frac{1}{e}e = 0$ ✓$g''(\lambda) = \alpha e^{-\lambda} = 1/e \cdot e =$

$1 \neq 0$ $\Rightarrow$ *multiplicity 2*

Case 4. $\alpha > 1/e$

Then:

$$-\alpha < -1/e \Rightarrow \lambda e^{\lambda} = -\alpha \text{ has no real solution}$$

So, no real roots exist.

Lemma 3. *(Oscillatory behavior conditions):*

- $0 < \alpha < \frac{\pi}{2}$: *All roots satisfy* $\Re(\lambda) < -\delta$ *for some* $\delta > 0$.
- $\alpha = \frac{\pi}{2}$: *Simple imaginary roots* $\lambda = \pm i\frac{\pi}{2}$.
- $\alpha > \frac{\pi}{2}$: *Roots with positive real parts exist* ⇒ *system becomes unstable.*

Proof: Let $\lambda = a + ib$. Substituting into:

$$\lambda + \alpha e^{-\lambda} = 0 \Rightarrow a + ib + \alpha e^{-a}(\cos b - i\sin b) = 0$$

Separate into real and imaginary parts:

$$a + \alpha e^{-a}\cos b = 0 \quad b - \alpha e^{-a}\sin b = 0$$

For stability ($\Re(\lambda) < 0$):

We analyze whether roots cross into $\Re(\lambda) > 0$. If $b \in [(\pi/2, \pi) + 2n\pi]$, then $\cos b < 0, \sin b > 0$. From imaginary part:

$$\sin b/b = e^{a}/\alpha \Rightarrow \text{since } \sin b/b \leq 2/\pi, \text{ we get: } \frac{1}{\alpha} \leq \frac{2}{\pi} \Rightarrow \alpha \geq \pi/2$$

Thus:

- If $\alpha < \pi/2$, then no complex roots with $\Re(\lambda) > 0$ exist.
- If $\alpha = \pi/2$, plug into (R) and (I) gives $\lambda = \pm i\pi/2$.
- If $\alpha > \pi/2$, a pair of complex roots exists with $\Re(\lambda) > 0$. □

Lemma 4. *For the delay differential equation:*

$$\frac{dk}{dt} = -\theta k(t - \tau)$$

Let $\alpha = \theta\tau$. *Then:*

- *If* $\theta < 0$, *then equilibrium* $k = 0$ *is unstable.*
- *If* $0 < \theta\tau < \pi/2$, *the zero solution is asymptotically stable.*
- *If* $\theta\tau = \pi/2$, $k(t) = sin(\pi t/2)$, $cos(\pi t/2)$ *are solutions.*
- *If* $\theta\tau > \pi/2$, *then* $k = 0$ *is unstable.*

Proof: from Equation (10), the characteristic equation is:

$$\lambda + \alpha e^{-\lambda} = 0$$

Using Lemma 3:

- If $\alpha < \pi/2$, then $\Re(\lambda) < 0$ for all roots ⇒ stable.
- If $\alpha > \pi/2$, then complex roots with $\Re(\lambda) > 0$ exist ⇒ unstable.
- $\alpha = \pi/2$ gives imaginary roots ⇒ critical delay.
- If $\theta < 0$, then $\alpha < 0$ and the unique real root is positive ⇒ unstable.

Therefore, stability depends directly on $\theta\tau$ with threshold $\pi/2$. □

Theorem 1. *(Oscillatory Solutions Criteria)*

For any $\theta > 0$ and $\tau > 0$, the following are equivalent:

1. *All solutions of (Equation (10)) are oscillatory.*
2. $\alpha = \theta\tau > \frac{1}{e}$

4. Numerical Application

The numerical findings provide us with information on the behavior of the system. This makes it more feasible for us to see how repercussions to the parameters or base points impact the dynamics of the business trade cycle. Furthermore, we have studied the stability of the Solow model. The parametric value is:

$$k = 0.9999$$

Result & Discussion Part

Figure 1 shows, when there is no delay $(\tau = 0)$ in the availability of labour and capital, the business trade cycle is always stable. Figure 2 represent the capital stock shows initial fluctuation in the business trade model. However, the limit cycle keeps decreasing in amplitude initially and become stable, if the delay parameter remains beneath the critical value, $\tau \geq 1.55999$. In Figure 3, the Business Trade Cycle represented by the Solow's model get trapped in an everlasting chaotic behaviour when $\tau \geq 1.55999$: Hopf-bifurcation is seen in the economic dynamics with limit cycle of same amplitude, same frequency, and in the same direction. Figures 4 and 5 shows the bar diagram and log scale view of business trade model when $\tau \geq 1.55999$ respectively.

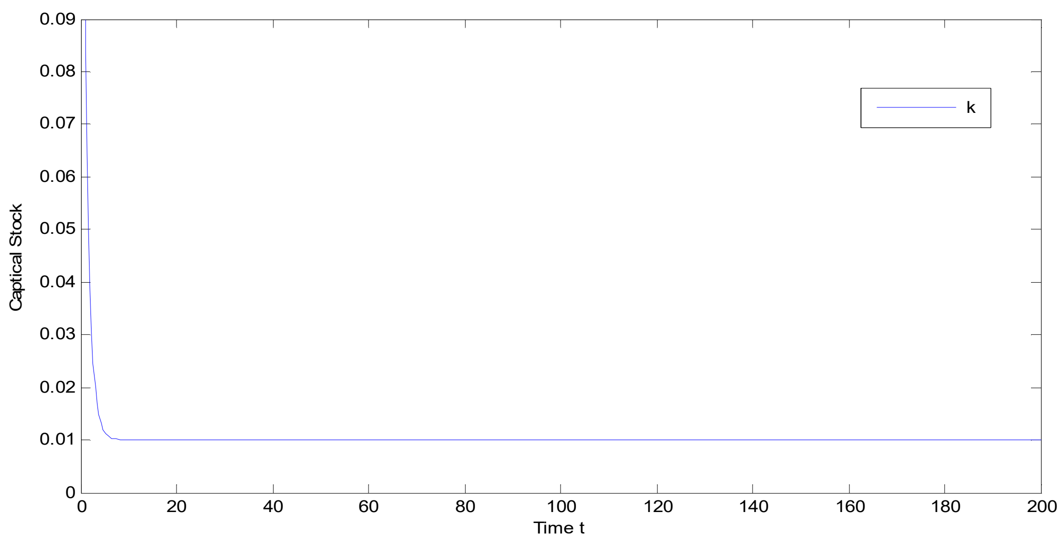


Figure 1. Represent the absolute stability of the business trade cycle in the absence of delay, i.e., $(\tau = 0)$.

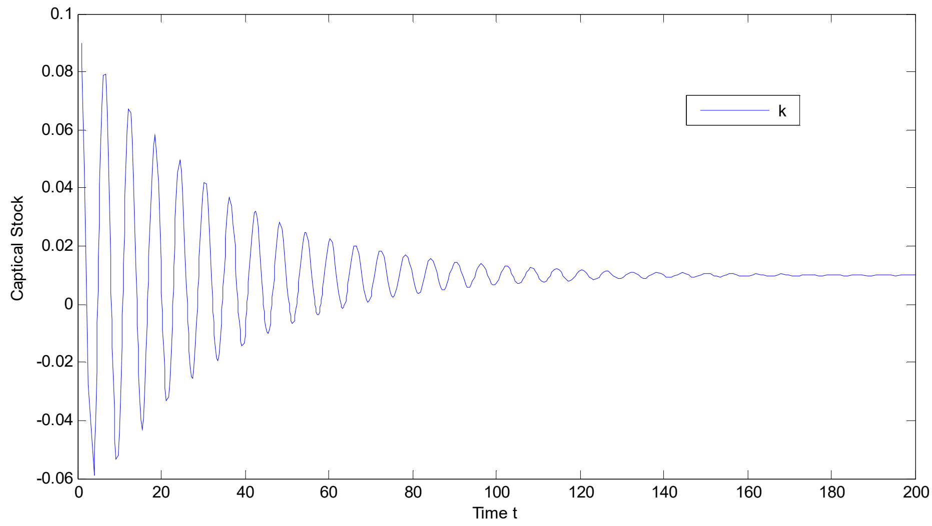


Figure 2. Shows the asymptotically stability of the business trade cycle when the value of delay parameter is less than the critical value, i.e., business trade cycle model when $\tau \leq 1.55999$.

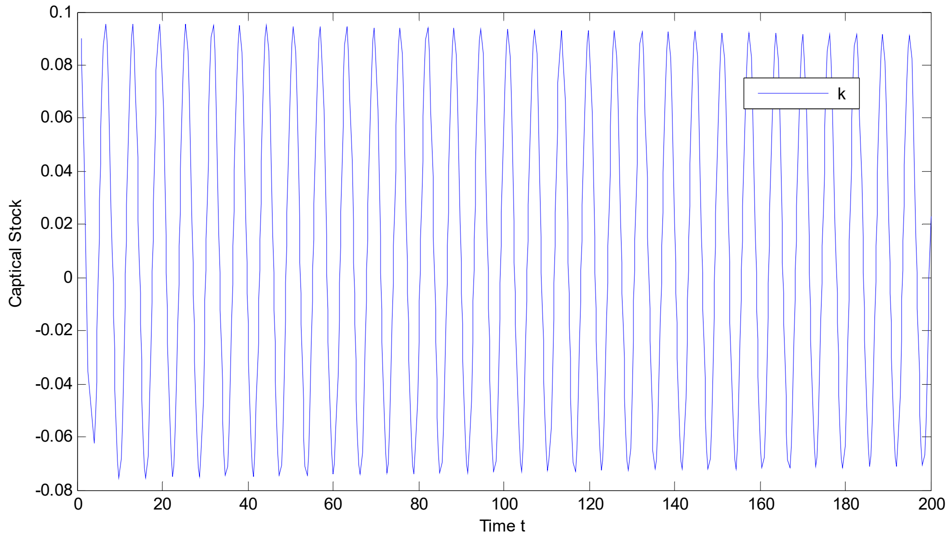


Figure 3. Shows the chaotic behaviour and bifurcation when the value of delay parameter is greater than the critical value, i.e., $\tau \geq 1.55999$.

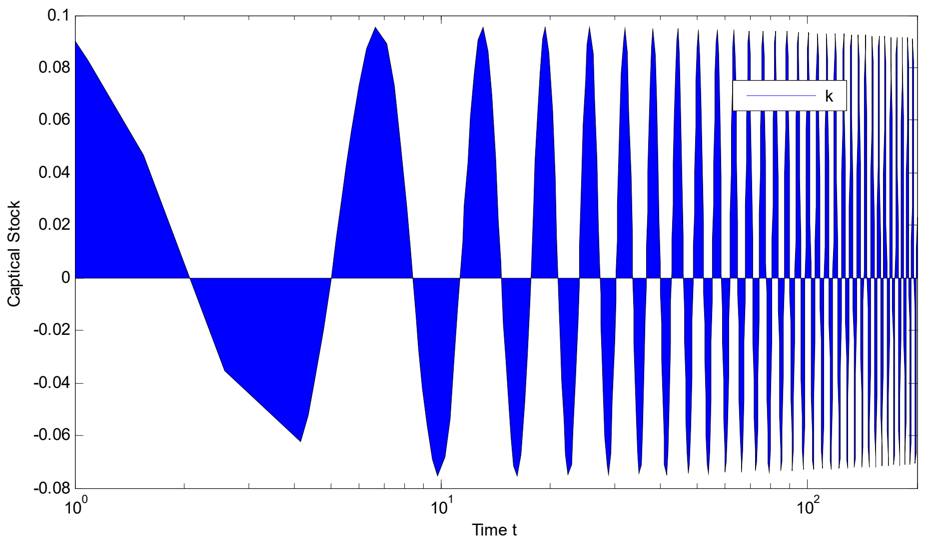


Figure 4. Bar diagram representation of business trade cycle model when $\tau \geq 1.55999$.

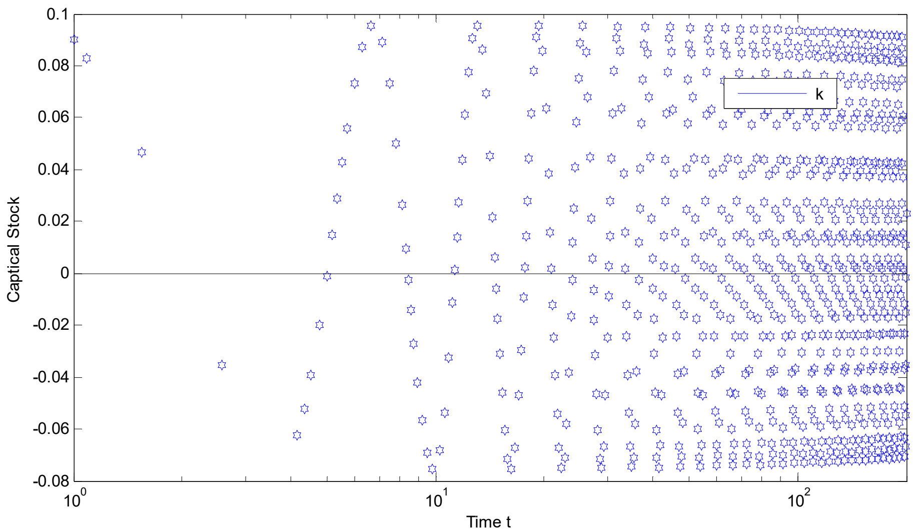


Figure 5. Shows the log scale view of business trade cycle model when $\tau \geq 1.55999$.

5. Conclusions

We present a mathematical model to investigate the stability of business trade model, using the Solow model. Delay parameter is introduced in the Solow model. The business trade cycle is stable when labour and capital is available (the capital stock is stable, $\tau = 0$). The capital stock losses the stability and shows the asymptotic stability when $\tau \leq 1.55999$ (The business trade cycle shows the limit cycle and then become stable). When $\tau \geq 1.55999$, the business trade cycle shows the chaotic behaviour and Hopf-bifurcation. The Solow model provides the insights into the critical threshold at which business trade cycle transition from stable to chaotic states.

The Solow model have several applications in the field of economic forecasting, policy development, risk management, investment strategies, and the supply chain management. The stakeholder can proactively manage and respond to economic fluctuations, enhancing overall stability and growth.

Author Contributions

Dipesh: Writing original manuscript, conceptualization, formal analysis, investigation, methodology, and software. P.K.: Formal analysis, and methodology, review & editing. All authors have read and agreed to the published version of the manuscript.

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

References

1. Samuelson, P.A. *Foundations of Economic Analysis*; Harvard University Press: Cambridge, MA, USA, 1983; Volume 197.
2. Gandolfo, G. *Economic Dynamics: Study Edition*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 1997.

3. Brock, W.A.; Malli, A.G. *Differential Equations, Stability and Chaos in Dynamic Economics*; North Holland: Amsterdam, The Netherlands, 1989.
4. Acemoglu, D. *Introduction to Modern Economic Growth*; Princeton University Press: Princeton, NJ, USA, 2008.
5. Acemoglu, D. Economic growth and development in the undergraduate curriculum. *J. Econ. Educ.* **2013**, *44*, 169–177.
6. Chen, Q.; Dipesh, K.P.; Baskonus, H.M. On the equilibrium point and Hopf-Bifurcation analysis of GDP-national debt dynamics under the delayed external investment: A new DDE model. *Alex. Eng. J.* **2024**, *91*, 510–515.
7. Chen, Q.; Dipesh, K.P.; Baskonus, H.M. Modeling and analysis of demand-supply dynamics with a collectability factor using delay differential equations in economic growth via the Caputo operator. *AIMS Math.* **2024**, *9*, 7471–7491.
8. Dmitriev, V.I.; Kurkina, E.S. The Inverse Problem of Determining the Parameters of a Mathematical Model Linking GDP with National Debt. *Comput. Math. Math. Phys.* **2020**, *31*, 75–95.
9. Fourie, D.J.; Blom, P.P.; Challenges, Strategies and Solutions to Manage Public Debt in South Africa. *Afr. J. Public Aff.* **2022**, *13*, 27–53.
10. Străchinariu, A.V. The Impact of Macroeconomic Indicators on Public Debt Dynamics. *Ann. Univ. Apulensis Ser. Oecon.* **2021**, *23*, 1–9.
11. Li, C.; Wan, J. The Influence of Enterprise Financing Structure in China on Business Performance—Simulation Analysis Based on the System Dynamics.In Proceedings of the 12th EAI International Conference, SIMUtools 2020, Guiyang, China, 28–29 August 2020; pp. 728–746.
12. Srinivasa, K.; Mundewadi, R.A. Wavelets approach for the solution of nonlinear variable delay differential equations. *Int. J. Math. Comput. Eng.* **2023**, *1*, 139–148.
13. Bilal, M.; Haris, H.; Waheed, A.; Faheem, M. The analysis of exact solitons solutions in monomode optical fibers to the generalized nonlinear Schrödinger system by the compatible techniques. *Int. J. Math. Comput. Eng.* **2023**, *1*, 149–170.
14. Kumar, A.; Kumar, S. Dynamic nature of analytical soliton solutions of the (1+1)-dimensional Mikhailov-Novikov-Wang equation using the unified approach. *Int. J. Math. Comput. Eng.* **2023**, *1*, 217–228.
15. Chen, Q.; Baskonus, H.M.; Gao, W.; Ilhan, E. Soliton theory and modulation instability analysis: The Ivancevic option pricing model in economy. *Alex. Eng. J.* **2022**, *61*, 7843–7851.
16. Chen, Q.; Sabir, Z.; Raja, M.A.Z.; et al. A fractional study based on the economic and environmental mathematical model. *Alex. Eng. J.* **2023**, *65*, 761–770.
17. Shananin, A.A.; Tarasenko, M.V.; Trusov, N.V. Mathematical modeling of household economy in Russia. *Comput. Math. Math. Phys.* **2021**, *61*, 1030–1051.
18. Gimaltdinov, I.F. Research of the demand for consumer loans and money. *Mat. Model.* **2012**, *24*, 84–98.
19. Mazloumfard, H.; Glantz, V. The influence of tax burden on the profit of banks in conditions of monopolistic competition: Economic-mathematical modeling. *Financ. Mark. Inst. Risks* **2017**, *1*, 28–36.
20. Tadmon, C.; Njike Tchaptchet, E.R. Financial crisis spread, economic growth and unemployment: A mathematical model. *Stud. Nonlinear Dyn. Econom.* **2023**, *27*, 147–170.
21. Arabov, N.; Nasimov, D.; Khuzhayorov, H.; et al. Modelling of commercial banks capitals competition dynamics. *Int. J. Early Child. Spec. Educ.* **2022**, 14, 4124–4132.
22. Wang, W.; Khan, M.A.; Kumam, P.; et al. A comparison study of bank data in fractional calculus. *Chaos Solitons Fractals* **2019**, *126*, 369–384.
23. Selyutin, V.V.; Rudenko, M.A. Mathematical model of the banking firm as tool for analysis, management and learning. *J. Inf. Technol. Educ.* **2013**, *16*, 170–177.
24. Comes, C.A. Banking system: Three level Lotka-Volterra model. *Procedia Econ. Financ.* **2012**, *3*, 251–255.
25. Marasco, A.; Picucci, A.; Romano, A. Market share dynamics using Lotka–Volterra models. *Technol. Forecast. Soc. Change* **2016**, *105*, 49–62.
26. Ruan, S. Absolute stability, conditional stability and bifurcation in Kolmogorov-type predator-prey systems with discrete delays. *Q. Appl. Math.* **2001**, *59*, 159–173.
27. Segura, J.; Franco, D.; Perán, J. Long-run economic growth in the delay spatial Solow model. *Spat. Econ. Anal.* **2023**, *18*, 158–172.
28. Kulikov, A.; Kulikov, D.; Radin, M. Periodic cycles in the Solow model with a delay effect. *Math. Model. Anal.* **2019**, *24*, 297–310.
29. Borges, M.J.; Fabião, F.; Teixeira, J. Long Cycles Versus Time Delays in a Modified Solow Growth Model. In *Eurasian Economic Perspectives*; Springer: Cham, Switzerland, 2020; pp. 375–383.
30. Tian, N.; Huang, D. Dynamical analysis of the Solow model with time delay and pollution. In Proceedings of the Second International Conference on Electrical, Electronics, and Information Engineering (EEIE 2023), Wuhan, China, 2–4 November 2023; pp. 651–656.
31. Ferrara, M.; Guerrini, L.; Mavilia, R. Modified neoclassical growth models with delay: A critical survey and perspectives. *Appl. Math. Sci.* **2013**, *7*, 4249–4257.

32. Harrod, R.F. An essay in dynamic theory. *Econ. J.* **1939**, *49*, 14–33.
33. Domar, E. Capital Expansion, Rate of Growth, and Employment. *Econometrica* **1946**, *14*, 137–147.
34. Samuelson, P.A. The stability of equilibrium: Comparative statics and dynamics. *Econometrica* **1941**, *9*, 97–120.
35. Solow, R.M. A contribution to the theory of economic growth. *Q. J. Econ.* **1956**, *70*, 65–94.
36. Inada, K.I. On a two-sector model of economic growth: Comments and a generalization. *Rev. Econ. Stud.* **1963**, *30*, 119–127.