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H_∞ Control for Multi-Agent Systems under Two-Way Amplify-and-Forward Relay Networks: The Event-Trigger Case

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Abstract: This paper addresses the finite-horizon H_∞ control problem for the two-way amplify-and-forward (AF) relay-assisted multi-agent system. To enhance communication quality and mitigate channel fading effects, a two-way AF relay protocol is introduced to coordinate data transmission among agents. Based on channel statistical characteristics and incorporating an event-triggered mechanism, this paper designs an H_∞ control protocol to ensure that the dynamic system satisfies the specified H_∞ performance metrics within a finite horizon. First, sufficient conditions are established for the system to satisfy the given H_∞ performance requirements. Then, by solving two coupled backward Riccati difference equations under the H_∞ performance constraints, the feedback gain matrix of the event-triggered controller is determined. Finally, the effectiveness of the proposed control strategy is verified through numerical simulations.

Keywords: multi-agent systems; event-triggered control; H_∞ control; two-way amplify-and-forward relays; coupled backward Riccati difference equations

1. Introduction

Since the 1980s, multi-agent systems (MASs) have emerged as a central and pivotal area of investigation within the field of distributed control systems. The principal objective of MASs is to coordinate the decentralized control of relatively simple agents to accomplish complex tasks. Owing to their high efficiency and scalability, MASs find widespread applications in domains such as robotics, power systems, distributed sensor networks, traffic management (see [1–6]). The capacity for coordinated control of a MAS is the foundation of the system's functionality. The aim of coordinated control of a MAS is to develop a control strategy for each agent and make decisions based on its own information and its neighboring agents. Based on the distinct objectives, coordinated control is classified into various types such as consensus control, formation control, rendezvous, containment control (see [7–14]).

In MASs, agents typically exchange information and coordinate their actions through communication networks. In practical scenarios, however, external disturbances, system uncertainties, communication noise, and channel fading can significantly degrade dynamic performance and coordination quality, and may even lead to instability. Owing to its capability to attenuate the influence of energy-bounded disturbances and noise on system outputs, H_∞ control has been widely adopted in distributed coordination and consensus control of MASs. By designing appropriate H_∞ control protocols, MASs can maintain desired performance levels in complex networked environments and achieve improved stability and reliability under uncertainties and extreme operating conditions. Therefore, investigating H_∞ control for MASs is of both theoretical and practical significance. Recent studies include a dynamic-observer-based H_∞ consensus control method in [15], an observer-based H_∞ fault-tolerant tracking control method in [16], an event-triggered H_∞ cooperative control method in [17], and an LMI-based H_∞ consensus control method in [18] and a nonfragile robust H_∞ containment control method in [19]. In [20], an H_∞

adaptive event-triggered control scheme is proposed for complex dynamical networks subject to deception attacks, time-varying coupling delays, and actuator failures.

Although communication networks facilitate the exchange of information among agents, they inevitably give rise to a series of network-induced phenomena. As one of the network-induced phenomena, signal attenuation directly leads to a decrease in the signal-to-noise ratio at the receiving end, which limits the channel's effective data transmission rate. This challenge becomes particularly pronounced when employing low-cost sensors that require wireless transmission. To ensure high-quality signal transmission over long distances, relays are typically employed to receive, process, and forward signals from source to destination nodes. The relay-assisted filtering and control paradigm has consequently emerged as a significant research focus in recent years. Current industrial practice predominantly employs several fundamental relay architectures, including decode-and-forward (DF), filter-and-forward, and amplify-and-forward (AF) relays (see [21–26]).

Compared to other relay types, the AF mechanism demonstrates unique technical advantages in industrial networks through its simplified approach that combines signal reception with amplification and forwarding. In particular, its low complexity and high reliability render it particularly amenable to deployment and maintenance in practical industrial environments. When integrated into two-way relay architectures, these advantages are further amplified. Research indicates that under equivalent data transmission requirements, two-way relay systems can significantly conserve total system transmission power by leveraging signal cooperative processing techniques [27]. This advantage primarily stems from the fact that two-way relay systems enable the collaboration and integration of information from two terminal nodes at the relay through advanced techniques such as physical layer network coding, thereby enhancing the combined efficiency of spectrum and power utilization.

Given these considerations, integrating the structurally simple and easily implementable AF mechanism with the efficient two-way relay architecture represents a highly promising research direction. In recent years, two-way relay networks have garnered extensive attention [28–31]. The AF and DF protocols originally developed for one-way relay networks have been successfully extended to half-duplex two-way relay networks and full-duplex two-way relay networks [32–36]. However, it is crucial to note that current research has not yet explored the application of two-way relay mechanisms in MASs with stringent requirements for transmission efficiency and signal quality. This paper aims to break through this research bottleneck and address the gap in this field.

In an ideal scenario, the communication bandwidth between neighboring agents is assumed to be infinite, and periodic triggering of communication is considered to have no impact on network load. In practical applications, the communication within MASs is constrained by available resources, and frequent signal transmission may result in adverse network-induced phenomena such as packet loss and disorder, potentially leading to system collapse. Therefore, employing event-triggered control strategies in MASs represents a more practical and efficient approach. Event-triggered control has garnered significant research interest in recent years, leading to diverse proposals of event-triggered control methods (see, e.g., [37–41]). Furthermore, event-triggered communication has been shown to enhance network bandwidth utilization and alleviate the communication burden among agents. However, limited attention has been paid to studying event-triggered control of relay-assisted MASs.

Motivated by the preceding discourse, this paper is dedicated to investigating event-triggered control for MASs assisted by two-way AF relays. Additionally, the H_∞ control strategy is utilized to mitigate the impact of external noise (including process noise, measurement noise, and channel noise) on the MAS. To address this problem, the following challenges need to be addressed: (1) how to model two-way relay-assisted MASs with random fading channels? (2) how to develop an H_∞ control strategy utilizing an event-triggered mechanism? and (3) how to derive sufficient conditions for the MAS to satisfy the predefined H_∞ performance requirement?

To tackle the above challenges, this paper develops a systematic solution framework. The main contributions are summarized as follows:

1. Compared with existing relay-assisted estimation/control studies that mainly focus on one-way relaying under fading channels (see, e.g., [23–25]), a two-way AF relay transmission model is developed to characterize the signal transmission process with random fading channels, where channel statistics are exploited to improve the reliability of measurement-feedback control.
2. Unlike conventional H_∞ formulations for MASs that typically neglect channel-induced communication noise (see, e.g., [15, 16]), an augmented H_∞ performance index is proposed by explicitly incorporating channel communication noise into the performance analysis, thereby providing a more comprehensive evaluation framework for relay-assisted MASs.
3. A novel approach utilizing coupled Riccati difference equations (RDEs) that incorporate event-triggered mechanisms and statistical information is developed to address the H_∞ control problem in relay-assisted MASs.

The remainder of this paper is organized as follows. Section 2 presents the graph theory framework for relay networks, formulates a mathematical model for two-way AF relay networks, and establishes an event-triggered control strategy for relay-assisted MASs. Section 3 proposes a pair of RDEs to address the challenges associated with maintaining H_∞ performance and designing event-triggered controllers for relay-assisted MASs. Section 4 presents a numerical example to verify the effectiveness of the proposed algorithm under various noise conditions. Finally, Section 5 concludes this paper.

Notations: The notation employed herein adheres to established conventions, unless otherwise specified. $\mathbb{E}\{x\}$ and $\mathbb{D}\{x\}$ denote the expectation and variance of the stochastic variable x, respectively. I_n denotes the n-dimensional identity matrix, and I is the abbreviated form with compatible dimension if no confusion is caused. A block-diagonal matrix is represented by the notation $\mathrm{diag}\{\cdots\}$. $\|x\|$ refers to the Euclidean norm of a vector x. $\|M\|_F$ denotes the Frobenius norm of the matrix M. $\otimes$ and $\circ$ denote the Kronecker product and the Hadamard product of matrices, respectively. $\mathrm{Tr}\{\cdot\}$ denotes the trace of a matrix.

2. Problem Formulation and Preliminaries

2.1. Graph with Two-Way AF Relays

Consider a two-way AF relay-assisted MAS with n agents and m relays. Let $\mathscr{G}(\mathscr{V},\mathscr{E},\mathscr{A})$ denote the communication topology of the system, where $\mathscr{V} \triangleq \mathscr{V}_a \cup \mathscr{V}_r$, $\mathscr{V}_a \triangleq \{v_1^a, v_2^a, \ldots, v_n^a\}$ and $\mathscr{V}_r \triangleq \{v_1^r, v_2^r, \ldots, v_m^r\}$ represent the finite set of agents and relays, respectively; $\mathscr{E}$ represents the set of edges between agents and relays; $\mathscr{A}$ is the adjacency matrix with non-negative elements. (v_j^a, v_s^r, v_i^a) denotes the indirect data flow from agent j through the s-th relay to agent i. The first element v_j^a is said to be the source agent and the other v_i^a to be the destination agent. The set of neighboring agents of v_i^a is defined as $\mathscr{N}_i \triangleq \{v_j^a \in \mathscr{V}_a \mid (v_j^a, v_k^r, v_i^a) \in \mathscr{E}\}$.

The adjacency matrix $\mathscr{A} \triangleq [a_{ij}] \in \mathbb{R}^{n\times n}$ of $\mathscr{G}$ is defined as

$$a_{ij} \triangleq \begin{cases} 1, & (v_i^a, v_k^r, v_j^a) \in \mathscr{E},\ i \neq j \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, $\mathscr{A}$ is a symmetric matrix in two-way AF based networks.

2.2. Plant Description

Consider a discrete time-varying MAS defined over a finite horizon $k \in [0, \tau-1]$. This MAS consists of n agents, with the set of agents denoted as $\mathscr{V}_a = \{1, 2, 3, \ldots, n\}$. For each agent $i \in \mathscr{V}_a$, the dynamic is defined as follows:

$$\begin{cases} x_{i,k+1} = A_k x_{i,k} + B_k u_{i,k} + D_k w_{i,k} \\ y_{i,k} = C_k x_{i,k} + E_k \nu_{i,k} \\ z_{i,k} = M_k x_{i,k} \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_x}$ and $u_{i,k} \in \mathbb{R}^{n_u}$ denote the state and the control input of agent i, respectively; $y_{i,k} \in \mathbb{R}^{n_y}$ and $z_{i,k} \in \mathbb{R}^{n_z}$ denote the measurement output and the controlled output of agent i, respectively; $w_{i,k} \in l_2([0,\tau-1];\mathbb{R}^{n_w})$ denotes the external disturbance of process and $\nu_{i,k} \in l_2([0,\tau-1];\mathbb{R}^{n_\nu})$ denotes the external disturbance of measurement; The matrices A_k, B_k, C_k, D_k, E_k and M_k are time-varying system parameters with compatible dimensions.

2.3. Two-Way AF Relays Network Model

Consider a general two-way relay network comprising $n+m$ nodes, where n agents and m relays are deployed. In this network architecture, each agent serves as a source node for its neighboring agents and establishes data transmission links through two-way relays. There is a one-to-one correspondence between each relay and the agents at both ends of the link. Let r_i^j denote the relay on link (v_i^a, v_j^a). The information exchange between any two agents occurs in two distinct phases: the agent-to-relay (A→R) phase and the relay-to-agent (R→A) phase, as illustrated in Figure 1.

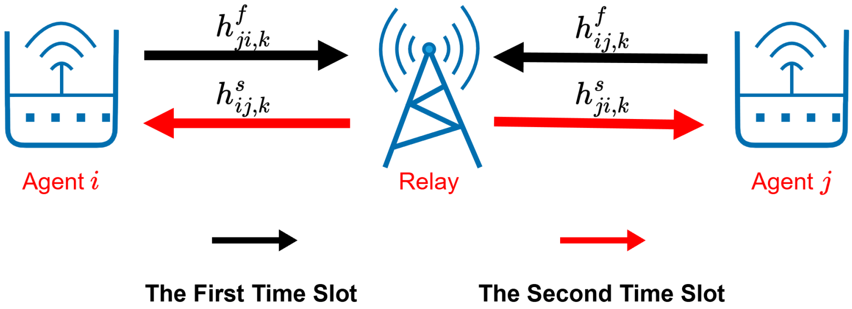

Figure 1. Two-way AF network.

(Phase 1) A→R : In the first phase, two adjacent agents simultaneously transmit their measurement data to the relay positioned between them, resulting in superimposed data at the relay. The signal received by the r_i^j relay located between agent i and agent j at time instant k can be expressed as

$$y_k^{r_i^j} = \sqrt{p_{ij}^f} h_{ij,k}^f y_{j,k}^{sou} + \sqrt{p_{ji}^f} h_{ji,k}^f y_{i,k}^{sou} + \delta_k^{r_i^j}$$

where p_{ij}^f and p_{ji}^f represent the first phase transmission power from agent j to i and from agent i to j, respectively; $y_{j,k}^{sou}$ and $y_{i,k}^{sou}$ denote the measurement data prepared for transmission by agent j and agent i, respectively; $y_k^{r_i^j}$ represents the superimposed measurement data received by the relay. Here, $\delta_k^{r_i^j}$ denotes the channel noise, which is mutually independent across different links and time instants, with $\mathbb{E}\{\delta_k^{r_i^j}\} = 0$ and $\mathbb{D}\{\delta_k^{r_i^j}\} = \varrho_1^2$ for $\forall i, j \in \mathscr{V}_a$.

(Phase 2) R→A : In the second phase, upon receiving the superimposed data, the r_i^j relay amplifies the received signal and broadcasts it to the two source agents connected to it. The data received by each agent can be expressed as

$$y_{ij,k}^{des} = \sqrt{p_{ij}^s} h_{ij,k}^s y_k^{r_i^j} + \delta_{ij,k}^a = \underbrace{\sqrt{p_{ij}^f p_{ij}^s} h_{ij,k}^f h_{ij,k}^s y_{j,k}^{sou}}_{data\ of\ agent\ j} + \underbrace{\sqrt{p_{ji}^f p_{ij}^s} h_{ji,k}^f h_{ij,k}^s y_{i,k}^{sou}}_{data\ of\ agent\ i} + \underbrace{\sqrt{p_{ij}^s} h_{ij,k}^s \delta_k^{r_i^j} + \delta_{ij,k}^a}_{channel\ noises},$$

$$y_{ji,k}^{des} = \sqrt{p_{ji}^s} h_{ji,k}^s y_k^{r_i^j} + \delta_{ji,k}^a = \underbrace{\sqrt{p_{ji}^f p_{ji}^s} h_{ji,k}^f h_{ji,k}^s y_{i,k}^{sou}}_{data\ of\ agent\ i} + \underbrace{\sqrt{p_{ij}^f p_{ji}^s} h_{ij,k}^f h_{ji,k}^s y_{j,k}^{sou}}_{data\ of\ agent\ j} + \underbrace{\sqrt{p_{ji}^s} h_{ji,k}^s \delta_k^{r_i^j} + \delta_{ji,k}^a}_{channel\ noises}$$

where $y_{ij,k}^{des}$ and $y_{ji,k}^{des}$ denote the measurement data at destination agents i and j, respectively; p_{ij}^s and p_{ji}^s represent the second phase transmission power from relay r_i^j to agents i and j, respectively; $\delta_{ij,k}^a$ and $\delta_{ji,k}^a$ denote the channel noises, which are independent of each other; $\mathbb{E}\{\delta_{ij,k}^a\} = 0$ and $\mathbb{D}\{\delta_{ij,k}^a\} = \varrho_2^2$ for $\forall i, j \in \mathscr{V}_a$.

Since each agent possesses knowledge of its own measurement data, by employing self-interference cancellation (SIC) technology, the data received by agent i from agent j can be expressed as

$$\begin{cases} y_{ij,k}^{des} = \sqrt{p_{ij}^f p_{ij}^s} h_{ij,k}^s h_{ij,k}^f y_{j,k}^{sou} + \sqrt{p_{ij}^s} h_{ij,k}^s \delta_k^{r_i^j} + \delta_{ij,k}^a \\ y_{ji,k}^{des} = \sqrt{p_{ji}^f p_{ji}^s} h_{ji,k}^s h_{ji,k}^f y_{i,k}^{sou} + \sqrt{p_{ji}^s} h_{ji,k}^s \delta_k^{r_i^j} + \delta_{ji,k}^a \end{cases} \tag{2}$$

In this article, the channel fading coefficients are mutually independent and exhibit the following statistical properties:

$$\mathbb{E}\{h_{ij,k}^f\} = \mathbb{E}\{h_{ij,k}^s\} = \mu, \qquad \mathbb{D}\{h_{ij,k}^f\} = \mathbb{D}\{h_{ij,k}^s\} = \sigma^2, \quad \forall i, j \in \mathscr{V}_a \tag{3}$$

Remark 1. *Specifically, the statistical characterization of fading captures the average effect of communication links without requiring exact instantaneous channel information at every time instant, whereas the independence assumption allows the expectation calculations and coupled Riccati derivations to be carried out in a closed and transparent form. In addition, since process disturbances, measurement noises, and relay-channel noises usually originate from different physical sources, these assumptions are also consistent with standard abstractions widely used in stochastic estimation and networked control.*

Remark 2. *In the second transmission slot, also known as the broadcast phase, the relay broadcasts the received information to the agents at both ends of the link, and each receiving agent contains the information it originally* *transmitted to the relay. Coding techniques can be employed to eliminate the impact of this self-interference information, a method known as SIC technology. For instance, in [42], the physical-layer network coding mapping (PNC) method addresses this issue by eliminating self-measurement data through bitwise XOR operations. Let S_A and S_B denote the signals after PNC mapping of the measurement data y_a and y_b from the source agents in the first time slot, respectively, and let S_R denote the PNC mapping signal received at the two-way relay (e.g., $S_R = S_A \oplus S_B$). Subsequently, the generated signal S_R is amplified by the relay and broadcast to the target agents. Since each target node is also a source node that holds its own measurement data transmitted in the first time slot, the target agent can eliminate the impact of its own measurement data by performing SIC operations when receiving S_R (e.g., $S_A \oplus S_R = S_A \oplus (S_A \oplus S_B) = S_B$). In practical two-way relay-assisted systems, residual self-interference inevitably exists because of imperfect channel knowledge and hardware non-idealities. In this paper, we assume that the SIC technique is perfectly implemented, namely, each agent is assumed to perfectly remove its own known signal from the received network-coded signal. Therefore, no residual self-interference or imperfect cancellation term is included in the received signal model. In other words, the analysis is conducted under an ideal perfect-cancellation assumption, which is used to keep the H_∞ control problem tractable.*

Remark 3. *In a two-way AF network, the channel fading factor demonstrates a property of reciprocity between the source agent and the relay node. That is, the channel fading coefficient for the forward link (from the source to the relay) is equal to that of the reverse link (from the relay back to the same source agent) at any given time instant. As shown in the preceding equations, this reciprocal relationship ensures that the channel fading characteristics remain unchanged between the two transmission phases.*

2.4. Controller Design

For the purpose of simplicity, the power matrices are denoted as $\mathsf{P}_f \triangleq [\sqrt{p^f_{ij}}]_{n\times n}$ and $\mathsf{P}_s \triangleq [\sqrt{p^s_{ij}}]_{n\times n}$, and the random channel fading coefficient matrices are denoted as $\mathsf{H}^f_k \triangleq [h^f_{ij,k}]_{n\times n}$ and $\mathsf{H}^s_k \triangleq [h^s_{ij,k}]_{n\times n}$.

Define

$$\delta^r_{i,k} \triangleq \text{col}\{\delta^{r^1_i}_{i1,k}, \delta^{r^2_i}_{i2,k}, \dots, \delta^{r^n_i}_{in,k}\}, \quad \delta^a_{i,k} \triangleq \text{col}\{\delta^a_{i1,k}, \delta^a_{i2,k}, \dots, \delta^a_{in,k}\},$$
$$\delta_k \triangleq \text{col}\{\delta^r_{1,k}, \delta^a_{1,k}, \delta^r_{2,k}, \delta^a_{2,k}, \dots, \delta^r_{n,k}, \delta^a_{n,k}\}, \quad y_k \triangleq \text{col}\{y_{1,k}, y_{2,k}, \dots, y_{n,k}\}.$$

According to (2), the aggregated information available to agent i is obtained by summing all pairwise received data from its neighbors, namely, $\widetilde{y}_{i,k} = \sum_{j=1}^n a_{ij} y^{des}_{ij,k}$. Therefore, the total received measurement is rewritten as

$$\begin{aligned}
\widetilde{y}_{i,k} &\triangleq \sum_{j=1}^n a_{ij} y^{des}_{ij,k} \\
&= \sum_{j=1}^n a_{ij}\Big(\sqrt{p^f_{ij}p^s_{ij}}h^s_{ij,k}h^f_{ij,k}y_{j,k} + \sqrt{p^s_{ij}}h^s_{ij,k}\delta^{r^j_i}_k + \delta^a_{ij,k}\Big) \\
&= \sum_{j=1}^n a_{ij}\big[\mathsf{P}_f \circ \mathsf{P}_s \circ \mathsf{H}^f_k \circ \mathsf{H}^s_k\big]_{ij} y_{j,k} + \big[(\mathsf{P}_s \circ \mathsf{H}^s_k \circ \mathscr{A}) \otimes I_\delta\big]_i \delta^r_{i,k} + \big[\mathscr{A} \otimes I_\delta\big]_i \delta^a_{i,k} \\
&\triangleq \big[H_k \otimes I_y\big]_i y_k + \big[G_k \otimes I_\delta\big]_i \delta_k
\end{aligned} \tag{4}$$

where

$$H_k \triangleq \mathsf{P}_f \circ \mathsf{P}_s \circ \mathsf{H}^f_k \circ \mathsf{H}^s_k \circ \mathscr{A}, \quad L_k \triangleq \begin{bmatrix}\mathsf{P}_s \circ \mathsf{H}^s_k \circ \mathscr{A} & \mathscr{A}\end{bmatrix},$$
$$G_k \triangleq \text{diag}\{[L_k]_1, [L_k]_2, \dots, [L_k]_n\},$$

and $[\cdot]_{ij}$ denotes the (i,j)-th entry of the matrix; $[\cdot]_i$ denotes the i-th block row of the matrix.

For the subsequent analysis, the following notations are defined: $\mathbb{E}\{\mathsf{H}^f_k\} \triangleq \bar{\mathsf{H}}^f \triangleq [\bar{h}^f_{ij}]_{n\times n}$, $\mathbb{E}\{\mathsf{H}^s_k\} \triangleq \bar{\mathsf{H}}^s \triangleq [\bar{h}^s_{ij}]_{n\times n}$, $\mathbb{E}\{H_k\} \triangleq \bar{H}$, and $\mathbb{E}\{G_k\} \triangleq \bar{G}$.

In this article, the output-feedback control strategy is adopted. The control protocol is constructed as follows:

$$u_{i,k} = K_k\Big(\sum_{j=1}^n a_{ij}y^{des}_{ij,k} - \sum_{j=1}^n a_{ij}y_{i,k}\Big) \triangleq K_k e_{i,k} \tag{5}$$

where $K_k \in \mathbb{R}^{n_x \times n_y}(k \in [0, \tau-1])$ is feedback gain matrix to be determined. $e_{i,k} \triangleq \sum_{j=1}^{n} a_{ij} y_{ij,k}^{des} - \sum_{j=1}^{n} a_{ij} y_{i,k}$ is the error signal of agent i at time k.

2.5. Event-Triggering Mechanism

To introduce the event-triggering scheduling mechanism, let the triggering instant sequence of agent i be denoted as $t_0^i = 0 < t_1^i < t_2^i < \cdots$. Consequently, the event-triggered MAS dynamics can be reformulated as follows:

$$\begin{cases} x_{i,k+1} = A_k x_{i,k} + B_k u_{i,k}^t + D_k \omega_{i,k} \\ u_{i,k}^t = K_k e_{i,k}^t \end{cases} \tag{6}$$

where, for simplicity, $e_{i,k}^t \triangleq e_{i,t_s^i}$ for $k \in [t_s^i, t_{s+1}^i)$, and $u_{i,k}^t$ denotes the control input of agent i at the triggering instant.

By denoting $\bar{e}_{i,k} \triangleq e_{i,k}^t - e_{i,k}$, the event-triggering functions $\Upsilon_i(\cdot)$ are designed as follows:

$$\Upsilon_i(\bar{e}_{i,k}, e_{i,k}) = \bar{e}_{i,k}^T \bar{e}_{i,k} - \rho^2 e_{i,k}^T e_{i,k}, \quad i = 1, 2, \ldots, n.$$

where $\rho > 0$ is a scalar parameter that prescribes the maximum allowable ratio between the accumulated deviation $\bar{e}_{i,k} = e_{i,k}^t - e_{i,k}$ and the current local control error $e_{i,k}$, thereby governing the triggering frequency. An event is triggered when the condition

$$\Upsilon_i(\bar{e}_{i,k}, e_{i,k}) > 0$$

is satisfied.

Obviously, the sequence of event-triggering instants is determined iteratively by

$$t_{s+1}^i = \inf\{k \in \mathbb{N} | k > t_s^i, \Upsilon_i(\bar{e}_{i,k}, e_{i,k}) > 0\}.$$

To facilitate the subsequent formulation, the following vectors are defined as

$$\begin{aligned} & x_k \triangleq \text{col}\{x_{1,k}, x_{2,k}, \ldots, x_{n,k}\}, \quad u_k \triangleq \text{col}\{u_{1,k}, u_{2,k}, \ldots, u_{n,k}\}, \\ & u_k^t \triangleq \text{col}\{u_{1,k}^t, u_{2,k}^t, \ldots, u_{n,k}^t\}, \quad e_k \triangleq \text{col}\{e_{1,k}, e_{2,k}, \ldots, e_{n,k}\}, \\ & e_k^t \triangleq \text{col}\{e_{1,k}^t, e_{2,k}^t, \ldots, e_{n,k}^t\}, \quad \omega_k \triangleq \text{col}\{\omega_{1,k}, \omega_{2,k}, \ldots, \omega_{n,k}\}, \\ & \nu_k \triangleq \text{col}\{\nu_{1,k}, \nu_{2,k}, \ldots, \nu_{n,k}\}. \end{aligned}$$

Based on the individual agent dynamics (6) and the previously defined vector notations, the compact form of the event-triggered MAS dynamics is given by

$$\begin{cases} x_{k+1} = \mathcal{A}_k x_k + \mathcal{B}_k u_k^t + \mathcal{D}_k \omega_k \\ y_k = \mathcal{C}_k x_k + \mathcal{E}_k \nu_k \\ z_k = \mathcal{M}_k x_k \end{cases}$$

where

$$\begin{aligned} & \mathcal{A}_k \triangleq I_n \otimes A_k, \quad \mathcal{B}_k \triangleq I_n \otimes B_k, \quad \mathcal{C}_k \triangleq I_n \otimes C_k, \\ & \mathcal{D}_k \triangleq I_n \otimes D_k, \quad \mathcal{E}_k \triangleq I_n \otimes E_k, \quad \mathcal{M}_k \triangleq I_n \otimes M_k. \end{aligned}$$

By defining $\widetilde{y}_k = \text{col}\{\widetilde{y}_{1,k}, \widetilde{y}_{2,k}, \ldots, \widetilde{y}_{n,k}\}$ and according to (4), we have

$$\widetilde{y}_k = (H_k \otimes I_y) y_k + (G_k \otimes I_\delta) \delta_k,$$

and, from the definition of $e_{i,k}$ in (5), it follows that

$$e_{i,k} = \sum_{j=1}^{n} a_{ij} y_{ij,k}^{des} - \sum_{j=1}^{n} a_{ij} y_{i,k} = \widetilde{y}_{i,k} - d_i y_{i,k},$$

where $d_i = \sum_{j=1}^{n} a_{ij}$. Stacking the above equations for all agents gives

$$\begin{aligned} e_k &= \widetilde{y}_k - (\mathsf{D} \otimes I_y) y_k \\ &= \big((H_k - \mathsf{D}) \otimes I_y\big) y_k + (G_k \otimes I_\delta)\delta_k \\ &= \big((H_k - \mathsf{D}) \otimes I_y\big)(\mathcal{C}_k x_k + \mathcal{E}_k \nu_k) + (G_k \otimes I_\delta)\delta_k \\ &\triangleq \mathcal{H}_k \mathcal{C}_k x_k + \mathcal{H}_k \mathcal{E}_k \nu_k + \mathcal{G}_k \delta_k. \end{aligned} \tag{7}$$

where $\mathcal{H}_k \triangleq (H_k - \mathsf{D}) \otimes I_y$, $\mathcal{G}_k \triangleq G_k \otimes I_\delta$, and $\mathsf{D} \triangleq \text{diag}\{d_1, d_2, \ldots, d_n\}$. Moreover, by stacking $u_{i,k}^t = K_k e_{i,k}^t$ for all agents, one has

$$u_k^t = (I_n \otimes K_k) e_k^t = \mathcal{K}_k e_k^t. \tag{8}$$

where $\mathcal{K}_k \triangleq I_n \otimes K_k$.

So far, a series of definitions and calculations have resulted in the event-triggered two-way AF relay-assisted MAS plant. Next, the closed-loop system needs to be constructed.

To facilitate the analysis, we denote

$$\mathbb{E}\{\mathcal{H}_k\} \triangleq \bar{\mathcal{H}}, \quad \mathbb{E}\{\mathcal{G}_k\} \triangleq \bar{\mathcal{G}}, \quad \check{\mathcal{H}}_k \triangleq \mathcal{H}_k - \bar{\mathcal{H}}, \quad \check{\mathcal{G}}_k \triangleq \mathcal{G}_k - \bar{\mathcal{G}}.$$

For simplicity, by denoting $\varpi_k \triangleq \text{col}\{\omega_k, \nu_k, \delta_k\}$ and noting that $\bar{e}_k = e_k^t - e_k$, substituting (7) and (8) into (6) yields

$$\begin{aligned} x_{k+1} &= \mathcal{A}_k x_k + \mathcal{B}_k \mathcal{K}_k e_k^t + \mathcal{D}_k \omega_k \\ &= (\mathcal{A}_k + \mathcal{B}_k \mathcal{K}_k \mathcal{H}_k \mathcal{C}_k) x_k + \mathcal{B}_k \mathcal{K}_k \bar{e}_k + \mathcal{B}_k \mathcal{K}_k \mathcal{H}_k \mathcal{E}_k \nu_k + \mathcal{D}_k \omega_k + \mathcal{B}_k \mathcal{K}_k \mathcal{G}_k \delta_k \\ &= (\mathcal{A}_k + \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{H}} \mathcal{C}_k + \mathcal{B}_k \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{C}_k) x_k + \mathcal{B}_k \mathcal{K}_k \bar{e}_k \\ &\quad + [\mathcal{D}_k,\ \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{H}} \mathcal{E}_k,\ \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{G}}] \varpi_k + [\mathbf{0},\ \mathcal{B}_k \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{E}_k,\ \mathcal{B}_k \mathcal{K}_k \check{\mathcal{G}}_k] \varpi_k. \end{aligned}$$

Thus, the dynamic of the closed-loop system in a simplified form is given by

$$\begin{cases} x_{k+1} = (\widetilde{\mathcal{A}}_k + \widetilde{\mathcal{B}}_k^r) x_k + \mathcal{B}_k \mathcal{K}_k \bar{e}_k + (\widetilde{\mathcal{D}}_k + \widetilde{\mathcal{D}}_k^r) \varpi_k \\ \quad z_k = \mathcal{M}_k x_k \end{cases} \tag{9}$$

where

$$\begin{aligned} &\widetilde{\mathcal{A}}_k \triangleq \mathcal{A}_k + \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{H}} \mathcal{C}_k, \quad \widetilde{\mathcal{B}}_k^r \triangleq \mathcal{B}_k \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{C}_k, \\ &\widetilde{\mathcal{D}}_k \triangleq [\mathcal{D}_k,\ \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{H}} \mathcal{E}_k,\ \mathcal{B}_k \mathcal{K}_k \bar{\mathcal{G}}], \quad \widetilde{\mathcal{D}}_k^r \triangleq [\mathbf{0},\ \mathcal{B}_k \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{E}_k,\ \mathcal{B}_k \mathcal{K}_k \check{\mathcal{G}}_k]. \end{aligned}$$

The event-trigger function is given by

$$\Upsilon(\bar{e}_k, e_k) = \bar{e}_k^T \bar{e}_k - \rho^2 e_k^T e_k. \tag{10}$$

Define the consensus error as $\bar{z}_{i,k} \triangleq z_{i,k} - (1/n) \sum_{j=1}^{n} z_{j,k}$ and $\bar{z}_k \triangleq z_k - \phi z_k$, where $\phi = (1/n)\, \mathbf{1}_n \mathbf{1}_n^T \otimes I$. We consider the finite-horizon H_∞ control problem in the presence of external disturbances. The objective is to design the feedback gain sequence $\{K_k\}_{k=0}^{\tau-1}$ for the two-way AF relay-assisted MAS in (1) such that the closed-loop system (9) satisfies the following H_∞ performance index:

$$J_1 \triangleq \mathbb{E}\Big\{ \sum_{k=0}^{\tau-1} \left(\|\bar{z}_k\|^2 - \gamma^2 \|\varpi_k\|^2 \right) - \gamma^2 x_0^T \mathcal{W} x_0 \Big\} < 0, \quad \forall \big(\varpi_k, x_0\big) \neq 0 \tag{11}$$

where $\gamma > 0$ denotes the prescribed disturbance attenuation level, and $\mathcal{W} = I_n \otimes W$ with $W = W^T > 0$ a weighting matrix.

Remark 4. *It is worth highlighting that the finite-horizon H_∞ performance index (11) proposed in this paper departs in an essential way from the H_∞ formulations commonly adopted in existing MAS literature (see, e.g., [15,16]). Specifically, the exogenous vector ϖ_k collects not only the external process and measurement disturbances, but also the channel-induced communication noise that originates from the two-way AF relay network and is* *typically overlooked in conventional H_∞ MAS settings. Imposing the attenuation level γ uniformly on all of these exogenous sources thereby furnishes a more comprehensive and physically faithful performance characterization for relay-assisted MASs.*

3. Main Result

Before presenting the main result, we introduce two fundamental lemmas that will be essential to the subsequent analysis.

Lemma 1. *Ref.* [43]. *Let $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ be known nonzero matrices with compatible dimensions. The optimal solution $\mathcal{X}^*$ to the minimization problem $\min_{\mathcal{X}} \|\mathcal{A}\mathcal{X}\mathcal{C} - \mathcal{B}\|_F$ is given by*

$$\mathcal{X}^* = \mathcal{A}^\dagger \mathcal{B} \mathcal{C}^\dagger,$$

where $\mathcal{A}^\dagger$ and $\mathcal{C}^\dagger$ denote the Moore-Penrose pseudoinverses of $\mathcal{A}$ and $\mathcal{C}$, respectively.

Lemma 2. *Ref.* [44]. *Let $\mathcal{X} \in \mathbb{R}^k$ be a k-dimensional random vector and $\mathcal{A} \in \mathbb{R}^{k\times k}$ be a constant symmetric matrix. If $\mathbb{E}(\mathcal{X}) = \mu$ and $\mathbb{D}(\mathcal{X}) = \Sigma$, then the expectation of the quadratic form satisfies*

$$\mathbb{E}(\mathcal{X}^T \mathcal{A} \mathcal{X}) = Tr(\mathcal{A}\Sigma) + \mu^T \mathcal{A} \mu.$$

The H_∞ performance is analyzed for the two-way AF relay-assisted MAS (9) in the following theorem.

Theorem 1. *Consider a time-varying MAS over two-way AF relay networks where $k \in [0, \tau - 1]$. Given a disturbance attenuation level $\gamma > 0$, two constants $\lambda > 0$ and $\rho > 0$, and a positive definite matrix W, the time-varying MAS (9) satisfies the H_∞ performance criterion for any disturbance sequence $\{\omega_k, \nu_k, \delta_k\}_{0\le k\le \tau-1}$ if there exists a sequence of non-negative definite matrices $\{\mathcal{P}_k\}_{0\le k\le\tau-1}$ that satisfy the following recursive backward RDE.*

$$\begin{aligned}
\mathcal{P}_k = {} & \widetilde{\mathcal{A}}_k^T \mathcal{P}_{k+1} \widetilde{\mathcal{A}}_k + \Omega_{1,k+1} + (\mathcal{M}_k - \phi\mathcal{M}_k)^T (\mathcal{M}_k - \phi\mathcal{M}_k) \\
& + (\widetilde{\mathcal{A}}_k^T \mathcal{P}_{k+1} \widetilde{\mathcal{D}}_k + \Omega_{2,k+1} + \rho^2\lambda^2\Phi_{3,k}) \Delta_{1,k+1}^{-1} (\widetilde{\mathcal{D}}_k^T \mathcal{P}_{k+1} \\
& \times \widetilde{\mathcal{A}}_k + \Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T) + \rho^2\lambda^2\Phi_{1,k} + \big(\mathcal{R}_{k+1}\widetilde{\mathcal{A}}_k \\
& + \mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \Delta_{1,k+1}^{-1} (\Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T)\big)^T \mathcal{B}_k \mathcal{K}_k \Delta_{2,k+1}^{-1} \\
& \times \mathcal{K}_k^T \mathcal{B}_k^T \big(\mathcal{R}_{k+1}\widetilde{\mathcal{A}}_k + \mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \Delta_{1,k+1}^{-1} (\Omega_{2,k+1}^T \\
& + \rho^2\lambda^2\Phi_{3,k}^T)\big)
\end{aligned} \tag{12}$$

subject to

$$\begin{cases}
\mathcal{P}_0 \le \gamma^2 \mathcal{W} \\
\mathcal{P}_\tau = \mathbf{0} \\
\Delta_{1,k+1} \triangleq \gamma^2 I - \widetilde{\mathcal{D}}_k^T \mathcal{P}_{k+1} \widetilde{\mathcal{D}}_k - \Omega_{3,k+1} - \rho^2\lambda^2\Phi_{2,k} > \mathbf{0} \\
\Delta_{2,k+1} \triangleq \lambda^2 I - \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{R}_{k+1} \mathcal{B}_k \mathcal{K}_k > \mathbf{0}
\end{cases} \tag{13}$$

where

$$\begin{aligned}
&\Phi_{1,k} \triangleq \mathcal{C}_k^T \bar{\mathcal{H}}^T \bar{\mathcal{H}} \mathcal{C}_k + \mathcal{C}_k^T \Lambda_{1,k} \mathcal{C}_k, \\
&\Phi_{2,k} \triangleq \begin{bmatrix}
\mathbf{0}_{(n\times n_w)\times(n\times n_w)} & \mathbf{0}_{(n\times n_w)\times(n\times n_v)} & \mathbf{0}_{(n\times n_w)\times(n\times n_\delta)} \\
\mathbf{0}_{(n\times n_v)\times(n\times n_w)} & \mathcal{E}_k^T \bar{\mathcal{H}}^T \bar{\mathcal{H}} \mathcal{E}_k + \mathcal{E}_k^T \Lambda_{1,k} \mathcal{E}_k & \mathcal{E}_k^T \bar{\mathcal{H}}^T \bar{\mathcal{G}} \\
\mathbf{0}_{(n\times n_\delta)\times(n\times n_w)} & \bar{\mathcal{G}}^T \bar{\mathcal{H}} \mathcal{E}_k & \bar{\mathcal{G}}^T \bar{\mathcal{G}} + \Lambda_{2,k}
\end{bmatrix}, \\
&\Phi_{3,k} \triangleq \begin{bmatrix} \mathbf{0}_{(n\times n_x)\times(n\times n_\delta)} & \mathcal{C}_k^T \bar{\mathcal{H}}^T \bar{\mathcal{H}} \mathcal{E}_k + \mathcal{C}_k^T \Lambda_{1,k} \mathcal{E}_k & \mathcal{C}_k^T \bar{\mathcal{H}}^T \bar{\mathcal{G}} \end{bmatrix}, \\
&\mathcal{R}_{k+1} \triangleq \mathcal{P}_{k+1} + \mathcal{P}_{k+1} \widetilde{\mathcal{D}}_k \Delta_{1,k+1}^{-1} \widetilde{\mathcal{D}}_k^T \mathcal{P}_{k+1}, \quad \Lambda_{1,k} \triangleq \mathbb{E}\{\check{\mathcal{H}}_k^T \check{\mathcal{H}}_k\}, \quad \Lambda_{2,k} \triangleq \mathbb{E}\{\check{\mathcal{G}}_k^T \check{\mathcal{G}}_k\}.
\end{aligned}$$

and

$$\begin{cases}\Omega_{1,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{B}}_k^r\} \\ \Omega_{2,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r\} \\ \Omega_{3,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{D}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r\}.\end{cases} \tag{14}$$

Proof. Defining the Lyapunov-like function as $V_k^{(1)} \triangleq x_k^T\mathcal{P}_k x_k$, then we obtain

$$\begin{aligned}\aleph_1 &\triangleq \mathbb{E}\big\{V_{k+1}^{(1)} - V_k^{(1)}|x_k\big\} \\ &= \mathbb{E}\Big\{\big(x_k^T(\widetilde{\mathcal{A}}_k + \widetilde{\mathcal{B}}_k^r)^T + \bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T + \varpi_k^T(\widetilde{\mathcal{D}}_k + \widetilde{\mathcal{D}}_k^r)^T\big) \\ &\quad \times \mathcal{P}_{k+1}\big((\widetilde{\mathcal{A}}_k + \widetilde{\mathcal{B}}_k^r)x_k + \mathcal{B}_k\mathcal{K}_k\bar{e}_k + (\widetilde{\mathcal{D}}_k + \widetilde{\mathcal{D}}_k^r)\varpi_k\big) - x_k^T\mathcal{P}_k x_k|x_k\Big\} \\ &= \mathbb{E}\Big\{x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + 2\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{B}}_k^r + \widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{B}}_k^r \\ &\quad - \mathcal{P}_k)x_k + \bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k + \varpi_k^T(\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \\ &\quad + 2\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r + \widetilde{\mathcal{D}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r)\varpi_k + 2x_k^T\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k \\ &\quad \times \mathcal{K}_k\bar{e}_k + 2x_k^T\widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k + 2x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \\ &\quad + \widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r + \widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r)\varpi_k \\ &\quad + 2\bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\varpi_k + 2\bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r\varpi_k|x_k\Big\}.\end{aligned}$$

Noticing that $\mathbb{E}\{\widetilde{\mathcal{B}}_k^r\} = \mathbf{0}$ and $\mathbb{E}\{\widetilde{\mathcal{D}}_k^r\} = \mathbf{0}$, one has

$$\begin{aligned}\aleph_1 = \mathbb{E}\Big\{&x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{B}}_k^r - \mathcal{P}_k)x_k + \bar{e}_k^T\mathcal{K}_k^T \\ &\times \mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k + \varpi_k^T(\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \widetilde{\mathcal{D}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r) \\ &\times \varpi_k + 2x_k^T\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k + 2x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \\ &+ \widetilde{\mathcal{B}}_k^{r,T}\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k^r)\varpi_k + 2\bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\varpi_k|x_k\Big\}.\end{aligned}$$

Adding the following zero term to the above equation:

$$\|\bar{z}_k\|^2 - \gamma^2\|\varpi_k\|^2 - \|\bar{z}_k\|^2 + \gamma^2\|\varpi_k\|^2 + \lambda^2\|\bar{e}_k\|^2 - \lambda^2\|\bar{e}_k\|^2 = 0$$

where λ is a known constant. Moreover, since $\bar{z}_k = z_k - \phi z_k$ and $z_k = \mathcal{M}_k x_k$, it follows that

$$\bar{z}_k = (\mathcal{M}_k - \phi\mathcal{M}_k)x_k, \qquad \|\bar{z}_k\|^2 = x_k^T(\mathcal{M}_k - \phi\mathcal{M}_k)^T(\mathcal{M}_k - \phi\mathcal{M}_k)x_k.$$

Together with $\Omega_{i,k+1}$ $(i = 1, 2, 3)$ in (14), we obtain

$$\begin{aligned}\aleph_1 = \mathbb{E}\Big\{&x_k^T\big(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{1,k+1} + (\mathcal{M}_k - \phi\mathcal{M}_k)^T(\mathcal{M}_k \\ &- \phi\mathcal{M}_k) - \mathcal{P}_k\big)x_k + \bar{e}_k^T(\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k - \lambda^2 I)\bar{e}_k \\ &+ \varpi_k^T(\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{3,k+1} - \gamma^2 I)\varpi_k + 2x_k^T\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1} \\ &\times \mathcal{B}_k\mathcal{K}_k\bar{e}_k + 2x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{2,k+1})\varpi_k + 2\bar{e}_k^T\mathcal{K}_k^T \\ &\times \mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\varpi_k - \|\bar{z}_k\|^2 + \gamma^2\|\varpi_k\|^2 + \lambda^2\|\bar{e}_k\|^2|x_k\Big\}.\end{aligned} \tag{15}$$

Next, we consider the term $\mathbb{E}\{||\bar{e}_k||^2|x_k\}$. According to the event-triggering function (10), at non-triggering instants, it holds that $\mathbb{E}\{||\bar{e}_k||^2|x_k\} = \mathbb{E}\{\bar{e}_k^T\bar{e}_k|x_k\} \leq \rho^2\mathbb{E}\{e_k^T e_k|x_k\}$. Moreover, from (7), it follows that

$$\begin{aligned}&\mathbb{E}\{e_k^T e_k|x_k\} \\ &= \mathbb{E}\big\{x_k^T\mathcal{C}_k^T\mathcal{H}_k^T\mathcal{H}_k\mathcal{C}_k x_k + \nu_k^T\mathcal{E}_k^T\mathcal{H}_k^T\mathcal{H}_k\mathcal{E}_k\nu_k + \delta_k^T\mathcal{G}_k^T\mathcal{G}_k\delta_k \\ &\quad + 2x_k^T\mathcal{C}_k^T\mathcal{H}_k^T\mathcal{H}_k\mathcal{E}_k\nu_k + 2x_k^T\mathcal{C}_k^T\mathcal{H}_k^T\mathcal{G}_k\delta_k + 2\nu_k^T\mathcal{E}_k^T\mathcal{H}_k^T\mathcal{G}_k\delta_k|x_k\big\} \\ &\triangleq \mathbb{E}\big\{x_k^T\Phi_{1,k}x_k + \varpi_k^T\Phi_{2,k}\varpi_k + 2x_k^T\Phi_{3,k}\varpi_k|x_k\big\}.\end{aligned}$$

Therefore, we have

$$\mathbb{E}\{||\bar{e}_k||^2|x_k\} \leq \rho^2\mathbb{E}\{x_k^T\Phi_{1,k}x_k + \varpi_k^T\Phi_{2,k}\varpi_k + 2x_k^T\Phi_{3,k}\varpi_k|x_k\}. \tag{16}$$

Substituting (16) into (15), we derive that

$$\begin{aligned}\aleph_1 \leq \mathbb{E}\Big\{& x_k^T\big(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{1,k+1} + (\mathcal{M}_k - \phi\mathcal{M}_k)^T(\mathcal{M}_k \\ & - \phi\mathcal{M}_k) - \mathcal{P}_k + \rho^2\lambda^2\Phi_{1,k}\big)x_k + \bar{e}_k^T(\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k \\ & - \lambda^2 I)\bar{e}_k + \varpi_k^T(\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{3,k+1} - \gamma^2 I + \rho^2\lambda^2 \\ & \times \Phi_{2,k})\varpi_k + 2x_k^T\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k + 2x_k^T(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \\ & + \Omega_{2,k+1} + \rho^2\lambda^2\Phi_{3,k})\varpi_k + 2\bar{e}_k^T\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\varpi_k \\ & - ||\bar{z}_k||^2 + \gamma^2||\varpi_k||^2|x_k\Big\}.\end{aligned}$$

First, we apply the completing square method to ϖ_k. Define $\Delta_{1,k+1} \triangleq \gamma^2 I - \widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k - \Omega_{3,k+1} - \rho^2\lambda^2\Phi_{2,k}$. Then we obtain

$$\begin{aligned}\aleph_1 \leq \mathbb{E}\Big\{& x_k^T\big(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{1,k+1} + (\mathcal{M}_k - \phi\mathcal{M}_k)^T(\mathcal{M}_k \\ & - \phi\mathcal{M}_k) + (\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{2,k+1} + \rho^2\lambda^2\Phi_{3,k})\Delta_{1,k+1}^{-1} \\ & \times (\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T) - \mathcal{P}_k + \rho^2\lambda^2 \\ & \times \Phi_{1,k}\big)x_k + \bar{e}_k^T(\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\Delta_{1,k+1}^{-1}\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k \\ & + \mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k - \lambda^2 I)\bar{e}_k - (\varpi_k - \varpi_k^*)^T\Delta_{1,k+1} \\ & \times (\varpi_k - \varpi_k^*) + 2x_k^T\big((\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{2,k+1} + \rho^2\lambda^2 \\ & \times \Phi_{3,k})\Delta_{1,k+1}^{-1}\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k + \widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\big)\bar{e}_k \\ & - ||\bar{z}_k||^2 + \gamma^2||\varpi_k||^2|x_k\Big\}\end{aligned}$$

where $\varpi_k^* \triangleq \Delta_{1,k+1}^{-1}(\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T)x_k + \Delta_{1,k+1}^{-1}\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k\bar{e}_k$.

Next, we apply the completing square method to $\bar{e}_k$. Define $\Delta_{2,k+1} \triangleq \lambda^2 I - \mathcal{K}_k^T\mathcal{B}_k^T\mathcal{R}_{k+1}\mathcal{B}_k\mathcal{K}_k$. It follows that

$$\begin{aligned}\aleph_1 \leq \mathbb{E}\Big\{& x_k^T\big(\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{1,k+1} + (\mathcal{M}_k - \phi\mathcal{M}_k)^T(\mathcal{M}_k \\ & - \phi\mathcal{M}_k) + (\widetilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k + \Omega_{2,k+1} + \rho^2\lambda^2\Phi_{3,k})\Delta_{1,k+1}^{-1} \\ & \times (\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\widetilde{\mathcal{A}}_k + \Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T) - \mathcal{P}_k + \rho^2\lambda^2 \\ & \times \Phi_{1,k} + \big(\mathcal{R}_{k+1}\widetilde{\mathcal{A}}_k + \mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\Delta_{1,k+1}^{-1}(\Omega_{2,k+1}^T + \rho^2\lambda^2 \\ & \times \Phi_{3,k}^T)\big)^T\mathcal{B}_k\mathcal{K}_k\Delta_{2,k+1}^{-1}\mathcal{K}_k^T\mathcal{B}_k^T\big(\mathcal{R}_{k+1}\widetilde{\mathcal{A}}_k + \mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k \\ & \times \Delta_{1,k+1}^{-1}(\Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T))\big)x_k + \bar{e}_k^T(\mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1} \\ & \times \widetilde{\mathcal{D}}_k\Delta_{1,k+1}^{-1}\widetilde{\mathcal{D}}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k + \mathcal{K}_k^T\mathcal{B}_k^T\mathcal{P}_{k+1}\mathcal{B}_k\mathcal{K}_k - \lambda^2 I) \\ & \times \bar{e}_k - (\varpi_k - \varpi_k^*)^T\Delta_{1,k+1}(\varpi_k - \varpi_k^*) - (\bar{e}_k - \bar{e}_k^*)^T \\ & \times \Delta_{2,k+1}(\bar{e}_k - \bar{e}_k^*) - ||\bar{z}_k||^2 + \gamma^2||\varpi_k||^2|x_k\Big\}\end{aligned}$$

where $\bar{e}_k^* \triangleq \Delta_{2,k+1}^{-1}\mathcal{K}_k^T\mathcal{B}_k^T\big(\mathcal{P}_{k+1}\widetilde{\mathcal{D}}_k\Delta_{1,k+1}^{-1}(\Omega_{2,k+1}^T + \rho^2\lambda^2\Phi_{3,k}^T) + \mathcal{R}_{k+1}\widetilde{\mathcal{A}}_k\big)x_k$.

Summing both sides from 0 to $\tau - 1$, and since the RDE (12) and the final condition $\mathcal{P}_\tau = 0$, it is deduced that

$$J_1 \leq \mathbb{E}\Big\{\sum_{k=0}^{\tau-1} -(\varpi_k - \varpi_k^*)^T\Delta_{1,k+1}(\varpi_k - \varpi_k^*)\Big\} + \mathbb{E}\Big\{\sum_{k=0}^{\tau-1} -(\bar{e}_k - \bar{e}_k^*)^T\Delta_{2,k+1}(\bar{e}_k - \bar{e}_k^*)\Big\} + x_0^T\big(\mathcal{P}_0 - \gamma^2\mathcal{W}\big)x_0.$$

Obviously, according to the aforementioned constraint (13), it can be obtained that $J_1 < 0$, which means the predefined H_∞ performance (11) is satisfied. □

Up to now, we have obtained and proven the condition for the time-varying MAS to satisfy the predefined H_∞ performance. Next, we design the controller parameter K_k in the worst case of the system, i.e., when $\bar{e}_k = \bar{e}_k^* \triangleq \Theta_{1,k} x_k$ and $\varpi_k = \varpi_k^* \triangleq \Theta_{2,k} x_k$ where

$$\begin{cases} \Theta_{1,k} \triangleq \Delta_{2,k+1}^{-1} \mathcal{K}_k^T \mathcal{B}_k^T \big(\mathcal{P}_{k+1} \widetilde{\mathcal{D}}_k \Delta_{1,k+1}^{-1} (\Omega_{2,k+1}^T + \rho^2 \lambda^2 \Phi_{3,k}^T) + \mathcal{R}_{k+1} \widetilde{\mathcal{A}}_k \big) \\ \Theta_{2,k} \triangleq \Delta_{1,k+1}^{-1} (\widetilde{\mathcal{D}}_k^T \mathcal{P}_{k+1} \widetilde{\mathcal{A}}_k + \Omega_{2,k+1}^T + \rho^2 \lambda^2 \Phi_{3,k}^T) + \Delta_{1,k+1}^{-1} \widetilde{\mathcal{D}}_k^T \mathcal{P}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} \end{cases}$$

According to the closed-loop system (9), by defining $\bar{u}_k \triangleq \mathcal{K}_k \bar{\mathcal{H}} \mathcal{C}_k x_k$, it can be rewritten as

$$x_{k+1} = \big(\mathcal{A}_k + \widetilde{\mathcal{B}}_k^r + \widetilde{\mathcal{D}}_k \Theta_{2,k} + \widetilde{\mathcal{D}}_k^r \Theta_{2,k} + \mathcal{B}_k \mathcal{K}_k \Theta_{1,k}\big) x_k + \mathcal{B}_k \bar{u}_k.$$

From (8), we obtain

$$\begin{aligned} u_k^t &= \mathcal{K}_k \bar{e}_k + \mathcal{K}_k \mathcal{H}_k \mathcal{C}_k x_k + \mathcal{K}_k \mathcal{H}_k \mathcal{E}_k \nu_k + \mathcal{K}_k \mathcal{G}_k \delta_k. \\ &\triangleq (\bar{\Pi}_{1,k} + \tilde{\Pi}_{1,k} + \bar{\Pi}_{2,k} \Theta_{2,k} + \tilde{\Pi}_{2,k} \Theta_{2,k}) x_k + \bar{u}_k \end{aligned}$$

where

$$\begin{aligned} &\bar{\Pi}_{1,k} \triangleq \mathcal{K}_k \Theta_{1,k}, \quad \bar{\Pi}_{2,k} \triangleq [\mathbf{0},\ \mathcal{K}_k \bar{\mathcal{H}} \mathcal{E}_k,\ \mathcal{K}_k \bar{\mathcal{G}}], \\ &\tilde{\Pi}_{1,k} \triangleq \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{C}_k, \quad \tilde{\Pi}_{2,k} \triangleq [\mathbf{0},\ \mathcal{K}_k \check{\mathcal{H}}_k \mathcal{E}_k,\ \mathcal{K}_k \check{\mathcal{G}}_k]. \end{aligned}$$

Thus, one has

$$\mathbb{E}\{u_k^{t,T} u_k^t | x_k\} \triangleq x_k^T \varPi_k x_k + \bar{u}_k^T \bar{u}_k \tag{17}$$

where

$$\begin{aligned} \varPi_k \triangleq\ & \bar{\Pi}_{1,k}^T \bar{\Pi}_{1,k} + \bar{\Pi}_{1,k}^T \bar{\Pi}_{2,k} \Theta_{2,k} + 2(\bar{\Pi}_{1,k}^T + \Theta_{2,k}^T \bar{\Pi}_{2,k}^T) \mathcal{K}_k \bar{\mathcal{H}} \mathcal{C}_k \\ & + \Theta_{2,k}^T \bar{\Pi}_{2,k}^T \bar{\Pi}_{1,k} + \Theta_{2,k}^T \bar{\Pi}_{2,k}^T \bar{\Pi}_{2,k} \Theta_{2,k} + \mathbb{E}\{\tilde{\Pi}_{1,k}^T \tilde{\Pi}_{1,k}\} \\ & + \mathbb{E}\{\tilde{\Pi}_{1,k}^T \tilde{\Pi}_{2,k}\} \Theta_{2,k} + \Theta_{2,k}^T \mathbb{E}\{\tilde{\Pi}_{2,k}^T \tilde{\Pi}_{2,k}\} \Theta_{2,k} \\ & + \Theta_{2,k}^T \mathbb{E}\{\tilde{\Pi}_{2,k}^T \tilde{\Pi}_{1,k}\}. \end{aligned}$$

Now, we are in the position to design a controller, with the following given cost function. The cost function is defined as

$$J_2 \triangleq \mathbb{E}\Big\{ \sum_{k=0}^{\tau-1} \Big(\|\bar{z}_k\|^2 + \varepsilon \|u_k^t\|^2 \Big) \Big\} \tag{18}$$

where $\varepsilon > 0$ is a known constant introduced for more flexibility in the controller parameter design.

Theorem 2. *Consider a class of time-varying MAS represented by (9) over two-way AF relay networks where $k \in [0, \tau - 1]$. Based on Theorem 1, given a positive scalar $\varepsilon > 0$, there exist controller parameters $\{K_k\}_{0 \le k \le \tau-1}$ for the controller such that, under the H_∞ performance guaranteed by Theorem 1, the cost functional (18) is minimized in the sense of the coupled RDEs below if there exist solutions $\{\mathcal{P}_k, \mathcal{Q}_k, K_k\}_{0 \le k \le \tau-1}$ satisfying the RDE (12) and the following recursive backward RDE*

$$\begin{aligned} \mathcal{Q}_k =\ & \mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \mho_{1,k+1} + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} + \Theta_{2,k}^T \\ & \times \mho_{4,k+1} \Theta_{2,k} + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mathcal{A}_k^T \mathcal{Q}_{k+1} \\ & \times \widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mho_{2,k+1} \Theta_{2,k} + \Theta_{2,k}^T \\ & \times \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \Theta_{2,k}^T \mho_{3,k+1} \\ & + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} \\ & + (\mathcal{M}_k - \phi \mathcal{M}_k)^T (\mathcal{M}_k - \phi \mathcal{M}_k) + \varepsilon \varPi_k + 2 \mathcal{C}_k^T \bar{\mathcal{H}}^T \\ & \times \mathcal{K}_k^T \mathcal{B}_k \mathcal{Q}_{k+1} (\widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{B}_k \mathcal{K}_k \Theta_{1,k}) - \mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \\ & \times \nabla_{k+1}^{-1} \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k \end{aligned} \tag{19}$$

subject to

$$\begin{cases} \mathcal{Q}_\tau = \mathbf{0} \\ \nabla_{k+1} \triangleq \varepsilon I + \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k > \mathbf{0} \end{cases} \tag{20}$$

with the controller parameters given as follows

$$K_k = -\mathcal{U}_k \mathcal{V}_k^\dagger \tag{21}$$

where

$$\begin{aligned} &\mathcal{U}_k \triangleq col\{\psi_k^1, \psi_k^2, \ldots, \psi_k^n\}, \quad \mathcal{V}_k \triangleq col\{\zeta_k^1, \zeta_k^2, \ldots, \zeta_k^n\}, \\ &\zeta_k^i \triangleq [\bar{\mathcal{H}}\mathcal{C}_k]_i, \quad \psi_k^i \triangleq [\nabla_{k+1}^{-1} \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k]_i, \quad i = 1, 2, \ldots, n, \end{aligned}$$

and

$$\begin{cases} \mho_{1,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{B}}_k^{r,T} \mathcal{Q}_{k+1} \widetilde{\mathcal{B}}_k^r\} \\ \mho_{2,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{B}}_k^{r,T} \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k^r\} \\ \mho_{3,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{D}}_k^{r,T} \mathcal{Q}_{k+1} \widetilde{\mathcal{B}}_k^r\} \\ \mho_{4,k+1} \triangleq \mathbb{E}\{\widetilde{\mathcal{D}}_k^{r,T} \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k^r\}. \end{cases} \tag{22}$$

Proof. By defining the Lyapunov-like function $V_k^{(2)} \triangleq x_k^T \mathcal{Q}_k x_k$, one has

$$\begin{aligned} \aleph_2 \triangleq\ & \mathbb{E}\Big\{V_{k+1}^{(2)} - V_k^{(2)} | x_k\Big\} \\ =\ & \mathbb{E}\Big\{x_{k+1}^T \mathcal{Q}_{k+1} x_{k+1} - x_k^T \mathcal{Q}_k x_k | x_k\Big\} \\ =\ & \mathbb{E}\big\{x_k^T (\mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \mho_{1,k+1} + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} \\ & + \Theta_{2,k}^T \mho_{4,k+1} \Theta_{2,k} + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mathcal{A}_k^T \\ & \times \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mho_{2,k+1} \Theta_{2,k} \\ & + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \Theta_{2,k}^T \\ & \times \mho_{3,k+1} + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \\ & \times \Theta_{2,k} - \mathcal{Q}_k) x_k + \bar{u}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \bar{u}_k + 2\bar{u}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \\ & \times (\mathcal{A}_k + \widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{B}_k \mathcal{K}_k \Theta_{1,k}) x_k | x_k\big\}. \end{aligned}$$

Similar to the proof of Theorem (1), by adding the zero term

$$\|\bar{z}_k\|^2 + \varepsilon \|u_k^t\|^2 - \|\bar{z}_k\|^2 - \varepsilon \|u_k^t\|^2 = 0$$

and combine with (17), we have

$$\begin{aligned} \aleph_2 =\ & \mathbb{E}\big\{x_k^T \big(\mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \mho_{1,k+1} + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} + \Theta_{2,k}^T \\ & \times \mho_{4,k+1} \Theta_{2,k} + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mathcal{A}_k^T \mathcal{Q}_{k+1} \\ & \times \widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{A}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \mho_{2,k+1} \Theta_{2,k} + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \\ & \times \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{2,k}^T \widetilde{\mathcal{D}}_k^T \mathcal{Q}_{k+1} \mathcal{B}_k \mathcal{K}_k \Theta_{1,k} + \Theta_{2,k}^T \mho_{3,k+1} \\ & + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k + \Theta_{1,k}^T \mathcal{K}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \widetilde{\mathcal{D}}_k \Theta_{2,k} \\ & + (\mathcal{M}_k - \phi \mathcal{M}_k)^T (\mathcal{M}_k - \phi \mathcal{M}_k) + \varepsilon \Pi_k + 2\mathcal{C}_k^T \bar{\mathcal{H}}^T \mathcal{K}_k^T \\ & \times \mathcal{B}_k \mathcal{Q}_{k+1} (\widetilde{\mathcal{D}}_k \Theta_{2,k} + \mathcal{B}_k \mathcal{K}_k \Theta_{1,k}) - \mathcal{Q}_k\big) x_k + \bar{u}_k^T (\mathcal{B}_k^T \\ & \times \mathcal{Q}_{k+1} \mathcal{B}_k + \varepsilon I) \bar{u}_k + 2\bar{u}_k^T \mathcal{B}_k^T \mathcal{Q}_{k+1} \mathcal{A}_k x_k \\ & - (||\bar{z}_k||^2 + \varepsilon ||u_k^t||^2) | x_k\big\}. \end{aligned}$$

Utilizing the completing squares method and summing both sides from 0 to $\tau - 1$, and according to (19) and the condition $\mathcal{Q}_\tau = 0$, it is deduced that

$$J_2 = \mathbb{E}\{\sum_{k=0}^{\tau-1}(\bar{u}_k + \bar{u}_k^*)^T \nabla_{k+1}(\bar{u}_k + \bar{u}_k^*)\} + x_0^T \mathcal{Q}_0 x_0$$
$$\leq \mathbb{E}\{\sum_{k=0}^{\tau-1}||\mathcal{K}_k\bar{\mathcal{H}}\mathcal{C}_k + \nabla_{k+1}^{-1}\mathcal{B}_k^T\mathcal{Q}_{k+1}\mathcal{A}_k||_F^2||x_k||^2\} + x_0^T\mathcal{Q}_0 x_0$$

where $\bar{u}_k^* \triangleq \nabla_{k+1}^{-1}\mathcal{B}_k^T\mathcal{Q}_{k+1}\mathcal{A}_k x_k$.

To minimize the cost, by means of Lemma 1, the controller parameter K_k is selected iteratively in a backward manner as follows

$$K_k = \underset{K_k}{\operatorname{argmin}}||\mathcal{K}_k\bar{\mathcal{H}}\mathcal{C}_k + \nabla_{k+1}^{-1}\mathcal{B}_k^T\mathcal{Q}_{k+1}\mathcal{A}_k||_F = -\mathcal{U}_k\mathcal{V}_k^{\dagger}.$$

The proof is complete. □

In summary, by observing the main results in the aforementioned theorems, we can derive the following two-way AF-assisted finite-horizon H_∞ control algorithm of the time-varying MAS (see Algorithm 1).

Algorithm 1: Two-Way AF-Assisted Finite-Horizon H_∞ Control

Step 1. Set initial conditions: $\mathcal{P}_\tau = \mathcal{Q}_\tau = 0, k = \tau - 1$. Givven the positive definite matrix W and positive sca--lars γ and ε.

Step 2. Calculate $\mho_{i,k+1}$, $i = \{1,2,3,4\}$ by (22), then calculate ∇_{k+1} by (20), if $\nabla_{k+1} > 0$, calculate control parameter K_k by (21), otherwise, jump to ***Step 5***.

Step 3. Calculate $\Omega_{i,k+1}$, $i = \{1,2,3\}$ by (14), then calculate $\Delta_{1,k+1}$ and $\Delta_{2,k+1}$ by (13). If $\Delta_{1,k+1} > 0$ and $\Delta_{2,k+1} > 0$ are not satisfied, jump to ***Step 5***. Continue.

Step 4. Obtain $\mathcal{P}_k$ and $\mathcal{Q}_k$ by solving the coupled backward RDEs of (12) and (19) respectively. If $k \neq 0$, set $k = k - 1$ and return to ***Step 2***, otherwise, continue.

Step 5. If any one of the conditions $\Delta_{1,k+1} > 0$, $\Delta_{2,k+1} > 0$, $\nabla_{k+1} > 0$, and $\mathcal{P}_0 \leq \gamma^2\mathcal{W}$ is not satisfied, this algorithm is infeasible. Stop.

Remark 5. *Three features distinguish our scheme from existing H_∞ MAS control studies. First, unlike the works in [15–18], which typically ignore channel-induced noise, the augmented H_∞ performance index in this paper explicitly incorporates channel noise and combines it with an event-triggering rule, thereby improving the physical fidelity of the AF-assisted MAS model while reducing unnecessary communication updates. Second, unlike the one-way relay designs in [23–25], the two-way AF channel completes a bidirectional exchange within a single time slot, and the coupled-RDE conditions use channel statistics instead of worst-case bounds, which helps reduce conservatism in the design. Finally, the proposed control strategy decouples analysis and synthesis through two coupled backward RDEs, which facilitates the computation of the time-varying gain K_k under the AF-assisted MAS framework.*

Remark 6. *This study investigates the finite-horizon H_∞ control problem for MASs under the two-way AF relay protocol. A tractable model is developed to characterize the behavior of two-way AF relay network. Based on this model and the event-triggering mechanism, an H_∞ control scheme is then derived through the dynamic analysis of control errors. Theorem 1 provides sufficient conditions to ensure the prescribed H_∞ performance, while Theorem 2 further presents the designs of the controller gain that guarantee this performance.*

Remark 7. *Compared to the existing literature, this paper makes the following contributions: (1) It proposes two-way AF relays to address signal attenuation in inter-agent data transmission, offering a novel approach within MAS frameworks that effectively mitigates channel fading effects and enhances communication reliability; and (2) To further reduce communication overhead while maintaining system performance, an event-triggered control strategy combined with the H_∞ performance measure is employed to design a MAS controller, which significantly reduces the communication frequency among agents without compromising the desired performance.*

4. A Numerical Example

In this section, we present a numerical example to verify the effectiveness of our proposed algorithm for the two-way AF relay-assisted MASs. The topology is shown in Figure 2.

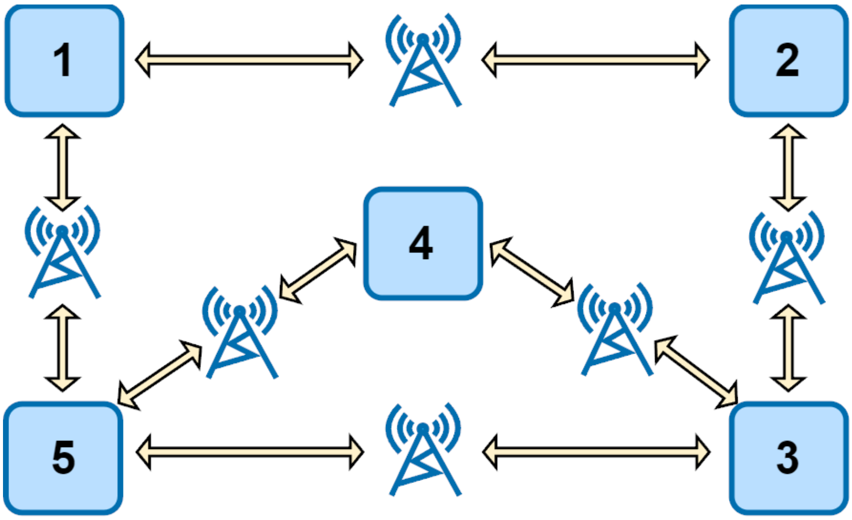

Figure 2. The topology of two-way AF relay-assisted MAS.

According to the topology, the corresponding adjacency matrix is given by

$$\mathscr{A}=\begin{bmatrix}0&1&0&0&1\\1&0&1&0&0\\0&1&0&1&1\\0&0&1&0&1\\1&0&1&1&0\end{bmatrix}.$$

Given the transmission power of phase A $\rightarrow$ R and phase R $\rightarrow$ A as $\mathsf{P}_f=\mathsf{P}_s=1.2\mathscr{A}$. The expectation and variance of the fading channel gains are set as $\mu=0.8$ and $\sigma^2=0.0025$, respectively.

Assume the external disturbances $\omega_{i,k}$ and $\nu_{i,k}$ as

$$\begin{aligned}&\omega_{1,k}=0.02\cos(1.2\mathrm{k}),\quad \omega_{2,\mathrm{k}}=0.02\sin(0.4\mathrm{k}),\quad \omega_{3,\mathrm{k}}=0.02\sin(1.2\mathrm{k}),\quad \omega_{4,\mathrm{k}}=0.02\cos(0.4\mathrm{k}),\\&\omega_{5,k}=0.01\sin(0.4\mathrm{k})+0.01\cos(0.8\mathrm{k}),\quad \nu_{1,\mathrm{k}}=0.02\sin(1.2\mathrm{k}),\quad \nu_{2,\mathrm{k}}=0.02\sin(0.6\mathrm{k}),\\&\nu_{3,k}=0.02\cos(0.2\mathrm{k}),\quad \nu_{4,\mathrm{k}}=0.02\cos(0.4\mathrm{k}),\quad \nu_{5,\mathrm{k}}=0.01\sin(0.5\mathrm{k})+0.01\cos(0.5\mathrm{k}).\end{aligned}$$

The channel disturbance is set as $\delta^{a}_{ij,k}=0.02\cos(1.2\mathrm{k})$ and $\delta^{r^j_i}_{k}=0.02\cos(1.2\mathrm{k})$ for $\forall i,j\in\mathscr{V}_a$ and $k\in[0,\tau-1]$.

In this numerical example, $\gamma=0.8$, $W=9.0I$, $\varepsilon=0.02$, $\tau=50$, and $\lambda=0.002$. The dynamic parameters of each agent are given as follows:

$$\begin{aligned}&A_k=\begin{bmatrix}0.4+0.06\sin(0.4k)&0.5-0.05\sin(0.2k)\\-0.5&0.5+0.06\cos(0.5k)\end{bmatrix},\quad B_k=\begin{bmatrix}0.5\\0.2+0.05\sin(0.4k)\end{bmatrix},\\&C_k=\begin{bmatrix}0.6&0.6\end{bmatrix},\quad D_k=\begin{bmatrix}0.2\\0.2\end{bmatrix},\quad E_k=0.2,\quad M_k=\begin{bmatrix}0.2&0.1\\0.1&0.2\end{bmatrix}.\end{aligned}$$

The initial states $x_{i,0}$, for $i\in\mathscr{V}_a$, are chosen as

$$x_{1,0}=[0.9;-1.2],\quad x_{2,0}=[0.7;-0.5],\quad x_{3,0}=[1.2;-1.2],\quad x_{4,0}=[1.3;-0.7],\quad x_{5,0}=[1.1;-0.9].$$

The simulation results are presented in Figures 3 and 4. Figure 3 depicts the event-triggering time instants of all agents over the simulation horizon. The resulting triggering sequences are aperiodic and agent-dependent, which suggests that control actions are updated only when the event-triggering rule is activated, instead of at every sampling instant. Consequently, unnecessary transmissions can be curtailed while the closed loop still relies on the most recently broadcast information. Figure 4 shows the time evolution of the H_∞ performance index.

Throughout the horizon, the index remains below the prescribed attenuation level γ, which aligns with the intended H_∞ performance under the considered process disturbances, measurement noise, and channel effects. Collectively, these results demonstrate that the proposed event-triggered two-way AF relay-assisted H_∞ control scheme attains the desired robustness with reduced communication burden.

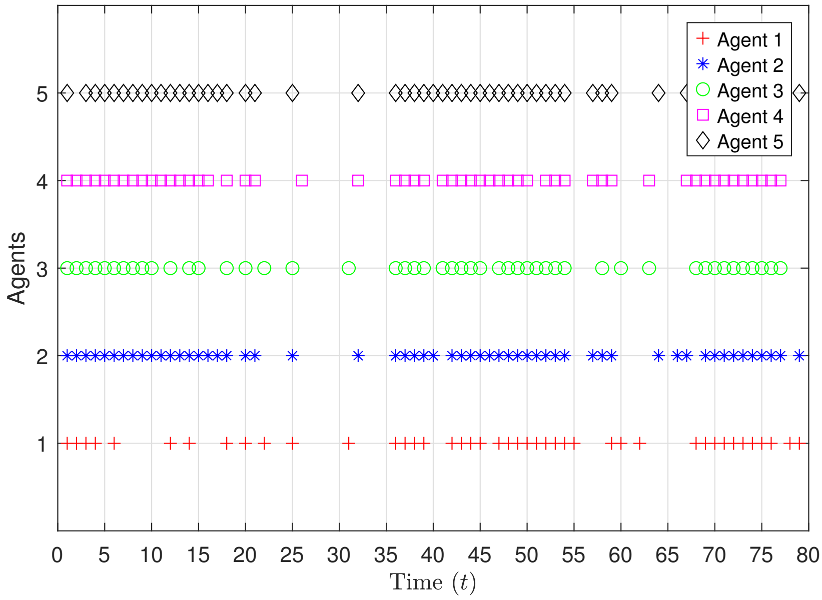

Figure 3. The event-triggering instants of each agent.

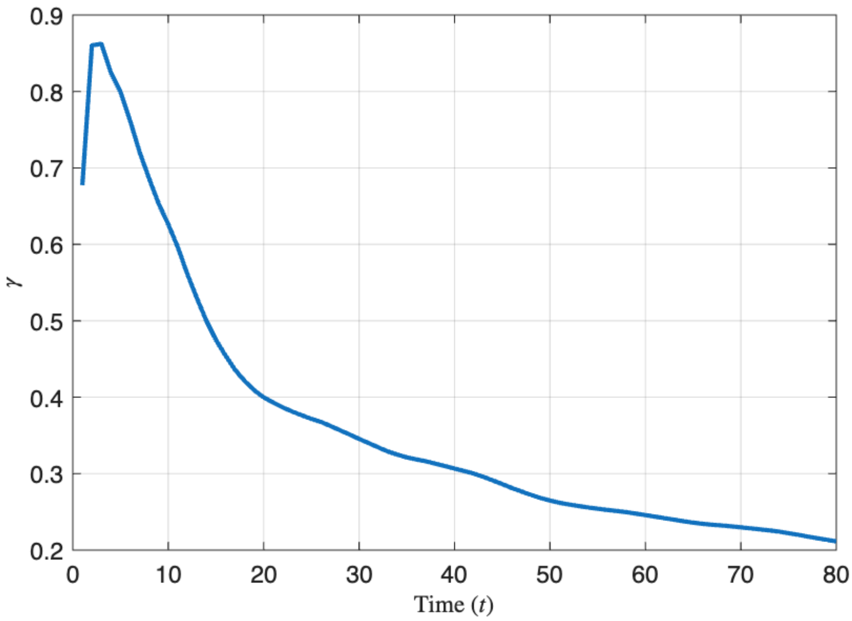

Figure 4. The curve of H_∞ performance index.

5. Conclusions

In this paper, the H_∞ consensus control problem for two-way AF relay-assisted time-varying MASs with an event-triggered strategy has been studied. A two-way AF protocol has been first introduced into MASs to improve communication quality and enhance communication efficiency. An H_∞ performance criterion incorporating channel noise has been proposed to enhance the system's robustness against communication noise. An advanced method based on coupled backward RDEs has been developed to design the MAS controller. Finally, numerical results demonstrate the effectiveness of the proposed method and the designed algorithm.

The potential future research directions of this paper are put forward as follows: (1) investigate the control problem for a MAS assisted by alternative relay protocols, such as DF, FF, and AF [22,26,45]; and (2) study the multi-agent consensus control problem of relay systems with energy harvesting (see [21,46,47]).

Author Contributions

J.C.: methodology, writing—original draft, software; Y.W.: methodology, writing—reviewing and editing, funding acquisition; F.W.: supervision, funding acquisition. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

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