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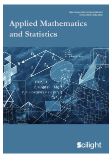

Article

Modeling and Analysis of the Spread of Racism in the Community with Optimal Control Strategy

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Abstract: The spread of racism in society affects all aspects of their lives. Hence, this study aims to analyze the control measures to reduce the spread of racism using a non-linear deterministic model with an optimal control strategy. To ensure the model is biologically and mathematically sound, we confirmed that its solutions are always non-negative and bounded under defined initial conditions. Additionally, we determined the basic reproductive number by applying the next-generation matrix method. Moreover, we analyze the stability of the equilibrium point using the Jacobian matrix and Lyapunov function techniques. Our analysis confirms that the racism-free equilibrium is locally asymptotically stable when $R_0 < 1$, meaning the behavior effectively dies out over time if the initial levels are low enough. Conversely, if $R_0 > 1$, the system shifts to a locally asymptotically stable endemic equilibrium, where the behavior persists and settles into a steady state within the population. Furthermore, the sensitivity analysis of the parametric value of the model is illustrated using the normalized forward sensitivity method. The optimal control strategy employed the application of Pontryagin's maximum principle, which was used to test the effectiveness of proposed control measures. The model was extended to an optimal control method, with the use of two time-dependent controls to assess the spread of racism in the community, namely, mass education campaigns using social media and teaching through religious institutions. Finally, numerical simulation of the optimal control model shows that a combination of mass education campaigns and religious teaching is the most effective strategy for reducing the spread of racism.

Keywords: racism; deterministic model; optimal control; sensitivity analysis; numerical simulation

1. Introduction

Racism is defined as a multifaceted system of beliefs, attitudes, and institutional structures that marginalize individuals based on physical traits or group identities [1]. It is more expansive than racial prejudice (which focuses on internal views) or racial discrimination (which focuses on outward actions and policies). Specifically, racism combines a discriminatory ideology with the exercise of social and systemic power to withhold the same rights, resources, and status from others that one's own group possesses [2].

The transmission of racism is modeled here as an infectious disease, progressing from susceptible individuals to exposed, then racist states via interpersonal contacts. However, while this framework captures core dynamics of ideological spread, social contagions differ markedly from biological pathogens. Unlike involuntary pathogen transmission, racist ideologies spread through voluntary adoption, amplified by network homophiles, and cultural reinforcement [3]. Our individual level compartments focus on interpersonal propagation but omit structural and institutional racism such as; policy biases or systemic inequities which sustain racism at societal scales and elevate baseline susceptibility independently of "infection" [4]. This bad consideration has an impact on rapid increase in racism propagation in the community [3]. Spread out of racism can lead to a variety of negative outcomes, including depression, hypertension, and coronary heart disease [4]. Therefore, the transmission of racism in complex diverse societies is expected to significantly affect all parts of their life.

Furthermore, racism acts as a corrosive force that fractures communities, isolating individuals from their social support networks and peers. This isolation often manifests in destructive behaviors, directed either inwardly as self-harm or outwardly toward perceived adversaries. Beyond its social impact, racism is increasingly recognized as a significant public health crisis, as it drives systemic health disparities [5]. To counter these problems, resilience is used as a system's capacity to maintain its integrity through internal reform, offering a path forward. This resilience is cultivated through social bonds and incremental change, making resilience theory an essential framework for tackling social inequities in the same way it is used to mitigate physical or environmental threats. The growing emphasis on community resilience in social policy and practice has highlighted the need for tools to evaluate resilience at the community level [5,6]. Communities can draw lessons from historical experiences of discrimination to enhance adaptability, primarily by advancing social equity and wealth distribution [7]. A key strategy for reducing racism is to bolster the community's resilience capacity [8]. Community resilience embodies a group's ability to navigate changes independently, with leadership serving as a core component of this resilience [9]. By strengthening these elements, communities can foster inclusive environments that dismantle systemic biases and promote equitable social interactions.

Mathematical modeling plays a significant role in studying the dynamics of real-world phenomena with different features. Mathematical modeling is widely applicable in ecology, epidemiology, physics, biology, and social sciences [3,7]. Developing a robust mathematical model to address racism holds profound theoretical value, as it allows us to map and obstruct the mechanisms through which these social "contagions" propagate [5,10]. Despite this potential, existing research features only a few quantitative studies on racism. Notable examples include Shahbazi et al. [11] bounded confidence model for opinion dynamics, which has been extended to simulate ideological polarization, social segregation, and cultural diffusion in subsequent studies. Additionally, epidemic-inspired models for social contagion, such as those by Green et al. [12] on rumor spreading, have been adapted to analyze hate crime propagation through network-based diffusion of violence and prejudice.

Mamo D.K. [6] explored the propagation of racism within digital environments by introducing the SEID model. This framework categorizes the population into four distinct groups based on their interaction with racist ideologies: Susceptible (Individuals who have not yet encountered the racist content), Exposed (Individuals who have been introduced to the ideology but remain in a neutral or undecided state, neither adopting nor rejecting it), Infected (Individuals who have embraced the ideology and are actively disseminating it across the network), and Denier (Individuals who are conscious of the ideology but consciously choose to oppose or refrain from spreading it). He derived the threshold value of the model and analyzed the stability of the equilibrium point. However, he did not apply the optimal control strategy to test the effectiveness of the control measures.

Mamo, D.K. [10] extended the racism model proposed by [6] considering the influence of public resilience on the transmission dynamics of racism. Thus, he formulated and analyzed the new mathematical model of the SEDRC which represents the transmission of propagation of racism in the community. He concluded that the spread of racism is under control where $R_0 < 1$, while persists in the community when $R_0 > 1$. A Sensitivity analysis of the parameter value of the model was conducted. The result of his study recommends that reducing the transmission rate and the rate of racial extremeness by improving social bonds and solidarity through community resilience could control the spread of racism. However, he did not consider the social influence on the racism denier individuals and also did not extend the model to the optimal control strategy.

The optimal control strategy in mathematical models is an imperative technique for understanding the dynamics of racism and for deciding on intervention programs for the control of racism. However, none of the above models considered an optimal control strategy. For this, we extend the work done by [10] considering the novelty of our study with the inclusion of social influence on people who deny being racist. Our work advances the field by developing a novel nonlinear deterministic compartmental model for racism spread, incorporating stability analysis like Jacobian matrices and Lyapunov functions, computation of the basic reproduction number, and sensitivity analysis of parameters. Most notably, we introduce optimal control problem through Pontryagin's maximum principle to derive time-dependent controls, providing a rigorous mathematical model for intervention strategies that extends beyond qualitative studies of racism dynamics.

By integrating optimal control, the model demonstrates the effectiveness of combined interventions such as mass education campaigns, social media and teaching through religious institutions in reducing racism prevalence. This offers actionable insights for policymakers, educators, and community leaders to mitigate social harms, fostering inclusivity and reducing losses from discrimination. Our numerical simulations validate that these controls can achieve significant reductions in racism spread, bridging theoretical modeling with real-world applications in social sciences.

The paper is organized as follows; Section (2) involves the model formulation process, flow diagram, and ordinary differential Equation representing the proposed model. Section (3) discusses the analysis of the model qualitatively, such as the boundedness of solutions, positivity of solutions, basic reproduction number, and the equilibrium point of the stability analysis. Section (4) presents the sensitivity analysis of the model parameter employing the normalized forward sensitivity index method. Section (5) discusses the analysis of the optimal control model. Section (6) presents the numerical simulation of the optimal control model to validate the analytical results. In addition, discussions and conclusions were made in Section (7).

2. Model Formulation

2.1. Model Formulation Process

The racism dynamics model is developed by extending the work done by Mamo, D. K. [10] by considering the social influence on anti-racist individuals. Based on the status of the racism they have, we divided the total population $N(t)$ into four compartments namely: Susceptible individuals $S(t)$, Exposed individuals $E(t)$, Racist individuals $R(t)$, and Anti-racist individuals $A(t)$ as described below:

(i) Susceptible $S(t)$: Individuals with no prior involvement in racism who are vulnerable to social influence. This group grows through natural population entry (births) and individuals returning from the anti-racist state. They exit the compartment by transitioning to the exposed stage.

(ii) Exposed $E(t)$: This compartment represents the latent exposure individuals (initial exposure to biased narratives without full endorsement or vocalization), drawing from psychological models of attitude formation. Transmission requires active racist status for ideological propagation. Individuals will come in from a susceptible compartment only. Though, they exit by either becoming racist or moving toward Anti-racist views.

(iii) Racist $R(t)$: These are individuals who frequently engage in racist practices and actively spread racism. This group is recruited solely from the exposed class. Some of them may eventually move out to join the anti-racist compartment.

(iv) Anti-racist $A(t)$*:* These are individuals who are aware of racism but against racism activities due to education campaigns and religious teaching. They enter this state from either the exposed or racist compartments and may transition back to the susceptible compartment over time.

The movement of individuals between compartments is defined by the following rates:

Movement of Susceptible individuals $S(t)$: Susceptible individuals obtained through an everyday recruitment rate of Λ and who are born with good behavior but vulnerable to racism. A susceptible individual can be exposed to racism at the rate of β after being contacted by racist individuals. If individuals are strongly learned about the impact of racism, then they might have stopped attending racist groups. The goal of creating awareness is to bring about interactive transformation in susceptible individuals. Therefore, some of susceptible individuals will turn into anti-racist compartments by the rate γ.

Movement of Exposed individuals $E(t)$: Individuals in the exposed compartment can transfer to the racist or anti-racist class at the rate of ω or δ respectively due to education campaigns.

Movement of Racist individuals $C(t)$: Individuals from the exposed class will transfer into the racist class at a rate of ωE. Racially racist individuals can move to anti-racist class at the rate of τ due to religious teaching.

Movement of Anti-racist individuals $A(t)$: Upon parting racism events, the individuals in the racist class will transfer to the anti-racist class at a rate of τ. Anti-racist individuals can be susceptible at the rate ψ due to social influence. μ is natural death rate from all compartments.

Moreover, we assumed that:

(a) Susceptible people are equally likely to be racist.

(b) Racist individuals motivate susceptible persons to participate in racism.

(c) The spread of racism in society is analogous to the spread of infectious diseases.

(d) Anti-racist individuals can relapse to a susceptible class due to social influence.

(e) All the model parameters are not negative.

2.2. Flow Diagram and Ordinary Differential Equation of the Model

Based on the description and assumptions mentioned above, we prepared the current diagram representation of the racism dynamics as displayed in Figure 1, below.

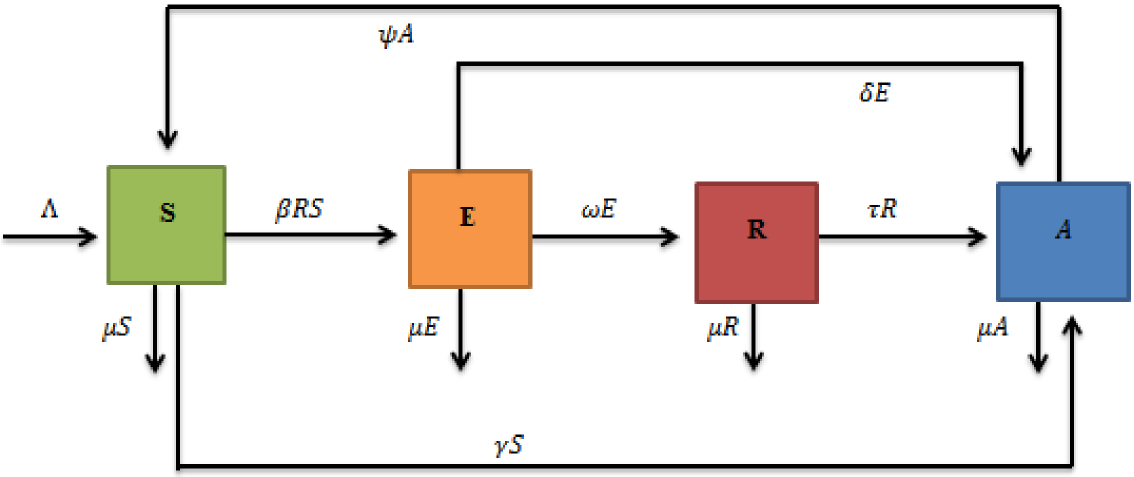

Figure 1. Flow-diagram of the model.

From the flow-diagram representation of the model displayed in the above figure, we obtain the subsequent system of the nonlinear ordinary differential Equation representing the transmission of racism dynamics.

$$\begin{cases} \dfrac{dS}{dt} = \Lambda + \psi A - \beta RS - (\gamma + \mu)S \\ \dfrac{dE}{dt} = \beta RS - (\omega + \delta + \mu)E \\ \dfrac{dR}{dt} = \omega E - (\tau + \mu) R \\ \dfrac{dA}{dt} = \gamma S + \delta E + \tau R - (\psi + \mu)A \end{cases} \quad (1)$$

With the initial conditions

$$S(0) > 0, \ E(0) \geq 0, \ R(0) \geq 0, \ A(0) \geq 0, \quad (2)$$

The description of the parameters and state variables of the model is given in Tables 1 and 2, respectively.

Table 1. Notation and description of parameters.

Parameters	Description
Λ	Recruitment rate
μ	Natural death rate
β	Rate change of racism transmission due to education campaigns
γ	The rate at which susceptible individuals move to the anti-racism class
δ	The rate at which exposed individuals transfer to the anti-racism class
ω	The rate at which exposed individuals join the racist population
τ	The rate at which racist individuals join the anti-racism compartment
ψ	The rate at which anti-racism individuals become susceptible

Table 2. Description of state variables.

Variables	Description
$S(t)$	Number of susceptible population at time t
$E(t)$	Number of exposed population at time t
$R(t)$	Number of racist population at time t
$A(t)$	Number of anti-racism population at time t

3. Qualitative Analysis

3.1. Basic Properties of the Model

This part is intended to prove the existence and uniqueness of the solution, non-negative solutions to state variables that represent human populations, and positively invariant regions of the model system (1).

Theorem 1. *(Existence and uniqueness of the solution) assume the initial conditions* $S(0) > 0, E(0) \geq 0, R(0) \geq 0$ *and* $A(0) \geq 0$ *exist in the region* R_+^4. *Then, the solution of state variables* $S(t),\quad E(t),\quad R(t)$ *and* $A(t)$ *exist for all time* t *and it remains in* R_+^4.

Proof. The right-hand side of system (1) can be expressed as

$$\begin{cases} f_1(S,\ E,\ R,\ A) = \Lambda + \psi A - \beta RS - (\gamma+\mu)S \\ f_2(S,\ E,\ R,\ A) = \beta RS - (\omega+\delta+\mu)E \\ f_3(S,\ E,\ R,\ A) = \omega E - (\tau+\mu)R \\ f_4(S,\ E,\ R,\ A) = \gamma S + \delta E + \tau R - (\psi+\mu)A \end{cases} \tag{3}$$

With the region $\Omega = \left\{(S(t),\ E(t),\ R(t),\ A(t)) \in \mathrm{R}_+^4 : 0 \leq N(t) \leq \frac{\Lambda}{\mu}\right\}$, using the Theorem in [13], the system (1) has a unique solution if $\frac{\partial f_i}{\partial x_j}$; $i, j = 1, 2, \ldots, 4$ are continuous and bounded in Ω. Hence, using the notations $x_1 = S, x_2 = E, x_3 = R, x_4 = A$, we show verification of continuity and boundedness as follows.

For f_1, we get $\left|\frac{\partial f_1}{\partial S}\right| = |-(\beta R+\gamma+\mu)| < \infty$; $\left|\frac{\partial f_1}{\partial E}\right| = 0 < \infty$; $\left|\frac{\partial f_1}{\partial R}\right| = |-\beta S| < \infty$ and $\left|\frac{\partial f_1}{\partial A}\right| = |\psi| < \infty$.

For f_2, we obtain $\left|\frac{\partial f_2}{\partial S}\right| = |\beta R| < \infty$; $\left|\frac{\partial f_2}{\partial E}\right| = |-(\omega+\delta+\mu)| < \infty$; $\left|\frac{\partial f_2}{\partial R}\right| = |\beta S| < \infty$ and $\left|\frac{\partial f_2}{\partial A}\right| = 0 < \infty$.

For f_3, we find $\left|\frac{\partial f_3}{\partial S}\right| = 0 < \infty$; $\left|\frac{\partial f_3}{\partial E}\right| = |\omega| < \infty$; $\left|\frac{\partial f_3}{\partial R}\right| = |-(\tau+\mu)| < \infty$ and $\left|\frac{\partial f_3}{\partial A}\right| = 0 < \infty$.

For f_4, we have $\left|\frac{\partial f_4}{\partial S}\right| = |\gamma| < \infty$; $\left|\frac{\partial f_4}{\partial E}\right| = |\delta| < \infty$; $\left|\frac{\partial f_4}{\partial C}\right| = |\tau| < \infty$ and $\left|\frac{\partial f_4}{\partial A}\right| = |-(\psi+\mu)| < \infty$.

Hence, from all the partial derivatives, $\frac{\partial f_i}{\partial x_j}$; $i, j = 1, 2, \ldots, 4$ we understand the solution of state the variables exists and they are continuous and bounded in Ω. Therefore, by the Lipchitz condition, the system (1) has a unique solution. □

Theorem 2. *(Positivity of solutions) if the set of initial conditions* $\{S(0) > 0, E(0) \geq 0, R(0) \geq 0, A(0) \geq 0\} \in R_+^4$ *then, the solutions of* $S(t),\quad E(t),\quad R(t),$ *and* $A(t)$ *are non-negative for all* $t \geq 0$.

Proof. Assume that all the state variables of the model are continuous. Consider the first Equation of the system (1) $\frac{dS}{dt} = \Lambda + \psi A - \beta RS - (\gamma+\mu)S \geq -\beta RS - (\gamma+\mu)S \Longrightarrow \frac{dS}{dt} \geq -(\beta R+\gamma+\mu)S$, integrating the expression and applying the initial condition yields;

$$S(t) > S(0)\ e^{-\int_0^t (\beta R+\gamma+\mu)dt} > 0. \tag{4}$$

Similarly, integrating the Equation and substituting initial values, we can obtain the following solutions for the remaining state variables.

$$\begin{aligned} E(t) &\geq E(0)\ e^{-(\omega+\delta+\mu)t} \geq 0 \\ R(t) &\geq R(0)e^{-(\tau+\mu)t} \geq 0 \\ A(t) &\geq A(0)e^{-(\psi+\mu)t} \geq 0 \end{aligned} \tag{5}$$

Therefore, the solutions of system (1) are non-negative for all $t \geq 0$. □

Theorem 3. *(Positively invariant region) the set of solutions of the Equation (1) with initial values* $S(0) > 0, E(0) \geq 0,\ R(0) \geq 0$ *and* $A(0) \geq 0$ *defined in* $\Omega = \left\{(S,\ E,\ R,\ A) \in R_+^4 : 0 \leq S(t) + E(t) + R(t) + A(t) \leq \frac{\Lambda}{\mu}\right\}$ *is positively invariant, where* $N(t) = S(t) + E(t) + R(t) + A(t)$, *and the inequality simplifies to* $0 \leq N(t) \leq \frac{\Lambda}{\mu}$, *as* $N(t)$ *is the total population derived from the state variables.*

Proof. By taking the time derivative of the total population Equation $N(t) = S(t) + E(t) + R(t) + A(t)$ and applying the differential equations in system (1), we arrive at;

$$\frac{dN}{dt} = \Lambda - \mu N \tag{6}$$

By applying the integrating factor $e^{\mu t}$ to the rearranged Equation (6), we arrive at the following simplified form.

$$N(\mathrm{t}) = \frac{\Lambda}{\mu} + \left(\mathrm{N}(0) - \frac{\Lambda}{\mu}\right)e^{-\mu t} \tag{7}$$

Since the total population, $\mathrm{N(t)}$ is positive for all time $t \geq 0$. Equation (7) is well defined, which $\lim_{t\to\infty} \mathrm{N(t)} \leq \frac{\Lambda}{\mu}$. Therefore, the invariant region Ω is positive that is, all solutions in Ω rests in Ω. This proves that the racism dynamics model given in system (1) is mathematically and epidemiologically well-posed. □

3.2. Stability Analysis

In this part, basic reproduction number and stability analyses of endemic and racism-free equilibrium points are discussed.

3.2.1. Racism-Free Equilibrium Point (RFE)

Equilibrium point of the model without racism is obtained by setting $E = 0,\quad R = 0$ in all equations. Based on the work of [14], the racism-free equilibrium point P_0 is given by

$$P_0 = (S^*,\ E^*,\ R^*, A^*) = \left(\frac{\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)}\ \ 0\ ,\ \ 0\ ,\ \ \frac{\gamma\Lambda}{\mu(\psi+\gamma+\mu)}\right) \tag{8}$$

3.2.2. Basic Reproduction Number

The basic reproduction number R_0 is defined averagely as the number of new people who are racist [15,16]. We compute using the next-generation matrix approach described by [15]. To analyze the dynamics of the system effectively, we reorganize the governing equations into a structured sequence. We first present the racism-active classes (those currently involved in the transmission or incubation of the ideology) for both populations E and R. This is followed by the racism-free classes S and A (those who are currently neutral or actively opposing the ideology).

$$\begin{cases} \dfrac{dE}{dt} = \beta RS\ - (\omega+\delta+\mu)E \\ \dfrac{dR}{dt} = \ \omega E - (\tau+\mu)\, R \\ \dfrac{dS}{dt} = \ \Lambda + \psi A - \ \beta RS - (\gamma+\mu) \\ \dfrac{dA}{dt} = \ \gamma S + \delta E + \tau R - (\psi+\mu)A \end{cases} \tag{9}$$

From model system (9) the infected compartments are as follows.

$$\begin{cases} \dfrac{dE}{dt} = \beta RS\ - (\omega+\delta+\mu)E \\ \dfrac{dR}{dt} = \ \omega E - (\tau+\mu)\, R \end{cases} \tag{10}$$

Now, from (10) F_i and V_i are given as follows

$$F_i = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} \beta RS \\ 0 \end{pmatrix}, \text{and } V_i = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (\omega+\delta+\mu)E \\ (\tau+\mu)\, R - \omega E \end{pmatrix}$$

Then, Jacobian matrix of F_i and V_i, by evaluating at the point of racism free P_0, is yielding the matrices F and V respectively, as follows:

$$F = \left(\frac{\partial F_i}{\partial x_j}(P_0)\right) = \begin{pmatrix} 0 & \frac{\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)} \\ 0 & 0 \end{pmatrix}, \text{and } V = \left(\frac{\partial V_i}{\partial x_j}(\ P_0)\right) = \begin{pmatrix} (\omega+\delta+\mu) & 0 \\ -\omega & (\tau+\mu) \end{pmatrix}$$

Now, the value of V inverse we have,

$$V^{-1} = \begin{pmatrix} \dfrac{1}{(\omega+\delta+\mu)} & 0 \\ \dfrac{\omega}{(\omega+\delta+\mu)(\tau+\mu)} & \dfrac{1}{(\tau+\mu)} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \dfrac{\omega\beta\Lambda(\psi+\mu)}{(\omega+\delta+\mu)(\tau+\mu)\mu(\psi+\gamma+\mu)} & \dfrac{\beta\Lambda(\psi+\mu)}{(\tau+\mu)\mu(\psi+\gamma+\mu)} \\ 0 & 0 \end{pmatrix}$$

The basic reproduction number R_0 of the model system (1) is the largest eigenvalue (spectral radius) of the next-generation matrix of FV^{-1} that is;

$$R_0 = \frac{\omega\beta\Lambda(\psi+\mu)}{(\omega+\delta+\mu)(\tau+\mu)\mu(\psi+\gamma+\mu)} \tag{11}$$

Theorem 4. *(Local stability of the racism-free equilibrium point) the racism-free equilibrium point* P_0 *of model system (1) is locally asymptotically if* $R_0 < 1$ *and otherwise unstable.*

Proof. To prove the local stability of the point of equilibrium without racism, we consider the Jacobian matrix of the system (1) at P_0 as follows

$$(P_0) = \begin{pmatrix} -(\gamma+\mu) & 0 & -\dfrac{\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)} & \psi \\ 0 & -(\omega+\delta+\mu) & \dfrac{\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)} & 0 \\ 0 & \omega & -(\tau+\mu) & 0 \\ \gamma & \delta & \tau & -(\psi+\mu) \end{pmatrix} \tag{12}$$

The characteristic Equation of (12) at racism-free equilibrium point P_0,

$$|J(P_0)-\lambda I_4| = \begin{vmatrix} -(\gamma+\mu)-\lambda & 0 & -\dfrac{\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)} & \psi \\ 0 & -(\omega+\delta+\mu)-\lambda & \dfrac{\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)} & 0 \\ 0 & \omega & -(\tau+\mu)-\lambda & 0 \\ \gamma & \delta & \tau & -(\psi+\mu)-\lambda \end{vmatrix} = 0$$

After some simplification, we obtain the following.

$$\begin{aligned} &\left((-(\omega+\delta+\mu)-\lambda)(-(\tau+\mu)-\lambda)-\frac{\omega\beta\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)}\right) \\ &\big((-(\gamma+\mu)-\lambda)(-(\psi+\mu)-\lambda)-\gamma\psi\big) = 0 \end{aligned} \tag{13}$$

Equation (13) can be

$$(\lambda^2+m_1\lambda+m_2)(\lambda^2+k_1\lambda+k_2)=0 \tag{14}$$

$$k_1=\gamma+\psi+2\mu,\, k_2=(\gamma+\psi+\mu)\mu,\ m_1=(\omega+\delta+\tau+2\mu) \text{ and}$$

$$m_2=(\tau+\mu)(\omega+\delta+\mu)(1-R_0).$$

Using the Routh-Hurwitz principles, the second expression of Equation (14) has all negative real part since $k_1=\gamma+\psi+2\mu>0$, & $k_2=(\gamma+\psi+\mu)\mu>0$. Furthermore, using the Routh-Hurwitz criteria, the first expression of Equation (14) has all negative real solution we require if $m_1>0$ and $m_2>0$. Using the assumption that all parameters of the model are non-negative, $m_1=(\omega+\delta+\tau+2\mu)>0$ and also $m_2=(\tau+\mu)(\omega+\delta+\mu)(1-R_0)>0$ for $R_0<1$.

Therefore, our model of system (1) at racism-free equilibrium (P_0) point gives all eigenvalues possessing negative real part, which is locally asymptotical stable when $R_0<1$.

Thus, the epidemiological suggestion of this theorem is that the spread of racism in the community is independent of the initial size of the populations. To ensure that the eradication of racism within the community does not depend on the specific starting numbers of each group, we must demonstrate that the racism-free point (P_0) is globally stable [17]. □

Theorem 5. *(Global stability of the racism-free equilibrium point (RFE)): The RFE point* P_0 *is GAS if* $R_0<1$ *and unstable otherwise.*

Proof. To verify the global stability, we choose the following appropriate Lyapunov function.

$$V=a_1E+a_2R \tag{15}$$

By differentiating the Lyapunov function of Equation (15) at time t, thus we get

$$\begin{aligned}\frac{\mathrm{dV}}{dt} &= a_1\frac{\mathrm{dE}}{dt} + a_2\frac{\mathrm{dR}}{dt}\\ &= a_1\left(\beta RS - (\omega+\delta+\mu)E\right) + a_2(\omega E - (\tau+\mu)R)\\ &= a_1\beta RS - a_2(\tau+\mu)R - a_1(\omega+\delta+\mu)E + a_2\omega E\end{aligned}$$

Now, take $a_1 = \frac{\omega a_2}{(\omega+\delta+\mu)}$, $a_2 = 1$and $S \leq S^*$, then after some simplification we have:

$$\frac{\mathrm{dV}}{dt} \leq Ra_2\left(\frac{\omega\beta S^*}{(\omega+\delta+\mu)} - (\tau+\mu)\right) \tag{16}$$

Using $S^* = \frac{\Lambda(\psi+\mu)}{\mu(\psi+\gamma+\mu)}$, $a_2 = 1$ into Equation (16) and after some simplification substitute the value of $R_0 = \frac{\omega\beta\Lambda(\psi+\mu)}{(\omega+\delta+\mu)(\tau+\mu)\mu(\psi+\gamma+\mu)}$, we get

$$\frac{\mathrm{dV}}{dt} \leq (\tau+\mu)(R_0-1)R \tag{17}$$

Since all model parameters are nonnegative $\frac{dV}{dt} < 0$ if $R_0 < 1$. The equality $\frac{dV}{dt} = 0$ holds if $R_0 = 1$ or $R = 0$. According to the LaSalle invariance principle [18], the equilibrium point without racism is globally asymptotically stable if $R_0 < 1$. This theorem implies that racism will be reduced regardless of the initial profile of the population for $R_0 < 1$. □

3.2.2. Racism-Endemic Equilibrium Point (REE)

The equilibrium point of the racism-endemic $P_1 = (S^{**}, E^{**}, R^{**}, A^{**})$ is a steady-state solution that occurs whenever racism exists in the community. It is solved by setting the system of Equation (1) equal to zero, as explained in [19]. Now, Equation (1) can be rewritten as

$$\begin{aligned}\Lambda + \psi A^{**} - \beta R^{**}S^{**} - (\gamma+\mu)S^{**} &= 0\\ \beta R^{**}S^{**} - (\omega+\delta+\mu)E^{**} &= 0\\ \omega E^{**} - (\tau+\mu)R^{**} &= 0\\ \gamma S^{**} + \delta E^{**} + \tau R^{**} - (\psi+\mu)A^{**} &= 0\end{aligned} \tag{18}$$

From the second, third, and fourth of Equation (18), we obtain the following.

$$\begin{cases} S^{**} = \dfrac{(\omega+\delta+\mu)(\tau+\mu)}{\beta\omega}\\ E^{**} = \dfrac{(\tau+\mu)R^{**}}{\omega}\\ A^{**} = \dfrac{\gamma(\omega+\delta+\mu)(\tau+\mu)}{\beta\omega} + \left(\dfrac{\delta(\tau+\mu)+\omega\tau}{\omega}\right)R^{**}\end{cases} \tag{19}$$

Substitute the value of S^{**} and A^{**} into the first Equation of (18), we obtain

$$R^{**} = \frac{(\omega+\delta+\mu)(\psi+\gamma+\mu)(R_0-1)}{\beta(\omega+\delta+2\mu)} \tag{20}$$

Since all parameters of the model are positive. Therefore, from Equation (20) the value of $R^{**} > 0$ if $R_0 > 1$. Thus, the system of racism dynamics model (1) has a unique endemic equilibrium point whenever $R_0 > 1$. □

Theorem 6. *(Global stability of the endemic equilibrium point): The endemic equilibrium point* (P_1) *of the model system (1) is globally asymptotically stable when* $R_0 > 1$.

Proof. We consider the following quadratic Lyapunov function constructed by [20] to investigate the conditions under which the endemic equilibrium is globally stable.

$$X(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n}\frac{1}{2}[x_i - x_i{}^*]^2,$$

where, x_i represents the population of the i^{th} class, while $x_i{}^*$ denotes the endemic equilibrium point. The specified function is positive definite with respect to model system (1).

$$V(S,E,R,A)=\sum_{i=1}^{4}\frac{1}{2}[x_i-x_i{}^{*}]^2 \tag{21}$$

Using the above Lyapunov function, the racism dynamics system is written as follows:

$$V=\frac{1}{2}[(S-S^{**})+(E-E^{**})+(R-R^{**})+(A-A^{**})]^2 \tag{22}$$

Obviously V is a smooth, continuous function, computing its instantaneous rate of change over time yields;

$$\frac{dV}{dt}=[(S-S^{**})+(E-E^{**})+(R-R^{**})+(A-A^{**})]$$

$$\frac{dV}{dt}=[S+E+R+A-(S^{**}+E^{**}+R^{**}+A^{**})]\frac{d}{dt}[S+E+R+A]$$

But

$$\frac{d}{dt}[S+E+R+A]=[\Lambda-\mu N] \tag{23}$$

And

$$\Lambda-\mu N^{**}=0,\Longrightarrow \Lambda-\mu[S^{**}+E^{**}+R^{**}+A^{**}]=0,$$

$$[S^{**}+E^{**}+R^{**}+A^{**}]=\frac{\Lambda}{\mu} \tag{24}$$

Substitute Equations (23) and (24) into $\frac{dV}{dt}$ gives

$$\frac{dV}{dt}\le\left[N(t)-\frac{\Lambda}{\mu}\right][\Lambda-\mu N],$$

$$\frac{dV}{dt}\le\left[N(t)-\frac{\Lambda}{\mu}\right]\left[-\mu\left(\mathrm{N(t)}-\frac{\Lambda}{\mu}\right)\right],$$

$$\frac{dV}{dt}\le-\mu\left[N(t)-\frac{\Lambda}{\mu}\right]\left[\left(\mathrm{N(t)}-\frac{\Lambda}{\mu}\right)\right],$$

$$\frac{dV}{dt}\le-\mu\left[N(t)-\frac{\Lambda}{\mu}\right]^2 \tag{25}$$

Therefore, from Equation (25) we have $\frac{dV}{dt}<0$. Therefore, using LaSalle's invariance principle [18], the racism-endemic equilibrium point P_0 is globally asymptotically stable if $R_0>1$. □

4. Sensitivity Analysis

In this section, we discuss the sensitivity study of system (1) towards determine the model constraints that affect the extent of racism and the number of reproductions. This strategy is used to identify the most significant parameters of model dynamics [21–23]. The normalized forward sensitivity index of the variable R_0 that depends on the differentiability of a parameter ρ_i is defined as;

$$\Upsilon_{p_i}^{R_0}=\frac{\partial R_0}{\partial p_i}\times\frac{p_i}{R_0} \tag{26}$$

For each parameter (p_i) in $R_0=\frac{\omega\beta\Lambda(\psi+\mu)}{(\omega+\delta+\mu)(\tau+\mu)\mu(\psi+\gamma+\mu)}$ from Equation (26) we have;

$$\Upsilon_{\beta}^{R_0}=1,\ \Upsilon_{\omega}^{R_0}=\frac{(\delta+\mu)}{(\omega+\delta+\mu)},\Upsilon_{\psi}^{R_0}=\frac{\gamma\psi}{(\psi+\mu)(\psi+\gamma+\mu)},\ \Upsilon_{\delta}^{R_0}=-\frac{\delta}{(\omega+\delta+\mu)},\ \Upsilon_{\tau}^{R_0}=-\frac{\tau}{(\tau+\mu)},\ \Upsilon_{\gamma}^{R_0}=-\frac{\gamma}{(\psi+\gamma+\mu)}$$

From the results of the sensitivity indices, some parameters are identified that have a great impact on the spread of racism in the community. Parameters with positive indices β, ω, and ψ are increase the basic reproduction number R_0 which increases the transmission of racism in the community when their value increases. However, parameters with negative indices δ,τ, and γ minimize the problem of racism in the community as their values increase.

Furthermore, the sensitivity of the parameters in model (1) has been displayed graphically for better and easier understanding. Figures 2 and 3 show that the rise of parameters β and ω facilitates to rapid increase in the basic reproduction number R_0. Similarly, Figures 4 and 5 show that the rise rate of parameters δ and τ decreases the basic reproduction number R_0.

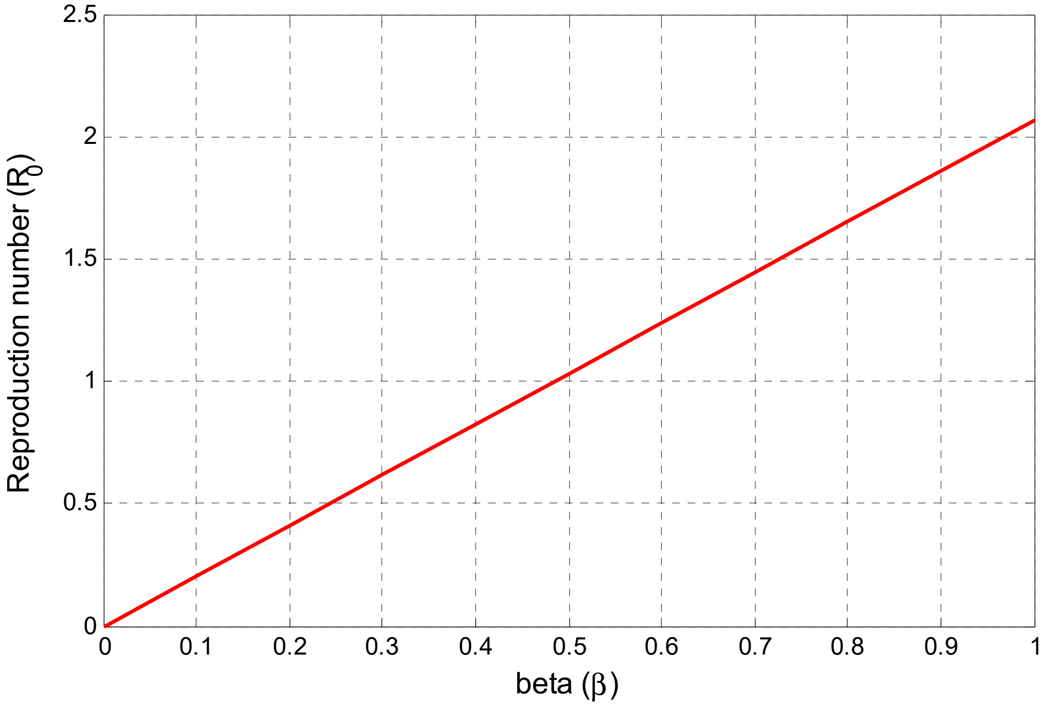

Figure 2. Reproduction number (R_0) versus parameter beta (β).

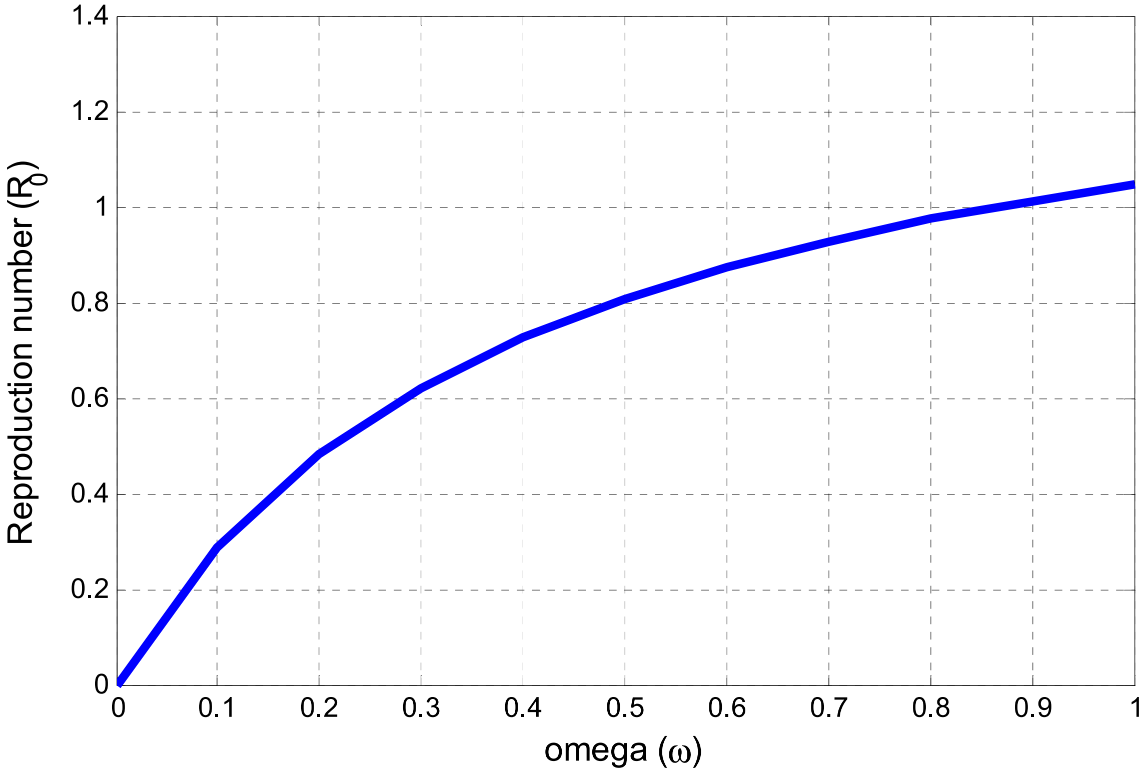

Figure 3. Reproduction number (R_0) versus parameter omega (ω).

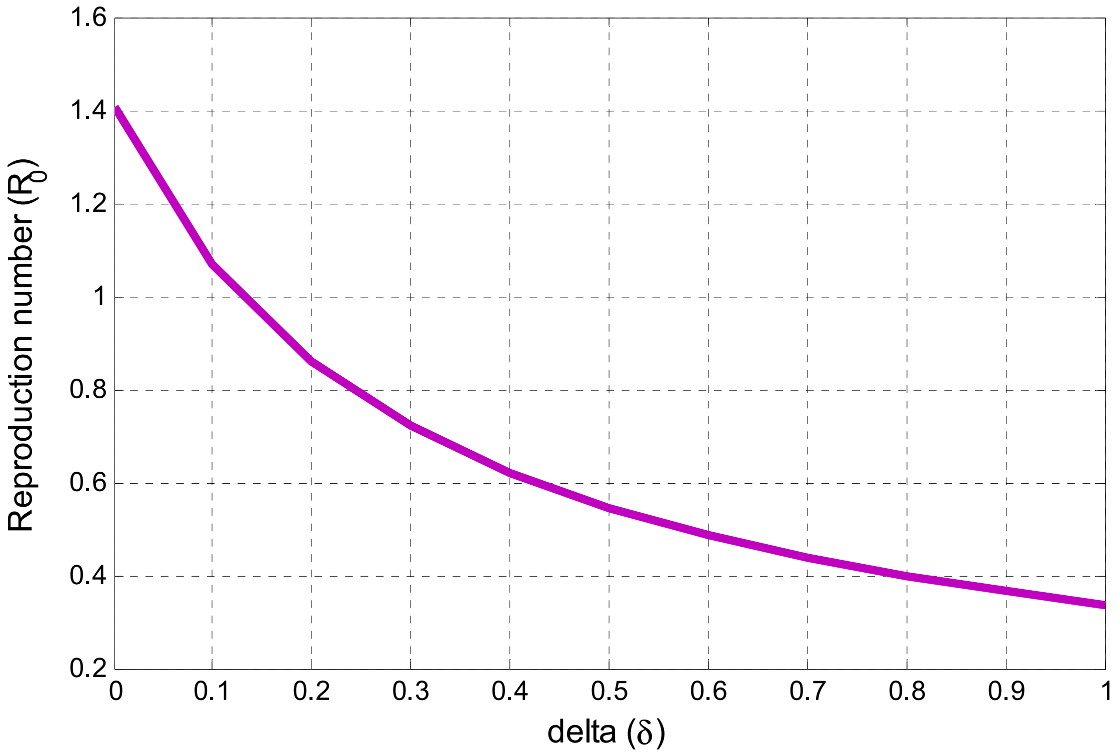

Figure 4. Reproduction number (R_0) versus parameter delta (δ).

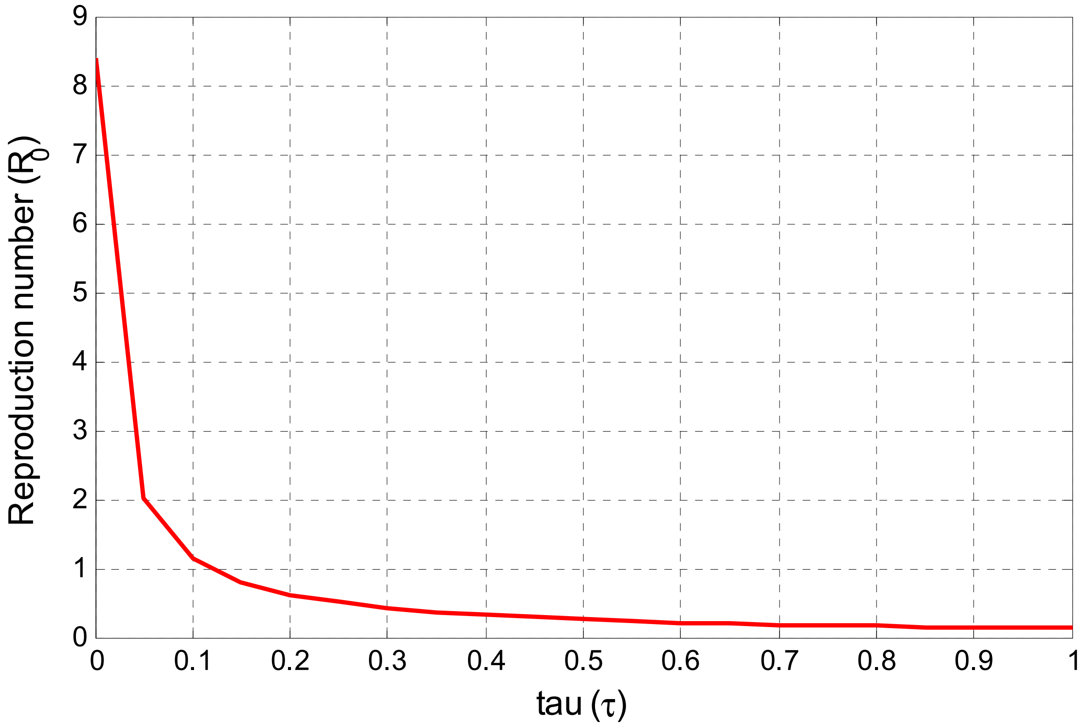

Figure 5. Reproduction number (R_0) versus parameter tau (τ).

Based on the sensitivity analysis, reducing the spread of racism from community requires stakeholders to lower positive index parameters while raising negative index parameters. Consequently, to determine the optimal strategy for combating racism, we will examine an optimal control strategy in the next section.

5. Analysis of the Optimal Control Problem

The optimal control framework employed here is grounded in Pontryagin's Maximum Principle (PMP), originally developed by [24] as a cornerstone of modern control theory for optimizing trajectories in dynamical systems. In epidemiology and social dynamics modeling, PMP has been widely applied to derive time-dependent interventions that minimize disease burden or social costs, such as vaccination campaigns or behavior modification strategies [25]. A recent comprehensive review by [26], synthesizes historical developments and modern extensions of PMP to nonlinear systems, including stability analysis of adjoint equations and bang-bang control characterization directly validating the Hamiltonian formulation, co-state dynamics, and transversality conditions. This approach ensures the derived controls are theoretically robust for the racism transmission model.

Although eliminating racism is not possible due to costs, social life and environmental reasons, we need to investigate the optimal control strategy to reduce transmission. Therefore, we have used two time-dependent control strategies $u_1(t)$ and $u_2(t)$ to decrease the extent of racism. The control u_1 denotes efforts expected to avoid racism using mass education campaigns like social media. The control u_2 represents efforts expected to reduce racist individuals using teaching through religious institutions. Thus, the racism dynamics model of system (1) with the two control strategies is given by the following non-linear system of ordinary differential equations:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda + \psi A - (1- u_1)\beta RS - (\gamma + \mu)S \\ \dfrac{dE}{dt} = (1- u_1)\beta RS - (\omega + \delta + \mu)E \\ \dfrac{dR}{dt} = \omega E - (u_2 + \tau + \mu) R \\ \dfrac{dA}{dt} = \gamma S + \delta E + (u_2 + \tau)R - (\psi + \mu)A \end{cases} \tag{27}$$

With the initial conditions

$$S(0) > 0,\ E(0) \geq 0,\ R(0) \geq 0,\ A(0) \geq 0 \tag{28}$$

For the nonlinear ordinary differential Equation (27), we consider the objective function as defined below based on the approach [27,28].

$$\mathrm{J}(u_1, u_2) = \int_0^{t_f} \left(b_1 E + b_2 R + \frac{1}{2} b_3 {u_1}^2 + \frac{1}{2} b_4 {u_2}^2\right) dt \tag{29}$$

where, constants b_1, b_2, b_3, and b_4 are non-negative constants that balance the price and the extent of racism at a time t, t_f is the final time. Our goal with the given objective function is to minimize the number of racist individuals while minimizing the cost of control $u_1(t)$ and $u_2(t)$. We seek an optimal control ${u_1}^*$ and ${u_2}^*$ such that:

$$\mathrm{J}({u_1}^*, {u_2}^*) = \min_{u_1, u_2 \in \mathcal{U}} \mathrm{J}(u_1, u_2) \tag{30}$$

where $\mathcal{U}$ is the set of measurable functions defined on the $[0, t_f]$. The necessary conditions of the optimal control model are satisfied by using Pontryagin's maximum principle as explained in [26].

The Hamiltonian function of an optimal control model is the combination of Equations (27) and (29) is given as

$$\mathcal{H} = \left[b_1 E + b_2 R + \frac{1}{2} b_3 {u_1}^2 + \frac{1}{2} b_4 {u_2}^2\right] + \lambda_1 \frac{dS}{dt} + \lambda_2 \frac{dE}{dt} + \lambda_3 \frac{dR}{dt} + \lambda_4 \frac{dA}{dt} \tag{31}$$

From Equation (31) the minimized Hamiltonian regarding the controls to u_1, u_2 is given by

$$\begin{aligned} \mathcal{H} = &\left[b_1 E + b_2 R + \frac{1}{2} b_3 {u_1}^2 + \frac{1}{2} b_4 {u_2}^2\right] \\ &+\lambda_1[\Lambda + \psi A - (1- u_1)\beta RS - (\gamma + \mu)S] \\ &+\lambda_2[(1- u_1)\beta RS - (\omega + \delta + \mu)E] \\ &+\lambda_3[\omega E - (u_2 + \tau + \mu) R] \\ &+\lambda_4[\gamma S + \delta E + (u_2 + \tau)R - (\psi + \mu)A] \end{aligned} \tag{32}$$

where $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are adjoint variables. The adjoint equations are calculated using Pontryagin's minimum principle [27–30]; we stated the theorem as follows:

Theorem 7. *Suppose that the control variables* $(u_1{}^*, u_2{}^*) \in \mathcal{U}$ *and a solution of* $(S,\ E,\ R,\ A)$ *of the corresponding state system (27) and (29) that minimizes* $J(u_1{}^*,\ u_2{}^*)$ *over* $\mathcal{U}$*. Then there exist adjoint variables* $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$ *and* $\lambda_4(t)$ *satisfying*

$$\left.\begin{aligned}
\frac{d\lambda_1}{dt} &= -\frac{\partial\mathcal{H}}{\partial S(t)} = \lambda_1[(1-u_1)\beta R + (\gamma+\mu)] - \lambda_2(1-u_1)\beta R - \lambda_4\gamma\\
\frac{d\lambda_2}{dt} &= -\frac{\partial\mathcal{H}}{\partial E(t)} = -b_1 + \lambda_2(\omega+\delta+\mu) - \lambda_3\omega - \lambda_4\delta\\
\frac{d\lambda_3}{dt} &= -\frac{\partial\mathcal{H}}{\partial R(t)} = -b_2 + \lambda_1(1-u_1)\beta S - \lambda_2(1-u_1)\beta S\\
&\qquad + \lambda_3(u_2+\tau+\mu) - \lambda_4(u_2+\tau)\\
\frac{d\lambda_4}{dt} &= -\frac{\partial\mathcal{H}}{\partial A(t)} = -\lambda_1\psi + \lambda_4(\psi+\mu)
\end{aligned}\right] \tag{33}$$

With transversality conditions:

$$\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = 0 \tag{34}$$

Moreover, for $t \in [0,\ t_f]$ *the optimal controls* $u_1{}^*$ *and* $u_2{}^*$ *are given by*

$$\begin{aligned}
u_1{}^* &= min\left\{1,\ \ max\left(0,\ \frac{(\lambda_2-\lambda_1)\beta RS}{b_3}\right)\right\}\\
u_2{}^* &= min\left\{1,\ \ max\left(0,\ \frac{(\lambda_3-\lambda_4)R}{b_4}\right)\right\}
\end{aligned} \tag{35}$$

Proof. The co-state equations and transversality conditions can be calculated using Pontryagin's maximum principle, as explained in [31–33]. Now, we use the Hamiltonian function of Equation (32) and distinguishing concerned to $S, E, R,$ and A separately, we obtain

$$\left.\begin{aligned}
\frac{d\lambda_1}{dt} &= -\frac{\partial\mathcal{H}}{\partial S(t)} = \lambda_1[(1-u_1)\beta R + (\gamma+\mu)] - \lambda_2(1-u_1)\beta R - \lambda_4\gamma\\
\frac{d\lambda_2}{dt} &= -\frac{\partial\mathcal{H}}{\partial E(t)} = -b_1 + \lambda_2(\omega+\delta+\mu) - \lambda_3\omega - \lambda_4\delta\\
\frac{d\lambda_3}{dt} &= -\frac{\partial\mathcal{H}}{\partial R(t)} = -b_2 + \lambda_1(1-u_1)\beta S - \lambda_2(1-u_1)\beta S\\
&\qquad + \lambda_3(u_2+\tau+\mu) - \lambda_4(u_2+\tau)\\
\frac{d\lambda_4}{dt} &= -\frac{\partial\mathcal{H}}{\partial A(t)} = -\lambda_1\psi + \lambda_4(\psi+\mu)
\end{aligned}\right] \tag{36}$$

Through transversality conditions

$$\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = 0 \tag{37}$$

Now, employing the optimality state, we obtain control functions $u_1{}^*(t)$ and $u_2{}^*(t)$ for $t \in [0,\ t_f]$ as follows

$$\begin{aligned}
&\frac{\partial\mathcal{H}}{\partial u_1} = 0,\ \ at\, u_1 = u_1{}^*,\\
&\qquad \frac{\partial\mathcal{H}}{\partial u_1} = \frac{\partial\mathcal{H}}{\partial u_1{}^*} = b_3u_1 + \lambda_1\,\beta RS - \lambda_2\,\beta RS = 0,\\
&\qquad u_1{}^* = \frac{(\lambda_2-\lambda_1)\beta RS}{b_3}\\
&\frac{\partial\mathcal{H}}{\partial u_2} = 0,\ \ at\, u_2 = u_2{}^*,\\
&\qquad \frac{\partial\mathcal{H}}{\partial u_2} = \frac{\partial\mathcal{H}}{\partial u_2{}^*} = b_4u_2 - \lambda_3R + \lambda_4R = 0,\\
&\qquad u_2{}^* = \frac{(\lambda_3-\lambda_4)R}{b_4}
\end{aligned} \tag{38}$$

Based on typical control constraints, we can easily determine the following expressions for $u_1{}^*(t)$ and $u_2{}^*(t)$.

$$\begin{aligned} u_1{}^* &= min\left\{1, \quad max\left(0, \ \frac{(\lambda_2 - \lambda_1)\beta RS}{b_3}\right)\right\} \\ u_2{}^* &= min\left\{1, \quad max\left(0, \ \frac{(\lambda_3 - \lambda_4)R}{b_4}\right)\right\} \end{aligned} \tag{39}$$

Hence, we have proved the characterization of the optimal control function. Furthermore, the simulation of an optimality scheme can be used to decide the greatest intervention approaches to reduce racism dynamics.

6. Numerical Simulation and Discussions

In this section, we investigate the effect of control strategies such as mass education campaigns and teaching through religious institutions to reduce the spread of racism in the community. We verify our work numerically using the fourth-order Runge-Kutta method. Based on the initial value of the state variables, we use the forward fourth-order Runge-Kutta method to solve the state variables of Equation (27). Similarly, the adjoint equations are solved using the fourth-order runge-Kutta method. The simulations are performed by adjusting key parameters as specified in Table 3 which is taken from the previous work and some assumptions. Furthermore, we assume that the weight constant values are: $b_1 = 0.7$ for susceptible via social media; $b_2 = 0.8$ for exposed individual progression; $b_3 = 0.6$ for racism reduction via religion, and $b_4 = 0.8$ for anti-racist recruitment are scaled relative costs effectiveness from socioeconomic data. The initial conditions are $S(0) = 5000,\ E(0) = 100,\ R(0) = 200,\ A(0) = 100.$

Table 3. Illustrative parameter values used in the baseline simulation.

Parameter	Standard Value	Source
Λ	0.018	[21]
μ	0.016	[21]
β	0.3	[10]
γ	0.001	[10]
δ	0.4	[10]
ω	0.3	[10]
τ	0.2	[10]
ψ	0.0021	[29]

These values are subjective estimates adjusted from prior social contagion and epidemic models [10,29], reflecting plausible dynamics for racism spread. They are not empirically derived and serve as a baseline for demonstration. Therefore, we have designed the following three strategies to determine the best intervention to minimize the spread of racism in the community.

6.1. Strategy (i): Using Only Mass Education Campaigns (u_1)

In this strategy, the optimality system was simulated by incorporating only the intervention from the mass education campaigns (u_1). Figure 6 shows that the numbers of racist individuals with the control method decreased related to those without the control policy. Therefore, we conclude that the use of a mass education campaign strategy plays an important role in reducing the spread of racism in the community in a specified time.

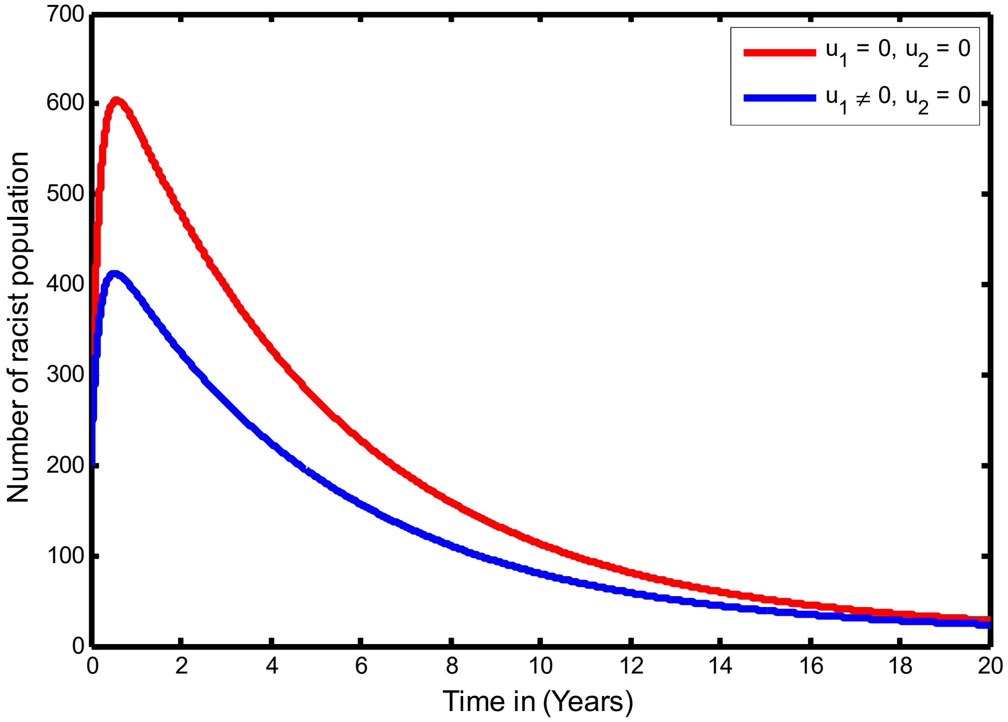

Figure 6. Effect of mass education campaigns (u_1).

6.2. Strategy (ii): Using Only Teaching through Religious Institutions (u_2)

In this strategy, the optimality system is simulated by incorporating only teaching through religious institutions (u_2) intervention. Figure 7 displays that the amount of racist persons with the control strategy decreased when we compared with those starved of the control strategy. Therefore, it is concluded that the community teaching approach through religious institutions theaters a significant role in decreasing the spread of racism over a specified period of time.

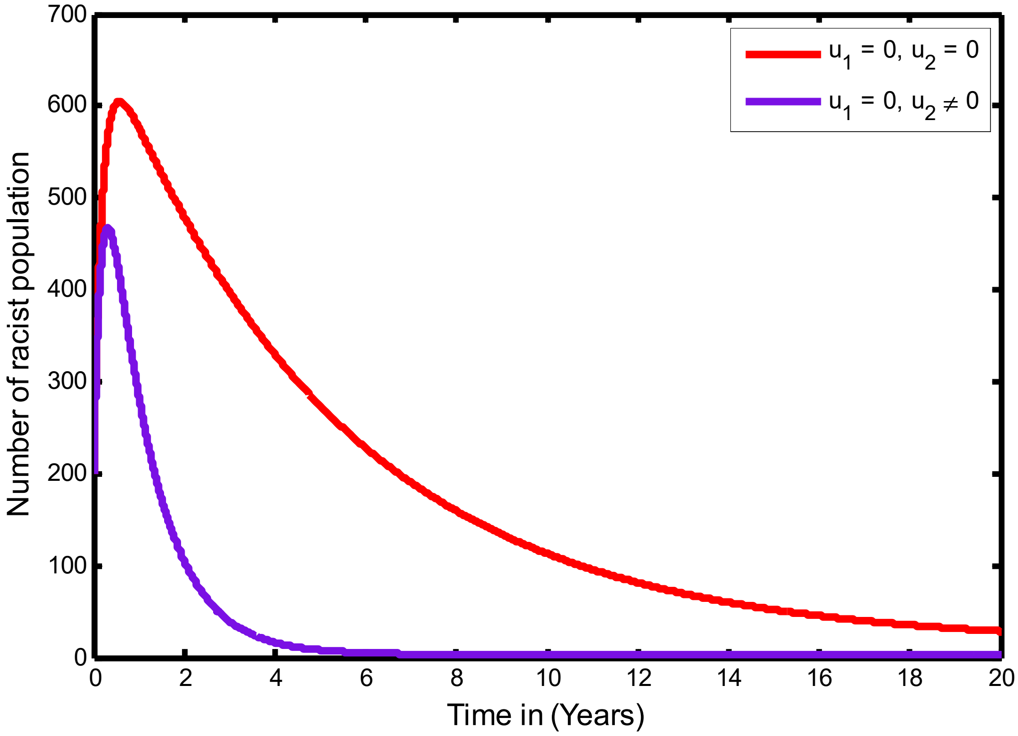

Figure 7. Effect of the teaching community through religious institutions (u_2).

6.3. Strategy (iii): Using Both Mass Education Campaigns (u_1) *and Teaching through Religious Institutions* (u_2).

Here, the model equations are simulated using a combination of mass education campaigns (u_1) and teaching through religious institutions (u_2) as intervention strategies to reduce the spread of racism in the community. Figure 8 shows that the amount of racist persons through the control strategy decreased when we compared it with those without the control strategy. Therefore, it is concluded that both these strategies are effective in reducing the spread of racism in the community in a specified time.

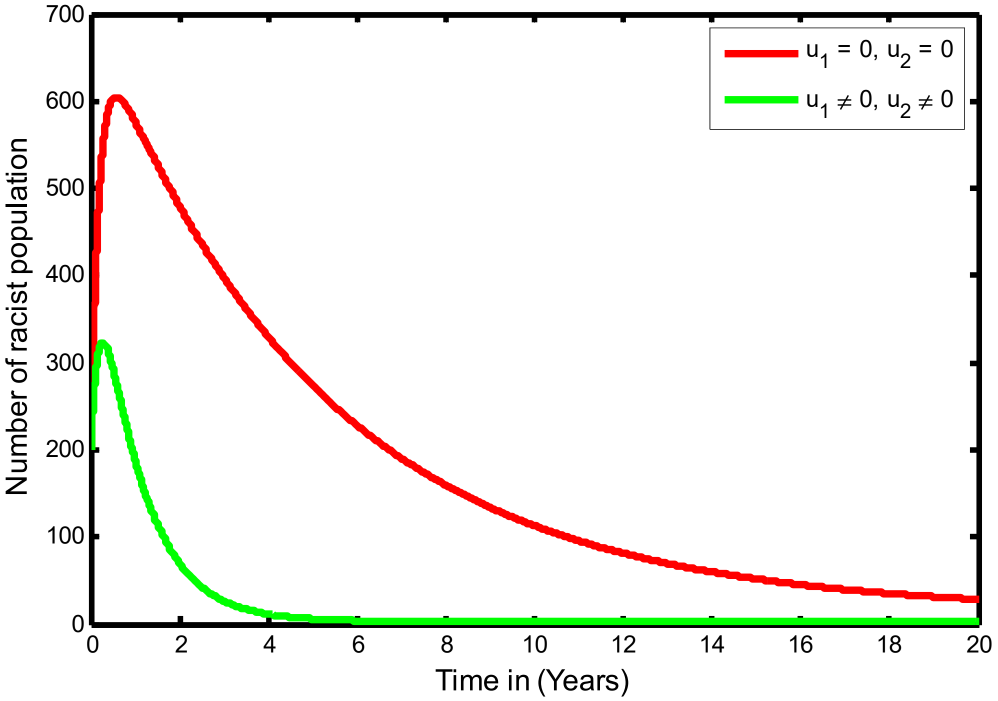

Figure 8. Effect of both mass education campaigns (u_1) and teaching through religious institutions (u_2).

7. Discussions and Conclusions

In this paper, we proposed and analyzed a mathematical model for the dynamics of spreading racism in the community. To assess the spread of racism in the community, we divided the total population of the model into four compartments: Susceptible individuals, Exposed individuals, Racist individuals, and Anti-racist individuals. For the proposed model, we have computed the basic properties of the model such as the existence and uniqueness of the solution, the positivity of the solution, and the positively invariant region. The basic reproduction number was derived using the next generation matrix method, and we investigated the existence of equilibrium points, as well as stability analysis. The analysis shows that the model has a locally asymptotically stable racism-free equilibrium point when the reproduction number $R_0 < 1$ and a locally asymptotically stable endemic equilibrium point when the reproduction number $R_0 > 1$.

Using the normalized forward sensitivity index approach, the most significant parameters of the racism model were identified. Sensitivity analysis shows that stakeholders are supposed to decrease positive index parameters and increase negative index parameters to reduce the spread of racism in the community. The optimal control model was then formulated by adding two time-dependent controls (mass education campaigns using social media and teaching through religious institutions). Numerical simulations of the optimal control model were performed using Runge-Kutta of order four. The results displayed in the graphs show that the amount of racist persons through the control strategy decreased when we compared with those without the control strategy. Therefore, this study recommends that both mass education campaigns and teaching through religious institutions should be applied at the same time, which can reduce racism in the community. A future study will evaluate the cost-effectiveness of the optimal strategy.

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Conflicts of Interest

The author declares no conflict of interest.

Use of AI and AI-Assisted Technologies

During the preparation of this work, the author used blackbox.ai to perform language editing and improve readability. After using this too, the author reviewed and edited the content as needed and take full responsibility for the content of the published article.

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