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Article

Analysis and Control of an Energy Harvesting Model

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Abstract: The energy harvesting process is highly nonlinear and one must understand the dynamics of this process in order to control it effectively. The main reason for this nonlinearity is the fact that the process is governed by second order nonlinear differential equations. In this work, these equations are modified into first order ODE and bifurcation and multiobjective nonlinear model predictive control (MNLMPC) calculations are performed on an energy harvesting model. MATCONT (a MATLAB program) was used for the bifurcation analysis while PYOMO was used for the MNLMPC calculations in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of a Hopf bifurcation point, which is eliminated using an activation factor involving the tanh function. The Hopf bifurcation causes limit cycles, which adversely affect both safety and productivity. The MNLMPC calculations provide the optimal control profiles for maximizing the voltage produced.

Keywords: bifurcation, optimization, control, energy, voltage, harvest, structure

1. Background

Energy harvesting involves capturing and converting ambient energy from the environment into electrical energy that can be used to power electronic devices or stored for later use. This has gained tremendous attention in recent decades due to the global push for sustainable energy solutions and the rapid growth of wireless, portable, and self-powered technologies. The idea of harvesting energy from the surroundings—whether from light, heat, mechanical vibrations, radio frequency waves, or even human motion—represents a shift toward reducing dependence on batteries and fossil fuels while promoting renewable and maintenance-free energy sources. As the demand for autonomous and low-power devices continues to rise, energy harvesting has become a cornerstone technology enabling long-term, self-sustaining operation.

The fundamental principle of energy harvesting lies in the conversion of one form of ambient energy into electrical power. Various natural and artificial sources are available in our environment that can serve this purpose. Solar energy, for example, is the most widely used and powerful source. Through photovoltaic cells, sunlight is directly converted into electricity using semiconducting materials that generate charge carriers when exposed to photons. Solar energy harvesting is efficient, reliable, and mature, making it suitable for outdoor applications like remote sensors, communication towers, and autonomous vehicles. However, solar energy depends heavily on environmental conditions such as sunlight intensity and availability, limiting its performance in shaded or indoor environments. To address these limitations, hybrid systems combining solar energy with other harvesting methods have been developed to ensure continuous power generation.

Another significant source of harvestable energy is mechanical energy, which can be captured from vibrations, movements, or pressure fluctuations. Mechanical energy harvesting technologies primarily rely on piezoelectric, electromagnetic, and electrostatic conversion. Piezoelectric materials generate an electric charge when subjected to mechanical stress, making them particularly effective for converting vibrations or mechanical motion into electrical energy. These materials have been integrated into various devices, from structural health monitoring systems in bridges and buildings to self-powered wearable sensors that capture energy from human motion. Electromagnetic energy harvesters use electromagnetic induction, where a changing magnetic field induces an electric current in a coil. These systems are common in environments with repetitive motion, such as rotating machinery or human activities like walking. Electrostatic harvesters, on the other hand, exploit changes in capacitance due to mechanical movement, making them suitable for microelectromechanical systems where size and scalability are critical.

Thermal energy harvesting, also known as thermoelectric energy conversion, utilizes temperature differences between two surfaces or environments. In thermoelectric generators, a temperature gradient across a material produces a voltage difference. This method is especially useful in industrial settings where waste heat from engines, exhaust systems, or manufacturing processes can be captured and converted into usable energy. Although the efficiency of thermoelectric generators remains relatively low, advances in materials science have resulted in the development of high-performance thermoelectric materials that can improve conversion rates. In space exploration, thermal energy harvesting plays a vital role in powering spacecraft and planetary probes, where radioisotope thermoelectric generators have provided consistent energy for decades.

Radio frequency (RF) energy harvesting represents another promising avenue, especially with the proliferation of wireless communication networks. Ambient electromagnetic waves generated by radio, television, mobile phones, and Wi-Fi routers carry energy that can be captured and rectified into usable direct current power. RF energy harvesting is particularly attractive for powering devices such as microcontrollers. Although the power density of RF signals is typically low, combining this approach with ultra-low-power electronics and efficient rectifier circuits can create fully autonomous systems that operate indefinitely without batteries. This has profound implications for the IoT revolution, enabling billions of interconnected devices to function seamlessly without manual recharging or battery replacement.

Biological and environmental energy harvesting also offer unique possibilities. For example, bioenergy harvesters can extract energy from biochemical reactions in living organisms, such as glucose oxidation in human blood. This concept has been explored for powering medical implants, wearable biosensors, and drug delivery systems, allowing for self-sustained operation within the human body. Similarly, environmental energy sources such as wind, ocean waves, and fluid flow have been harnessed on larger scales for sustainable energy generation, but micro-scale adaptations of these technologies are emerging for localized and low-power applications. The development of micro wind turbines or triboelectric nanogenerators that convert frictional contact into electricity exemplifies how diverse and innovative energy harvesting approaches can be.

The challenge in energy harvesting lies in the intermittent and variable nature of ambient energy sources. The amount of energy available from environmental sources fluctuates widely depending on time, location, and external conditions. To overcome this issue, efficient energy storage systems are often integrated with harvesters to maintain a stable power supply. Supercapacitors, microbatteries, and advanced solid-state storage devices are used to store harvested energy and release it when needed. The efficiency of the power management circuitry, including voltage regulators and charge controllers, is equally critical to ensure that the harvested energy is used effectively. The design of these circuits must balance low power consumption with the ability to handle very small and irregular input power levels.

Miniaturization and integration are key trends driving the evolution of energy harvesting technology. The development of micro and nanogenerators, made possible by advancements in materials science and microfabrication techniques, has enabled the creation of compact, flexible, and lightweight energy harvesters suitable for integration into wearables, sensors, and implantable medical devices. Smart textiles, for example, can incorporate piezoelectric fibers or thermoelectric threads to generate power from motion or body heat, creating clothing that powers small electronic devices. Similarly, structural materials embedded with energy harvesting components can convert mechanical stress or vibrations into electricity, transforming ordinary surfaces into power-generating entities.

Energy harvesting is closely tied to the rise of the Internet of Things, where billions of interconnected devices require continuous power for sensing, computation, and communication. Traditional batteries present major limitations in such systems due to their finite lifespan, environmental impact, and maintenance requirements. Energy harvesting offers a sustainable alternative, enabling self-powered IoT networks capable of operating indefinitely with minimal human intervention. This not only enhances device reliability but also supports the development of smart cities, intelligent infrastructure, and remote monitoring systems that can function in hard-to-reach or hazardous environments.

Despite its promise, energy harvesting still faces several technological and economic challenges. Conversion efficiency remains a major hurdle, as many ambient energy sources provide only microwatts to milliwatts of power, which can be insufficient for high-demand applications. Moreover, the cost of materials and fabrication for certain types of harvesters, such as piezoelectric or thermoelectric devices, can be relatively high compared to conventional batteries. Reliability and durability are additional concerns, particularly for systems exposed to harsh environmental conditions or mechanical wear. Continued research is focused on developing new materials, improving transducer efficiency, and integrating advanced power management systems that can maximize the utility of harvested energy.

Energy harvesting represents a transformative step toward sustainable and autonomous power generation. By converting readily available environmental energy into electricity, it minimizes dependence on traditional power supplies, reduces waste, and opens new possibilities for wireless and self-sustaining technologies. Its applications span across industries—from wearable electronics and biomedical devices to industrial sensors and smart infrastructure—illustrating its broad impact on modern society. Although challenges in efficiency, cost, and scalability persist, ongoing advancements in materials science, electronics, and energy storage are rapidly improving the practicality of these systems. As the world continues to seek cleaner, renewable, and decentralized energy solutions, energy harvesting stands at the forefront of innovation, embodying the vision of a future where devices are seamlessly powered by the environment itself.

Rome et al. (2005) [1] researched electricity generation with loads. Elvin et al. (2006) [2] discussed structural monitoring with sensors that are powered by vibrations. Stephen (2006) [3] investigated energy harvesting from ambient vibration. Donelan (2008) [4] researched generating electricity during walking minimizing user effort. Mitcheson et al. (2008) [5] conducted research on energy harvesting from human and machine motion for wireless electronic devices. Scruggs et al. (2009) [6], investigated issues about harvesting ocean wave energy. Erturk et al. (2009) [7] developed a piezomagnetoelastic structure for vibration energy harvesting. Wang et al. (2010) [8] discussed piezoelectric energy harvesting from flow-induced vibration. Cammarano et al. (2010) [9] tuned a resonant energy harvester with an electrical load. Mann et al. (2010) [10] performed investigations of a nonlinear energy harvester with a potential well. Stanton et al. (2010) [11], investigated the dynamics of broadband energy harvesting involving a piezoelectric inertial generator. Litak et al. (2010) [12] discussed the magnetopiezoelastic energy harvesting using excitations. Masana et al. (2011) [13] discussed the relative performance of a vibratory energy harvester in bi-stable potentials. Galchev et al. (2011) [14] researched harvesting vibrations for structural monitoring of bridges. Erturk and Inman (2011) [15] investigated broadband piezoelectric power generation on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling. Karami et al. (2012) [16] showed how to power pacemakers from heartbeat vibrations using linear and nonlinear energy harvesters. Daqaq et al. (2012) [17], worked on using stiffness nonlinearities for energy harvesting under white Gaussian excitations. Szarka et al. (2012) [18] performed a review of power conditioning for kinetic energy harvesting systems. Dicken et al. (2012) [19], developed circuits for energy harvesters in min low-power applications. Halvorsen (2013) [20] discussed some issues in nonlinear wideband-vibration energy harvesting. Zhao and Erturk (2013) [21] worked on the excitation of electroelastic power generators and demonstrated advantages and tradeoffs in a physical system. Green et al. (2013) [22] provided an in-depth evaluation of current nonlinear energy harvesting solutions. Hosseinloo et al. (2014) [23] demonstrated the existence of fundamental limits to nonlinear energy harvesting. Daqaq et al. (2014) [24] discussed the role of nonlinearities in vibratory energy harvesting. Dagdeviren et al. (2014) [25] researched energy harvesting and storage from heart, lung, and diaphragm motions. Hosseinloo et al. (2015) [26] investigated energy harvesting via an adaptive bistable potential. Hosseinloo et al. (2015) [27] performed optimal control calculations for efficient energy harvesting from ambient vibration.

In this work, bifurcation analysis and multiobjective nonlinear model predictive control calculations are done for an energy harvesting model (Hosseinloo et al. (2015) [27]). The model equations are presented, followed by the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results, discussion and conclusions are then presented.

2. Model Equations

In this model, xv_1 is the non-dimensional displacement of the structure, xv_2 is the derivative of xv_1 with time, xv_3 is the non-dimensional displacement of the harvester relative to the structure, xv_4 is the derivative of xv_3 with time, and xv_5 is the non-dimensional electrical voltage. ue is the non-dimensional electrical current, um is the non-dimensional control force, ξt is the exogenous excitation acting on the structure, ξh is the harvester modal damping ratio, k^2 represents the non-dimensional electromechanical coupling, ξs is the structure modal damping ratio, and λ is the ratio of the natural frequency of the structure to that of the harvester. The ratio between the mechanical and electrical time constants is α. The equations representing the model are

$$\begin{aligned}
\frac{d(xv_1)}{dt} &= xv_2 \\
\frac{d(xv_2)}{dt} &= \xi t - 2\xi s\lambda xv_2 - \lambda^2 xv_1 \\
\frac{d(xv_3)}{dt} &= xv_4 \\
\frac{d(xv_4)}{dt} &= um - \xi t + 2\xi s\lambda xv_2 + \lambda^2 xv_1 \ - 2\xi h xv_4 - xv_3 - k^2 xv_5 \\
\frac{d(xv_5)}{dt} &= ue + xv_4 - \alpha xv_5
\end{aligned} \tag{1}$$

The base parameter values are $\lambda = 5$; $\xi s = 0.01$; $\xi h = 0.01$; $k = 0.6$; $\alpha = 10$; $\xi t = 10^{-6}$; $um = 1$; $ue = 1$.

The exogenous excitation acting on the structure ξt causes the variables xv_2 and xv_4 (which are the derivatives of xv_1 and xv_3); to increase and decrease with time and this causes the nonlinearity in the model. All the equations and parameter values were taken from Hosseinloo et al. (2015) [27].

3. Bifurcation Analysis

The MATLAB software MATCONT [28,29] that detects Limit points (LP), branch points (BP), and Hopf bifurcation points (H) is used for the bifurcation analysis. For a set of

$$\frac{dx}{dt} = f(x, \alpha) \tag{2}$$

$x \in R^n$ with the bifurcation parameter α, the gradient is orthogonal to the tangent vector and the tangent plane at any point $w = [w_1, w_2, w_3, w_4, \ldots. w_{n+1}]$ satisfies

$$Aw = 0 \tag{3}$$

where

$$A = [\partial f/\partial x \quad |\partial f/\partial \alpha] \tag{4}$$

$\partial f/\partial x$ is the Jacobian matrix. At the limit and branch points, the Jacobian matrix $J = [\partial f/\partial x]$ is singular.

There is only one tangent at the limit point singularity. At this singular point, there is a non-zero vector, *y*, where $Jy = 0$. This vector is of dimension n. Since there is only one tangent the vector $y = (y_1, y_2, y_3, y_4, \ldots y_n)$ must align with $\hat{w} = (w_1, w_2, w_3, w_4, \ldots w_n)$. Since

$$J\hat{w} = Aw = 0 \tag{5}$$

This implies that the $n + 1$ th component of the tangent vector $w_{n+1} = 0$.

For a branch point, there are two tangents at the singularity. Let the two tangents be *z* and *w*. This implies that

$$\begin{gathered} Az = 0 \\ Aw = 0 \end{gathered} \tag{6}$$

Consider a vector *v* that is orthogonal to the tangent (*w*). *v* can be expressed as a linear combination of z and w $(v = \alpha z + \beta w)$. Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal, $w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, for a branch point (BP) the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular.

At a Hopf bifurcation point,

$$\det(\, 2f_x(x, \alpha) @ I_n) = 0 \tag{7}$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in [30–32].

Hopf bifurcations cause limit cycles. Limit cycles cause equipment damage and make control tasks difficult. Additionally, they result in less beneficial products. The tanh activation function (where a control value *u* is replaced by) $(u\, tanh\, u\, /\varepsilon)$ is used to eliminate spikes in the optimal control profiles [33–36]. Sridhar (2024) [37] demonstrated that the tanh activation factor eliminates the limit-cycle causing Hopf bifurcation points. This was because the tanh function increases the oscillation time period in the limit cycle.

Multiobjective Nonlinear Model Predictive Control (MNLMPC)

The multiobjective nonlinear model predictive control (MNLMPC) method originally developed by Flores Tlacuahuaz et al. (2012) [38] was used. For a problem where $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ (j = 1, 2…n) have to be optimized simultaneously subject to

$$\frac{dx}{dt} = F(x,u) \tag{8}$$

t_f being the final time value, and n the total number of objective variables and u the control parameter. the single objective optimal control problem is solved optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$leading to the values q_j^*. Then, we solve the multiobjective optimal control (MOOC) problem

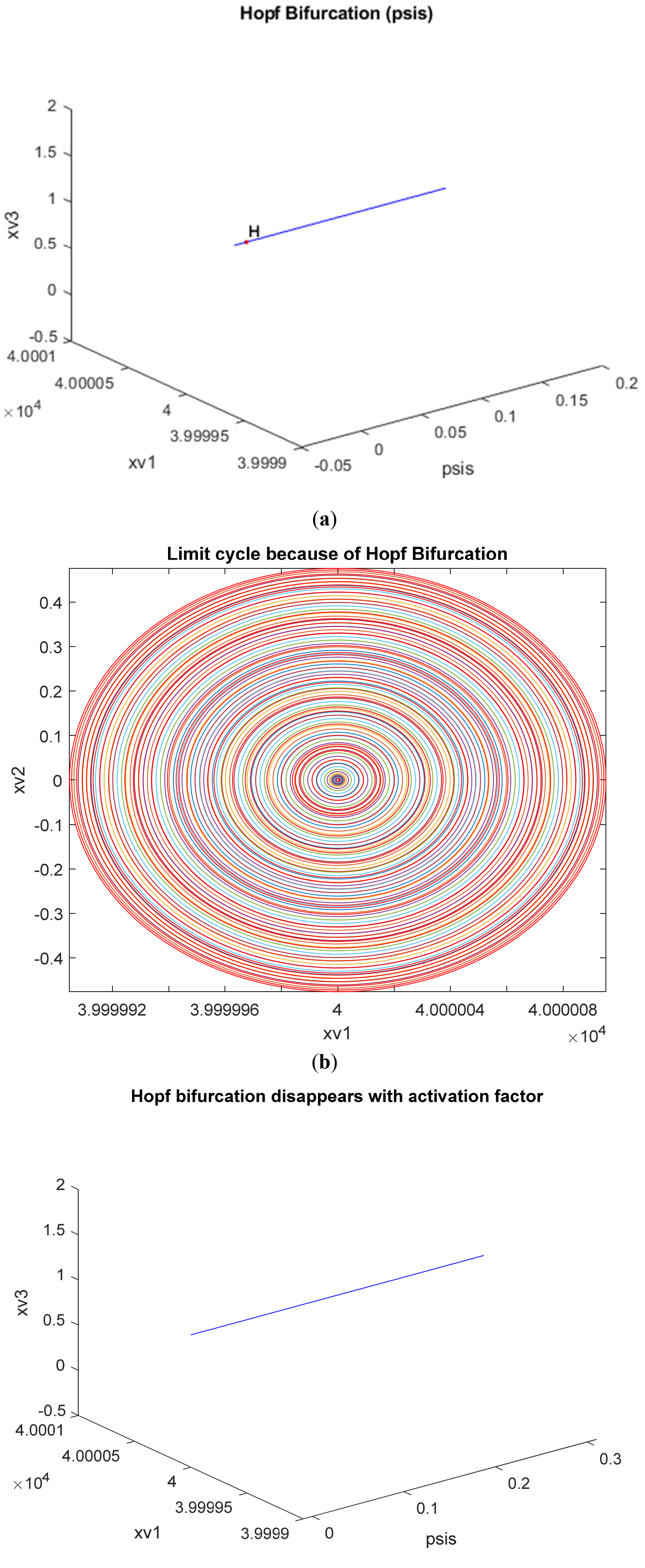
$$\begin{aligned} &min(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2 \\ &subject\ to\ \ \frac{dx}{dt} = F(x,u); \end{aligned} \tag{9}$$

This will provide the control variable values (u) at different times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained. Pyomo [39] is used for these calculations in conjunction with IPOPT [40] and confirmed as a global solution with BARON [41]. This multiobjective nonlinear model predictive control (MNLMPC) is very rigorous because it does not involve weighting factors or additional constraints.

3. Results and Discussion

When ξs is the bifurcation parameter a Hopf bifurcation point was found at ($xv_1, xv_2, xv_3, xv_4, xv_5,\ \xi s$) values of (40,000, 0, 0.964, 0, 0.1, 0). (Figure 1a). The limit cycle caused by this Hopf Bifurcation point is seen in Figure 1b. The limit cycle caused by this Hopf Bifurcation is shown in Figure 1b. When ξs is modified to $\xi_s anh(\xi_s)/10^8$ the Hopf Bifurcation disappears (Figure 1c). The use of an activation factor involving the tanh function removes the unwanted limit cycle, causing Hopf bifurcations, which validates the hypothesis of Sridhar (2024) [37]. The Hopf bifurcation is supercritical. There is no significant reduction in the resulting voltage as a result of this activation factor.

For the MNLMPC calculations, *ue* and *um* are the control parameters, and $\sum_{t_{i=0}}^{t_i=t_f} xv5(t_i)$ was maximized, leading to a value of 3.43198. $\sum_{t_{i=0}}^{t_i=t_f} ue(t_i), \sum_{t_{i=0}}^{t_i=t_f} um(t_i)$ were minimized individually, and each of them led to a value of 0. The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} xv5(t_i) - 3.43198)^2 + (\sum_{t_{i=0}}^{t_i=t_f} ue(t_i) - 0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} um(t_i) - 0)^2$ subject to the equations governing the model. This led to a value of 1.0488. The MNLMPC values of the control variables *ue* and *um* were 0.99 9 and 7.8×10^{-7}. The MNLMPC profiles are shown in Figure 2a–e.

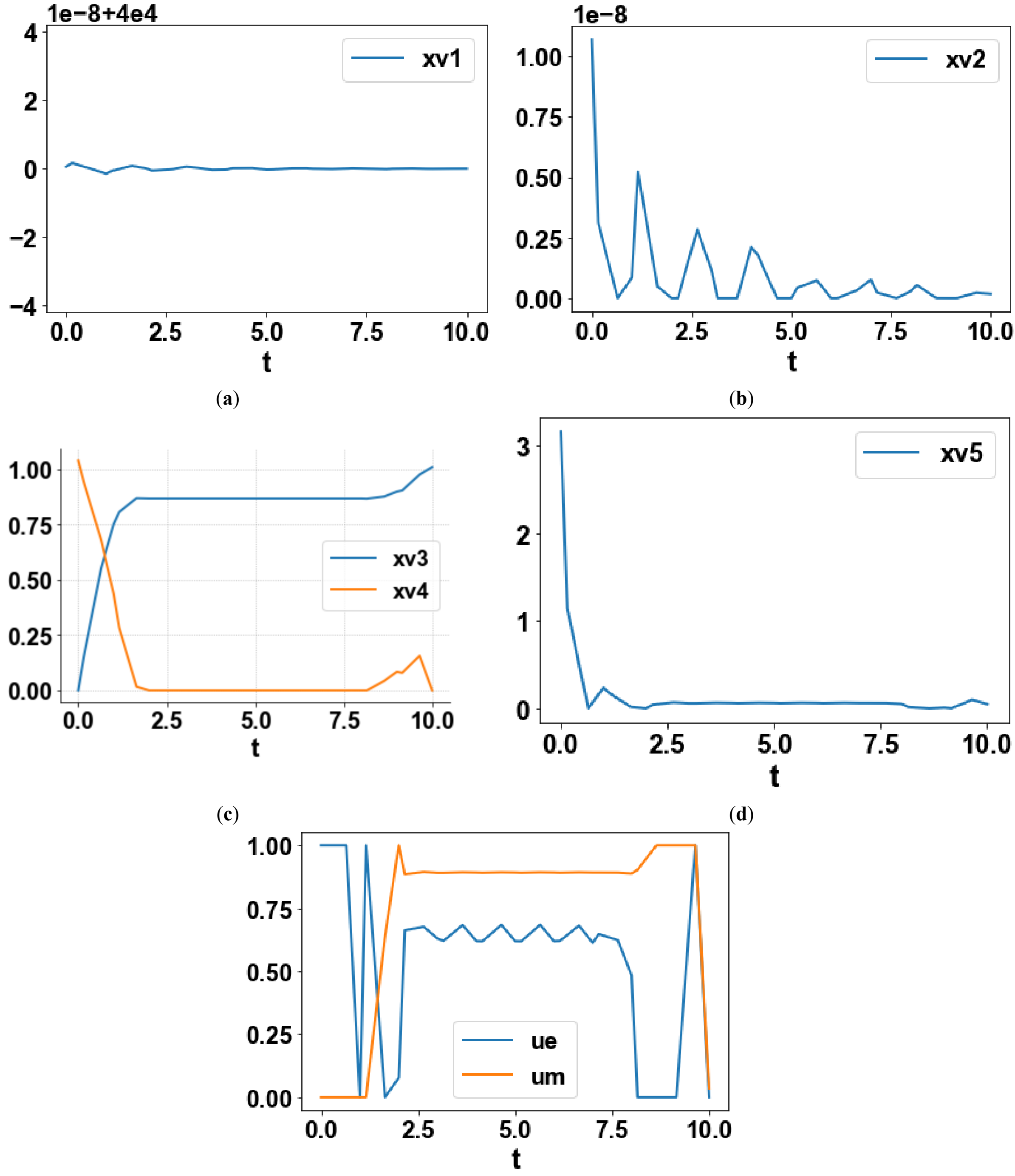


(**a**)

(**b**)

(**c**)

Figure 1. (**a**) Hopf Bifurcation when ξs is the bifurcation parameter; (**b**) Limit Cycle caused by Hopf Bifurcation. (**c**) Hopf Bifurcation disappears when ξs is modified to $\xi_s anh(\xi_s)/10^8$.



(**a**) (**b**) (**c**) (**d**) (**e**)

Figure 2. (**a**) MNLMPC xv_1 vs. t; (**b**) MNLMPC xv_2 vs. t; (**c**) MNLMPC xv_3 xv_4 vs. t; (**d**) MNLMPC xv_5 vs. t; (**e**) MNLMPC *um*, *ugn* vs. t.

4. Conclusions

Bifurcation analysis and multi-objective nonlinear control (MNLMPC) studies are conducted on an energy-harvesting model. The bifurcation analysis revealed the existence of a Hopf bifurcation point The Hopf bifurcation point, which causes an unwanted limit cycle, is eliminated using an activation factor involving the tanh function. This tanh function is normally applied to optimal control problems, and this work is the first attempt to incorporate this tanh function in bifurcation analysis, and that is the advantage linkage between bifurcation analysis and MNLMPC. The MNLMPC calculations resulted in optimal control profiles. This should encourage engineers to incorporate this activation factor into the energy-harvesting process. The most important takeaway from this research is that an activation factor must always be used to prevent limit cycles in energy harvesting. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for an energy harvesting model is the main contribution of this paper.

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Data Availability Statement

All data used is presented in the paper

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Conflicts of Interest

The author declares no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were used in this paper.

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