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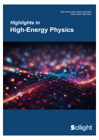

Article

Quantum Indiscernibility and Perspectivalism

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Abstract: It has often been suggested that similar quantum particles may be a counterexample to the principle of the Identity of Indiscernibles. In this article, I discuss the status of indiscernibility in a perspectival approach to quantum mechanics. I argue that adopting an internal physical perspective can have the effect of making entities discernible even if they are not discernible relative to external reference frames, and thus perspectivalism offers a new way to understand the physical meaning of weak discernibility. I discuss whether this approach is circular, and I consider whether the reference frames involved are physically meaningful. I conclude that the Identify of Indiscernibles can probably be maintained in this context, though there are outstanding questions pertaining to the meaning of the quantum state and the way in which a system should be represented in its own reference frame.

Keywords: indiscernibility; quantum mechanics; quantum reference frames

1. Introduction

In the context of quantum mechanics, ‘similar’ particles are indistinguishable from one another in a particularly strong and philosophically interesting way. This feature has given rise to much debate around issues of identity and discernibility. In particular, questions have been raised about whether similar quantum particles may represent a counterexample to Leibniz’ principle of the Identity of Indiscernibles.

Evidently the resolution of this question may depend to some extent on our chosen interpretation of quantum mechanics—for example, van Fraassen [1] argues that our answer will depend on whether or not we countenance hidden variables. In this article, I will consider the impact of a different kind of interpretative move. Recently there has been increasing interest in thinking about quantum mechanics as a relational or perspectival theory [2–5]. In order to be concrete, in this article, I will focus on a moderate form of relationalism that I refer to as moderate physical perspectivalism [6], which maintains that empirically meaningful descriptions must always be expressed relative to a perspective. Here, a perspective is associated with a physical system relative to which we may offer a description. I will argue that perspectivalism has a significant impact on issues surrounding discernibility, because physically meaningful claims about discernibility now have to be relativized to a perspective.

We will see that adopting a physical perspective can have the effect of making entities discernible even if they are not discernible relative to some other reference frame. In particular, in the case of a set of similar quantum particles, adopting the reference frame of one particle from that set often appears to render these particles discernible. We will ultimately see that the way in which this should be interpreted, and in particular what it means for the distinction between symmetric and antisymmetric similar particles, may depend quite sensitively on how we think about the issue of whether or not a system can be assigned a quantum state in its own reference frame.

I will begin in Section 2 by giving more details on moderate physical perspectivalism. Then in Section 3 I will discuss the debate over the discernibility of similar particles. In Section 4 I will introduce the quantum reference frame formalism and consider what it has to say about collections of similar particles. I will also consider how this approach to discernibility relates to the ‘heterodox’ approach which has been developed in recent literature on the subject. Finally in Section 5 I will discuss what these results mean for the Identity of Indiscernibles and the discernibility question.

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2. Relationalism and Reference Frames

The idea that quantum mechanics has some relational or perspectival aspects has a long history, dating back to Bohr's Copenhagen approach [7] and to the work of Everett and his notion of a 'relative state' [8]. One well-known relational view is Rovelli's Relational Quantum Mechanics or RQM [5]; similar perspectival ideas appear in Brukner's neo-Copenhagenism [2], as well as in Healey's pragmatist approach [4] and QBism [9].

An important reason for taking relational or perspectival approaches to quantum mechanics seriously is the fact that relational observables appear to play a particularly important role in General Relativity; it has been suggested that such relational and perspectival features may be the common thread between the two theories which will ultimately allow us to fully unify them [10]. For example, since we use 'reference frames' to codify relational descriptions in the context of relativity, there has recently been significant interest in developing a quantum reference frame (QRF) formalism for quantum mechanics [11–16].

For definiteness, in this article I will focus on a moderate form of physical perspectivalism which maintains that empirically meaningful descriptions must always be relativized to some physical system [6]. This system plays the role of a reference frame relative to which we can describe physical properties. Here I am using the term 'empirically meaningful' to refer to a description of some part of reality in terms of phenomena of the kind that could be directly experienced by a realistic observer. Note that much of the discussion in this article would probably still apply if we were to adopt a stronger form of perspectivalism, such as the view that all physical facts are relativized to a physical system, so the conclusions here are likely to be relevant beyond the specific context of moderate physical perspectivalism; but I will focus on moderate physical perspectivalism here because this weaker view is already adequate to draw some interesting conclusions about the connections between discernibility and perspectivalism.

I emphasize that moderate physical perspectivalism is a *metaphysical* position, not an epistemic one. For example, the point of the claim that 'spin' must be relativized to a physical system is not to say that observers like us must observe spin from some standpoint, but rather that 'spin' is simply not a meaningful notion in isolation. The category 'empirically meaningful' is of course somewhat anthropocentric since it is about the things that observers like us can observe, but it is used here to demarcate a category of entities and properties and to make a claim about their metaphysical nature, not to say anything about our knowledge of them.

The word 'moderate' is used here to express a contrast with stronger forms of perspectivalism which maintain that all physical facts must be relativized to a physical system. Moderate physical perspectivalism is more 'moderate' than this, since it allows that there can be some non-relative physical facts as long as they do not express direct empirical content. But it should be noted that 'moderate physical perspectivalism' might still look quite radical, because it suggests that a number of features of the world that we intuitively see as absolute should in fact be understood as relational—for example, in special relativity facts about simultaneity turn out to depend on a choice of reference frame, and in this article we will see that the quantum reference frame formalism reveals an even more dramatic degree of relativity.

With regard to discernibility, it seems natural to say that facts about discernibility are empirically meaningful—to discern two entities is to carry out an empirical procedure which produces an empirically observable result. Moreover, a striking result within the quantum reference frame formalism is that even the way in which we decompose systems into subsystems is dependent on a reference frame: tensor product decompositions are not invariant under quantum reference frame transformations, so degrees of freedom may be shifted from one subsystem to another under such transformations [17]. These results suggest that even facts about individuality and objecthood should be understood as the kind of 'empirically meaningful' facts which must be understood as perspectival in nature, and if so then it clearly would not make sense to expect facts about discernibility to hold in a perspective-independent manner. So moderate physical perspectivalism suggests that questions about identity and discernibility can only meaningfully be posed relative to a reference frame, and in this article I will try to understand what this tells us about ongoing discussions regarding the discernibility or otherwise of similar quantum particles.

Before moving on, a note about the relation of this project to quantum field theory. Given that elementary non-relativistic quantum mechanics has been superseded by quantum field theory, one might wonder why we should worry about discernibility in elementary quantum mechanics—why not just defer all such issues to QFT?

Caulton [18] offers two reasons. First, non-relativistic elementary quantum mechanics is still useful in many physical applications, so we should be able to interpret it in its own domain without going beyond it to a successor theory. Second, conclusions drawn in the non-relativistic regime may help us better understand QFT. To these, I would add that these questions about discernibility are a good starting point to start to understand the philosophical consequences of physical perspectivalism and the reference frame formalism. In particular, the notion of discernibility relative to reference frames is clearly related to the notion of objecthood relative to reference frames, so it is important to address these questions if we hope to understand the sense in which the familiar physical world which appears to us to be full of distinct individuated objects can emerge relative to some reference frame.

Moreover, we probably have little choice but to begin this endeavour with non-relativistic elementary quantum mechanics, because it is not obvious how to apply something like the QRF formalism in the context of QFT, given that the formalism seems to require well-defined individual systems which can be used as reference frames. Indeed, stable reference frames may only emerge in certain regimes, so insofar as perspectivalists are committed to saying that a concept such as discernibility must be relativized to a reference frame, they are possibly also committed to saying that such a concept is only meaningful in the regimes in which reference frames exist. Thus from the perspectivalist point of view, studying discernibility in the regime of elementary quantum mechanics makes sense, because it may well turn out that questions about discernibility cannot even be posed in other regimes of QFT.

It should be acknowledged that the QRF formalism is still under active development and that there are significant ongoing discussions around its interpretation and conceptual significance – I will discuss some of these issues in greater detail in Section 4.1. However, in this article I will not present arguments for the correctness of the formalism, and nor will I seek to solve its conceptual problems. The conclusions of this article are therefore conditional on the assumption that the QRF formalism is ultimately coherent and that it is a meaningful expression of the perspectivalist viewpoint; I think this is a reasonable project to pursue because it seems to me that the QRF research program has now reached a level of sophistication and seriousness that makes it deserving of philosophical consideration, even while admitting that not all of the questions have been answered yet. Moreover, I would argue that the project of applying the formalism to open questions in the foundations of quantum mechanics is a useful way to get a better handle on its conceptual implications, which in turn may help us think more clearly about the ongoing discussions internal to the formalism.

3. Discernibility

What does it mean to say that two physical objects are distinct? One common answer to this question is based on a principle due to Leibniz known as the Identity of Indiscernibles (IOI) [19,20] which posits that no two distinct entities can have all of their properties in common (The term 'Leibniz's Principle' has sometimes been used in quantum foundations to refer to the idea that procedures which lead to the same operational predictions should be represented in the same way at the ontic level [21]. But unlike the Identity of Indiscernibles, this is an epistemic principle rather than a metaphysical one, so the issues at stake are quite different. I will not discuss this version of the principle in this article).

In order to apply this principle it is necessary to clarify what counts as a 'property'. In particular, most people writing on the subject accept that the term 'properties' must include relational properties, since intuitively it seems that objects with the same intrinsic properties can nonetheless be distinguished by their relational properties [22]. With such clarifications, IOI seems quite compelling. In particular, as emphasized by Dieks [23], it satisfies '*the empiricist desire to ground individuality in empirically accessible qualitative features*'.

A famous challenge to IOI was posed by Black [24], who imagined a world containing nothing but a pair of spheres made of pure iron, each exactly a mile in radius. The spacetime in this world is purely relational, so there are no absolute positions which can distinguish the spheres. Thus these spheres have all the same intrinsic and relational properties, and therefore IOI says they cannot possibly be distinct—and yet a world containing two such spheres seems at least conceptually possible, so this looks like a counterexample to IOI. Note that I will return to this example in Section 3.2, arguing that it can be resolved by clarifying the perspective from which the description of the spheres is stated.

It has also been argued that quantum mechanics poses challenges for IOI. In particular, the symmetrization postulate [25] tells us that the state of a set of quantum particles of the same type must always be completely symmetric (if the particles are 'bosons') or antisymmetric (if the particles are 'fermions'). This means that if we obtain the reduced density operator of one of the particles in the set by tracing out the states of all of the other particles, then we will always find the same reduced density operator for any one of the particles. As a consequence, the probability distribution over the outcomes for any possible single-particle measurement will be the same for all of the particles in the set. Thus it seems that all of these particles have exactly the same properties and stand in exactly the same relations: there is nothing that could possibly distinguish one of them from the others [26]. Following Saunders [27], I will refer to particles which exhibit this form of indistinguishability as 'similar.' Similar particles appear to be indiscernible, and yet we normally suppose that they are nonetheless distinct from each other, so they appear to constitute a counterexample to IOI.

There are three possible ways in which we could respond to this quandary. First, we could accept that IOI is wrong, or at least it does not apply in the context of quantum mechanics. Second, we could argue that similar quantum particles are not individuals and/or not distinct from each other, so they are not a counterexample to IOI since they are indiscernible and identical. Finally, we could argue that similar quantum particles are in fact discernible, so they are not a counterexample to IOI since they are discernible and distinct. In this article I will largely focus on this final response.

Progress can be made on these putative counterexamples to IOI by distinguishing between various different kinds of discernibility. This classification is relativized to a langauge, with the caveat that the language does not contain any individual names (since that would render all objects in the domain trivially discernible). Of course, as emphasized by Bigaj [28], in order to use these definitions to decide whether we have physically meaningful discernibility, we must '*accept additional assumptions regarding the interpretation of primitive predicates of our language*'. For now I will simply assume that the relevant predicates can be mapped to physically meaningful properties and relations in the way one would naturally expect, but in Section 3.2 we will look more closely at this assumption. Here I will employ a classification originally based on work by Quine [29,30], later expanded and clarified by Saunders [31]:

- **Absolute Discernibility:** X and Y are absolutely discernible iff there exists a predicate ϕ in the language such that $\phi(X) \neq \phi(Y)$
- **Relative Discernibility:** X and Y are relatively discernible iff there exists a relation $R(,)$ in the language such that $R(X,Y)$ and $\sim R(Y,X)$
- **Weak Discernibility:** X and Y are weakly discernible iff there exists a relation $R(,)$ in the language such that $R(X,Y)$ and $\sim R(X,X)$

In a relational context, it is natural to think that objects might also be discernible in virtue of standing in different relations to some object or objects. Thus, following Ladyman and Bigaj [22], we can add the following definition:

- **Relational Discernibility:** X and Y are relationally discernible iff there exists a n-argument relation R such that it is not the case that $R(...X...)$ is true of exactly the same objects as $R(...Y...)$.

Given this definition, relative discernibility is a special case of relational discernibility.

In fact, as shown by Ladyman and Bigaj [22], relational discernibility is equivalent to weak discernibility, since relational discernibility just requires that X and Y stand in different relations to some object or objects, and if they are weakly discernible then they necessarily stand in different relations to X itself. So if we want to preserve the notion of relational discernibility without making it equivalent to weak discernibility, we might consider a modification which requires that X and Y are differently related to some entity which is not identical to either of them:

- **External Relational Discernibility:** X and Y are relationally discernible iff there exists some object Z and some relation R such that $R(X,Z)$ and $\sim R(Y,Z)$ and Z is either absolutely or relatively discernible from both X and Y

With this classification in mind, what can we say about the putative counterexamples to the IOI described above?

First, in Black's case the spheres do not exhibit absolute, relative or external relational discernibility. But they *do* exhibit weak discernibility, since they stand in the irreflexive relation of being some distance apart, e.g., three miles apart. Thus as long as we admit weak discernibility as a kind of discernibility which is relevant to IOI, we can say that the spheres are both distinct and discernible.

Similarly, in the quantum case, Saunders [32], and Saunders and Muller [27] have argued that at least when the similar particles are fermions, there always exists an irreflexive relation between them which renders them weakly discernible. In the simplest case, where we have a single pair of fermions with a spin degree of freedom, we can think of the relation in question as simply the relation of 'having opposite spin.' Since a particle cannot have opposite spin to itself, this relation is irreflexive and thus weakly discerns the particles. Thus if we admit this kind of weak discernibility as a kind of discernibility which is relevant to IOI, we can say that fermions are both distinct and discernible.

This still leaves the question of how to think about bosons; in this case we cannot use the relation 'having opposite spin' to discern the particles, but it has been argued that nonetheless there are other physically reasonable relations which weakly discern bosons [27,33,34]. If we admit this kind of weak discernibility as a kind of discernibility which is relevant to IOI, we can say that bosons too are both distinct and discernible.

3.1. Quantum Discernibility

The use of weak discernibility to defend the IOI in the quantum context has been challenged in various different ways. Here I will focus on the challenge posed by Dieks [23], who object to the use of weak discernibility on the grounds that irreflexive relations are not always physically meaningful. For example, it is possible to define irreflexive relations between Euros in a bank account, even though individual Euros in a bank account are not discernible and are not individuals. Thus they argue that '*Such relations can only be trusted to be significant for the individuality issue if they are of the sort to connect actual relata*'. That is, we cannot accept just *any* irreflexive relations as conferring discernibility in any robust sense on their relata. We need some assurance that the irreflexive relations which weakly discern the particles are 'scientifically respectable' as defined by the theory in which they appear: '*Whether relations occurring in scientific theories connect and determine actual relata is decided by their meaning, reflected by the role they play within the theory in question*' [23].

How could such an assurance be given? Here there is some disagreement. Ref. [27] offer concrete criteria for physical meaningfulness, arguing that we should require that the relation 'should be transparently defined in terms of physical states and operators that correspond to physical magnitudes', by which they mean magnitudes which appear as eigenvalues associated with eigenstates of some operator, and that it should obey permutation invariance. They show that the relations they invoke in the demonstration of weak discernibility do indeed satisfy these criteria. However, others have argued that this kind of construction might not be adequate, since the notorious interpretational difficulties associated with quantum mechanics make it less than straightforward to argue that such magnitudes always correspond to occurrent properties of the corresponding systems [23]. Thus it has been suggested that we need some more concrete and operational way of showing the meaningfulness of a relation.

For example, one possibility suggested by Dieks [26] is that these irreflexive relations are scientifically respectable if they are the kind of relations which could, under other circumstances, be used to discern entities in a more straightforward way. So for example we might try to imagine an alternative situation which is less symmetric but which involves the same kinds of properties and relations; if we find that '*the breaking of the symmetry is physically possible, does not involve any change in the type of properties assigned, and results in a situation with distinguishable objects*' we have thereby demonstrated that this is the kind of relation which typically relates physically meaningful and discernible relata, and therefore it is reasonable to think the relata are discernible in a physically meaningful sense in the symmetric case as well.

For example, Dieks [26] notes that '*in an arbitrary configuration of classical particles the mutual distances will unambiguously characterize each individual particle. Changing the configuration so that it becomes more symmetrical will change the values of the distances, but not the actuality of the objects: the description of the situation furnished by classical theories will not change as far as the actuality of the particles is concerned, when we approach a Blackean spheres-type configuration*'. That is, because everyone agrees that relations of relative distance can be used to discern classical objects in asymmetric scenarios, we should allow that the irreflexive relation 'being three miles apart' can also discern objects in symmetric scenarios, so we can respond to Black's spheres counterexample by simply saying the spheres are both distinct and discernible.

Meanwhile, Ref. [26] argues that this method does not succeed in the case of quantum particles, because the symmetrization postulate [25] ensures that fermions and bosons can only exist in antisymmetric or symmetric states respectively, and thus we can't appeal to an alternative case with less symmetry since no such case is nomically possible. Therefore Ref. [26] contends that the kind of relations discussed by Saunders are not adequate to make the fermions discernible in a physically meaningful sense, so we can't respond to this counterexample by saying the fermions are both distinct and discernible.

However, the purpose of breaking the symmetry by moving to a counterfactual case was simply to show that the relations in question are 'scientifically respectable.' Thus we should be willing to consider other possible ways of breaking the symmetry which might similarly be used to establish the scientific meaningfulness of these relations: for example, we might instead break the symmetry by moving to an alternative reference frame. In particular, I will shortly argue that the fact that fermions are weakly discerned by a relation 'having opposite spin', means that a pair of similar fermions will have different spin relative to the reference frame defined by one of those fermions. Moreover, in the context of moderate physical perspectivalism, any form of discernibility ultimately amounts to having different properties relative to some reference frame. Thus I contend that in the context of moderate physical perspectivalism, 'having different spin relative to a reference frame' is clearly the kind of 'scientifically respectable' relation which can be used to discern entities. So in this context, the relation of 'having different properties relative to a reference frame', must be capable of discerning entities in the sense relevant to IOI.

It should be noted that there is an alternative approach to quantum discernibility based on quasisets [35,36], which involves adopting an alternative non-classical logic—in particular, a non-reflexive logic, where the classical theory of identity does not hold in full. This is argued to be a better way of thinking about collections of indiscernible objects. In this paper I am focusing on the weak discernibility approach, because as we will see in the next section there seems to be a clear connection between weak discernibility and perspectivalism, whereas it seems less evident that perspectivalism will shed new light on the quasi-set approach. In addition, if we conclude that in the perspectivalist setting quantum particles are discernible after all, it may not be necessary in that setting to appeal to an alternative logic which is suitable for indiscernible objects. However, it would certainly be interesting to explore in more detail how the quasiset approach might be altered by moving to a perspectivalist picture, and I hope to consider this matter in future work.

3.2. Discernibility and Reference Frames

Consider again Black's universe with two identical spheres [24]. When one tries to visualize or conceptualize Black's universe, it is natural to imagine looking at it from the point of view of an external observer. Thus we run into a puzzle: to the external observer, the two spheres seem clearly distinct, and yet there is nothing to distinguish them.

I suggest that the conceptual puzzle is down to the fact that we are implicitly adopting an external point of view on this hypothetical world whilst simultaneously denying the possibility of any external reference frame. To see this, note that Black explicitly specifies that there is no absolute spacetime in this world; it is a purely relational world. And one natural way to cash out this claim is to adopt some form of moderate physical perspectivalism, meaning that descriptions with direct empirical content must always be relativized to a reference frame attached to an object in that world. In Black's world, that means such a description must be attached to one of the spheres, since there is nothing else it could be attached to.

Yet the characterization of 'a world containing two identical spheres' is apparently given from the outside: this is the world as an external observer would see it. From a perspectivalist point of view, such an external description is not really meaningful—the only possible external description of this kind of world would be a perspective-neutral description which would not have direct empirical content, meaning that we shouldn't be trying to visualize it or use it to judge empirical matters like discernibility. Moderate physical perspectivalism suggests that the only description of Black's world which is legitimate for these purposes would involve saying something like 'You are a sphere, and there is another sphere three miles away from you'—and if you are given that description there is surely no doubt that the two spheres are both discernible and distinct.

So intuitively we can see that the possibility of adopting an internal reference frame should be relevant to the issue of discernibility. Similarly, imagine that I am in a perfectly symmetric universe, so I am not absolutely, relatively or relationally discernible from my twin on the other side of the universe. I am weakly discernible from my twin in virtue of being some distance away from her, but is that enough to ground the claim that we are distinct? Here is one way to argue that it is: suppose that I approach the universe's axis of symmetry and see my twin approaching me from the other side. From a third-person point of view, there is nothing that distinguishes me from my twin, but nonetheless I can discern myself from my twin: one of them is me and the other is not. So relative to my internal frame of reference there is no doubt about the discernibility of the twins, even though externally they cannot be distinguished.

This suggests that in the context of a perspectival view, the various definitions of discernibility given above should be rewritten to refer explicitly to reference frames. In particular, in Section 3 we provisionally assumed that predicates in a language can be mapped directly to physically meaningful properties and relations. But this assumption becomes problematic in the context of a perspectival view, because one important function of a 'reference frame' is that it turns a relation into a property. For example, right now my cat stands in the relation to me of being one metre away from me on my right-hand side. If we now adopt a reference frame centered on my body and with axes defined by my orientation in space, we can assign my cat coordinates $(1, 0, 0)$, and these coordinates can be thought of as defining something like an 'absolute position' relative to this fixed reference frame. So adopting this reference frame has (at least superficially) turned the relation of relative distance into a property of absolute position. Thus we should not necessarily expect that the actual physical relation of relative distance can be mapped to one single predicate in some language, since the structure I have just described indicates that it would require either a one-place or a two-place predicate depending on whether or not we are using a reference frame and which reference frame we are using. Therefore in the remainder of this article I will focus on giving definitions not in terms of predicates of a language but directly in terms of objects and properties.

As a first pass, the following definitions seem sensible, where we aim to define discernibility for two physical subsystems X and Y, which will be identified by means of some labelling relative to a given reference frame but which we may also be able to track into other reference frames using reference frame transformations. Note though, that reference frame transformations can change subsystem decompositions, so we should not expect that we can sensibly track X and Y into *all* frames of reference, and thus the definitions below can be applied only in cases where X and Y still exist as distinct subsystems with respect to the relevant reference frames.

- Perspectival absolute discernibility: X and Y have different intrinsic properties, and these properties exist without relativization to any reference frame.

- Perspectival relative discernibility: there exists some property P such that X has property P relative to the reference frame of Y, but Y does not have property P relative to the reference frame of X.
- Perspectival relational discernibility: there exists some object Z, and relative to the reference frame defined by Z, X and Y have different properties.
- Perspectival external relational discernibility: there exists some object Z which is absolutely or relatively discernible from X and Y, and relative to the reference frame defined by Z, X and Y have different properties.
- Perspectival weak discernibility: relative to the reference frame defined by X and/or Y, X and Y have different properties.

Note that just as relative discernibility is a special case of relational discernibility, so perspectival relative discernibility is a special case of perspectival relational discernibility.

However, we have seen already that in a fully fleshed-out version of moderate physical perspectivalism it is plausible that no physical object will have any intrinsic categorical properties: all empirically meaningful properties, and indeed objecthood itself, must be relativized to a reference frame. So in that context it is likely that nothing at all, or at least no physical object, can exhibit perspectival absolute discernibility as defined above. This suggests that either the notion of absolute discernibility for physical objects is empty in the context of physical perspectivalism, or perspectival absolute discernibility should in fact be relativized to a reference frame.

But if perspectival absolute discernibility is relativized to a reference frame, then it just amounts to the statement that there exists some object Z, and relative to the reference frame defined by Z, X and Y have different properties—that is, it becomes equivalent to perspectival relational discernibility. Moreover, we have already noted that relational discernibility is equivalent to weak discernibility, so it looks like absolute discernibility collapses into weak discernibility in the context of moderate physical perspectivalism. In that case, we had better accept perspectival weak discernibility as a criterion for discernibility in the sense relevant to IOI if we want to say that there exists any kind of physically meaningful discernibility at all!

Now, one approach one might adopt to try to block this conclusion would involve insisting that the correct analogue of absolute discernibility for a perspectival context is perspectival external relational discernibility, rather than simply perspectival relational discernibility, and in that case the further equivalence to perspectival weak discernibility does not obtain (Note that if we make this move we presumably cannot appeal to absolute discernibility in the definition of external relational discernibility, on pain of vicious circularity; so in this case we would have to appeal to appeal to only relative and not absolute discernibility in the definition of external relational discernibility).

But even if we make this move, it is clear that perspectival absolute discernibility and perspectival weak discernibility will still be very closely related, so really the only reasonable way to block the use of perspectival weak discernibility to affirm IOI would be to argue that there is some specific issue preventing us from using the reference frame of X itself when establishing the discernibility of X and Y. Yet at least in ordinary cases it seems perfectly reasonable to use the reference frame of X for this purpose: for example, when I perceive the world around me I am in effect representing it to myself relative to my own reference frame, and I consider that representation perfectly adequate to establish that I am distinct from my cat. But let us now look in more detail at the case of similar particles, to see if there is anything special about this case which would justify rejecting the use of a particle's own reference frame to establish discernibility.

4. The QRF Formalism

In what follows, I will use the QRF formalism [11,14,16,37] to characterize the kinds of relational descriptions we might use to arrive at claims about various kinds of perspectival discernibility. The QRF formalism provides a set of reference frame transformations which can be used to switch from the reference frame of one system to another, even in cases where systems may be in quantum superpositions relative to one another. As we will shortly see, formally speaking this notion of 'switching into a reference frame of a system', simply amounts to using the system in question as the origin of a coordinate system of some kind. But the process is sometimes described more suggestively as yielding something like the 'perspective' of the system in question. There are still conceptual questions to be resolved here—in particular, one might have questions about what a term like 'perspective' really means in the case where the system is not a conscious observer, and indeed one might perhaps think that generic reference frames really represent something more like a proto-perspective which only becomes a true perspective in an appropriate limit. But nonetheless, evidently within any form of physical perspectivalism it is clear that perspectival facts about the world relative to a reference system must ultimately emerge in some way or other, and thus for the sake of concreteness I will assume throughout this article the correct way of doing this involves something like the QRF notion of switching into a reference frame.

In the QRF formalism, any physical system can in principle act as a reference frame. For example, if we are interested in reference frames for position, we simply choose our reference system and then we take advantage of the fact that the overall state should be invariant under global translations and rotations; this freedom allows us to rewrite our models such that '*the position and momentum of the reference frame are not dynamical variables when considered from the reference frame itself ... the reference frame is not a degree of freedom in its own description, but external systems to it are*' [14]. This can be thought of as a generalization of the standard classical procedure in which we use the location of a physical system as the origin of a classical coordinate system. The procedure can also be extended to variables other than position and momentum—we can construct reference frames for any variable V as long as there exists a symmetry group G such that transformations in G typically change the value of V but the overall state is invariant under transformations in G [38]. The existence of this symmetry group is what ensures the existence of nicely behaved transformations between reference frames, because these transformations just turn out to be symmetry transformations [37].

Note that in the classical case, there are certain limitations on a good choice of of reference system—in general it works better to define a classical coordinate system using some heavy, extended body. And we will see that exactly the same is true in the quantum case: although any system can in principle furnish some kind of reference frame, for small or limited reference systems the resulting description may be unstable or contain little information. A system may also be a better reference frame for some variables than others—for example de la Hammete et al. [38] argue that transformations between reference systems will be well-behaved only if the systems carry the left and right regular representations of the group, so systems may be better behaved for some groups than others.

The main purpose of the QRF formalism is to define the transformations between reference frames. To understand where the transformations come from, consider the following simple example. Suppose that we have systems A, B, and C and we begin from a two-dimensional coordinate system defined relative to A, i.e., A is by stipulation at the origin. Let us stipulate that in this coordinate system B is at position $(1, 1)$. Now, first consider a case where C is at position $(2, 0)$; it can be seen that if we switch to a coordinate system with C at the origin while retaining the original orientation of the axes, then the position of B in this new coordinate system must be $(-1, 1)$. Then consider a case where C is at position $(3, 0)$; then if we switch to a coordinate system with C at the origin while retaining the original orientation of the axes, the position of B in this new coordinate system must be $(-2, 1)$.

Finally consider a case where C is originally in an equal superposition of these two positions relative to A, i.e., the position of C is described by a quantum state of the form $\frac{1}{\sqrt{2}}(|2, 0\rangle + |3, 0\rangle)$. As argued by Giacomini et al. [14], a 'natural procedure is to make use of the linearity of quantum mechanics and 'coherently translate' the state of B relative to the position of A.' Mathematically, we can achieve this by using a 'quantized' version of the usual generator of spatial translations, which will look something like this: $U = e^{\frac{i}{\hbar}\vec{x}_C\vec{p}_B}$, where $\vec{x}_C$ is the position operator of the system C relative to A and $\vec{p}_B$ is the momentum operator of the system B relative to A. The effect is as one might expect based on the assumption of linearity: the final position of B relative to C will be $\frac{1}{\sqrt{2}}(|-1, 1\rangle + |-2, 1\rangle)$, a superposition of two positions of B relative to C corresponding to the two possible positions of C relative to B.

Obviously some extrapolations are being made here, particularly with regard to the assumption of linearity and the coherence of the transformation, but we may be encouraged by the fact that intuitively this seems like the right result: if B has a well-defined position relative to A and meanwhile C has an indeterminate position relative to A, then it seems natural to think there should be some kind of transitivity principle such that C also has an indeterminate position relative to B. Moreover, saying that C has an indeterminate position relative to B is equivalent to saying that their relative displacement is indeterminate, which suggests that this relation ought to be reflexive, and so it is natural to expect that in this case B will have an indeterminate position relative to C; this is exactly what the transformation described above achieves. Thus we can see that at least in simple cases, the QRF approach to deriving reference frame transformations matches what we might intuitively expect.

Note also that this example the reference frames are related in a relatively straightforward way, so the transformations described above allow us to track the 'same' entity across different reference frames—that is, the transformations directly connect subsystems in one reference frame to subsystems in another, so we can sensibly assign the same label to both subsystem and thus ask if an entity initially labelled in one subsystem might be discernible relative to another subsystem. However, it should be kept in mind that more complex reference frame transformations can change the tensor product decompositions so we should not necessarily expect that a given subsystem can meaningfully be tracked across all possible reference frames.

As set out by Vanrietvelde et al. [16], there is also a more 'first-principles' way of deriving these transformations, following closely the method of canonical quantization as in quantum gravity. Here we again consider a theory which is subject to some kind of physical symmetry, such as a translation symmetry. The standard quantization method involves first describing the theory in a ‘kinematical’ Hilbert space where we ignore the symmetry, and then moving to a ‘physical’ Hilbert space by imposing constraints enforcing that states related by symmetry transformations are identical. The resulting physical Hilbert space furnishes a kind of ‘perspective-neutral’ description of the world which incorporates some redundancy since it can be described in a variety of ways, but we can remove this redundancy by choosing a physical system and describing the rest of the world relative to that system. It can be shown that the various relational descriptions we obtain by this procedure are related to one another by the same transformations as we derive using the intuitive method described above.

It is informative to say something at this juncture about how the reference frame formalism relates to a position known as ‘factorism’ [18] which has played an important role in discussions of indistinguishability. Factorism is the view that the indices used in the Hilbert space formalism ‘have physical meaning in that they refer to particles’ [39]. Recently, it has sometimes been suggested that factorism is true for distinguishable particles but not for indistinguishable particles, and that this plays an important role in resolving worries about the identity of indiscernibles in quantum mechanics [18,39].

By contrast, in a perspectival view such as the reference frame formalism, the Hilbert space formalism has meaning only when relativized to some reference frame. Moreover, as noted earlier, in this formalism subsystem decompositions vary across different reference frames [17], so the decomposition of the Hilbert space suitable for one reference frame is not necessarily the same as the one suitable for another reference frame. There is thus no expectation in this formalism that Hilbert space indices will remain meaningful or refer consistently to the same thing as we change between reference frames. So in a perspectival picture it is likely that nothing that we would normally label to as a ‘particle’, whether distinguishable or not, truly refers to an individual that exists in an absolute sense, since both kinds of particles are defined in the first place only relative to a reference frame. It’s not clear that there are any ‘individuals’ in this picture at all, except possibly the subsystems defined by the decomposition of the underlying kinematical Hilbert space from which the relational descriptions can be derived, but typically neither distinguishable nor indistinguishable particles of the kind that feature in empirically meaningful descriptions of the world will correspond in any direct way to these kinematical Hilbert space subsystems [17].

Therefore the moderate physical perspectivalist will most likely have to say that from an absolute point of view factorism is false both for distinguishable particles and also for indistinguishable particles, since there do not exist any particles to which the indices could possibly refer. Thus any labels we use for these particles are meaningful only insofar as they serve as meaningful ‘handles’ with which to describe the world from within some perspective, and this is equally true for distinguishable and indistinguishable particles. Thus in this context, any question we pose about the ‘discernibility’ of a labelled entity cannot be about whether some real underlying individual is discernible from others in an absolute sense, but rather about whether there is a physically meaningful perspective with respect to which that label points to a subsystem which is discernible in one of the relational senses set out in Section 3.2 (Note that a ‘perspectivalist’ approach to the denial of factorism has been suggested by Bigaj [40]. However, here the individuation is relativized not to a reference frame but rather to a choice of basis, since different bases give rise to different individuations. Although this is clearly distinct from the view under discussion here, it would be interesting in future work to see if any connections between these approaches can be established).

4.1. Limitations

The QRF program remains in active development, and there are open issues surrounding its interpretation. I think the formalism is by now sufficiently well-developed that it is a worthwhile project to see what it tells us about discernibility, but the interpretational issues serve as caveats to this discussion, and there is one issue that I particularly want to highlight.

Since several authors argue that questions about ‘discernibility’ come down to questions about what we can empirically access and distinguish, it should be emphasized that the operational meaning of the ‘reference frame’ of an individual quantum particle is not at all clear. When I specify a classical reference frame and describe physics relative to it, this physics has a clear operational meaning: it predicts the outcomes that would be obtained if we performed measurements using instruments located ‘in’ that reference frame, e.g., measuring rods and clocks moving at the same speed and in the same direction as the reference frame. But although the QRF formalism allows us to make predictions for measurements performed relative to individual quantum systems, in most cases there is no straightforward way to actually put instruments into the reference frame of an individual quantum system. For example, Mikusch et al. [41] suggest that an experiment to implement such a thing might involve using a vibrating wire as a QRF: ‘*this reference frame can be extended by an atom placed on it, which can “measure” external systems (including parts of laboratory devices such as mirrors) by absorption and emission of photons*’. But even if such an experiment could be performed, we would not be able to personally enter the reference frame of the wire and check what things look like relative to it, so we will still have to make assumptions about how the relevant variables transform into our own reference frame.

Mikusch et al. [41] thus distinguish between an 'active' QRF transformation, which would amount to actually performing measurements in a different reference frame, versus a 'passive' QRF transformation, in which we simply '*take mentally the point of view of a reference frame and analyze other systems mathematically as if these systems were actually observed from that reference frame*'. As noted above there are ongoing conceptual questions about how literally we should think about this notion that a reference frame has a 'point of view', but for Mikusch et al. [41] it has quite a concrete meaning—to take the point of view of a reference frame just means using that reference frame to do calculations. And thus although active QRF transformations cannot currently be tested empirically, we can test passive ones—for example, by expressing an empirical question in the laboratory frame, then transforming the problem into the reference frame of a particle, doing the calculation in that reference frame, and then transforming back into the laboratory frame to obtain a testable empirical prediction. This is a useful procedure since sometimes the calculation may be much easier in the reference frame of the particle rather than the laboratory frame—for example, Giacomini et al. [42] use this technique to calculate appropriate observables to measure the angular momentum of a relativistic particle. Testing predictions of this kind offers an indirect way of verifying the validity of the quantum reference frame paradigm, but because it is only indirect one might still have questions about how literally we ought to take the physical description relativized to such a reference frame.

Thus although I will argue here that adopting the reference frame of a particle counts as a way of 'discerning' the particles, it should be emphasized that adopting the reference frame of a particle is not a physical operation we can currently perform, and it may well be the case that we will never be able to perform such operations; thus what we get from this move is discernibility in principle rather than in practice. I will discuss the significance of this point in Section 4.4.

4.2. Spin Reference Frames

Let us now consider what the QRF formalism has to say about the scenario relevant to the discernibility dispute: a collection of similar quantum particles. For the moment, I will consider a system of only two fermions, and I will focus on the spin degree of freedom, since that is the variable which is relevant to the paradigmatic case of a pair of particles which are weakly discerned by the relation 'having opposite spin'. For now I will disregard the possibility that systems could be weakly discerned by some other kind of relation, but I will return to that issue in Section 4.6.

Mikusch et al. [41] derives the appropriate QRF description for the case where two spin systems, A and B, are entangled in their spin degree of freedom from the point of view of a third system C, which could for example represent the laboratory frame. A and B together appear to be in a superposition of different spin states with respect to C. For example, consider a pair of fermions with the following relative state:

$$\psi^F_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B)$$

What happens when we switch into the reference frame of A? Well, by stipulation A always has a well-defined spin in its own reference frame, since the spin of A is what *defines* the reference frame. To avoid redundancy we can choose a labelling convention such that the spin of a system always points up along the X axis in its own reference frame. Then note that in the state ψ^F_{AB}, although neither A nor B has a definite spin, it is nonetheless definitely the case that A has the opposite spin to B. That relational fact is invariant under symmetry transformations and thus it is preserved by the reference frame transformations, so when we go into the reference frame of A it still must be the case that the spin of B is oriented opposite to that of A, which means that B now has a well-defined spin pointing down along the X axis. So by adopting the reference frame of A, we have brought into being a well-defined state of spin for B, where previously none existed.

Meanwhile, C will now appear to be in a superposition of spins relative to A. This claim can be arrived at using a linearity argument similar to the one in Section 4. Again, we can see why it is intuitively the right result: the state ψ^F_{AB} above describing A relative to C indicates that the relation between the spin of A and the spin of C is indefinite, and so when we switch into a reference frame where the spin of A is definite, e.g., the reference frame of A itself, the spin of C must be indefinite because the relation 'is indefinite relative to' is as symmetric.

Mikusch et al. [41] are particularly concerned with the case where A is significantly bigger than B. However, for the discernibility dispute we are interested in the case where A and B are indistinguishable, meaning they must have the same dimension. So what changes if both A and B are both spin-1/2 systems? Well, in that scenario A on its own can define only one axis, and thus we will not be able to express generic spins with respect to A: we will be able to write a spin vector as a linear sum of the directions associated with A only if that vector happens to be parallel or antiparallel to the single spin of A. So we might think of A as defining a quasi-reference frame; it's certainly possible to describe some things relative to A, but it is too simple to allow a relational description of all possible spin states relative to it.

But fortunately, for the scenario of two indistinguishable spin-1/2 particles we are specifically dealing with cases where the systems have the same or opposite spin to each other. Clearly any vector lying entirely up or down along a single axis can be written as a linear combination of any other vector lying entirely along that axis, so although a spin-1/2 particle doesn't have enough resources to act as a generic spin reference frame for any possible system, it does have enough resources to act as a spin reference frame for a set of particles which are guaranteed to be aligned or antialigned with it. For now, let us assume that this is a physically meaningful reference frame; I will re-examine that assumption in Section 4.4. Then, assuming that the transformation to this reference frame works in a similar way to the full transformations for spin reference frames described by Mikusch et al. [41], it follows that if A and B are initially in the antisymmetric state ψ^F_{AB} given above, then relative to the reference frame of A, B must be in the state $|\downarrow\rangle$, for the same reasons as discussed earlier.

However, some care must be taken here. Formally speaking, in the QRF methodology when we switch into the reference frame of A we remove A from the description entirely, so technically A does not have a spin relative to its own reference frame. Thus rather than A and B having two different values of spin in the reference frame of A, in fact A has no spin at all in its own reference frame and meanwhile B has spin down. Still, one might think that either way the description in A's reference frame is adequate to ground discernibility: 'no spin at all' versus 'spin down' are still different properties, so it looks like we still get perspectival weak discernibility.

With that said, it is possible that one might interpret the removal of A from its own reference frame as meaning that A simply is not represented at all in its own reference frame, in which case one might think that A and B cannot have different properties in the reference frame of A since A does not even exist in that reference frame. Thus the question of discernibility in this scenario may depend sensitively on the way in which we interpret the removal of A from its own reference frame.

What about the bosonic case? In this scenario the result depends on the specific state. One possible symmetrized spin state for a pair of bosons is as follows:

$$\psi^{B1}_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$$

In this case, when we switch into the reference frame of X, we get the same result as before: using the conventions described above, relative to the reference frame of A, B must be in the state $|\downarrow\rangle$, since the state ψ^{B1}_{AB} still tells us that the spin of B is opposite to that of A. The relative phase plays no role in determining the spin of B relative to A, so the change in sign between ψ^F_{AB} and ψ^{B1}_{AB} makes no difference here. This is consistent with the results obtained by Saunders and Muller [27], who note that the relation they construct to weakly discern fermions depends only on the existence of anticorrelations and not the relative phase, so since ψ^{B1}_{AB} still exhibits anticorrelations, it can be weakly discerned by the same relations.

However, another possible symmetrized spin state for a pair of bosons is as follows:

$$\psi^{B2}_{AB} = |\uparrow\rangle_A|\uparrow\rangle_B$$

In this case, when we switch into the reference frame of A, we find that relative to the reference frame of A, B must be in the state $|\uparrow\rangle$. That is, in this scenario it is still true that in the reference frame of A, B has a well-defined spin, but that spin is now the same as the spin of A; if we think of spins as living in a mathematical space, then the spin of A of course lies at the origin of the spin space defined by the reference frame of A, and the spin of B will also lie at the origin. Thus even in the reference frame of A the particles A and B don't have different spins, so it seems that relative to A's reference frame, the two entities do not exhibit perspectival weak discernibility or perspectival relational discernibility.

With that said, recall that in the standard formalism A does not actually have a spin in its own reference frame. If we take that stipulation seriously, then it may appear that A and B do in fact have different intrinsic properties in the reference frame of A, since B has a spin in the reference frame of A whereas A does not have a spin in this reference frame. This suggests that once again, the discernibility question in this scenario depends sensitively on how we interpret the stipulation that A does not have a spin in its own reference frame.

Thus I see three possibilities for what we should say about the discernibility of particles A and B in a two-particle symmetrized spin state, depending on how seriously we take the idea that A does not have properties in its own reference frame:

1. Because A is not represented at all in the reference frame of A, we cannot say that A and B have different properties in the reference frame of A, and hence A and B do not exhibit perspectival weak discernibility in virtue of their spins in any state for either bosons or fermions.
2. Because A has no spin in the reference frame of A while B has a well-defined spin in the reference frame of A, the two particles have different properties relative to that reference frame, and thus they exhibit perspectival weak discernibility for all possible states for either bosons or fermions.
3. For all fermions, and for bosons in a state like ψ^{B1}_{AB}, B has a different spin from A relative to the reference frame of A, so A and B exhibit perspectival weak discernibility. However, for bosons in a state like ψ^{B2}_{AB}, B has the same spin as A relative to the reference frame of A, so A and B do not exhibit perspectival weak discernibility in virtue of their spins (although as we will see in Section 4.6, there remains the possibility that A and B could exhibit perspectival weak discernibility in this state in virtue of some other kind of variable).

For now I will remain neutral on the question of whether removing X from the representation is the right thing to do when switching into the reference frame of X. For in any case, even if this is the right way to proceed, it seems reasonable to me that when interpreting the contents of a reference frame, we should be allowed to acknowledge the existence of the physical system defining the reference frame, regardless of whether or not we explicitly represent that system in the relevant reference frame. For example, when I judge the relative distance between me and my cat, I do not understand this in terms of the distance between her and the origin of an abstractly-defined reference frame; I think of this as the distance to me, even though I arguably do not model my own position as a degree of freedom in my reference frame. So it seems plausible that either option (2) or option (3) is correct, in which case we have indeed shown that there exists a physically significant kind of discernibility for similar particles in the fermionic case and for at least some states in the bosonic case.

4.3. Circularity

At this point, it is natural to worry about circularity. Indeed, this already a common criticism of Saunders' weak discernibility argument: Dieks and Versteegh [23] worry that it is impossible to apply the notion of weak discernibility without presupposing the existence of separate individual objects. And if the motivation for seeking to preserve the principle of Identity of Indiscernibles is indeed '*the empiricist desire to ground individuality in empirically accessible qualitative features, by means of (IOI)*' [23] then we surely can't use a prior assumption about the individuality of the particles to arrive at the conclusion that they are discernible, since that would be assuming exactly what we hoped to prove.

Similar objections might be made to the use of reference frames. For in the two-particle case, I have suggested that by adopting the reference frame of one of the particles, we can render them discernible. But one might think the claim that it is possible to adopt the reference frame of just one of the particles already presupposes that there exist two distinct particles, which is a circularity.

Now, there would indeed be a circularity here if the formalism required us to first pick out one of the particles, and then transform into its reference frame. For in that case we would need to already have access to some physical description which picks out a particle uniquely before we can perform the transformation, and we know that no such thing can possibly exist relative to any external reference frame, since the particles have identical properties relative to all external reference frames. However, the formalism does not in fact require this. We can simply choose some arbitrary labelling of the particles—as emphasized by Saunders [27], at a formal level this can always be done regardless of whether or not the particles are distinguishable—and then switch into the reference frame of the particle with some arbitrary label: the description of the rest of the world relative to particle i will always be exactly the same as the description relative to particle j for any pair i, j so the choice of labels makes no difference to the description we obtain.

One might worry that even if we don't have to distinguish the particles before performing the reference frame transformation, nonetheless the very process of adopting a reference frame assumes that there are two distinct reference frames present, so there is still a circularity. However, we do not have to assume in advance that there are two distinct reference frames present. There is nothing to stop us from starting out with the assumption that there is only one entity and that the reference frame transformation simply takes us into the reference frame associated with that single entity; in that case, we will interpret the process of assigning labels to the particles as simply a mathematical convenience which does not reflect genuine numerical distinctness. But nonetheless, when we then try to move into the reference frame of this single entity, what we will find is that it contains a representation of two distinct particles—or rather, we will find that it contains a representation of one particle which is, at least in the antisymmetric case, clearly distinct from the particle to which the reference frame is attached. So we can make whatever assumptions we like about the number of particles and reference frames from the external point of view, but when we actually adopt one of the reference frames, we may encounter something which gives us reason to question those assumptions; if we thought there was only one particle present, then at least in the antisymmetric case, when we try to adopt the reference frame of that one particle we will discover that it actually looks as though there are two particles present.

Now, one might object that this does not really address the circularity objection, because if the particles are not already known to be distinct, then the procedure described above is a meaningless mathematical exercise. However, the point that must be emphasized is that if we are serious about adopting the perspectivalist stance then it would seem that all judgements about discernibility must take a similar form, and thus the procedure used here certainly cannot be written off as a meaningless mathematical exercise in general: if all physically meaningful discernibility is relative to a reference frame, then using a reference frame to establish discernibility is not only legitimate but necessary. To many people it seems natural that, as Marcus [43] puts it, 'individuals must be there before they enter into any relations, even relations of self-identity' and thus it is common to object to the idea that objects can be discerned by relations on the grounds of circularity [44]. But perspectivalism turns this idea on its head: it suggests that objects can only exist in the first place relative to reference frames, and thus it is impossible for individuals to 'be there' before they enter into any relations. Therefore in this setting, the physical meaningfulness of discernibility by means of relations seems hard to deny. And since the procedure of adopting the reference frame of a similar particle and judging discernibility relative to it is both in principle and practice very closely aligned with the procedure that we would use for any other kind of system, then following similar reasoning as Dieks [26], it is reasonable to presume that the procedure still yields a physically meaningful judgement about discernibility in the case of similar particles, unless some strong reason can be offered for why it should not.

4.4. Is This a Physically Reasonable Reference Frame?

I have argued that by shifting into the reference frame of one of the similar particles, we can see that the particles have different properties relative to such a reference frame. But one might object to this argument on the grounds that a single spin-half particle cannot adequately define a reference frame, and thus shifting into its 'reference frame' is illegitimate. To address this concern, we will need to understand better the difference between the spin-half particle and the infinite spin reference frame employed by Mikusch et al. [41].

The first main difference is that the spin-half particle has a well-defined angular momentum quantum number along only one axis in space, whereas the spin reference frame is stipulated to have a well-defined angular momentum quantum number along three orthogonal axes. As noted above, this means that a 'reference frame' defined by the spin-half particle cannot properly represent spins in directions orthogonal to the single axis parallel to the particle's; however, since we will not need to represent such spins in the case of similar particles, this should not be a problem.

The second main difference is that the angular momentum quantum number of the spin-half particle is 1/2, whereas the spin reference frame is stipulated to have an infinite angular momentum quantum number along each axis. Mikusch et al. [41] make this stipulation because the reference frame is required to have '*unlimited resources for measuring orientations*'. It isn't necessarily obvious that this issue is relevant to discernibility, because our goal here is not to actually perform any measurements but simply to write down a meaningful physical description. However, for the sake of argument let us suppose that in order to say that particle B is discernible from A relative to the reference frame of A, it must be possible to distinguish between the case in which B has the same properties as A and the case in which B has different properties as A, by means of an operational procedure in which A is used as a reference. Note that I am not assuming that discernibility should be understood as an operational matter based on the outcomes of measurements; rather my point here is to show that even if someone does think the practical concerns raised in this section about the quality of a reference frame are relevant to the question of discernibility, in fact we can arrive at a meaningful operational distinction between these two kinds of cases.

So what exactly is the problem with small reference frames, from an operational point of view? It is twofold. The first issue that every time a measurement is performed on a reference frame, it 'degrades'—that is, its spin will tend to move towards the spin of the system being measured, and thus over time it changes in such a way that, relative to a hypothetical external reference frame, it would not be considered to represent the same direction of spin any longer. And smaller reference frames will degrade much faster than large ones, which limits their utility as reference systems. However, this seems mostly a practical issue; if we are just concerned about whether two possibilities are in principle discernible relative to a given reference frame, it is surely enough to show that there exists some single-shot measurement which can distinguish them, so we need not worry about multiple measurements.

The second issue is that in general quantum measurements aiming to establish some feature of B relative to the reference frame of A will not be perfectly accurate, and as the reference system becomes closer in size to the measured system, the results become less accurate. However we cannot reasonably demand that a quantum reference frame allows perfect accuracy, because the absence of perfect reference frames is in fact a characteristic feature of quantum mechanics. As Poulin [45] explains, 'In a classical relational theory, it is often possible to choose an arbitrary system as a reference frame and recover a non-relational theory, e.g., by working in the rest frame of a specific particle. In a quantum setting however, switching from the relational to the non-relational description will always require some sort of approximation since reference frames defined with respect to quantum systems are subject to quantum fluctuations [AK84a, Rov91b, Tol97a, Maz00a]. These quantum effects can be made arbitrarily small by increasing the mass of the reference system. However, in practical situations reference systems have finite masses' (Poulin's citations are to [46–49]). Thus no real quantum reference frame will always be able to perfectly distinguish any two cases, so unless we are willing to completely rule out the possibility that any quantum system could act as a reference frame, we will have to accept imperfect reference frames.

Thus I will suppose here that in order for us to meaningfully say that particles A and B having different spins relative to the reference frame of A grounds a physically meaningful kind of discernibility, we need only establish that it is possible to measure their relative spin in such a way that we can at least sometimes distinguish the case where A and B have different spins from the case where A and B have the same spin. For if there is no measurement of relative spin which can distinguish 'the same spin' and 'different spin', then it seems that the reference frame of A is simply too limited to meaningfully capture the fact that A and B have different spins relative to that reference frame. But if there is a measurement of relative spin which can distinguish 'the same spin' and 'different spin', then the reference frame of A is capable of capturing the fact that A and B have different spins relative to that reference frame. Even if the probability of distinguishing 'the same spin' and 'different spin' is small, I presume we would not want to say that two particles are indiscernible if there is any non-zero probability of discerning them. Thus we need only show that it is possible to perform a joint measurement on A and B with two outcomes, 'parallel' and 'antiparallel', such that $P(antiparallel|A \neq B) > (antiparallel|A = B)$, where $P(antiparallel|A \neq B)$ is the probability of the antiparallel outcome when A and B are in fact antiparallel, and $P(antiparallel|A = B)$ is the probability of the antiparallel outcome when A and B are in fact parallel.

Here I will follow the treatment of this problem suggested by Poulin [45]. We begin using a (fictitious) external reference frame, adopting the convention that $|\uparrow\rangle$ represents spin along the z direction in this reference frame. Now consider a scenario in which a spin-1/2 particle P is in a state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ and a second system of spin G, which we will use as our reference system, is in a state of angular momentum G along the z direction. Since we are trying to work in a fully relational context, Poulin [45] now recommends averaging over unphysical degrees of freedom associated with the fictitious reference frame. Using the resulting averaged state, we can now imagine performing a measurement to see whether or not the spins are parallel; and in fact they are found to be parallel with the following probability:

$$P(parallel) = |\alpha|^2 + \frac{|\beta|^2}{2G+1}$$

And they are found to be antiparallel with the following probability

$$P(antiparallel) = \frac{2G|\beta|^2}{(2G+1)}$$

Thus it can be seen that as G becomes large, the probabilities asymptote towards $|\alpha|^2$ and $|\beta|^2$, indicating that we can think of this procedure as something like a classical 'measurement' of the spin of the particle P, with G playing the role of a macroscopic measuring device which asymptotically yields the two possible outcomes with exactly the probabilities predicted by standard quantum theory.

But in the case we are interested in, $G = 1/2$, and either $\alpha = 1$ (if the spins are parallel) or $\alpha = 0$ (if they are antiparallel). In the parallel case, we find that $P(parallel|A = B) = 1$, $P(antiparallel|A = B) = 0$ regardless of the size of G. So if the spins are parallel the measurement will always correctly tell us that they are parallel, and it does not matter how large the reference system is. Meanwhile, if the spins are antiparallel then $P(antiparallel|A \neq B) = 1/2$. Thus $P(antiparallel|A \neq B) > (antiparallel|A = B)$, which means we have a least some non-zero probability of distinguishing the parallel case from the antiparallel case.

From an operational perspective, using a spin-half system as a reference system is clearly not ideal: we always get the right result if the systems are parallel, but in the case where they are pointing in opposite directions we get the right outcome only half the time. So if we know the spins are either parallel or antiparallel, then the 'antiparallel' answer tells us conclusively that they are in fact antiparallel, whereas the 'parallel' answer could mean either parallel or antiparallel. However, the point here is not to actually use this reference frame as a practical tool, but simply to see if the reference frame of A has enough resources in principle to distinguish the kinds of facts we aim to distinguish; and the fact that $P(antiparallel|A \neq B) > (antiparallel|A = B)$ seems to give us grounds to say that A and B are discernible relative to the reference frame of A. Our ability to distinguish these cases is admittedly not as good as one would like, but 'discernible' is not a graded term: we can distinguish these cases with probability better than a random guess, and thus the fact that the particles have different spin is detectable, so it seems reasonable to say that they exhibit perspectival weak discernibility.

4.5. The Heterodox Approach

A different kind of worry about the meaningfulness of the reference frames employed here comes from a way of thinking about the quantun indiscernibility problem which is known as the 'heterodox approach'. The heterodox approach is based on the denial of factorism for indistinguishable particles [18,39,50], as discussed in Section 4. The idea is that for such particles it is a mistake to identify the particles by means of the labels on the tensor product Hilbert spaces: we should instead individuate particles qualitatively, which means the individuation criteria should be associated with projectors which represent qualitative properties. It can be shown that once we identify our particles in this way, all fermions are absolutely discernible and bosons are absolutely discernible in all but a minority of states [18].

Now, a proponent of the heterodox approach might reasonably object to the procedure described above for estbalishing discernibility in a relational context that when we performed the transformation into the reference frame of A we treated the labels A, B attached to the factor Hilbert spaces as physically meaningful, whereas in fact these labels do not refer to particles at all. Indeed it would be natural for proponents of the heterodox approach to argue that we should only be making reference frame transformations into subsystems identified according to the qualitative individuation criteria that they regard as physically meaningful.

However, as noted in Section 4, in a perspectival context it is unlikely that there exist any 'particles' to which labels could refer under either the Hilbert space index approach or the qualitative individuation approach, and thus if we deny the meaningfulness of transformations into systems labelled by Hilbert space indices on the grounds that they do not refer to particles in an absolute sense, then we should probably also deny the meaningfulness of transformations into systems identified by qualitative individuations. What descriptions would we actually be permitted if we limit ourselves to descriptions relativized to systems labelled in such a way that the label refers to something which exists in an absolute sense?

To make sense of this, we would probably have to adopt the first-principles approach described in Section 4 where we start from the kinematical Hilbert space and then move to the physical Hilbert space. In this picture there will usually be some subsystem decomposition associated with the underlying kinematical Hilbert space, so one might be tempted to say that the 'real' or 'absolute' individuals in this picture are those associated with the decomposition in the kinematical space, and then insist that meaningful relational descriptions must involve relativization to systems labelled in such a way that the label ultimately points to a subsystem in the kinematical Hilbert space. However, this actually seems like quite an odd approach, because a subsystem in the kinematical Hilbert space need not correspond to what we would naturally think of as a subsystem in the physical Hilbert space within which our empirically meaningful descriptions are posed—in particular, the notion of subsystem that we get within the physical Hilbert space will often be nonlocal with respect to the kinematical tensor product decomposition [17]. So this way of thinking about identity and individuality would make it strangely divorced from any of the empirical and operational considerations for which it is typically relevant.

One way of responding to this concern would be to adopt a strong form of perspectivalism which denies that it is valid to adopt the kind of third-person, perspective-independent description which makes questions about the independent reality of the reference frames seem meaningful. From this point of view, we must always start 'from the inside', and thus the right way to identify valid reference frames is simply to look from your own perspective, note the subsystems around you as they appear to you, and then use the reference frame transformations to move into their perspectives in turn. In such a strong perspectivalist view, it seems natural to say that as long as the indices in a Hilbert space description can be associated with a well-defined reference frame transformation, they do refer to physically meaningful reference frames because this is the only way we can ever get any reference frames at all.

However, even in a weaker form of perspectivalism such as moderate physical perspectivalism, where we do admit the existence of some kind of third-person description, the considerations raised above suggest that it is perhaps not wise to insist that all descriptions must be relativized to subsystems appearing within that absolute third-person description. Here it is important to remember that in the reference frame formalism, a reference frame does not have to be associated with consciousness or with any metaphysically special kind of system; a description relative to a reference frame is simply a way of picking out empirically meaningful structures from within the abstract, perspective-neutral description. Thus as long as there is a well-defined transformation which takes us into the reference frame of the 'particle' associated with some index, that is a perfectly legitimate way of picking out relationally-defined structure—the 'physicality' of the reference system comes precisely from the fact that we can write down a meaningful set of facts relativized to it, so we do not have to establish its physicality before implementing the reference frame transformation. And there does not seem to be any particular reason why this would be different in the case of indistinguishable particles; some reference frames may be better than others, in the sense that descriptions relative to them are richer or more stable, but that does not mean the other ones are not physically meaningful at all. Thus from a conceptual point of view, once we agree that labels in general do not in general point to individuals that exist in an absolute sense, it's not clear that there is any compelling reason to deny the meaningfulness of transformations making use of Hilbert space indices.

But perhaps more importantly, it should be noted that at least with respect to judgements of distinguishability it plausibly does not actually matter whether we use the coordinate labels or the qualitative individuation recommended by the heterodox view. I will illustrate this by an example, which is owed to Bräutigam [39] and Muller and Leegwater [50]. Consider the following state for two indistinguishable particles:

$$\psi_{12} = \frac{1}{\sqrt{2}}(|R\rangle_1|\uparrow\rangle_1|L\rangle_2|\downarrow\rangle_2 + |L\rangle_1|\downarrow\rangle_1|R\rangle_2|\uparrow\rangle_2)$$

Here, $|R\rangle$ and $|L\rangle$ refer to two positions which I will take to be separated by a distance of 1 in some chosen unit, and $|\uparrow\rangle, |\downarrow\rangle$ refer to up and down spin along the Z axis, and 1 and 2 are the Hilbert space labels.

Now, naïvely this state appears to describe two indistinguishable particles both associated with a combination of right and left positions and up and down spins. However, as shown by Muller and Leegwater [50] the heterodox view proposes an alternative factorization and a basis change which allows the state to be rewritten as follows:

$$\psi_{LR} = |\uparrow\rangle_L|\downarrow\rangle_R$$

This state now describes one particle on the left with spin up, and another particle on the right with spin down. That is, rather than identifying particles by their Hilbert space labels 1 and 2 we are now identifying particles by their position in space, with the result that the particles now self-evidently exhibit absolute discernibility.

Now suppose we try to use these two states respectively to transform into the reference frame of the associated particles. If we use the state ψ_{12}, then, applying the same kind of reasoning as set out in Section 4, it would seem the resulting state of one particle relative to another is given by $\frac{1}{\sqrt{2}}|\downarrow\rangle \otimes (|-1\rangle + |+1\rangle)$ (the relative state is the same for 2 relative to 1 and 1 relative to 2). Whereas if we use the state ψ_{LR} and transform into the reference frame of one of the particles, the state of the other is given by $|\downarrow\rangle$ (again, the relative state is the same for L relative to R and R relative to L). That is, if we ignore the position degree of freedom then we get the same relative state for both ψ_{12} and ψ_{LR} and thus we reach the same conclusion either way—the particles exhibit perspectival discernibility because they have opposite spin. Indeed, the difference between the two relative states here really just concerns whether or not we are treating position as a dynamical degree of freedom, and plausibly there is no fact of the matter about whether we ought to do that—treating position as a dynamical degree of freedom might be useful for some applications and not for others.

Moreover, the behaviour exhibited in this example can be expected to generalize. We got the same relative spin state from both ψ_{12} and ψ_{LR} because both states encode anticorrelations between the spins of the systems, and a relative state is ultimately just a way of encoding the (anti)correlations between systems. Moreover, the point of the heterodox approach is precisely to take advantage of these kinds of anticorrelations in order to arrive at a qualitative individuation of particles according to which they are clearly discernible, so we can expect that whatever anticorrelations are present will be preserved in some form in at least one of the new states that we arrive at after shifting to a qualitative individuation recommended by the heterodox approach (in general the choice is not unique). Thus, at least when we focus on the variable for which the anticorrelations apply, the relative state that we get from performing a reference frame transformation using the coordinate description ought to be closely related to the relative state we would get from performing the transformation using some qualitative individuation, and thus we would most likely get the same conclusions about discernibility either way.

In this sense the reference frame approach and the heterodox approach may be viewed as complementary: both are, in effect, ways of leveraging the existence of anticorrelations to argue that indistinguishable particles are in fact discernible in a strong and meaningful sense that goes beyond just weak discernibility. The heterodox approach does this by rearranging the factorization from the 'outside' so to speak, whereas the reference frame approach instead looks from the 'inside', but both are ultimately appealing to the same kind of relational properties.

Of course, the case above is just one example and it would be another matter to show formally that the two formalisms continue to agree in the relevant way in all cases. It is beyond the scope of this paper to demonstrate such a thing (and indeed it is not clear that the reference frame formalism currently has adequate tools to do so in general), but this would certainly be an interesting direction for further investigation.

4.6. More General Cases

I have suggested that for pairs of particles, the relation ‘having opposite spin’ grounds perspectival relational discernibility, which is the strongest kind of discernibility available in a perspectival picture. If this is accepted, it ensures that fermions are always discernible. Meanwhile, bosons sometimes exhibit this relation and sometimes do not, so bosons will definitely sometimes be discernible, but there is still a question mark about the cases where they do not exhibit the relation ‘having opposite spin’.

So now there are two further questions to address. First, is there some other relation which could give us perspectival weak discernibility for the other bosonic states? And second, what happens when we extend to states of more than two particles?

The key to establishing weak discernibility in the case above was to identify some basis in which the single-particle states exhibit anticorrelations. This is because anticorrelations have a straightforward physical interpretation in the reference frame formalism: if the states of two particles are anticorrelated, then when we use one of the particles as a reference frame, the state of the other particle relative to that reference frame is different from the reference state. So it appears that in order to demonstrate perspectival weak discernibility for particles in some joint state we need only show that this state involves anticorrelations between single-particle states.

Now, for fermions this is easy to achieve because the way the symmetrization postulate works for fermions means that the single-particle states must be anticorrelated in *any* possible basis: ‘*this is ‘easy’ because of Pauli exclusion: in any basis the occupation number for any single particle state never exceeds one*’ [33]. But for bosons things are not so straightforward, because the boson single-particle particle states will not be anticorrelated in every basis—for example, in the state ψ_{AB}^{B2} the particles are perfectly correlated in the spin basis, and thus as noted above, using one of the particles as a spin reference frame does not yield different states for the two particles.

However, the reference frame formalism is not limited to the spin basis; as described in Section 4, we can use systems as reference frames for any variable which is associated with a suitable symmetry group, although a given system might not be an equally good reference frame for all types of variables. So in principle, if we want to show discernibility for a pair of bosons, we need only identify some basis in which the bosons are anticorrelated—then, as long as the variable relevant to that basis can be associated with a suitable symmetry group in the way described in Section 4, making it susceptible to the standard reference frame treatment, then we will be able to use one of the bosons as a reference frame for this variable and we will then find that the bosons have different properties relative to that reference frame.

Caulton [33] shows that it is always possible to identify such a basis. In fact, that basis is simply the position basis, because the way in which the Hilbert space is defined for systems of many particles does not allow that wavefunctions are infinitely peaked at points of the configuration space, so it is impossible to define a wavefunction which is nonzero only where the positions of the particles are equal, which means their positions must exhibit at least a small anticorrelation. And the position basis is indeed associated with a suitable symmetry group, i.e., symmetry under global translations. Moreover, Caulton notes that even if we want to work in some idealized version of the theory which does allow infinitely peaked wavefunctions, then it can be shown that a given state cannot be infinitely peaked in both relative position and also relative momentum, so if the particles are not anticorrelated in the position basis, they will necesssarily be anticorrelated in the momentum basis. The momentum basis is also associated with a suitable symmetry group, i.e., symmetry under global rotations. Thus it appears that we will always be able to establish perspectival weak discernibility by using one of our bosons as either a position reference frame or a momentum reference frame, since the anticorrelations ensure that the other particle will always have a different state to the reference particle in that reference frame.

Furthermore, Caulton [33] also shows that this result extends to states of more than two particles: for a many-particle bosonic or fermionic state, we can select any two particles and they will always be anticorrelated in some basis. So we should always be able to use one of the particles as a reference frame such that the other particle has a different state relative to that reference frame. Therefore Caulton’s approach appears to resolve both of the questions above: for all boson states and for any number of fermions or bosons we will be able to find a basis exhibiting anticorrelations and thus we will be able to find a reference frame in which the particles have different states.

Of course, there may be further worries to contend with. The result of Caulton [33] proves that there must be anticorrelations, but presumably in some cases the anticorrelations might be very small. We have seen already that in the fermionic case, even when the fermions are perfectly anticorrelated they are still not operationally ideal reference frames for one another; so one might worry that things could get signigicantly worse for bosonic states in which the anticorrelations may be weaker. Thus depending on how seriously one takes the kinds of operational considerations described in Section 4.4, one might worry that some bosonic states with only weak anticorrelations might not be discernible in a physically meaningful way.

There is also a worry here related to the issue of factorism. Caulton [18] argues that the theorem of Caulton [33] does not actually ground the distinguishability of bosons in all states, because given a pair of bosons in the spin state $\psi^{B2}_{AB} = |\uparrow\rangle_A|\uparrow\rangle_B$, 'to talk about the sort of anti-correlations of which Caulton's (2013) Theorem 2 takes advantage, we would have to shift our individuation criteria to that basis, but by doing so we would thereby be talking about a different collection of bosons' [18]. In the reference frame formalism, this concern translates to a question about what exactly we are doing when we consider a single system as a reference frame for various different types of property. For example, we might initially think of using bosons as a reference frame for spin, but when we attempt to do that using bosons in the spin state ψ^{B2}_{AB} we have no anticorrelations which could ground perspectival discernibility. We can instead switch to using the bosons as a reference frame for position, in which case we do get anticorrelations which ground perspectival discernibility. But there does not seem to be any clear sense in which the boson that we first tried to use as reference frame for spin is the same boson as the boson that we then use as a reference frame for position—since subsystem decompositions are relative to a reference frame in any case, that kind of identity claim could only be made relative to a given reference frame, but what kind of reference frame? It is not clear that either a spin or a position reference frame could be used to ground claims about identity across these bases, particularly in the case where there are no interesting correlations between them, so such an assertion simply fails to have any clear physical content in a perspectival context.

That is, it is commonly noted that there is no way to track indistinguishable quantum particles across time [51], but given the limited physical significance of particle labels, there is a similar problem with tracking indistinguishable quantum particles across different bases. Indeed, this is perhaps not so surprising in the perspectival context, because in that setting it is already the case that particles do not exist in an absolute sense as individual entities to which properties can be ascribed—'particles' are simply degrees of freedom which have been collected together in a certain kind of way from the point of view of some reference frame, and which might be collected together differently from the point of view of another reference frame. So for types of properties that must be relativized to different reference frames, showing that we get discernibility when we consider the particles as reference frames for a certain kind of variable does not necessarily show that we have discernibility in any meaningful sense when we are focusing on a different kind of variable, and thus although we might take this result to show that bosons are in some sense always discernible, it does not necessarily follow that the specific set of bosons that appear to feature in a spin state such as ψ^{B2}_{AB} are discernible.

5. Discussion

To sum up, we have seen that for a set of similar quantum particles, it is possible to adopt the reference frame of one of those particles, and when we do so we find that at least in the antisymmetric case, the particles do seem to have different properties relative to this reference frame. Thus such particles exhibit perspectival weak discernibility and also perspectival relational discernibility.

In order to establish whether or not these particles constitute a counterexample to IOI, then, we must decide whether or not perspectival weak discernibility and perspectival relational discernibility should count as a physically meaningful kind of discernibility for the sake of IOI. In fact, there seem to be good reasons to think that it does, for in the context of moderate physical perspectivalism, all physically meaningful properties of the kind that could be used for discerning systems and even the subsystem decompositions themselves are relativized to some reference frame, so in fact 'absolute discernibility' ultimately reduces to discernibility relative to some reference frame, and thus becomes equivalent to perspectival relational discernibility. Thus in the context of moderate physical perspectivalism, we certainly must accept that perspectival relational discernibility is at least sometimes the kind of discernibility which is relevant to IOI. So the only remaining question is whether there is some good reason to forbid the use of the reference frames of one of the systems concerned when seeking to settle questions of discernibility—i.e., is there any reason to insist on perspectival external relational discernibility, or is any kind of relational discernibility enough to ground the distinctness of the systems concerned?

Throughout this article, we have seen that there does not appear to be any obvious reason to forbid the use of the reference frames of the systems concerned. In particular, we have seen that although spin-half particles and other small systems are not ideal reference frames, their symmetry properties mean that they will typically have the right features to serve as reference frames for one another. It is true that at an operational level our ability to distinguish the parallel and antiparallel cases is not particularly good, but it seems reasonable to think that any non-zero probability of distinguishing them should render the particles technically discernible.

One might perhaps think we should forbid the use of the reference frames of one of the systems concerned on the grounds that in the case of similar quantum particles it will never be possible for agents like us to actually adopt such a reference frame or perform measurements in it, so the fact that the particles have different properties relative to that reference frame does not imply that there is any practical way for *us* to distinguish them. This might be motivated by the kind of view taken by Ladyman and Bigaj [22], who argue that the goal of discriminating objects physically is '*to ensure the existence of a possible physical procedure that could yield one outcome when applied to one object and a different outcome when applied to the other one. This procedure does not have to be practically executable, but it should be available to us in principle*'.

If we accept this view then we might think that the kind of discernibility relevant to IOI must involve a procedure which is available to us in principle—but still, much will then depend on what exactly is meant by 'in principle'. It is unlikely that there is any remotely feasible way in which a human observer can literally adopt the reference frame of a fermion, and if there were some way of doing this then it would likely involve altering the system in such a way that the relevant symmetries would be broken and thus the particles would not be similar any longer. However, as discussed in Section 4.1, it is possible to empirically test passive reference frame transformations, which looks like at least indirect evidence for the meaningfulness of the description relative to the reference frames of the fermions themselves. It is also possible for an external observer to perform the measurement described in Section 4.4, thus externally distinguishing the parallel and antiparallel cases. So even though we cannot literally jump into the reference frame of a fermion and observe directly that the fermions have different properties relative to that reference frame, there are reasons to take the description relative to that reference frame seriously, and thus one might think there is a sense in which we should say the relevant procedure is available 'in principle'.

In addition, insisting on an overly strict understanding of the term 'in principle' has strange consequences in this particular case. For similar quantum particles happen to be a special sort of physical system which are only discernible from within—that is, in effect you have to be one of the particles in order to discriminate them. And if we take take 'in principle' to be making some claim about what is true in other possible worlds, then many natural ways of fixing cross-world identities would have the consequence that I myself could not even in principle be something other than a human, or at least I could not be something so different from a human as an indidividual fermion. This would suggest that it is not possible even in principle for me to discriminate similar quantum particles. But this way of thinking about what is possible 'in principle' gives oddly parochial answers as to the significance of weak discernibility. It implies that it is possible 'in principle' to discriminate the two twins in the symmetric universe described in Section 3.2, since I could possibly be one of those twins, but it is not possible 'in principle' to discriminate Black's spheres, since I could not possibly be an iron sphere. Indeed, arguably it would be possible 'in principle' for me to discriminate the twins only if the twins are not only human but qualitatively similar enough to me for them to count as my counterparts. These do not seem like considerations which should be relevant to the question of discernibility or distinctness, so I think we should not understand 'in principle' in a way that makes perspectival weak discernibility physically relevant only for systems sufficiently similar to us. Therefore we should be willing to admit that the perspectival weak discernibility of fermionic particles is the kind of discernibility relevant to the Identity of Indiscernibles, meaning that fermions at least do not constitute a counterexample to IOI.

The other possible obstacle we should consider is the fact that according to the current formalism, systems are not directly represented in their own reference frame—or at least, the degrees of freedom associated with the specific type of reference frame are not represented (if you are using a system as a position reference frame, you remove its position degree of freedom, if you are using it as a momentum reference frame, you remove its momentum degree of freedom, and so on). This issue is relevant only in scenarios where we judge discernibility relative to the reference frame of one of the systems that we are trying to discern, i.e., for perspectival weak discernibility, so it might potentially be considered as a reason to rule out perspectival weak discernibility in particular.

But the force of such an objection depends on how seriously we take this particular feature of the formalism; it's not necessarily obvious whether the removal of the degrees of freedom of a system from its own reference frame is just a conventional choice that the originators of this formalism have made for convenience, or whether it reflects some deep fact about the nature of quantum reference frames. Mikusch et al. [41] explain this choice on the basis that it helps '*to avoid 'self-reference problems*'', but it's not necessarily obvious that we should be avoiding self-reference. For example, Ismael [52] argues that self-reference is an essential feature of physical reality that plays a particularly important role in defining the nature of agency, and if one buys that argument one might think that removing the degrees of freedom from its own reference frame is not always the right way to proceed.

Ultimately, the question of whether systems should have a quantum state in their own reference frame probably depends quite sensitively on how we understand the meaning of the quantum state. So interestingly these questions about discernibility of particles and the role of symmetry appear to be closely tied to questions about the interpretation of quantum mechanics in a way that may not have been obvious without using the reference frame formalism. Thus it would be useful for future work on perspectivalism to examine further the status of the quantum state and its connection to objecthood and individuality.

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