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Singularity-Free Prescribed-Time Distributed Optimization for Nonlinear Multi-Agent Systems with Time-Varying Cost Functions

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Abstract: This paper proposes a singularity-free prescribed-time distributed optimization algorithm for nonlinear multi-agent systems with time-varying cost functions and dynamic communication topologies. The proposed algorithm avoids singularity, and it does not require the local cost functions to have identical Hessian matrices. First, a novel time-varying scaling function based on time-space deformation theory is designed to address the singularity issue in the proposed distributed optimization algorithm. Second, a distributed prescribed-time optimization estimator by introducing an intermediate driving variable to counteract the inconsistency disturbances is designed to track the average of the global gradients, global Hessian matrices, and the global partial derivatives of gradients, respectively. Furthermore, under this estimator, all local cost functions need not have identical Hessians. Third, refined theoretical analysis demonstrates that our algorithm converges to the globally optimal trajectory. Meanwhile it is also shown to avoid the singularity problem and achieve prescribed-time estimation. Finally, a UAV formation experiment verifies the effectiveness of our proposed algorithm, including its singularity-free property, prescribed-time convergence, and optimality.

Keywords: nonlinear multi-agent systems; distributed optimization; singularity-free; prescribed-time convergence; time-varying cost functions

1. Introduction

Over the past few decades, the distributed optimization of multi-agent systems has received substantial attention. Its strengths in parallel computing, scalability, and robustness have facilitated successful applications in various fields, e.g., resource allocation [1], smart grids [2], distributed sensor networks [3] and multi-robot formation control [4]. In distributed optimization systems, multiple agents are interconnected via a communication network topology. They collaborate to achieve consensus while minimizing a global objective function, with each agent only utilizing its local information and information from neighboring nodes.

Early research in distributed optimization primarily focuses on discrete-time algorithms [5–10] due to their relative ease of deployment and implementation. However, recent research focuses on continuous-time techniques, e.g., subgradient method [11,12], gradient tracking techniques [13], proportional-integral-based approaches [14], zero-gradient-sum (ZGS) algorithms [15, 16]. Each of these methods can achieve the optimal solution in a distributed optimization framework, though they differ in convergence rate and robustness. Nevertheless, most of these algorithms exhibit exponential or asymptotic convergence, which means that the optimal solution is only attained as time approaches infinity, thus limiting their practical utility. This motivates the development of finite-time and fixed-time convergent algorithms [17–19], which provide bounded convergence time. However, the establishment of a finite-time upper bound depends on the initial state, whereas although the convergence time upper bound of fixed-time algorithms depends on system parameters and is independent of the initial state, it involves complex parameter calculations.

Prescribed-time convergence mechanisms attract widespread attention in recent years due to their ability to preset convergence time, which is an ideal property in control systems. Consequently, a large number of prescribed-time convergence algorithms have been proposed [20–25]. For instance, Wang et al. [20] propose an algorithm based on a time-varying scaling function that can solve distributed consensus problems within a prescribed-time. Under the zero-gradient-sum (ZGS) framework, Chen et al. [21] develop a dynamic event-triggered strategy that solves both unconstrained and constrained optimization problems within a prescribed-time. De et al. [22] introduce an algorithm that guarantees prescribed-time convergence even in the presence of switching networks and external disturbances. It is noteworthy that these algorithms are predominantly addressed to static optimization problems, characterized by time-invariant agent cost functions and a fixed optimal solution.

Nevertheless, practical implementations often involve time-varying cost functions, thereby transforming the optimal solution into a dynamic trajectory. For such time-varying problems, Deng [26] proposed a distributed online algorithm for noncooperative games over unbalanced digraphs, where the average dynamic regret asymptotically approaches zero as time goes to infinity. Rahili et al. [27] studied distributed continuous-time convex optimization with time-varying cost functions for single-integrator and double-integrator dynamics, achieving asymptotic convergence to the optimal trajectory. Sun et al. [28] extended the results to account for time-varying nonlinear inequality and linear equality constraints, achieving asymptotic convergence to the optimal solution or its vicinity using a Hessian-based approach. Meanwhile, Shi et al. [29] developed finite-time convergent algorithms for time-varying distributed optimization, including unconstrained consensus optimization and resource allocation problems, without requiring identical Hessians. Prescribed-time optimization has also been explored for time-varying cost functions in several recent works [30–32]. For instance, Zhang et al. [30] introduce a sliding manifold to drive local gradients to zero within a prescribed-time frame, which enables convergence to the time-varying optimal trajectory in dynamic environments with time-varying cost functions. Nevertheless, this approach requires all local cost functions to possess identical time-varying Hessian matrices. To address this limitation, Cui et al. [31] relax the requirement for Hessian matrix equivalence by developing a distributed estimator that reconstructs global gradient information from local data. It should be noted that this method achieves prescribed-time convergence through fixed-time lemma backtracking, which establishes a conceptual link between prescribed-time and fixed-time algorithms. Nevertheless, the complex norm operations involved in the algorithm impose a relatively high computational load. Chen et al. [32] propose an algorithm featuring a hybrid update scheme with constant and time-specific update rates, which decouples the optimization process into three cascaded subsystems. These subsystems sequentially address optimization estimation, consensus convergence, and optimality pursuit across three consecutive time intervals.

However, most of the aforementioned prescribed-time algorithms suffer from a singularity issue caused by time-varying scaling functions, where the time-varying gain tends to infinity as the system time approaches the prescribed settling time, ultimately leading to divergence. To address this problem, several singularity-free methods for achieving prescribed-time stability have been developed. For instance, Ye et al. [33] design a bounded feedback controller that avoids singularity by imposing saturation constraints on the time-varying high gain. However, this method only guarantees convergence to a compact set within the prescribed-time. To achieve exact prescribed-time convergence without singularities, Ding et al. [34] propose an alternative solution based on periodic delayed feedback. Nevertheless, this approach introduces delays into the multi-agent communication network. In contrast, Orlov et al. [35] introduce a singularity-free prescribed-time consensus algorithm based on time-space deformation, which operates without communication delays and guarantees precise convergence. It inspires us to develop distributed singularity-free prescribed-time optimization algorithms.

Motivated by these observations, we propose a distributed optimization algorithm with singularity-free prescribed-time convergence based on a distributed optimization estimator. Through the designed estimator, global Hessian matrix information is indirectly obtained and utilized in the optimization process, thereby relaxing the prevailing requirement in existing time-varying cost function algorithms that all local cost functions must share identical Hessian matrices. Furthermore, by introducing a singularity-free time-varying scaling function based on time-space deformation theory, the algorithm achieves prescribed-time convergence while avoiding singularity issues. The proposed method is applicable to nonlinear multi-agent systems with time-varying cost functions and dynamic communication topologies. The three main contributions of this work are summarized as follows:

(1) We propose a singularity-free prescribed-time distributed optimization algorithm for nonlinear multi-agent systems. This algorithm is founded on a novel time-varying scaling function derived from time-space deformation theory, addressing the singularity issue prevalent in existing prescribed-time optimizers. To the best of our knowledge,

this represents the first integration of a singularity-free scaling function into a prescribed-time optimization framework for systems with time-varying cost functions. Unlike previous prescribed-time approaches [25,30,32] that suffer from singularity issues, our algorithm guarantees prescribed-time convergence while avoiding singularity. Furthermore, the use of an edge-based protocol extends its applicability to time-varying undirected connected graphs, offering broader applicability than the node-based algorithm in [31].

(2) We design a distributed prescribed-time optimization estimator for the proposed algorithm by introducing an intermediate driving variable. This estimator counteracts inconsistent disturbances and is designed to track the averages of the global gradients, global Hessian matrices, and the global partial derivatives of gradients, respectively. This approach relaxes the requirement of identical local Hessian matrices, which is necessary in some time-varying cost function algorithms [27,30,31]. On this basis, our algorithm drives all agents to converge to the globally optimal trajectory within a prescribed-time. Moreover, by incorporating a disturbance rejection compensation term, the proposed method enhances robustness against nonlinear disturbances compared to [25,32].

(3) We design a dedicated Lyapunov function for each of the three stages: estimation, consensus, and optimal trajectory tracking. It is rigorously proved that the system achieves prescribed-time convergence within each independent time interval, with guaranteed boundedness of inter-stage outputs. The convergence time is proven to be independent of initial conditions and robust to time-varying communication topologies and external disturbances. The unmanned aerial vehicle (UAV) formation experiment further validates the singularity-free property, prescribed-time convergence, and optimality of the proposed algorithm under time-varying cost functions.

Notation: We use $\mathbb{R}$, $\mathbb{R}_{>0}$, and $\mathbb{R}^n$ to represent the sets of real numbers, positive real numbers, and n-dimensional real vectors, respectively. For any vector $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in \mathbb{R}^n$, the signed power function is defined as $\text{sig}(x_i)^s = [\text{sig}(x_{i1})^s, \text{sig}(x_{i2})^s, \ldots, \text{sig}(x_{in})^s]^T$, where $\text{sig}(z)^s = \text{sign}(z)|z|^s$ for $z \in \mathbb{R}$. The algebraic connectivity of a matrix $\mathcal{L}$ is denoted by $\lambda_2(\mathcal{L})$, corresponding to its second smallest eigenvalue. The p-norm of x_i is given by $\|x_i\|_\xi = \left(\sum_{k=1}^n |x_{ik}|^\xi\right)^{1/\xi}$ with $\xi > 0$. Furthermore, the fractional power of the vector x_i is defined component-wise as $x_i^\varepsilon = [x_{i1}^\varepsilon, x_{i2}^\varepsilon, \ldots, x_{in}^\varepsilon]^T \in \mathbb{R}^n$, where $\varepsilon \in \mathbb{R}$ is a constant.

2. Problem Formulation and Preliminaries

2.1. Graph Theory

The interaction topology of the N agents is represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the communication edge set. The existence of an edge $(v_i, v_j) \in \mathcal{E}$ indicates that agents i and j can directly exchange state information. A time-varying weighted adjacency matrix $A(t) = [a_{ij}(t)]_{N\times N}$ is defined such that $a_{ij}(t) > 0$ if $(v_i, v_j) \in \mathcal{E}$, and $a_{ij}(t) = 0$ otherwise. The associated Laplacian matrix $\mathcal{L}(t) = [\ell_{ij}(t)]_{N\times N}$ is given by $\ell_{ij}(t) = -a_{ij}(t)$ for $i \neq j$, with the diagonal entries defined as $\ell_{ii}(t) = \sum_{j\neq i} a_{ij}(t)$. This work focuses on time-varying undirected connected graphs, which implies $a_{ij}(t) = a_{ji}(t)$ for all agent pairs.

2.2. Problem Formulation

Consider a network of N agents, denoted by an undirected graph $\mathcal{G}$, where each agent $i \in \mathcal{V}$ possesses nonlinear dynamics evolving according to:

$$\dot{x}_i(t) = u_i(t) + d_i(t, x_i), t \in \mathbb{R}_{\geq 0}, \forall i \in \mathcal{V}. \tag{1}$$

Here, $x_i \in \mathbb{R}^n$ is agent i's local decision vector, $u_i \in \mathbb{R}^n$ is the control input, and $d_i(t, x_i) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ models an unknown nonlinear disturbance. This disturbance is bounded such that:

$$\|d_i(t, x_i)\| \leq \|\theta_i(t)\| \, \|\phi_i(t, x_i)\|, \tag{2}$$

where $\phi_i(t, x_i)$ is a known nonlinear function, and $\theta_i(t) \in \mathbb{R}^n \times \mathbb{R}^n$ is a time-varying parameter with an unknown instant value but a known upper bound $\|\theta_i(t)\| \leq \eta$, where η is a positive constant.

Each agent is associated with a private time-varying cost function $f_i(x_i, t) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$. The collective objective of the multi-agent system is to solve the following optimization problem in a distributed manner:

$$F_{\min} = \min \sum_{i\in\mathcal{V}} f_i(x_i, t) \quad \text{s.t.} \quad x_i = x_j. \tag{3}$$

Denote the optimal trajectory that solves (3) as $x^*(t)$. The control objective is to design a distributed input $u_i(t)$ for each agent, using only local information and neighbor communication such that all agents achieve consensus and converge to the global optimal trajectory $x^*(t)$ within a prescribed-time frame.

Definition 1. *Ref.* [32]. *Consider the time-varying system (1) with an equilibrium point at the origin, i.e.,* $f_i(0,t) = 0, \forall t \geq 0$. *The origin is said to be globally prescribed-time stable if for any initial time* $t_0 \in \mathbb{R}_{\geq 0}$, *there exists a settling time* $T_a > 0$ *such that for each agent* $i \in \mathcal{V}$, *its solution* $x_i(t)$ *satisfies* $x_i(t) = 0, \forall t > t_0 + T_a$, *where the constant* T_a *is the prescribed convergence time and is independent of the initial conditions.*

Definition 2. *Ref.* [23]. *Consider system (1) with initial time* $t_0 \in \mathbb{R}_{\geq 0}$. *The time-varying scaling functions are defined as:*

$$\mathcal{P}_1(t;t_0,T_1) = \begin{cases} \left(\frac{T_1}{t_0+T_1-t}\right)^{h_1}, & \mathrm{t} \in [t_0, t_0+T_1) \\ 1, & \text{otherwise} \end{cases} \tag{4}$$

$$\mathcal{P}_2(t;t_1,T_2) = \begin{cases} \left(\frac{T_2}{t_1+T_2-t}\right)^{h_2}, & \mathrm{t} \in [t_1, t_1+T_2) \\ 1, & \text{otherwise,} \end{cases} \tag{5}$$

where $h_1, h_2 > 2$ *are design parameters,* $T_1, T_2 > 0$ *represent the duration of each phase, and* $t_1 = t_0 + T_1$ *denotes the initial time of the first and second phases, respectively.*

2.3. Assumptions and Lemmas

The subsequent theoretical analysis relies on the following fundamental assumptions and supporting lemmas.

Assumption 1. *The communication network maintains time-varying connectivity through an undirected graph* $\mathcal{G}(t)$.

Assumption 2. *Each agent i receives information only from its neighboring agents* $j \in \mathcal{N}_i$.

Assumption 3. *The optimization problem for the global time-varying cost function* $F = \sum_{i\in\mathcal{V}} f_i(x_i,t)$ *possesses a unique optimal trajectory* $x^*(t)$.

Assumption 4. *Ref.* [32]. *For each agent, the local cost function* $f_i(x_i,t)$ *and the global cost function* $\sum_{i\in\mathcal{V}} f_i(x_i,t)$ *remain continuous in time t and are twice continuously differentiable in state x. Furthermore, the gradient* $\nabla f_i(x_i,t)$, *its partial time derivative* $\frac{\partial \nabla f_i(x_i,t)}{\partial t}$, *and the Hessian matrix* $H_i(x_i,t)$ *are uniformly bounded, with* $H_i(x_i,t)$ *being invertible at all times.*

Lemma 1. *Ref.* [31]. *Consider the system (1). Suppose there exists a radially unbounded, continuous positive definite function* $V(x(t)) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ *such that its time derivative along the trajectories satisfies:*

$$\dot{V}(x) \leq -\frac{V(x)^{\mu}}{(1-\mu)\zeta T} - \frac{V(x)^{\nu}}{(\nu-1)(1-\zeta)T}, \tag{6}$$

where the parameters $0 < \mu < 1$, $\nu > 1$ *and* $0 < \zeta < 1$. *Then, the system is globally prescribed-time convergent, with the settling time bounded by* T.

Lemma 2. *Ref.* [30]. *For any non-negative scalars* $\kappa_1, \kappa_2, \cdots, \kappa_N \geq 0$, *the inequality* $\sum_{i\in\mathcal{V}} \kappa_i^a \geq \left(\sum_{i\in\mathcal{V}} \kappa_i\right)^a$ *holds for* $0 < a \leq 1$, *whereas the inequality* $\sum_{i\in\mathcal{V}} \kappa_i^a \geq N^{1-a}\left(\sum_{i\in\mathcal{V}} \kappa_i\right)^a$ *holds for* $a > 1$.

Lemma 3. *Ref.* [31]. *Let* $\mathcal{L} \in \mathbb{R}^n \times \mathbb{R}^n$ *denote the Laplacian matrix of an undirected and connected graph* $\mathcal{G}$, *with its eigenvalues ordered as* $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$. *For any real vector* $x \in \mathbb{R}^n$, *the quadratic form admits the decomposition:*

$$x^T L x = \frac{1}{2}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)(x_i - x_j)^2. \tag{7}$$

Moreover, for all vectors x satisfying $\mathbf{1}_N^T x = 0$, *the following spectral bound holds:*

$$x^T L x \geq \lambda_2 x^T x, \tag{8}$$

where λ_2 *is the second smallest eigenvalue of* $\mathcal{L}$.

Lemma 4. *Ref.* [25]. *Let $\mathcal{G} = (A, \mathcal{V})$ be an undirected graph with time-varying adjacency matrix $A(t) = [a_{ij}(t)]$. Given an odd function $w : \mathbb{R}^n \to \mathbb{R}^n$ satisfying $w(z) = -w(-z)$ for all $z \in \mathbb{R}^n$, and considering arbitrary vector sets $\{x_i\}_{i\in\mathcal{V}}$ and $\{e_i\}_{i\in\mathcal{V}}$, define the pairwise differences $x_{ij} = x_i - x_j$ and $e_{ij} = e_i - e_j$. Then, the following identity holds:*

$$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} a_{ij}(t)e_i^T w(x_{ij}) = \frac{1}{2}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} a_{ij}(t)e_{ij}^T w(x_{ij}). \tag{9}$$

3. Main Results

This section presents a new prescribed-time distributed optimization algorithm with consensus controller and distributed estimator to address the time-varying optimization problem in system (1), where the control input u_i and distributed estimator z_i for the ith agent are designed as follows:

$$u_i = \begin{cases} -\mathcal{T}_2\mathcal{X}(\mu_2,\nu_2,x_i) - \varepsilon \sum\limits_{j\in\mathcal{N}_i} sgn(x_i - x_j) - \hat{\mathcal{Z}}_{2i}^{-1}(\psi(\mu_3,\nu_3,\hat{\mathcal{Z}}_{1i}) + \hat{\mathcal{Z}}_{3i}), & \text{if } (x_i \neq x_j, j \in \mathcal{N}_i) \\ -\mathcal{X}(\mu_2,\nu_2,x_i) - \hat{\mathcal{Z}}_{2i}^{-1}(\psi(\mu_3,\nu_3,\hat{\mathcal{Z}}_{1i}) + \hat{\mathcal{Z}}_{3i}), & \text{otherwise} \end{cases} \tag{10a}$$

$$\mathcal{Z}_i = z_i + \delta_i, z_i(0) = 0, \dot{\delta}_i = \varphi_i = \left[\nabla f_i(x_i,t); H_i(x_i,t); \frac{\partial \nabla f_i(x_i,t)}{\partial t}\right], \hat{\mathcal{Z}}_i = \dot{\mathcal{Z}}_i, \tag{10b}$$

$$\dot{z}_i = \begin{cases} -\mathcal{T}_1\mathcal{S}(\mu_1,\nu_1,\mathcal{Z}_i) - \gamma \sum\limits_{j\in\mathcal{N}_i} sgn(\mathcal{Z}_i - \mathcal{Z}_j)\|\varphi_i - \varphi_j\|, & \text{if } (\mathcal{Z}_i \neq \mathcal{Z}_j, j \in \mathcal{N}_i) \\ -\mathcal{S}(\mu_1,\nu_1,\mathcal{Z}_i), & \text{otherwise,} \end{cases} \tag{10c}$$

where $\mathcal{T}_1 = k_1\frac{\dot{P}_1(t;t_0,T_1)}{P_1(t;t_0,T_1)}$ and $\mathcal{T}_2 = k_2\frac{\dot{P}_2(t;t_1,T_2)}{P_2(t;t_1,T_2)}$ represent the durations of the time-varying gain parameters over the expected designated time intervals T_1 and T_2 respectively, with z_i being an intermediate auxiliary variable. Moreover, the estimator in (10c) tracks the global averages of gradients, Hessian matrices, and partial derivatives of gradients. Denote $\hat{\mathcal{Z}}_i = \dot{\mathcal{Z}}_i = [\hat{\mathcal{Z}}_{1i}; \hat{\mathcal{Z}}_{2i}; \hat{\mathcal{Z}}_{3i}]$ as these tracked values, whose components correspond to the three optimization terms, respectively. These tracked values are then utilized in the optimization term of the control law in (10a), which drives all agents to reach consensus and converge to the optimal trajectory. Furthermore, the following functions are employed to facilitate convergence.

$$\begin{aligned} \mathcal{S}(\mu_1,\nu_1,\mathcal{Z}_i) &= \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(\mathcal{Z}_i - \mathcal{Z}_j)^{\mu_1} + \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(\mathcal{Z}_i - \mathcal{Z}_j)^{\nu_1}, \\ \mathcal{X}(\mu_2,\nu_2,x_i) &= \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(x_i - x_j)^{\mu_2} + \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(x_i - x_j)^{\nu_2}, \\ \psi(\mu_3,\nu_3,\mathcal{Z}_{1i}) &= -\alpha sig(\hat{\mathcal{Z}}_{1i})^{\mu_3} - \beta sig(\hat{\mathcal{Z}}_{1i})^{\nu_3}, \end{aligned}$$

where $0 < \mu_1,\mu_2,\mu_3 < 1$, $\nu_1,\nu_2,\nu_3 > 1$, and α, β are constants to be designed.

Remark 1. *The signum function is introduced into the control input algorithm to compensate for inconsistent time-varying optimization variables, enabling all agents to track the time-varying optimal trajectory within prescribed time. Since the designed algorithm involves a piecewise differentiable signum function, the solutions of system (3) are considered in the sense of Filippov.*

Theorem 1. *Suppose Assumptions 1–4 hold, and the parameters are selected as $\gamma > \frac{1}{N}$, $\varepsilon \geq \frac{\eta}{N}\max\limits_{i\in\mathcal{V}}\|\phi_i\|$, $\alpha = \frac{1}{2(1-\mu_3)\zeta N^{1-\mu_3}T_3}$ and $\beta = \frac{1}{2(\nu_3-1)(1-\zeta)N^{1-\nu_3}T_3}$. For the optimization problem (3), the system (1) achieves the following three objectives in sequence:*

(i) Within time interval T_1,complete the consensus estimation of the global optimization term, i.e., $\lim\limits_{t\to t_0+T_1} \mathcal{Z}_i = \mathcal{Z}_j$;

(ii) Within time interval T_2, achieve consensus convergence among the agents, i.e., $\lim\limits_{t\to t_1+T_2} x_i = x_j = x_c$;

(iii) Within time interval T_3, all agents reach the optimal value x^*, i.e., $\lim\limits_{t\to t_2+T_3} x_c = x^*$, where T_1, T_2, T_3 are preset time intervals, and $t_1 = t_0 + T_1, t_2 = t_1 + T_2$.

Proof. The prescribed-time convergence framework allows the optimization problem (3) to be decomposed into three sequentially executed subtasks: global optimization estimation, consensus convergence, and optimal trajectory tracking. The detailed proofs for each stage follow.

We first prove Phase (i). To demonstrate that the distributed estimator achieves global optimization estimation within T_1 without singularity, we analyze the behavior of the time-varying scaling function via a space-time transformation. This analysis ensures that the estimator remains bounded and avoids divergence at the prescribed time boundary. We begin by introducing a time-rescaling variable: $\tau_1(t;T_1) = -T_1 \ln(\frac{T_1+t_0-t}{T_1}) : [t_0, T_1+t_0) \to \mathbb{R}_{\geq 0}$, thereby mapping the prescribed convergence time interval $t \in [t_0, T_1+t_0)$ to a new interval $\tau_1(t;T_1) \in [0, \infty)$. Furthermore, its inverse mapping with respect to time t is given by: $t(\tau_1;T_1) = t_0 + T_1(1 - e^{-\frac{\tau_1}{T_1}}) : \mathbb{R}_{\geq 0} \to [t_0, T_1+t_0)$, then, we transform the original state variable $\mathcal{Z}(t)$ into a new variable on the transformed time scale:

$$\sigma_i(\tau_1) = \mathcal{Z}_i(t(\tau_1;T_1)). \tag{11}$$

Differentiating Equation (11) with respect to time τ_1 under the condition $\mathcal{Z}_i \neq \mathcal{Z}_j$:

$$\dot{\sigma_i}(\tau_1) = \frac{d\mathcal{Z}_i(t(\tau_1;T_1))}{dt(\tau_1;T_1)} \frac{dt(\tau_1;T_1)}{d\tau_1} = (\dot{z}_i + \dot{\delta}_i)e^{-\frac{\tau_1}{T_1}}. \tag{12}$$

Substituting (10b) and (10c) into Equation (12) yields:

$$\begin{aligned}
\dot{\sigma_i}(\tau_1) &= \left(-k_1 \frac{\dot{\mathcal{P}}_1(t;t_0,T_1)}{\mathcal{P}_1(t;t_0,T_1)} S(\mu_1,\nu_1,\mathcal{Z}_i) \ -\gamma \sum_{j\in\mathcal{N}_i} sgn(\mathcal{Z}_i - \mathcal{Z}_j) \left\|\varphi_i - \varphi_j\right\| + \varphi_i \right) e^{-\frac{\tau_1}{T_1}} \\
&= \left(-k_1 \cdot \frac{h_1}{T_1} \mathcal{P}_1^{\frac{1}{h_1}} \mathcal{S}(\mu_1,\nu_1,Z_i) - \gamma \sum_{j\in\mathcal{N}_i} sgn(Z_i - Z_j)\|\phi_i - \phi_j\| + \varphi_i \right) \mathcal{P}_1^{-\frac{1}{h_1}} \\
&= -\frac{k_1 h_1}{T_1} S(\mu_1,\nu_1,\mathcal{Z}_i) - \gamma \mathcal{P}_1^{-\frac{1}{h_1}} \sum_{j\in\mathcal{N}_i} sgn(\mathcal{Z}_i - \mathcal{Z}_j) \left\|\varphi_i - \varphi_j\right\| + \mathcal{P}_1^{-\frac{1}{h_1}} \varphi_i.
\end{aligned} \tag{13}$$

In the second equality of (13), the time-varying gain $T_1 = k_1 \frac{\dot{\mathcal{P}}_1}{\mathcal{P}_1}$ and the exponential term $e^{-\frac{\tau_1}{T_1}}$ can be derived from $\mathcal{P}_1(t;t_0,T_1) = \left(\frac{T_1}{t_0+T_1-t}\right)^{h_1}$. From this definition, we have $t_0 + T_1 - t = T_1\mathcal{P}_1^{-\frac{1}{h_1}}$, and hence $e^{-\frac{\tau_1}{T_1}} = \mathcal{P}_1^{-\frac{1}{h_1}}$. Meanwhile, differentiating $\mathcal{P}_1$ with respect to time gives $\dot{\mathcal{P}}_1 = \frac{h_1}{t_0+T_1-t}\mathcal{P}_1$. Using the relation $t_0 + T_1 - t = T_1\mathcal{P}_1^{-\frac{1}{h_1}}$, we obtain $\frac{\dot{\mathcal{P}}_1}{\mathcal{P}_1} = \frac{h_1}{T_1}\mathcal{P}_1^{\frac{1}{h_1}}$.

Then, Substituting the state transformation of Equation (11) into Equation (13) yields:

$$\dot{\sigma_i}(\tau_1) = -\frac{k_1 h_1}{T_1} S(\mu_1,\nu_1,\sigma_i) - \gamma \mathcal{P}_1^{-\frac{1}{h_1}} \sum_{j\in\mathcal{N}_i} sgn(\sigma_i - \sigma_j) \left\|\varphi_i - \varphi_j\right\| + \mathcal{P}_1^{-\frac{1}{h_1}} \varphi_i. \tag{14}$$

Next, under the new time mapping τ_1, we define an error variable: $\vartheta_i = \sigma_i - \frac{1}{N}\sum_{j\in\mathcal{V}} \sigma_j$ then select the Lyapunov function as: $V_1 = \sum_{i\in\mathcal{V}} \|\vartheta_i\|^2$ and differentiate it to obtain:

$$\begin{aligned}
\dot{V}_1 &= \sum_{i\in\mathcal{V}} \vartheta_i^T (\dot{\sigma}_i(\tau_1) - \frac{1}{N}\sum_{j\in\mathcal{V}} \dot{\sigma}_j(\tau_1)) \\
&= \sum_{i\in\mathcal{V}} {\vartheta_i}^T (\dot{\sigma}_i - \frac{1}{N}\mathcal{P}_1^{-\frac{1}{h_1}} \sum_{j\in\mathcal{V}} \varphi_j) \\
&= -\frac{k_1 h_1}{2T_1} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)(\|\vartheta_i - \vartheta_j\|^{\mu_1+1} + \|\vartheta_i - \vartheta_j\|^{\nu_1+1}) \\
&\quad - \frac{\gamma}{2}\mathcal{P}_1^{-\frac{1}{h_1}} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \|\vartheta_i - \vartheta_j\| \left\|\varphi_i - \varphi_j\right\| \\
&\quad + \frac{1}{2N}\mathcal{P}_1^{-\frac{1}{h_1}} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} (\vartheta_i - \vartheta_j)(\varphi_i - \varphi_j) \\
&\leq -\frac{k_1 h_1}{2T_1} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)(\|\vartheta_i - \vartheta_j\|^{\mu_1+1} + \|\vartheta_i - \vartheta_j\|^{\nu_1+1}) \\
&\quad + \mathcal{P}_1^{-\frac{1}{h_1}}(-\frac{\gamma}{2} + \frac{1}{2N}) \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \|\vartheta_i - \vartheta_j\| \left\|\varphi_i - \varphi_j\right\|.
\end{aligned} \tag{15}$$

By setting $(\frac{1}{2N} - \frac{1}{2}\gamma) < 0$, we obtain $\gamma > \frac{1}{N}$, and then let $\tilde{k}_1 = \frac{k_1 h_1}{T_1}$ therefore, Equation (15) can be simplified as:

$$\begin{aligned}\dot{V}_1 \leq& -\frac{\tilde{k}_1}{2}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)\|\vartheta_i - \vartheta_j\|^{\mu_1+1} - \frac{\tilde{k}_1}{2}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)\|\vartheta_i - \vartheta_j\|^{\nu_1+1} \\ \leq& -\frac{\tilde{k}_1}{2}(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(a_{ij}^{\frac{2}{\mu_1+1}}(t)\|\vartheta_i - \vartheta_j\|^2)^{\frac{\mu_1+1}{2}} + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(a_{ij}^{\frac{2}{\nu_1+1}}(t)\|\vartheta_i - \vartheta_j\|^2)^{\frac{\nu_1+1}{2}} \\ \leq& -\frac{\tilde{k}_1}{2}(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}^{\frac{2}{\mu_1+1}}(t)\|\vartheta_i - \vartheta_j\|^2)^{\frac{\mu_1+1}{2}} - \frac{\tilde{k}_1}{2}N^{\frac{1-\nu_1}{2}}(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}^{\frac{2}{\nu_1+1}}(t)\|\vartheta_i - \vartheta_j\|^2)^{\frac{\nu_1+1}{2}} \\ \leq& -\frac{\tilde{k}_1}{2}(2\lambda_2(\mathcal{L}_B))^{\frac{\mu_1+1}{2}}V^{\frac{\mu_1+1}{2}} - \frac{\tilde{k}_1}{2}N^{\frac{1-\nu_1}{2}}(2\lambda_2(\mathcal{L}_C))^{\frac{\nu_1+1}{2}}V^{\frac{\nu_1+1}{2}},\end{aligned} \tag{16}$$

where $\mathcal{L}_B$ and $\mathcal{L}_C$ represent the Laplacian matrices of graphs $\mathcal{G}_B = \left[a_{ij}^{\frac{2}{\mu_1+1}}(t)\right]$ and $\mathcal{G}_C = \left[a_{ij}^{\frac{2}{\nu_1+1}}(t)\right]$ respectively, and $\lambda_2(\cdot)$ denotes the smallest non-zero eigenvalue of the matrix. According to Lemma 2.1 of [31] there exists a time upper bound $\tau_1^* > 0$ such that $\lim_{\tau_1\to\tau_1^*}\sum_{i\in\mathcal{V}}\vartheta_i(\tau_1) = 0$ and $\sum_{i\in\mathcal{V}} E_i \equiv 0$ for $\forall\tau_1 \geq \tau_1^*$. Furthermore, it can be derived that $\lim_{\tau_1\to\tau_1^*}\sigma_i(\tau_1) = \sigma_j(\tau_1), \forall\tau_1 \geq \tau_1^*$. Under the mapping transformation (11), it can be concluded that $\lim_{t\to t_0+T_1^*}\mathcal{Z}_i(t) = \mathcal{Z}_j(t)$ and $\mathcal{Z}_i(t) \equiv \mathcal{Z}_j(t), \forall t \geq t_0 + T_1^*$. Based on the above, we obtain:

$$\hat{\mathcal{Z}}_i(t) = \dot{\mathcal{Z}}_i(t) = \frac{1}{N}\sum_{i\in\mathcal{V}}\dot{\mathcal{Z}}_i(t) = \frac{1}{N}\sum_{i\in\mathcal{V}}\dot{z}_i(t) + \frac{1}{N}\sum_{i\in\mathcal{V}}\varphi_i(t). \tag{17}$$

Since $\sum_{i\in\mathcal{V}}\dot{z}_i(t) \equiv 0$, it follows that: $\hat{\mathcal{Z}}_{1i}(t) = \frac{1}{N}\sum_{i\in\mathcal{V}}\varphi_{1i}(t) = \frac{1}{N}\sum_{i\in\mathcal{V}}\nabla f_i(x_i, t)$ and similarly, $\hat{\mathcal{Z}}_{2i}(t) = \frac{1}{N}\sum_{i\in\mathcal{V}} H_i(x_i, t)$, $\hat{\mathcal{Z}}_{3i}(t) = \frac{1}{N}\sum_{i\in\mathcal{V}}\frac{\partial\nabla f_i(x_i,t)}{\partial t}$. Thus, all optimization terms achieve accurate tracking within time T_1. It is noteworthy that the time-varying gain $\mathcal{T}_1$ is active during the interval $(t_0, t_0 + T_1^*)$ and switches to a constant value for $t \geq t_0 + T_1^*$, thereby ensuring that the entire estimator remains free from singularities.

Next, we prove Phase (ii). Based on the global optimization estimation achieved in Phase (i), we now show that all agents reach state consensus within the prescribed interval T_2. Building upon the distributed estimator in Stage (i), which has achieved global information estimation within time T_1, we now prove that all agents reach consensus within time T_2. Inspired by [32] we decompose u_i into the consensus term u_i^c and the optimization term g_i as detailed below:

$$\begin{aligned} u_i &= u_i^c - g_i, g_i = (\hat{\mathcal{Z}}_{2i}^{-1}(\psi(\mu_3, \nu_3, \hat{\mathcal{Z}}_{1i}) + \hat{\mathcal{Z}}_{3i}), \\ u_i^c &= -\mathcal{T}_2\mathcal{X}(\mu_2, \nu_2, x_i) - \varepsilon\sum_{j\in\mathcal{N}_i} sgn(x_i - x_j). \end{aligned} \tag{18}$$

Subsequently, analogous to the proof in Stage (i), we define a new time mapping as follows:

$$y_i(\tau_2) = x_i(t(\tau_2; T_2)). \tag{19}$$

Similar to Step (13), we set $\tilde{k}_2 = \frac{k_2 h_2}{T_2}$. Then, differentiating Equation (19) yields:

$$\begin{aligned}\dot{y}_i(\tau_2) =& \dot{x}_i(t(\tau_2; T_2))\frac{dt(\tau_2; T_2)}{d\tau_2} = \dot{x}_i(t(\tau_2; T_2))\mathcal{P}_2^{-\frac{1}{h_2}} \\ =& -\tilde{k}_2(\sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(x_i - x_j)^{\mu_2} + \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(x_i - x_j)^{\nu_2}) \\ &- \varepsilon\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{j\in\mathcal{N}_i} sgn(x_i - x_j) + \mathcal{P}_2^{-\frac{1}{h_2}}(d_i + g_i) \\ =& -\tilde{k}_2(\sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(y_i - y_j)^{\mu_2} + \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(y_i - y_j)^{\mu_2}) \\ &+ \mathcal{P}_2^{-\frac{1}{h_2}}(-\varepsilon\sum_{j\in\mathcal{N}_i} sgn(y_i - y_j) + d_i + g_i).\end{aligned} \tag{20}$$

Then, we define the error variable $e_i = y_i - \frac{1}{N}\sum\limits_{j\in\mathcal{V}} y_j$, and select a Lyapunov function $V_2 = \sum\limits_{i\in\mathcal{V}} \|e_i\|^2$. By differentiating V_2, we obtain:

$$\begin{aligned}
\dot{V}_2 &= \sum_{i\in\mathcal{V}} {e_i}^T(\dot{y_i}(\tau_2) - \frac{1}{N}\sum_{j\in\mathcal{V}}\dot{y_j}(\tau_2)) \\
&\le \sum_{i\in\mathcal{V}} e_i^T(-\tilde{k}_2(\sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(y_i-y_j)^{\mu_2} + \sum_{j\in\mathcal{N}_i} a_{ij}(t)sig(y_i-y_j)^{\nu_2}) \\
&\quad + \sum_{i\in\mathcal{V}} e_i^T\left(-\varepsilon\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{j\in\mathcal{N}_i} sgn(y_i-y_j)\ + \mathcal{P}_2^{-\frac{1}{h_2}}d_i - \frac{1}{N}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{j\in\mathcal{V}} d_j\right) \\
&\quad + \sum_{i\in\mathcal{V}} e_i^T(\mathcal{P}_2^{-\frac{1}{h_2}}g_i - \frac{1}{N}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{j\in\mathcal{V}} g_j).
\end{aligned} \tag{21}$$

The disturbance term d_i in Equation (21) can be simplified as follows:

$$\begin{aligned}
&\mathcal{P}_2^{-\frac{1}{h_2}}\frac{1}{N}\sum_{j\in\mathcal{V}}(d_i-d_j) \le \mathcal{P}_2^{-\frac{1}{h_2}}\frac{1}{N}\sum_{j\in\mathcal{V}}\|d_i-d_j\| \\
&\le \mathcal{P}_2^{-\frac{1}{h_2}}\frac{1}{N}\sum_{j\in\mathcal{V}}(\|d_i\|+\|d_j\|) \le \mathcal{P}_2^{-\frac{1}{h_2}}\frac{\eta}{N}\sum_{j\in\mathcal{V}}(\|\phi_i\|+\|\phi_j\|).
\end{aligned} \tag{22}$$

Since the consistency estimation of the optimization terms has been achieved within time T_1 in Stage (i), it follows that $g_i = g_j$. Therefore, the last two terms in Equation (21) cancel each other out. Then, substituting Equation (22) into Equation (21) yields:

$$\begin{aligned}
\dot{V}_2 &\le -\tilde{k}_2\left(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\mu_2} + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\nu_2}\right) \\
&\quad - \varepsilon\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} e_i sgn(y_i-y_j) + \frac{\eta}{N}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} e_i(\|\phi_i\|+\|\phi_j\|) \\
&\le -\tilde{k}_2\left(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\mu_2} + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\nu_2}\right) \\
&\quad - \frac{\varepsilon}{2}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(e_i-e_j)sgn(e_i-e_j) + \frac{\eta}{2N}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(e_i-e_j)\|\phi_i\| \\
&\le -\tilde{k}_2\left(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\mu_2} + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)e_i sig(y_i-y_j)^{\nu_2}\right) \\
&\quad - \frac{\varepsilon}{2}\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\|e_i-e_j\| + \frac{\eta}{2N}\max_{i\in\mathcal{V}}\|\phi_i\|\mathcal{P}_2^{-\frac{1}{h_2}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\|e_i-e_j\| \\
&\le -\frac{\tilde{k}_2}{2}(\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)\|e_i-e_j\|^{\mu_2+1} + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(t)\|e_i-e_j\|^{\nu_2+1}),
\end{aligned} \tag{23}$$

where the last term in the second inequality is obtained by using $\sum_{i\in\mathcal{V}} e_i = 0$ to eliminate the part involving $\|\phi_j\|$, (i.e.,$\frac{\eta}{N}\mathcal{P}_2^{-\frac{1}{h_2}}\sum\limits_{i\in\mathcal{V}}\sum\limits_{j\in\mathcal{N}_i} e_i\|\phi_j\| = 0$).

Then, by choosing the parameter $\varepsilon \ge \frac{\eta}{N}\max\limits_{i\in\mathcal{V}}\|\phi_i\|$, the last two terms in the second inequality of Equation (21) cancel each other out. Then, following a similar approach to the proof of (16), we arrive at the following inequality:

$$\dot{V}_2 \le -\frac{\tilde{k}_2}{2}(2\lambda_2(\mathcal{L}_D))^{\frac{1+\mu_2}{2}}V_2^{\frac{1+\mu_2}{2}} - \frac{\tilde{k}_2}{2}N^{\frac{1-\nu_2}{2}}(2\lambda_2(\mathcal{L}_E))^{\frac{1+\nu_2}{2}}V_2^{\frac{1+\nu_2}{2}}, \tag{24}$$

where $\mathcal{L}_D$ and $\mathcal{L}_E$ represent the Laplacian matrices of graphs $\mathcal{G}_D = \left[a_{ij}^{\frac{2}{\mu_2+1}}(t)\right]$ and $\mathcal{G}_E = \left[a_{ij}^{\frac{2}{\nu_2+1}}(t)\right]$ respectively. Therefore, there exists a finite time upper bound τ_2^* such that:

$$\lim_{\tau_2 \to \tau_2^*} \sum_{i \in \mathcal{V}} e_i(\tau_2) = \lim_{t \to t_1 + T_2^*} \sum_{i \in \mathcal{V}} e_i(t_2) = 0, \tag{25}$$

and for all $\forall t \geq t_1 + T_2^*$, $\sum\limits_{i \in \mathcal{V}} e_i \equiv 0$. Consequently, it follows that all states $x_i(t)$ achieve consensus within the prescribed time T_2, i.e., $x_i(t) = x_j(t), \forall t \geq T_2$.

Finally, we prove Phase (iii). To show convergence to the global optimum $x^*(t)$ within T_3, we consider the Lyapunov function: $V_3 = (\sum\limits_{i \in \mathcal{V}} \nabla f_i(x_c, t))^T \sum\limits_{i \in \mathcal{V}} \nabla f_i(x_c, t)$ and differentiate it to obtain the equation:

$$\begin{aligned}
\dot{V}_3 &= \frac{d}{dt}\left[\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)\right] \\
&= 2\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \frac{d}{dt}\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right) \\
&= 2\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \sum_{i \in \mathcal{V}} \left(H_i(x_c, t)\dot{x}_c + \frac{\partial \nabla f_i(x_i, t)}{\partial t}\right).
\end{aligned} \tag{26}$$

Based on the conclusions from the proof of phase (i), all optimization terms have converged to the global average (i.e., $\hat{\mathcal{Z}}_{1i} = \frac{1}{N}\sum\limits_{j \in \mathcal{V}} \nabla f_j(x_c, t)$, $\hat{\mathcal{Z}}_{2i} = \frac{1}{N}\sum\limits_{j \in \mathcal{V}} H_j(x_c, t)$, $\hat{\mathcal{Z}}_{3i} = \frac{1}{N}\sum\limits_{j \in \mathcal{V}} \frac{\partial \nabla f_i(x_i, t)}{\partial t}$). Therefore, the expression $\sum\limits_{i \in \mathcal{V}} (H_i(x_c, t)\dot{x_c} + \frac{\partial \nabla f_i(x_i, t)}{\partial t})$ can be simplified as follows:

$$\begin{aligned}
&\sum_{i \in \mathcal{V}} \left(H_i(x_c, t)\dot{x}_c + \frac{d\nabla f_i(x_c, t)}{dt}\right) \\
&= \sum_{i \in \mathcal{V}} H_i(x_c, t)\left(-N(\sum_{j \in \mathcal{V}} H_j(x_c, t))^{-1}\left(\alpha sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\mu_3} + \beta sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\nu_3}\right.\right. \\
&\quad \left.\left. + \frac{1}{N}\sum_{j \in \mathcal{V}} \frac{\partial \nabla f_i(x_i, t)}{\partial t}\right)\right) + \sum_{i \in \mathcal{V}} \frac{\partial \nabla f_i(x_i, t)}{\partial t} \\
&= -\alpha N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\mu_3} - \beta N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\nu_3} - \sum_{i \in \mathcal{V}} \frac{\partial \nabla f_i(x_i, t)}{\partial t} + \sum_{j \in \mathcal{V}} \frac{\partial \nabla f_i(x_i, t)}{\partial t} \\
&= -\alpha N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\mu_3} - \beta N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\nu_3}.
\end{aligned} \tag{27}$$

Substituting Equation (27) into Equation (26) yields the equality:

$$\begin{aligned}
\dot{V}_3 &= 2\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \left(-\alpha N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\mu_3} - \beta N sig\left(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right)^{\nu_3}\right) \\
&= 2\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \left(-\alpha N sign(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t))\left\|\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right\|^{\mu_3}\right. \\
&\quad \left. -\beta N sign(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t))\left\|\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right\|^{\nu_3}\right) \\
&= 2\left(\sum_{i \in \mathcal{V}} \nabla f_i(x_c, t)\right)^T \left(sign(\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t))\right)\left(-\alpha N\left\|\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right\|^{\mu_3}\right. \\
&\quad \left. -\beta N\left\|\frac{1}{N}\sum_{j \in \mathcal{V}} \nabla f_j(x_c, t)\right\|^{\nu_3}\right).
\end{aligned} \tag{28}$$

Note that $sign\left(\frac{1}{N}\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right)$ has the same sign as $\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)$. Thus, their inner product is always positive, and we have:

$$\left(\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right)^T sign\left(\frac{1}{N}\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right) = \left\|\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right\|. \tag{29}$$

Then, substituting Equation (29) into Equation (28) yields:

$$\begin{aligned}
\dot{V}_3 &= 2\left\|\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right\|^T\left(-\alpha N\left\|\frac{1}{N}\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right\|^{\mu_3} - \beta N\left\|\frac{1}{N}\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right\|^{\nu_3}\right)\\
&\le 2\left\|\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right\|^T\left(-\alpha N^{1-\mu_3}\left\|\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right\|^{\mu_3} - \beta N^{1-\nu_3}\left\|\sum_{j\in\mathcal{V}}\nabla f_j(x_c,t)\right\|^{\nu_3}\right)\\
&= -2\alpha N^{1-\mu_3}\left\|\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right\|^{\mu_3+1} - 2\beta N^{1-\nu_3}\left\|\sum_{i\in\mathcal{V}}\nabla f_i(x_c,t)\right\|^{\nu_3+1}\\
&= -2\alpha N^{1-\mu_3}V_3^{\frac{\mu_3+1}{2}} - 2\beta N^{1-\nu_3}V_3^{\frac{\nu_3+1}{2}}.
\end{aligned} \tag{30}$$

According to Lemma 1, by appropriately selecting the control parameters $\alpha = \frac{1}{2(1-\mu_3)\zeta N^{1-\mu_3}T_3}$ and $\beta = \frac{1}{2(\nu_3-1)(1-\zeta)N^{1-\nu_3}T_3}$, all agents can reach the optimum within the time interval T_3. This completes the entire proof. □

Remark 2. *Although the time-varying gains in (10) may appear to diverge in form, the proposed algorithm ensures practical singularity-free operation via a finite-time switching mechanism: once the distributed estimator achieves global optimization estimation (i.e., $\mathcal{Z}_i = \mathcal{Z}_j$) at $t = t_0 + T_1^* < t_0 + T_1$ and the agent states reach consensus (i.e., $x_i = x_j$) at $t = t_1 + T_2^* < t_1 + T_2$, the gains $\mathcal{T}_1$ and $\mathcal{T}_2$ are switched from time-varying forms to constant values. This mechanism guarantees that the control gains remain bounded throughout execution, thereby entirely avoiding the singularity issue caused by the divergence of time-varying scaling functions at the prescribed time boundaries.*

Remark 3. *As discussed in [30], the theoretical foundation of our algorithm relies on a Lyapunov function independent of the communication topology. Therefore, by selecting the smallest algebraic connectivity of the Laplacian matrix across all possible topologies for the proof, the convergence results can be extended to time-varying topologies.*

Remark 4. *The control protocol exhibits discontinuities between different phases, yet this does not compromise the overall system stability. Since all control inputs remain bounded and the output of each preceding stage only serves as the initial condition for the subsequent one, stability in the previous stage ensures that the system can maintain stability in the next phase based on these initial values. Thus, the overall stability of the system is preserved.*

Table 1 highlights several distinctive advantages of the proposed work compared with existing distributed optimization algorithms. It is worth noting that entries marked as N/A (static cost) or N/A (no scaling) indicate that the corresponding criterion (Hessian identity or singularity analysis) is not applicable, as those algorithms are designed for time-invariant problems or do not employ time-varying scaling functions, respectively.

For time-varying cost functions, online learning approaches [26] evaluate performance via dynamic regret, which guarantees that the average regret asymptotically approaches zero as the time horizon goes to infinity. Constrained optimization methods [28] extend the results to handle inequality and equality constraints, yet they only guarantee asymptotic convergence to the optimal solution or its vicinity. Algorithms [29] guarantee convergence within a finite settling time, but this settling time depends on initial conditions and system parameters, lacking the flexibility of user-assignable convergence time.

Table 1. Comparisons of several distributed optimization algorithms.

Method	Convergent Form	Time-Varying Cost Function	Hessian Identical	Singularity-Free
Liu et al. [7]	Asymptotic	No	N/A (static cost)	N/A (no scaling)
Carnevale et al. [13]	Exponential	No	N/A (static cost)	N/A (no scaling)
Shi et al. [10]	Finite-time	No	N/A (static cost)	N/A (no scaling)
Wu et al. [15]	Finite-time/FxT	No	N/A (static cost)	N/A (no scaling)
Zhang et al. [17]	FxT	Yes	Yes	N/A (no scaling)
Zhou et al. [23]	PT	No	N/A (static cost)	Yes(time-space)
Gong et al. [25]	PT	No	No	No
Deng [26]	N/A(Dynamic regret)	Yes	No	N/A (no scaling)
Rahili et al. ([27] Theorem 3.9)	Exponential	Yes	Yes	N/A (no scaling)
Rahili et al. ([27] Theorem 4.3)	Exponential	Yes	Yes	N/A (no scaling)
Rahili et al. ([27] Theorem 4.9)	Asymptotic	Yes	Yes	N/A (no scaling)
Sun et al. [28]	Exponential	Yes	No	N/A (no scaling)
Shi et al. [29]	Finite-time	Yes	No	N/A (no scaling)
Zhang et al. [30]	PT	Yes	Yes	Yes
Cui et al. [31]	PT	Yes	No	Yes
Chen et al. [32]	PT	Yes	No	No
Ye et al. [33]	PT	N/A	N/A	Yes (bounded gain)
Ding et al. [34]	PT	N/A	N/A	Yes (delay)
Orlov et al. [35]	PT	N/A	N/A	Yes (time-space)
Proposed Method	PT	Yes	No	Yes (time-space)

In contrast, our work achieves prescribed-time convergence independent of initial conditions, which allows the original complex optimization problem to be decoupled into multiple subtasks completed sequentially. This decoupling simplifies both the algorithm design and the theoretical analysis, as each subtask can be addressed independently with a dedicated Lyapunov function while ensuring overall convergence within the prescribed time. Moreover, our approach relaxes the requirement of identical local Hessian matrices, which is necessary in the fixed-time algorithm [17], prescribed-time algorithms [25, 30], and the asymptotic approach in [27]. Although [28,29,31] do not require identical Hessians, the algorithm in [31] achieves prescribed-time convergence through fixed-time lemma backtracking with complicated norm operations and higher computational cost, while [28,29] focus on asymptotic or finite-time convergence rather than user-assignable prescribed-time convergence.

In terms of singularity avoidance, unlike prescribed-time algorithms [25,32] that also employ time-varying scaling functions, our design based on time-space deformation theory fundamentally avoids the singularity issue caused by gain divergence at the prescribed time horizon. Furthermore, unlike the stabilization control approaches in [33–35] which achieve singularity-free prescribed-time convergence in a non-optimization context, our work successfully integrates this key technique into a distributed optimization framework.

In addition, compared to methods that are limited to static topologies [17,25,31], the proposed edge-based protocol extends its applicability to time-varying undirected connected graphs, which enhances its robustness and practicality in dynamic network environments.

4. Simulation Study

In this section, we validate the algorithm's singularity-free nature, consistency, and optimality by designing an optimal formation problem for UAVs. First, consider a UAV system consisting of six agents, where the switching topology is governed by the switching signal shown in Figure 1.

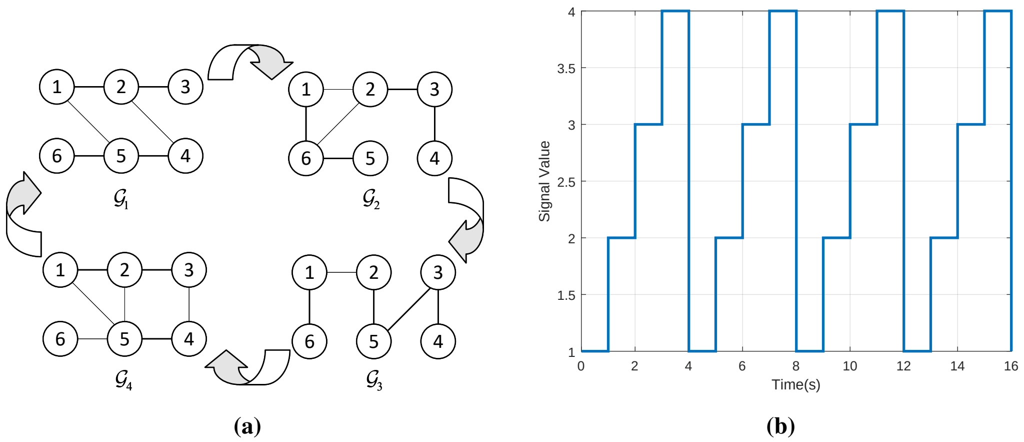


Figure 1. (**a**): Switching communication topology of the multi-agent system, (**b**): Communication switching signal.

Each agent is associated with a supply station whose position varies with time. The time-varying position coordinates of the supply station are modeled as:

$$M_i = \begin{bmatrix} 4\cos(t) + 2\cos(\frac{2\pi(i-1)}{6}) \\ 4\sin(t) + 2\sin(\frac{2\pi(i-1)}{6}) \end{bmatrix}.$$

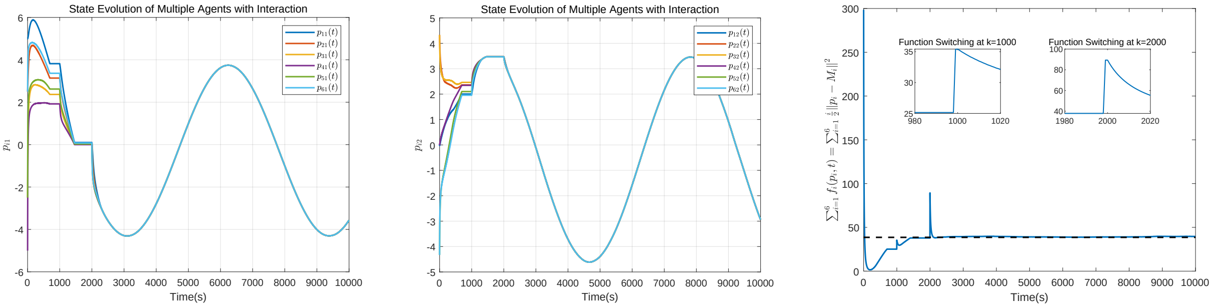
The supply stations maintain a regular hexagonal formation with a radius of 2, whose center point undergoes circular motion with a radius of 4. Each agent needs to converge within a specified time to the point that minimizes the sum of distances to the six supply stations and maintain the formation shape. Therefore, the objective function is defined as: $f_i(p_i, t) = \frac{i}{2}\|p_i - M_i\|^2$, where $p_i = x_i - h_i$ and $h_i = \left[\cos(\frac{2\pi(i-1)}{6}), \sin(\frac{2\pi(i-1)}{6})\right]$ $i \in \mathcal{V}$. Here, h_i represents the designed formation shape. Through calculation, we obtain:

$$\begin{aligned} \nabla f(p_i, t) &= \begin{bmatrix} i(p_{i1} - 4\cos(t) - 2\cos(\frac{2\pi(i-1)}{6})) \\ i(p_{i2} - 4\sin(t) - 2\sin(\frac{2\pi(i-1)}{6})) \end{bmatrix}, \\ \frac{\partial \nabla f(p_i, t)}{\partial t} &= [4i\sin(t); -4i\cos(t)], \\ H_i^{-1}(p_i, t) &= \begin{bmatrix} \frac{1}{i} & 0 \\ 0 & \frac{1}{i} \end{bmatrix}. \end{aligned}$$

The system parameters are set as: $\mu_1 = \mu_2 = \mu_3 = \frac{2}{5}, \nu_1 = \nu_2 = \nu_3 = \frac{8}{5}$, $k_1 = k_2 = 1, \zeta = 0.5$, $h_1 = h_2 = 2$,$\gamma = 3$ and the initial values of the agents are set as: $x_i(0) = \left[6\cos(\frac{2\pi(i-1)}{6}), 6\sin(\frac{2\pi(i-1)}{6})\right]$. The nonlinear function is represented by: $d_i = [0.1\sin(it); 0.1\cos(it)]$, hence we set the parameter $\varepsilon = 0.2$. The planned convergence times for the three sub-phases are: $T_1 = T_2 = T_3 = 1$, therefore, the parameters α and β are calculated as follows: $\alpha = 0.5688, \beta = 4.8836$.

As demonstrated in Figure 2a,b the algorithm successfully accomplishes its three designated subtasks within their respective 1-s (1000 iterations) intervals: global optimization estimation in T_1, consensus convergence in T_2, and optimal trajectory tracking in T_3. Moreover, it is particularly important to note that the control inputs of our algorithm remain bounded during each switching phase. The spikes observed in Figure 2c at the switching instants are caused by the parameter variation when the control protocol switches from a time-varying gain to a constant. Furthermore, this transient phenomenon does not cause singularity issues. The control input for each phase is computed based on the bounded terminal states from the preceding phase, which prevents the time-varying scaling function from growing unbounded. The system ultimately converges to the theoretical optimum of 38.5714 as indicated by the black dashed line.

Figures 3 and 4 depict the evolution of the UAV formation over time in 3D and 2D views, respectively. It can be observed that all agents robustly accomplish the formation task within the prescribed-time and successfully track the optimal trajectory (black dashed circle), despite the presence of external nonlinear disturbances.



(a) **(b)** **(c)**

Figure 2. (**a**): Evolution trajectory of p_{i1}, (**b**): Evolution trajectory of p_{i2}, and (**c**): Evolution trajectory of the total cost function.

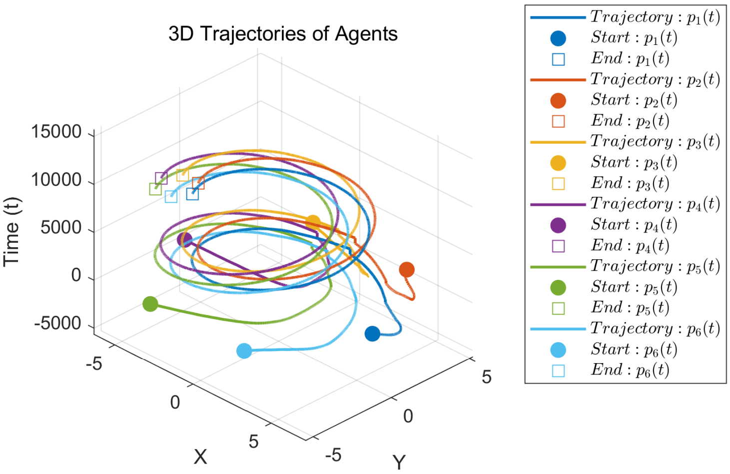


Figure 3. State evolution diagrams: The time-varying trajectory evolution of all intelligent agents in 3D graph.

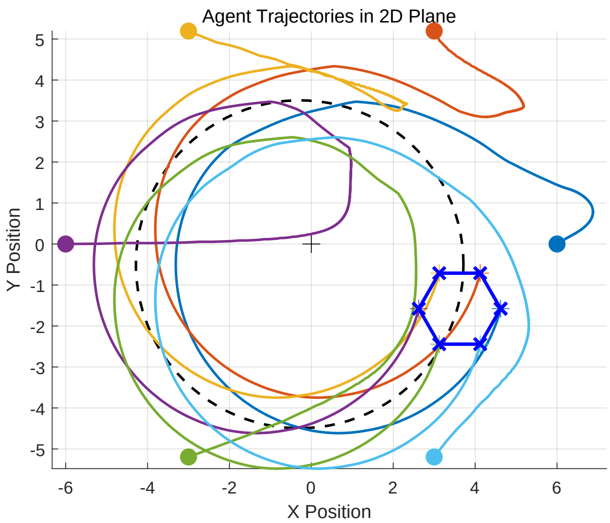


Figure 4. State evolution diagrams: The time-varying trajectory evolution of all intelligent agents in 2D graph.

5. Conclusions and Future Work

This paper presents a distributed optimization algorithm with prescribed-time convergence and singularity-free characteristics for nonlinear multi-agent systems under time-varying cost functions and dynamic communication topologies. The proposed approach eliminates the singularity issues commonly induced by time-varying scaling functions and relaxes the restriction in existing methods that requires all local cost functions to have identical Hessian matrices. Based on the designed distributed optimization estimator, global information including the gradient, Hessian matrix, and the time partial derivative of the gradient can be indirectly obtained. This information

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is then used in the optimization process to drive all agents to converge to the globally optimal trajectory. Moreover, rigorous theoretical proof is provided through the construction of multiple Lyapunov functions and the incorporation of time-space deformation theory, thereby demonstrating global average estimation, optimal trajectory convergence, and singularity-free performance. Finally, we validate the algorithm's singularity-free property, prescribed-time convergence, and optimality through a UAV formation experiment. Future work will focus on reducing the reliance on second-order partial derivatives and extending the algorithm to more complex higher-order systems.

Author Contributions

H.W.: conceptualization, methodology, software, writing—original draft preparation; T.D.: writing—original draft preparation; Y.Y.: writing—review and editing, supervision; T.L.: writing—review and editing, supervision; A.W.: writing—review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

During the preparation of this work, the authors used DeepSeek to polish the language of the manuscript. After using this tool, the authors reviewed and edited the content as needed and took full responsibility for the content of the published article.

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