International Journal of
Network Dynamics and Intelligence (IJNDI)

Article

Recursive Filtering for Complex Networks with Filter-and-Forward Relays Subject to Fading Channels

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Received: 15 July 2025; Revised: 25 November 2025; Accepted: 7 January 2026; Published: 6 March 2026

Abstract: In this paper, we study the filtering issue for a class of complex networks (CNs) with filter-and-forward relays (FFR) under fading channels. To facilitate transmission, the FFRs deployed between the sensor and filter are employed to orchestrate the signal communication. Here, the phenomena of fading channels are considered in both the sensor-relay and relay-filter channels, which are depicted by two sets of random variables. The aim of this paper is to jointly design the recursive filters for the FFRs and the underlying CNs in the presence of fading channels. Initially, the inductive method is utilized to obtain upper bounds on the filtering error covariances which, by following this, are minimized by the design of suitable filter gains. Finally, an example is given to showcase the effectiveness of the filtering strategy.

Keywords: recursive filtering; complex networks; relay networks; fading channels

1. Introduction

For decades, research interest in complex networks (CNs) has surged owing mainly to its effective ability to describe various large-scale systems including smart grid systems, transportation systems, social networks and biological engineering [1–6]. The defining feature of CN lies in that it composes a vast number of interconnected nodes, whose dynamics are influenced by their neighboring nodes within a given topology. In particular, the complex dynamics and intricate couplings of CN nodes have generated escalating attention yielding many excellent findings related to stability, control, synchronization and filtering (or estimation), see e.g., [7–11].

For CNs, the information of node states has been well recognized to be practically significant for accomplishing certain synchronization/tracking tasks. Unfortunately, due mainly to the resource constraints, obtaining/computing all state information is usually challenging, especially in huge-scale networks. In response, over the years, the filtering issue, aiming at offering estimates of node states with help of partial measurements, has appeared as a prominent research topic with many effective filtering algorithms proposed in the literature, see e.g., [12–16]. For example, in [13, 17, 18], the variance-constrained filtering method, the H_∞ state estimation strategy and set-membership filtering techniques have been put forward for CNs with respect to Gaussian-type noises, energy bounded disturbances and unknown-but-bounded noises, respectively.

It is noteworthy that, most existing results on the filtering issue have been obtained based on the foundational presumption that sensors are capable of transmitting their signals to the filter across arbitrary distances without limitation. This presumption, however, proves unrealistic in the field of wireless communication particularly in long-distance scenarios, when considering the technical/physical constraints of the energy-limitation sensors. To tackle the transmission concerns, the relay-aided techniques have been extensively employed in practice, whose essence is to receive/process the signal from sensor and then forward it to the distant filter with aim to enhance the reliability of long-distance transmission. In most recent, the use of relay-based strategies in filtering issues has been explored in [14,19–22] for various networked systems.

Broadly saying, common relays can be classified into three types, i.e. compress-and-forward relays, amplify-and-forward relays, and filter-and-forward relays (FFRs), see [23–28]. In FFRs, the relay performs a linear filtering on the incoming signal and immediately retransmits the filtered output to the destination. A distinct benefit of FFRs lies in its ability to realize a trade-off between the system performance and the computational complexity [29]. Owing to this advantage, the FFRs have been widely embraced within the signal processing community, see e.g., [30,31]. On the other hand, as a kind of resource that impairs the signal quality in wireless communication, fading inevitably arise from the limited bandwidth, shadowing or multiple-path propagation. Accordingly, research on the control/filtering issues under fading effects has emerged as a hot topic in the last years with some elegant results displayed in [32–36]. Notably, FFRs have not yet been examined in the filtering issue for CNs, especially not in cases under the fading effects. This unaddressed gap promotes the main motivation of the current study.

Building upon the above discussions, this paper delves into the study of estimation of CNs under the FFR framework. The key contributions can be summarized as follows: (1) FFRs are, for the first time, introduced into the design of recursive filters of CNs, hence providing an insight closer to practical applications; (2) Upper bounds on the filtering error covariances(FECs) are obtained by solving two recursive equations, which are seen as an important index for evaluating the filtering performance; and (3) The desired filters are jointly designed for both the FFRs and the CNs to minimize the obtained upper bounds, which ensures the efficiency of the estimation process.

Notations. In this paper, the notation is fairly standard. $\mathbb{R}^n$ and $\text{diag}\{\cdots\}$ mean the n-dimensional Euclidean space and the block diagonal matrix. For a matrix $\eth$, let $(\eth)^{-1}$, $(\eth)^T$, $\lambda_{\max}(\eth)$, $\mathbb{E}\{\eth\}$ and $\text{tr}\{\eth\}$ be the inverse, the transpose, the largest singular value, the mathematical expectation and the trace of $\eth$. For symmetric matrix $\eth$, $\eth > 0$ ($\eth \geq 0$) represents that $\eth$ is positive (semi-positive).

2. Problem Formulation

Consider a CN having Q nodes whose dynamics is described as

$$\begin{aligned} x^p_{\iota+1} &= A^p_\iota x^p_\iota + \sum_{q=1}^{Q} \omega_{pq}\Gamma x^q_\iota + B^p_\iota w_\iota \\ y^p_\iota &= C^p_\iota x^p_\iota + D^p_\iota \upsilon_\iota \end{aligned} \tag{1}$$

for $p = 1, \ldots, Q$, the state and measurement of node p are denoted by $x^p_\iota \in \mathbb{R}^{n_x}$ and $y^p_\iota \in \mathbb{R}^{n_y}$, respectively. $W = W^T = (\omega_{pq})_{Q\times Q}$ is the outer-coupled matrix with $\omega_{pp} = -\sum_{q=1,q\neq p}^{Q} \omega_{pq}$ and $\omega_{pq} \geq 0$ $(p \neq q)$ but not all elements zeros. $\Gamma = \text{diag}\{o_1, \ldots, o_{n_x}\}$ represents an inner coupling matrix. A^p_ι, B^p_ι, C^p_ι and D^p_ι are known matrices. $w_\iota \in \mathbb{R}^{n_w}$ and $\upsilon_\iota \in \mathbb{R}^{n_\upsilon}$ are noises obeying Gaussian distributions with mean 0 and variances $\mathcal{W}_\iota$ and $\mathcal{V}_\iota$. The initial values of state x^p_0 is a Gaussian distributed variable with $\mathbb{E}\{x^p_0\} = \bar{x}_p$ and $\mathbb{E}\{(x^p_0 - \bar{x}_p)(x^p_0 - \bar{x}_p)^T\} = \Xi_p$.

In this paper, for each node p, its measurement output is transmitted in accordance with an FFR with aim to improve the signal quality especially in long-distance wireless transmission. The FFR first receives the data from sensor and then generates an estimate by its own filter. Meanwhile, the relay forwards such an estimate to the remote filter. Here, denoted $\widehat{y}^p_\iota$ by the received signal of the p-th FFR with

$$\widehat{y}^p_\iota = \sqrt{E_p}\tau^p_\iota y^p_\iota + \vartheta_\iota \tag{2}$$

where E_p is the average transmission energy of sensor node p. $\vartheta_\iota \in \mathbb{R}^{n_y}$ is the channel noise obeying Gaussian distribution with mean 0 and variances Φ_ι. $\tau^p_\iota \in \mathbb{R}$ is the stochastic fading coefficient in the sensor-to-relay channel with $\mathbb{E}\{\tau^p_\iota\} = \bar{\tau}_p$ and $\mathbb{E}\{(\tau^p_\iota - \bar{\tau}_p)^2\} = \vec{\tau}_p$.

Based on the received data $\widehat{y}^p_\iota$, the filter in the relay side is construed as follows:

$$\psi^p_\iota = A^p_{\iota-1}\psi^p_{\iota-1} + K^p_\iota(\widehat{y}^p_\iota - \sqrt{E_p}\bar{\tau}_p C^p_\iota A^p_{\iota-1}\psi^p_{\iota-1}) \tag{3}$$

where ψ^p_ι is the estimate signal of state x^p_ι in the relay p. K^p_ι is the filter gain to be designed.

Afterward, let $\widetilde{\psi}^p_\iota = C^p_\iota \psi^p_\iota$ be the signal delivered by the relay to the remote filter. The real signal received by the remote filter is

$$\widetilde{y}^p_\iota = \sqrt{F_p}\mu^p_\iota \widetilde{\psi}^p_\iota + \nu_\iota \tag{4}$$

where F_p is the average energy delivered by the relay p. $\nu_\iota \in \mathbb{R}^{n_y}$ is the channel noise obeying Gaussian distribution with mean 0 and variances Ψ_ι. $\mu^p_\iota \in \mathbb{R}$ stands for the stochastic fading coefficient in the relay-to-filter channel with $\mathbb{E}\{\mu^p_\iota\} = \bar{\mu}_p$ and $\mathbb{E}\{(\mu^p_\iota - \bar{\mu}_p)^2\} = \vec{\mu}_p$.

Assumption 1. *All these random variables x^p_0, w_ι, υ_ι, ϑ_ι, ν_ι, τ^p_ι and μ^p_ι are auto-uncorrelated and mutually uncorrelated.*

Remark 1. *In this paper, we consider the relay-based communication between the sensor and filter for the purpose of enhance the signal quality by splitting the long transmission path into two shorter ones. Specifically, the FFR (2)–(4) has been deployed due to its ability to suppress the effects of channel noises on the delivered signal* *by developing a suitable filtering strategy in the relay node, thereby guaranteeing a satisfied communication performance. In addition, note that the fading phenomenon is also considered in both the sensor-relay channel and the relay-filter channel to account for the possibly unreliable wireless network.*

With the local data $\widetilde{y}_\iota^p$, the following filter is constructed for CN (1):

$$\begin{aligned} \widehat{x}_{\iota+1|\iota}^p =& A_\iota^p \widehat{x}_{\iota|\iota}^p + \sum_{q=1}^{Q} \omega_{pq} \Gamma \widehat{x}_{\iota|\iota}^q \\ \widehat{x}_{\iota+1|\iota+1}^p =& \widehat{x}_{\iota+1|\iota}^p + G_{\iota+1}^p \big(\widetilde{y}_{\iota+1}^p - \sqrt{F_p} \bar{\mu}_p C_{\iota+1}^p \widehat{x}_{\iota+1|\iota}^p \big) \end{aligned} \tag{5}$$

where, at instant ι, the one-step prediction of x_ι^p is denoted by $\widehat{x}_{\iota+1|\iota}^p$ and the estimate of x_ι^p is denoted by $\widehat{x}_{\iota|\iota}^p$. The gain of the filter designed later is denoted by G_ι^p.

Defining $\overrightarrow{x}_\iota^p \triangleq x_\iota^p - \psi_\iota^p$, it is known from (1) and (3) that

$$\begin{aligned} \overrightarrow{x}_{\iota+1}^p =& (A_\iota^p - \sqrt{E_p} \bar{\tau}_p K_{\iota+1}^p C_{\iota+1}^p A_\iota^p) \overrightarrow{x}_\iota^p + (I - \sqrt{E_p} \bar{\tau}_p K_{\iota+1}^p C_{\iota+1}^p) \sum_{q=1}^{Q} \omega_{pq} \Gamma x_\iota^q \\ & - \sqrt{E_p} \widetilde{\tau}_{\iota+1}^p K_{\iota+1}^p C_{\iota+1}^p A_\iota^p x_\iota^p - \sqrt{E_p} \widetilde{\tau}_{\iota+1}^p K_{\iota+1}^p C_{\iota+1}^p \sum_{q=1}^{Q} \omega_{pq} \Gamma x_\iota^q \\ & + (I - \sqrt{E_p} \bar{\tau}_p K_{\iota+1}^p C_{\iota+1}^p) B_\iota^p w_\iota - \sqrt{E_p} \widetilde{\tau}_{\iota+1}^p K_{\iota+1}^p C_{\iota+1}^p B_\iota^p w_\iota \\ & - \sqrt{E_p} \bar{\tau}_p K_{\iota+1}^p D_{\iota+1}^p \upsilon_{\iota+1} - \sqrt{E_p} \widetilde{\tau}_{\iota+1}^p K_{\iota+1}^p D_{\iota+1}^p \upsilon_{\iota+1} - K_{\iota+1}^p \vartheta_{\iota+1} \end{aligned} \tag{6}$$

with $\widetilde{\tau}_{\iota+1}^p = \tau_{\iota+1}^p - \bar{\tau}_p$.

Defining $\widetilde{x}_{\iota+1|\iota}^p \triangleq x_{\iota+1}^p - \widehat{x}_{\iota+1|\iota}^p$ and $\widetilde{x}_{\iota+1|\iota+1}^p \triangleq x_{\iota+1}^p - \widehat{x}_{\iota+1|\iota+1}^p$ by the prediction error and the filtering error, according to (1) and (5), it is easy to obtain the following error dynamics:

$$\begin{aligned} \widetilde{x}_{\iota+1|\iota}^p =& A_\iota^p \widetilde{x}_{\iota|\iota}^p + \sum_{q=1}^{Q} \omega_{pq} \Gamma \widetilde{x}_{\iota|\iota}^q + B_\iota^p w_\iota \\ \widetilde{x}_{\iota+1|\iota+1}^p =& (I - \sqrt{F_p} \bar{\mu}_p G_{\iota+1}^p C_{\iota+1}^p) \widetilde{x}_{\iota+1|\iota}^p - \sqrt{F_p} \widetilde{\mu}_{\iota+1}^p G_{\iota+1}^p C_{\iota+1}^p x_{\iota+1}^p \\ & + \sqrt{F_p} \widetilde{\mu}_{\iota+1}^p G_{\iota+1}^p C_{\iota+1}^p \overrightarrow{x}_{\iota+1}^p + \sqrt{F_p} \bar{\mu}_p G_{\iota+1}^p C_{\iota+1}^p \overrightarrow{x}_{\iota+1}^p - G_{\iota+1}^p \nu_{\iota+1} \end{aligned} \tag{7}$$

where $\widetilde{\mu}_{\iota+1}^p = \mu_{\iota+1}^p - \bar{\mu}_p$.

Now, denote the FECs as $\mathfrak{X}_{\iota|\iota}^p \triangleq \mathbb{E}\{\widetilde{x}_{\iota|\iota}^p \widetilde{x}_{\iota|\iota}^{pT}\}$ and $\mathfrak{Z}_\iota^p \triangleq \mathbb{E}\{\overrightarrow{x}_\iota^p \overrightarrow{x}_\iota^{pT}\}$. We are aimed at designing filters of forms (3) and (5) such that upper bounds on FECs are ensured and then minimized by designing appropriate gain K_ι^p and G_ι^p, respectively.

3. Main Results

The following three helpful lemmas are given to obtain our main results.

Lemma 1. *Given matrices $\digamma$ and $\daleth$, and scalar $\pi > 0$, the following holds:*

$$\digamma \daleth^T + \daleth \digamma^T \leq \pi \digamma \digamma^T + \pi^{-1} \daleth \daleth^T.$$

Lemma 2. *Denote $\Re_\iota^p \triangleq \mathbb{E}\{x_\iota^p x_\iota^{pT}\}$. For a given scalar a_1, if there exists a set of positive definite matrices $\overline{\Re}_\iota^p$ satisfying*

$$\overline{\Re}_{\iota+1}^p = (1 + a_1) A_\iota^p \overline{\Re}_\iota^p A_\iota^{pT} + B_\iota^p \mathcal{W}_\iota B_\iota^{pT} + (1 + a_1^{-1}) \widehat{\omega} \sum_{q=1}^{Q} |\omega_{pq}| \Gamma \overline{\Re}_\iota^q \Gamma \tag{8}$$

with initial value $\overline{\Re}_0^p = \Xi_p + \bar{x}_p \bar{x}_p^T$ and $\widehat{\omega} = \sum_{q=1}^{Q} |\omega_{pq}|$, then $\overline{\Re}_\iota^p$ is an upper bound of $\Re_\iota^p$, i.e. $\Re_\iota^p \leq \overline{\Re}_\iota^p$.

Proof. It follows from (1) that

$$
\begin{aligned}
\Re^{p}_{\iota+1} =& \mathbb{E}\{x^{p}_{\iota+1}x^{pT}_{\iota+1}\} \\
=&\mathbb{E}\Big\{\Big[A^{p}_{\iota}x^{p}_{\iota}+\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota}+B^{p}_{\iota}w_{\iota}\Big]\Big[A^{p}_{\iota}x^{p}_{\iota}+\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota}+B^{p}_{\iota}w_{\iota}\Big]^{T}\Big\} \\
\leq&(1+a_1)\mathbb{E}\Big\{A^{p}_{\iota}x^{p}_{\iota}x^{p}_{\iota}A^{pT}_{\iota}\Big\}+(1+a_1^{-1})\mathbb{E}\Big\{\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota} \\
&\times\Big[\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota}\Big]^{T}\Big\}+B^{p}_{\iota}\mathbb{E}\Big\{w_{\iota}w^{T}_{\iota}\Big\}B^{pT}_{\iota}
\end{aligned} \tag{9}
$$

by using Lemma 1.

Furthermore, we deduce that

$$
\begin{aligned}
&\mathbb{E}\Big\{\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota}\Big[\sum_{q=1}^{Q}\omega_{pq}\Gamma x^{q}_{\iota}\Big]^{T}\Big\} \\
=&\frac{1}{2}\mathbb{E}\Big\{\sum_{q=1}^{Q}\sum_{l=1}^{Q}\omega_{pq}\omega_{pl}\Gamma\Big[x^{q}_{\iota}x^{lT}_{\iota}+x^{l}_{\iota}x^{qT}_{\iota}\Big]\Gamma^{T}\Big\} \\
\leq&\frac{1}{2}\mathbb{E}\Big\{\sum_{q=1}^{Q}\sum_{l=1}^{Q}|\omega_{pq}||\omega_{pl}|\Gamma\Big[x^{q}_{\iota}x^{qT}_{\iota}+x^{l}_{\iota}x^{lT}_{\iota}\Big]\Gamma^{T}\Big\} \\
\leq&\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\Re^{q}_{\iota}\Gamma^{T}.
\end{aligned} \tag{10}
$$

Inserting (10) into (9) results in

$$
\begin{aligned}
\Re^{p}_{\iota+1}\leq&(1+a_1)A^{p}_{\iota}\Re^{p}_{\iota}A^{pT}_{\iota}+B^{p}_{\iota}\mathcal{W}_{\iota}B^{pT}_{\iota} \\
&+(1+a_1^{-1})\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\Re^{q}_{\iota}\Gamma^{T}.
\end{aligned} \tag{11}
$$

Clearly, we can confirm $\Re^{p}_{\iota}\leq\overline{\Re}^{p}_{\iota}$ based on the mathematical induction. □

Lemma 3. *For given scalars a_2 and a_3, if there exists a set of positive definite matrices $\overline{\mathfrak{Z}}^{p}_{\iota}$ satisfying*

$$
\begin{aligned}
\overline{\mathfrak{Z}}^{p}_{\iota+1}=&(1+a_2)(A^{p}_{\iota}-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1}A^{p}_{\iota})\overline{\mathfrak{Z}}^{p}_{\iota}(A^{p}_{\iota}-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1}A^{p}_{\iota})^{T} \\
&+(1+a_2^{-1})(I-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1})\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\Re}^{q}_{\iota}\Gamma^{T}\Big)(I-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1})^{T} \\
&+(1+a_3)E_p\vec{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1}A^{p}_{\iota}\overline{\Re}^{p}_{\iota}A^{pT}_{\iota}C^{pT}_{\iota+1}K^{pT}_{\iota+1}+(1+a_3^{-1})E_p\vec{\tau}_pK^{p}_{\iota+1} \\
&\times C^{p}_{\iota+1}\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\Re}^{q}_{\iota}\Gamma^{T}\Big)C^{pT}_{\iota+1}K^{pT}_{\iota+1}+(I-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1})B^{p}_{\iota}\mathcal{W}_{\iota} \\
&\times B^{pT}_{\iota}(I-\sqrt{E_p}\bar{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1})^{T}+E_p\vec{\tau}_pK^{p}_{\iota+1}C^{p}_{\iota+1}B^{p}_{\iota}\mathcal{W}_{\iota}B^{pT}_{\iota}C^{pT}_{\iota+1}K^{pT}_{\iota+1} \\
&+E_p(\bar{\tau}_p^{2}+\vec{\tau}_p)K^{p}_{\iota+1}D^{p}_{\iota+1}\mathcal{V}_{\iota+1}D^{pT}_{\iota+1}K^{pT}_{\iota+1}+K^{p}_{\iota+1}\Phi_{\iota+1}K^{pT}_{\iota+1}
\end{aligned} \tag{12}
$$

with initial value $\overline{\mathfrak{Z}}^{p}_{0}=\Xi_p$, then $\overline{\mathfrak{Z}}^{p}_{\iota}$ is an upper bound of $\mathfrak{Z}^{p}_{\iota}$, i.e. $\mathfrak{Z}^{p}_{\iota}\leq\overline{\mathfrak{Z}}^{p}_{\iota}$.

Proof. Referring to (6), we have

$$
\begin{aligned}
\mathfrak{Z}^p_{\iota+1} =&\mathbb{E}\big\{\overrightarrow{x}^p_{\iota+1}\overrightarrow{x}^{pT}_{\iota+1}\big\}\\
=&\mathbb{E}\bigg\{\Big[(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)\overrightarrow{x}^p_\iota+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\\
&-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}A^p_\iota x^p_\iota-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\\
&\times B^p_\iota w_\iota-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}B^p_\iota w_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}D^p_{\iota+1}\upsilon_{\iota+1}-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}D^p_{\iota+1}\upsilon_{\iota+1}\\
&-K^p_{\iota+1}\vartheta_{\iota+1}\Big]\Big[(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)\overrightarrow{x}^p_\iota+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\\
&-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}A^p_\iota x^p_\iota-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\\
&\times B^p_\iota w_\iota-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}C^p_{\iota+1}B^p_\iota w_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}D^p_{\iota+1}\upsilon_{\iota+1}-\sqrt{E_p}\tilde{\tau}^p_{\iota+1}K^p_{\iota+1}D^p_{\iota+1}\upsilon_{\iota+1}\\
&-K^p_{\iota+1}\vartheta_{\iota+1}\Big]^T\bigg\}\\
\leq&(1+a_2)(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)\mathbb{E}\Big\{\overrightarrow{x}^p_\iota\overrightarrow{x}^{pT}_\iota\Big\}(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)^T\\
&+(1+a_2^{-1})(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\mathbb{E}\bigg\{\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\Big[\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\Big]^T\bigg\}\\
&\times(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})^T+(1+a_3)\vec{\tau}_pE_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota\mathbb{E}\Big\{x^p_\iota x^{pT}_\iota\Big\}A^{pT}_\iota C^{pT}_{\iota+1}K^{pT}_{\iota+1}\\
&+(1+a_3^{-1})\vec{\tau}_pE_pK^p_{\iota+1}C^p_{\iota+1}\mathbb{E}\bigg\{\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\Big[\sum_{q=1}^{Q}\omega_{pq}\Gamma x^q_\iota\Big]^T\bigg\}C^{pT}_{\iota+1}K^{pT}_{\iota+1}\\
&+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})B^p_\iota\mathcal{W}_\iota B^{pT}_\iota(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})^T+E_p\vec{\tau}_pK^p_{\iota+1}C^p_{\iota+1}B^p_\iota\\
&\times\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1}K^{pT}_{\iota+1}+E_p(\bar{\tau}_p^2+\vec{\tau}_p)K^p_{\iota+1}D^p_{\iota+1}\mathcal{V}_{\iota+1}D^{pT}_{\iota+1}K^{pT}_{\iota+1}+K^p_{\iota+1}\Phi_{\iota+1}K^{pT}_{\iota+1}.
\end{aligned}
\tag{13}
$$

Similar to (10), it contributes to

$$
\begin{aligned}
\mathfrak{Z}^p_{\iota+1}\leq&(1+a_2)(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)\mathfrak{Z}^p_\iota(A^p_\iota-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota)^T\\
&+(1+a_2^{-1})(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\mathfrak{R}^q_\iota\Gamma^T\Big)(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})^T\\
&+(1+a_3)E_p\vec{\tau}_pK^p_{\iota+1}C^p_{\iota+1}A^p_\iota\mathfrak{R}^p_\iota A^{pT}_\iota C^{pT}_{\iota+1}K^{pT}_{\iota+1}+(1+a_3^{-1})E_p\vec{\tau}_pK^p_{\iota+1}\\
&\times C^p_{\iota+1}\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\mathfrak{R}^q_\iota\Gamma^T\Big)C^{pT}_{\iota+1}K^{pT}_{\iota+1}+(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})B^p_\iota\mathcal{W}_\iota\\
&\times B^{pT}_\iota(I-\sqrt{E_p}\bar{\tau}_pK^p_{\iota+1}C^p_{\iota+1})^T+E_p\vec{\tau}_pK^p_{\iota+1}C^p_{\iota+1}B^p_\iota\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1}K^{pT}_{\iota+1}\\
&+E_p(\bar{\tau}_p^2+\vec{\tau}_p)K^p_{\iota+1}D^p_{\iota+1}\mathcal{V}_{\iota+1}D^{pT}_{\iota+1}K^{pT}_{\iota+1}+K^p_{\iota+1}\Phi_{\iota+1}K^{pT}_{\iota+1}.
\end{aligned}
\tag{14}
$$

Bearing in mind Lemma 2, it is not difficult to see that $\mathfrak{Z}^p_\iota\leq\overline{\mathfrak{Z}}^p_\iota$ holds by means of mathematical induction. □

Lemma 4. *For given scalars c_l, c_2 and c_3, if there exists a set of positive definite matrices $\overline{\mathfrak{X}}^p_{\iota|\iota}$ satisfying*

$$
\begin{aligned}
\overline{\mathfrak{X}}^p_{\iota+1|\iota+1}=&(1+c_2)(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})\overline{\mathfrak{X}}^p_{\iota+1|\iota}(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})^T+(1+c_2^{-1})\\
&\times F_p\bar{\mu}_p^2G^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+(1+c_3)F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{R}}_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}\\
&+(1+c_3^{-1})F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+G^p_{\iota+1}\Psi_{\iota+1}G^{pT}_{\iota+1}
\end{aligned}
\tag{15}
$$

with $\overline{\mathfrak{X}}_{0|0}=\mathfrak{X}_{0|0}=\Xi_p$ and $\overline{\mathfrak{X}}^p_{\iota+1|\iota}=(1+c_1)A^p_\iota\overline{\mathfrak{X}}^p_{\iota|\iota}A^{pT}_\iota+B^p_\iota\mathcal{W}_\iota B^{pT}_\iota+(1+c_1^{-1})\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{X}}^q_{\iota|\iota}\Gamma$, then $\overline{\mathfrak{X}}^p_{\iota|\iota}$ is an upper bound of $\mathfrak{X}^p_{\iota|\iota}$, i.e. $\mathfrak{X}^p_{\iota|\iota}\leq\overline{\mathfrak{X}}^p_{\iota|\iota}$.

Proof. Bearing in mind the initial condition, assume that $\mathfrak{X}^p_{\iota|\iota} \leq \overline{\mathfrak{X}}^p_{\iota|\iota}$. If we verify $\mathfrak{X}^p_{\iota+1|\iota+1} \leq \overline{\mathfrak{X}}^p_{\iota+1|\iota+1}$ as follows, then this theorem will be proved in light of mathematical induction.

First, denote $\mathfrak{X}^p_{\iota+1|\iota} \triangleq \mathbb{E}\Big\{\widetilde{x}^p_{\iota+1|\iota}\widetilde{x}^{pT}_{\iota+1|\iota}\Big\}$ by the prediction covariance. It is determined from (7) that

$$\begin{aligned}\mathfrak{X}^p_{\iota+1|\iota} =&\mathbb{E}\Big\{\Big[A^p_\iota\widetilde{x}^p_{\iota|\iota}+\sum_{q=1}^{Q}\omega_{pq}\Gamma\widetilde{x}^q_{\iota|\iota}+B^p_\iota w_\iota\Big]\Big[A^p_\iota\widetilde{x}^p_{\iota|\iota}+\sum_{q=1}^{Q}\omega_{pq}\Gamma\widetilde{x}^q_{\iota|\iota}+B^p_\iota w_\iota\Big]^T\Big\}\\ \leq&(1+c_1)\mathbb{E}\big\{A^p_\iota\widetilde{x}^p_{\iota|\iota}\widetilde{x}^{pT}_{\iota|\iota}A^{pT}_\iota\big\}+\mathbb{E}\big\{B^p_\iota w_\iota w^T_\iota B^{pT}_\iota\big\}\\ &+(1+c_1^{-1})\mathbb{E}\Big\{\sum_{q=1}^{Q}\omega_{pq}\Gamma\widetilde{x}^q_{\iota|\iota}\Big[\sum_{q=1}^{Q}\omega_{pq}\Gamma\widetilde{x}^q_{\iota|\iota}\Big]^T\Big\},\end{aligned}\tag{16}$$

which, according to (10), can be further obtained as

$$\mathfrak{X}^p_{\iota+1|\iota} \leq(1+c_1)A^p_\iota\overline{\mathfrak{X}}^p_{\iota|\iota}A^{pT}_\iota+B^p_\iota\mathcal{W}_\iota B^{pT}_\iota+(1+c_1^{-1})\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{X}}^q_{\iota|\iota}\Gamma.\tag{17}$$

Then, we have

$$\mathfrak{X}^p_{\iota+1|\iota}\leq\overline{\mathfrak{X}}^p_{\iota+1|\iota}.\tag{18}$$

Next, the local FEC is calculated as

$$\begin{aligned}\mathfrak{X}^p_{\iota+1|\iota+1} =&\mathbb{E}\Big\{\widetilde{x}^p_{\iota+1|\iota+1}\widetilde{x}^{pT}_{\iota+1|\iota+1}\Big\}\\ =&\mathbb{E}\Big\{\big[(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})\widetilde{x}^p_{\iota+1|\iota}-\sqrt{F_p}\widetilde{\mu}^p_{\iota+1}G^p_{\iota+1}C^p_{\iota+1}x^p_{\iota+1}\\ &+\sqrt{F_p}\widetilde{\mu}^p_{\iota+1}G^p_{\iota+1}C^p_{\iota+1}\overrightarrow{x}^p_{\iota+1}+\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overrightarrow{x}^p_{\iota+1}-G^p_{\iota+1}\nu_{\iota+1}\big]\\ &\times\big[(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})\widetilde{x}^p_{\iota+1|\iota}-\sqrt{F_p}\widetilde{\mu}^p_{\iota+1}G^p_{\iota+1}C^p_{\iota+1}x^p_{\iota+1}\\ &+\sqrt{F_p}\widetilde{\mu}^p_{\iota+1}G^p_{\iota+1}C^p_{\iota+1}\overrightarrow{x}^p_{\iota+1}+\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overrightarrow{x}^p_{\iota+1}-G^p_{\iota+1}\nu_{\iota+1}\big]^T\Big\}\\ \leq&(1+c_2)(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})\mathfrak{X}^p_{\iota+1|\iota}(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})^T+(1+c_2^{-1})\\ &\times F_p\bar{\mu}^2_pG^p_{\iota+1}C^p_{\iota+1}\mathfrak{Z}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+(1+c_3)F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\mathfrak{R}_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}\\ &+(1+c_3^{-1})F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\mathfrak{Z}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+G^p_{\iota+1}\Psi_{\iota+1}G^{pT}_{\iota+1}\\ \leq&(1+c_2)(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})\overline{\mathfrak{X}}^p_{\iota+1|\iota}(I-\sqrt{F_p}\bar{\mu}_pG^p_{\iota+1}C^p_{\iota+1})^T+(1+c_2^{-1})\\ &\times F_p\bar{\mu}^2_pG^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+(1+c_3)F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{R}}_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}\\ &+(1+c_3^{-1})F_p\vec{\mu}_pG^p_{\iota+1}C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}G^{pT}_{\iota+1}+G^p_{\iota+1}\Psi_{\iota+1}G^{pT}_{\iota+1}\end{aligned}\tag{19}$$

which implies that

$$\mathfrak{X}^p_{\iota+1|\iota+1}\leq\overline{\mathfrak{X}}^p_{\iota+1|\iota+1}.\tag{20}$$

The proof is complete now. □

Theorem 1. *The upper bounds obtained in (12) and (15) could achieve their minimum values by designing $K^p_{\iota+1}$ and $G^p_{\iota+1}$ as*

$$G^p_{\iota+1}=\Lambda^p_{\iota+1}(\Omega^p_{\iota+1})^{-1},\ K^p_{\iota+1}=\aleph^p_{\iota+1}(\Upsilon^p_{\iota+1})^{-1}\tag{21}$$

where

$$\begin{aligned}
\Lambda^p_{\iota+1} =&(1+c_2)\sqrt{F}_p\bar{\mu}_p\overline{\mathfrak{X}}^p_{\iota+1|\iota}C^{pT}_{\iota+1},\\
\Omega^p_{\iota+1} =&(1+c_2)F_p\bar{\mu}_p^2C^p_{\iota+1}\overline{\mathfrak{X}}^p_{\iota+1|\iota}C^{pT}_{\iota+1}+(1+c_2^{-1})F_p\bar{\mu}_p^2C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}\\
&+(1+c_3)F_p\vec{\mu}_pC^p_{\iota+1}\overline{\mathfrak{R}}_{\iota+1}\bar{C}^{pT}_{\iota+1}+(1+c_3^{-1})F_p\vec{\mu}_pC^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}+\Phi_{\iota+1},\\
\aleph^p_{\iota+1} =&(1+a_2)\sqrt{E}_p\bar{\tau}_pA^P_\iota\overline{\mathfrak{Z}}^p_\iota A^{PT}_\iota C^{pT}_{\iota+1}+(1+a_2^{-1})\sqrt{E}_p\bar{\tau}_p\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)C^{pT}_{\iota+1}\\
&+\sqrt{E}_p\bar{\tau}_pB^p_\iota\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1},\\
\Upsilon^p_{\iota+1} =&(1+a_2)E_p\bar{\tau}_p^2C^p_{\iota+1}A^P_\iota\overline{\mathfrak{Z}}^p_\iota A^{PT}_\iota C^{pT}_{\iota+1}+(1+a_2^{-1})E_p\bar{\tau}_p^2C^p_{\iota+1}\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)\\
&\times C^{pT}_{\iota+1}+(1+a_3)E_p\vec{\tau}_pC^p_{\iota+1}A^p_\iota\overline{\mathfrak{R}}^p_\iota A^{pT}_\iota C^{pT}_{\iota+1}+(1+a_3^{-1})E_p\vec{\tau}_pC^p_{\iota+1}\\
&\times\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)C^{pT}_{\iota+1}+E_p(\bar{\tau}_p^2+\vec{\tau}_p)C^p_{\iota+1}B^p_\iota\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1}\\
&+E_p(\bar{\tau}_p^2+\vec{\tau}_p)D^p_{\iota+1}\mathcal{V}_{\iota+1}D^{pT}_{\iota+1}+\Psi_{\iota+1}.
\end{aligned}\tag{22}$$

Proof. Noting(12) and (15), taking the partial derivatives of the traces of $\overline{\mathfrak{Z}}^p_{\iota+1}$ and $\overline{\mathfrak{X}}^p_{\iota+1|\iota+1}$ with respect to $K^p_{\iota+1}$ and $G^p_{\iota+1}$, respectively, we have

$$\begin{aligned}
&\frac{\partial}{\partial K^p_{\iota+1}}\mathrm{tr}(\overline{\mathfrak{Z}}^p_{\iota+1})\\
=&-2(1+a_2)\sqrt{E}_p\bar{\tau}_pA^P_\iota\overline{\mathfrak{Z}}^p_\iota A^{PT}_\iota C^{pT}_{\iota+1}-2(1+a_2^{-1})\sqrt{E}_p\bar{\tau}_p\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)C^{pT}_{\iota+1}\\
&-2\sqrt{E}_p\bar{\tau}_pB^p_\iota\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1}+2K^p_{\iota+1}\Big((1+a_2)E_p\bar{\tau}_p^2C^p_{\iota+1}A^P_\iota\overline{\mathfrak{Z}}^p_\iota A^{PT}_\iota C^{pT}_{\iota+1}\\
&+(1+a_2^{-1})E_p\bar{\tau}_p^2C^p_{\iota+1}\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)C^{pT}_{\iota+1}+(1+a_3)E_p\vec{\tau}_pC^p_{\iota+1}A^p_\iota\overline{\mathfrak{R}}^p_\iota A^{pT}_\iota C^{pT}_{\iota+1}\\
&+(1+a_3^{-1})E_p\vec{\tau}_pC^p_{\iota+1}\Big(\widehat{\omega}\sum_{q=1}^{Q}|\omega_{pq}|\Gamma\overline{\mathfrak{R}}^q_\iota\Gamma\Big)C^{pT}_{\iota+1}+E_p(\bar{\tau}_p^2+\vec{\tau}_p)C^p_{\iota+1}B^p_\iota\mathcal{W}_\iota B^{pT}_\iota C^{pT}_{\iota+1}\\
&+E_p(\bar{\tau}_p^2+\vec{\tau}_p)D^p_{\iota+1}\mathcal{V}_{\iota+1}D^{pT}_{\iota+1}+\Psi_{\iota+1}\Big)=-2\aleph^p_{\iota+1}+K^p_{\iota+1}\Upsilon^p_{\iota+1}
\end{aligned}\tag{23}$$

and

$$\begin{aligned}
&\frac{\partial}{\partial G^p_{\iota+1}}\mathrm{tr}(\overline{\mathfrak{X}}^p_{\iota+1|\iota+1})\\
=&-2(1+c_2)\sqrt{F}_p\bar{\mu}_p\overline{\mathfrak{X}}^p_{\iota+1|\iota}C^{pT}_{\iota+1}+2G^p_{\iota+1}\Big((1+c_2)F_p\bar{\mu}_p^2C^p_{\iota+1}\overline{\mathfrak{X}}^p_{\iota+1|\iota}C^{pT}_{\iota+1}\\
&+(1+c_2^{-1})F_p\bar{\mu}_p^2C^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}+(1+c_3)F_p\vec{\mu}_pC^p_{\iota+1}\overline{\mathfrak{R}}_{\iota+1}\bar{C}^{pT}_{\iota+1}\\
&+(1+c_3^{-1})F_p\vec{\mu}_pC^p_{\iota+1}\overline{\mathfrak{Z}}^p_{\iota+1}C^{pT}_{\iota+1}+\Phi_{\iota+1}\Big)\\
=&-2\Lambda^p_{\iota+1}+2G^p_{\iota+1}\Omega^p_{\iota+1}
\end{aligned}\tag{24}$$

where $\Lambda^p_{\iota+1}$, $\Omega^p_{\iota+1}$, $\aleph^p_{\iota+1}$ and $\Upsilon^p_{\iota+1}$ are defined in (22). Setting (23) and (24) to be zero, $\overline{\mathfrak{X}}^p_{\iota+1|\iota+1}$ and $\overline{\mathfrak{Z}}^p_{\iota+1}$ could achieve their minimums when $K^p_{\iota+1}$ and $G^p_{\iota+1}$ are designed as (21). □

Remark 2. *In this paper, a thorough examination has been provided for the recursive filtering over FFR networks subject to fading channels. Comparing to existing results, the novelty of this paper lies in the following three key aspects: (1) the proposal of the FFRs within the long-distance transmission with aim to enhance the signal quality; and (2) the designation of recursive filters that effectively quantify the effects from the deployed FFRs and fading channels.*

4. Numerical Example

The simulation parameters of CN (1) with 3 nodes are given by:

$$A_\iota^1 = A_\iota^2 = A_\iota^3 = \begin{bmatrix} 0.8\sin(2\iota) & 0 \\ 0.1 & 0.5 \end{bmatrix},\ \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad \omega_{ij} = \begin{cases} -0.2, & i=j, \\ 0.1, & i \neq j. \end{cases}$$

$$B_\iota^1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\quad B_\iota^2 = \begin{bmatrix} 0.12\cos(2\iota) \\ -0.1 \end{bmatrix},\ B_\iota^3 = \begin{bmatrix} -0.15 \\ 0.1\cos(2\iota) \end{bmatrix}.$$

The measurement is considered under FFR-based communications (2)–(4) with the following parameters:

$$\begin{aligned} & C_\iota^1 = \begin{bmatrix} 1+0.01\cos(2\iota) & 0.9 \end{bmatrix},\ C_\iota^2 = \begin{bmatrix} 0.9 & 0.8-0.01\sin(2\iota) \end{bmatrix}, \\ & C_\iota^3 = \begin{bmatrix} 0.8 & 0.8+0.01\cos(2\iota) \end{bmatrix},\ D_\iota^1 = 0.15,\ D_\iota^2 = 0.1,\ D_\iota^3 = 0.2, \\ & E_1 = E_2 = E_3 = 1,\ F_1 = F_2 = F_3 = 1, \end{aligned}$$

and the channel coefficients satisfy $\bar{\tau}_1 = \bar{\tau}_2 = \bar{\tau}_3 = 0.99$, $\bar{\mu}_1 = \bar{\mu}_2 = \bar{\mu}_3 = 0.9$, $\tilde{\tau}_1 = \tilde{\tau}_2 = \tilde{\tau}_3 = 0.01$ and $\tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3 = 0.01$. Moreover, the introduced parameters are selected as $a_1 = a_2 = a_3 = c_1 = c_2 = c_3 = 1$.

For node p, the following mean-squared errors (MSEs) are used:

$$\text{MSE1}_\iota^p = \frac{1}{P}\sum_{\iota=1}^{P}\sum_{s=1}^{2}\left(x_\iota^{p(s)} - \widehat{x}_{\iota|\iota}^{p(s)}\right)^2,\ \text{MSE2}_\iota^p = \frac{1}{P}\sum_{\iota=1}^{P}\sum_{s=1}^{2}\left(x_\iota^{p(s)} - \psi_\iota^{p(s)}\right)^2 \tag{25}$$

where $x_\iota^{p(s)}$, $\psi_\iota^{p(s)}$ and $\widehat{x}_{\iota|\iota}^{p(s)}$ is the sth element of x_ι^p, ψ_ι^p and $\widehat{x}_{\iota|\iota}^p$, $P = 300$ is the Monte Carlo trials number.

The covariances of the random noises are given as $\mathcal{W}_\iota = 0.5$, $\mathcal{V}_\iota = 0.1$, $\Phi_\iota = 0.1$, $\Upsilon_\iota = 0.1$. The initial value of network state has mean $\bar{x}_p = 0$ and covariance $\Xi_p = 0.2$. According to these given parameter settings, the filter gains and the minimal upper bounds can be obtained from Theorem 1. The simulation results are given in Figures 1–6. The network states and their corresponding estimates both in the relay side and filter side are illustrated in Figures 1–3. In Figures 4–6, the traces of the minimal upper bounds $\overline{\mathfrak{X}}_{\iota|\iota}^p$ and $\overline{\mathfrak{Z}}_\iota^p$ as well as the actual MSEs are plotted. It is observed that the proposed filtering method is capable of effectively estimating the real network states, even when the measurement signals are subjected to the FFRs and fading channels.

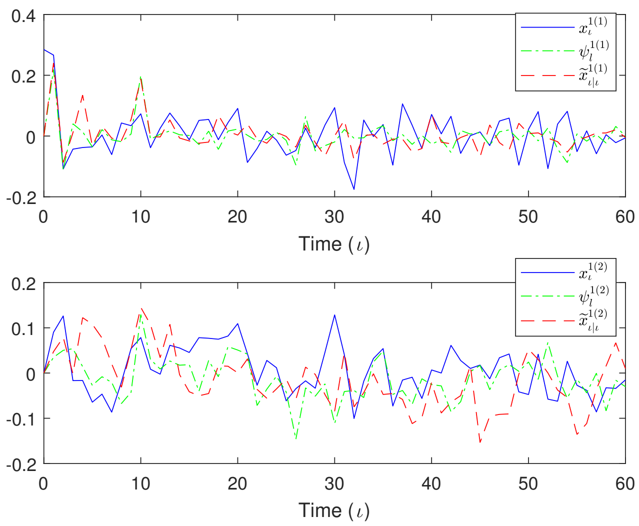

Figure 1. State x_ι^1 and its estimates both in relay side and filter side.

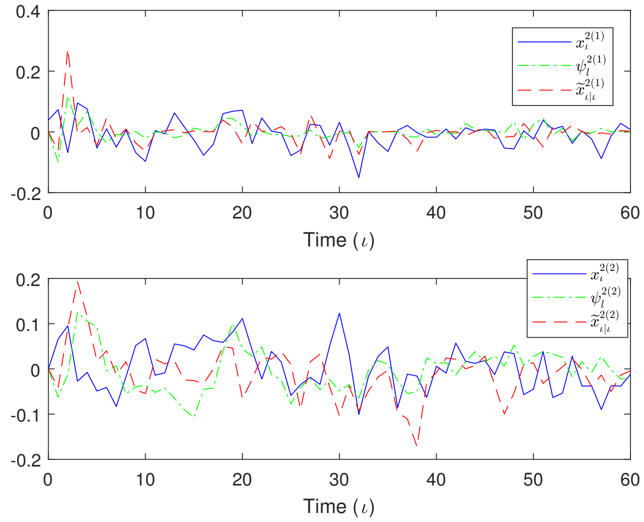

Figure 2. State x_ι^2 and its estimates both in relay side and filter side.

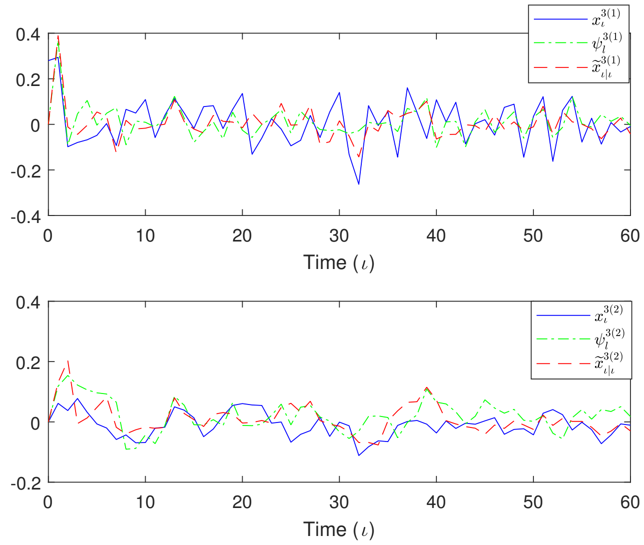

Figure 3. State x_ι^3 and its estimates both in relay side and filter side.

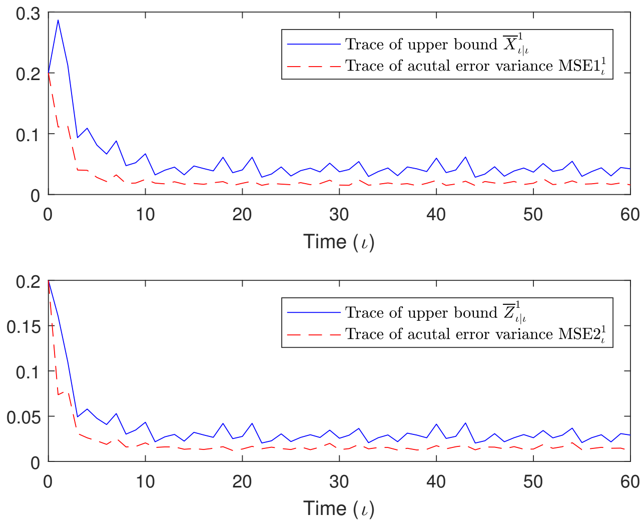

Figure 4. Trace of error variance and their upper bounds for node 1.

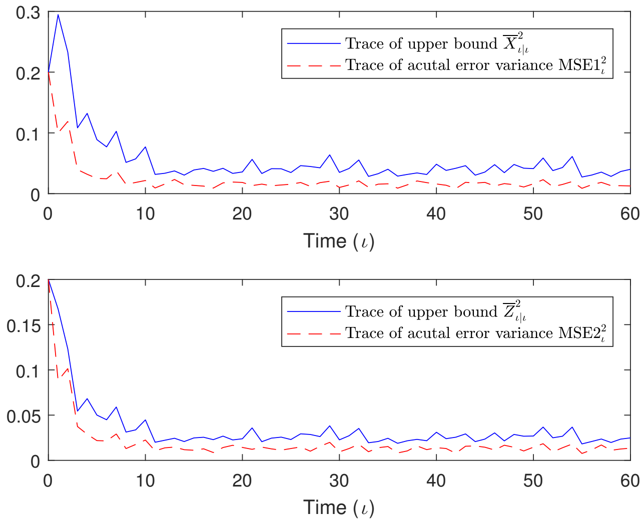

Figure 5. Trace of error variance and their upper bounds for node 2.

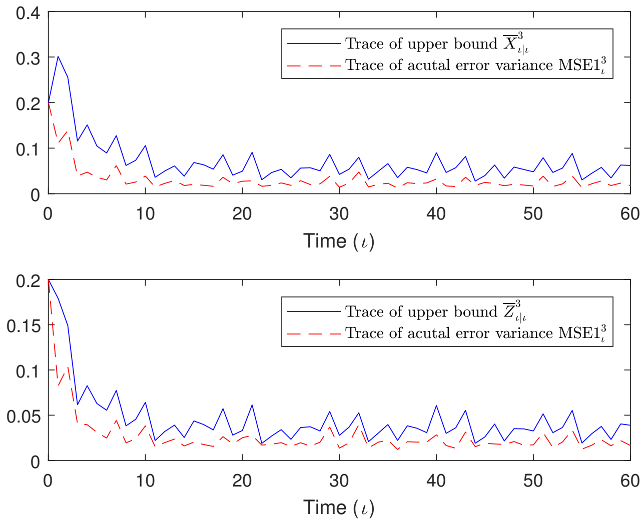

Figure 6. Trace of error variance and their upper bounds for node 3.

5. Conclusions

This paper has been dedicated to designing a recursive filter for CNs with FFRs under fading channels. In order to facilitate transmission, the FFRs have been deployed between the sensor and filter to orchestrate the signal communication. Further, the phenomena of fading channels have been considered in both the sensor-relay and relay-filter channels, which are depicted by two sets of random variables. Making good use of the mathematical induction, the corresponding filtering algorithm has been proposed for the FFRs and the underlying CNs through solving a minimization issue based on the upper bounds of the FECs. Finally, an example is given to showcase the effectiveness of the filtering strategy.

Author Contributions: Q.L.: Investigation, Methodology, Writing; M.G.: Visualization, Software, Editing and reviewing.

Funding: This work was supported in part by the Zhejiang Provincial Natural Science Foundation under Grant LY24F030006.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies: No AI tools were utilized for this paper.

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