Scilight

Journal of Particle Physics and Cosmology
https://www.sciltp.com/journals/jppc

Article

Scheme Dependence of the One-Loop Domain Wall Tension

Jarah Evslin [1,2,*] and Hui Liu [3]

[1] Institute of Modern Physics, Lanzhou 730000, China
[2] University of the Chinese Academy of Sciences, Beijing 100049, China
[3] Yerevan Physics Institute, Yerevan 0036, Armenia
* Correspondence: jarah@impcas.ac.cn (J.E.); hui.liu@yerphi.am (H.L.)

How To Cite: Evslin, J.; Liu, H. Scheme Dependence of the One-Loop Domain Wall Tension. *Journal of Particle Physics and Cosmology* **2026**, *1*(1), 1.

Received: 18 November 2025
Accepted: 26 December 2025
Published: 7 January 2026

Abstract: The one-loop tension of the domain wall in the 3+1 dimensional ϕ^4 double-well model was derived long ago using dimensional regularization. The methods used can only be applied to solitons depending on a single dimension. In the past few months, domain wall tensions have been recalculated using spectral methods with Born subtractions and also linearized soliton perturbation theory, both of which may be generalized to arbitrary solitons. It has been shown that the former agrees with the results of Rebhan et al. In the present work, we argue that, if the same renormalization scheme is chosen, both new results agree.

Keywords: quantum soliton; domain wall; renormalization

1. Introduction

The one-loop quantum correction to the tension of the kink in the 1+1 dimensional ϕ^4 double-well model was computed half a century ago in Ref. [1]. It was not long before the result was generalized to the domain wall in the corresponding 3+1 dimensional model, in Ref. [2]. These results were confirmed in later studies [3,4]. However, some studies found incompatible answers [5,6].

Spurred on by recent interest in cosmology [7,8], the one-loop corrections to the domain wall tension was recomputed in Ref. [9] using a new method that, unlike the spectra methods above, can be extended to multiple loops. However, the result could not be directly compared with previous results due to a different renormalization scheme. Recently, Ref. [10] attempted to fix this shortcoming by repeating the calculation using various schemes and comparing with the literature. It found agreement with Refs. [2–4] but claimed disagreement with Refs. [5,6,9].

In the present note, we observe that the scheme used by Ref. [10] is not quite that of Ref. [9], and that this mismatch in scheme is precisely responsible for the disagreement in the results. Choosing the same scheme, we show that Refs. [9, 10] are consistent. Namely, while both use a scheme in which quantum corrections to the three-point coupling vanish, the three-point coupling itself is defined in different vacua in the two studies. We provide a simple, analytical formula for the change in the one-loop tension correction resulting from this change in scheme, which agrees with the mismatch in the results of these references in all dimensions considered.

We begin in 1+1 dimensions in Section 2, reviewing the one-loop quantum correction to the kink mass calculated in Ref. [11]. This will be more convenient than the approach of Ref. [1] because it uses a normal-ordered Hamiltonian, which removes all ultraviolet divergences in 1+1 dimensions and, at one loop, even in 2+1 dimensions. In Section 3 we consider an arbitrary renormalization scheme, characterized by a renormalization of the meson mass and coupling. In this context, we define a one-loop mass shift. In Section 4 we calculate the potentially-divergent contributions to the amputated two-point and three-point functions. Finally, in Section 5, we consider two renormalization schemes, which tie together the loop diagrams of Section 4 with the arbitrary renormalization of Section 3. The first scheme leads to the results of Ref. [9] and the second to those of Ref. [10]. We then lift the argument from 1+1 to 2+1 and 3+1 dimensions, which turns out to be rather trivial and reproduces, analytically, the differences reported between Refs. [9, 10].

2. Cahill, Comtets and Glauber

Consider the ϕ^4 double-well model in 1+1 dimensions. The Hamiltonian is

$$H = \int dx \left[\frac{:\pi^2: + :(\partial_x \phi)^2:}{2} + \frac{\lambda_0}{4} : \left(\phi^2 - \frac{m_0^2}{2\lambda_0} \right)^2 : \right] \tag{1}$$

where :: is the usual Schrodinger picture normal ordering in terms of a plane wave decomposition, and $\phi(x)$ and $\pi(x)$ are a Schrodinger picture field and its conjugate momentum. λ_0 and m_0 are the bare coupling and bare meson mass.

Cahill, Comtets and Glauber long ago derived the one-loop (order $O(\lambda_0^0)$) kink mass in this model [11]

$$Q = Q_0 + Q_1 \quad Q_0 = \frac{m_0^3}{3\lambda_0} \quad Q_1 = \left(\frac{1}{4\sqrt{3}} - \frac{3}{2\pi} \right) m_0. \tag{2}$$

3. Arbitrary Redefinitions

Let us define two parameters λ and m and also the differences

$$\delta\sqrt{\lambda} = \sqrt{\lambda} - \sqrt{\lambda_0} \quad \delta m^2 = m^2 - m_0^2. \tag{3}$$

Consider the dimensionless ratio λ/m^2 small and positive. Then we may expand in powers of λ/m^2

$$\delta\sqrt{\lambda} = \sum_i \delta\sqrt{\lambda}_i \quad \delta m^2 = \sum_i \delta m_i^2 \tag{4}$$

where we define $\delta\sqrt{\lambda}_i$ and δm_i^2 to be the terms in the expansion of order $O(\lambda^i)$.

We will demand that the leading terms are $\delta\sqrt{\lambda}_{3/2}$ and δm_1^2. To avoid clutter, we will then drop the subscripts. Then, up to order $O(\lambda^0)$, we may rewrite the kink mass as

$$Q = Q_0 + Q_1 + O(\lambda/m^2) \quad Q_0 = \frac{m_0^3}{3\lambda_0} \quad Q_1 = \left(\frac{1}{4\sqrt{3}} - \frac{3}{2\pi} \right) m. \tag{5}$$

Here Q_1 is the same quantity as appeared in Ref. [1], although the mass renormalization there was different.

We are interested in the quantity

$$\Delta Q = Q - \frac{m^3}{3\lambda}. \tag{6}$$

Note that while Q is an observable mass, ΔQ depends on our choice of parameters m and λ which later will depend on our choice of renormalization scheme.

Using our expansion, we find, up to corrections of order $O(\lambda)$

$$\begin{aligned} \Delta Q &= \frac{m_0^3}{3\lambda_0} - \frac{m^3}{3\lambda} + Q_1 = \frac{m^3}{3\lambda} \left(1 - \frac{\delta m^2}{m^2} \right)^{3/2} \left(1 - \frac{\delta\sqrt{\lambda}}{\sqrt{\lambda}} \right)^{-2} - \frac{m^3}{3\lambda} + Q_1 \\ &= \frac{m^3}{\lambda} \left(\frac{2\delta\sqrt{\lambda}}{3\sqrt{\lambda}} - \frac{\delta m^2}{2m^2} \right) + \left(\frac{1}{4\sqrt{3}} - \frac{3}{2\pi} \right) m. \end{aligned} \tag{7}$$

This is our master formula for ΔQ, valid for any renormalization scheme.

4. Renormalization

4.1. Coupling Constant Renormalization

Following Equation (34) of Ref. [10], the amputed three point function is

$$\Gamma_3(0,0) = \Gamma_{3a} + \Gamma_{3b} + \Gamma_{3c}. \tag{8}$$

Our notation is related that of Ref. [10] by

$$\lambda = \frac{\lambda_{GW}}{2} \quad m = \mu_{GW} \tag{9}$$

where the subscript GW refers to Ref. [10].

Then the first term is

$$\Gamma_{3a} = \frac{9}{4}\frac{m\,2^{3/2}\lambda^{3/2}}{4\pi}\int_0^1 d\alpha[m^2]^{-1} = \frac{9\lambda^{3/2}}{4\sqrt{2}\pi m}. \tag{10}$$

The second is

$$\Gamma_{3b} = -\frac{9im^3}{2}2\sqrt{2}\lambda^{3/2}\int\frac{d^2l}{(2\pi)^2}\frac{1}{(l^2-m^2+i\epsilon)^3} = -9\sqrt{2}im^3\lambda^{3/2}\left(-\frac{i}{8\pi m^4}\right) = -\frac{9\lambda^{3/2}}{4\sqrt{2}\pi m}. \tag{11}$$

The third arises from the counterterm c_1, to be defined below, and is

$$\Gamma_{3c} = \frac{c_1}{2}\frac{m}{\sqrt{2\lambda}}. \tag{12}$$

In Equation (35) of Ref. [10] the renormalization condition $\Gamma_3(0,0) = 0$ is imposed and so we find

$$c_1 = 0. \tag{13}$$

4.2. Mass Renormalization

In Ref. [10] the mass renormalization involves two diagrams, one arising from a single interaction and one from two interactions. The single interaction diagram vanishes in the case at hand because our Hamiltonian (1) is normal ordered. The remaining polarization function consists of two terms

$$\Pi = \Pi_1 + \Pi_2. \tag{14}$$

The first arises from a two-vertex loop diagram and is

$$\Pi_1 = \frac{9m^2\,2\lambda}{16\pi}\int_0^1 d\alpha\frac{1}{m^2(1-\alpha(1-\alpha))} = \frac{9\lambda}{8\pi}\frac{2\pi}{3\sqrt{3}} = \frac{\sqrt{3}\lambda}{4}. \tag{15}$$

The second is the counterterm

$$\Pi_2 = \frac{c_2}{2} + \frac{m^2}{4\lambda}c_1. \tag{16}$$

Using (13), the renormalization condition $\Pi = 0$ leads to

$$c_2 = -\frac{\sqrt{3}}{2}\lambda. \tag{17}$$

5. The Kink Mass and Domain Wall Tension

We have determined ΔQ as a function of our choices δm^2 and $\delta\sqrt{\lambda}$. We also evaluated c_1 and c_2, which at this point are simply expressions for loop corrections. The choice of renormalization condition is a choice of identification of the loop corrections c_1 and c_2 with the counterterm coefficients δm^2 and $\delta\sqrt{\lambda}$. We will now consider two such choices.

5.1. Scheme A

The classical ground states of H are $\phi = \pm m_0/\sqrt{2\lambda_0}$. Let us decompose the Schrodinger picture quantum field about one of the ground states

$$\phi(x) = \frac{m_0}{\sqrt{2\lambda_0}} + \eta(x). \tag{18}$$

Our Hamiltonian can then be written in terms of $\eta(x)$

$$H = \int dx\left[\frac{:\pi^2(x): + :(\partial_x\eta)^2:}{2} + \frac{\lambda_0}{4}:\eta^2\left(\eta + m_0\sqrt{\frac{2}{\lambda_0}}\right)^2:\right]. \tag{19}$$

The cubic interaction corresponds to the term

$$H_3 = m_0\sqrt{\frac{\lambda_0}{2}}\int dx :\eta^3: = m\sqrt{\frac{\lambda}{2}}\left(1 - \frac{\delta m^2}{2m^2} - \frac{\delta\sqrt{\lambda}}{\sqrt{\lambda}}\right)\int dx :\eta^3: . \tag{20}$$

Equation (12) is the amputated three-point function for η if we define the counterterms using this η decomposition.

In other words, we will define c_1 by

$$H_3 = m\sqrt{\frac{\lambda}{2}}\int dx : \eta^3 : -\Gamma_{3c}\int dx : \eta^3 : \tag{21}$$

so that

$$c_1 = \lambda\left(\frac{\delta m^2}{m^2} + 2\frac{\delta\sqrt{\lambda}}{\sqrt{\lambda}}\right). \tag{22}$$

Similarly, the mass term in Equation (19) is

$$H_2 = \frac{m_0^2}{2}\int dx : \eta^2 := \frac{m^2}{2}\int dx : \eta^2 : -\frac{\delta m^2}{2}\int dx : \eta^2 : . \tag{23}$$

Identifying

$$H_2 = \frac{m^2}{2}\int dx : \eta^2 : -\Pi_2\int dx : \eta^2 : \tag{24}$$

We obtain

$$c_2 + \frac{m^2}{2\lambda}c_1 = \delta m^2. \tag{25}$$

Now our results (13) and (17) lead to

$$\frac{\delta m^2}{m^2} = -\frac{\sqrt{3}}{2}\frac{\lambda}{m^2} \qquad \frac{\delta\sqrt{\lambda}}{\sqrt{\lambda}} = \frac{\sqrt{3}}{4}\frac{\lambda}{m^2}. \tag{26}$$

Substituting these into our master formula (7) we obtain

$$\Delta Q = \left(\frac{\sqrt{3}}{2} - \frac{3}{2\pi}\right)m \tag{27}$$

in agreement with Ref. [9].

5.2. Scheme B

The counterterms corresponding to the parameters c_1 and c_2 are given in Equation (31) of Ref. [10]

$$H_{ct} = \int dx\left[-\frac{c_1}{8} : \left(\phi^2 - v^2\right)^2 : -\frac{c_2}{2} : \left(\phi^2 - v^2\right) :\right]. \tag{28}$$

We have included normal ordering, which was not used in Ref. [10]. The definition of v is somewhat subtle, as the expectation value of ϕ on a state depends on the state and the renormalization conditions. However, the fact that c_2 does not appear in the amputated three-point function suggests that

$$v = \frac{m}{\sqrt{2\lambda}}. \tag{29}$$

We will assume this. Any other choice does not affect the kink mass, but does affect our renormalization conditions by changing the definition of the three-point coupling. This is because the three-point coupling is defined to be the third derivative evaluated at a certain value of the classical field, and so it is necessarily sensitive to this choice. Therefore a change in v will change our renormalization scheme and so will change $\delta\sqrt{\lambda}$.

With this caveat, we write a Hamiltonian as

$$H = H_{ct} + \int dx\left[\frac{:\pi^2: + :(\partial_i\phi)^2:}{2} + \frac{\lambda}{4} : \left(\phi^2 - \frac{m^2}{2\lambda}\right)^2 :\right]. \tag{30}$$

The potential terms in the classical Hamiltonian density are

$$U(\phi) = \frac{\lambda - \frac{c_1}{2}}{4}\left(\phi^2 - \frac{m^2}{2\lambda}\right)^2 - \frac{c_2}{2}\left(\phi^2 - \frac{m^2}{2\lambda}\right). \tag{31}$$

This of course is equal to the original potential $(\lambda_0/4)(\phi^2 - m_0^2/(2\lambda_0))^2$ and so

$$c_1 = 4\sqrt{\lambda}\delta\sqrt{\lambda} \quad c_2 = m^2\left(\frac{\delta\sqrt{\lambda}}{\sqrt{\lambda}} - \frac{\delta m^2}{2m^2}\right). \tag{32}$$

Now the loop corrections (13) and (17) lead to

$$\delta\sqrt{\lambda} = 0 \quad \delta m^2 = \sqrt{3}\lambda \tag{33}$$

and so

$$\Delta Q = \left(-\frac{5\sqrt{3}}{12} - \frac{3}{2\pi}\right)m \sim -1.199153m \tag{34}$$

consistent with the result of Ref. [10].

5.3. Domain Walls

As shown in Ref. [12], the Cahill, Comtets and Glauber formula applies to the domain wall tension Q in any dimension

$$Q = Q_0 + Q_1 \quad Q_0 = \frac{m_0^3}{3\lambda_0}. \tag{35}$$

In the case of the ϕ^4 double-well model, the formula for the classical tension Q_0 is independent of the dimension. On the other hand, the one-loop correction Q_1 does depend on the dimension. However, critically, it is independent of the scheme. As a result, in any dimension our master formula may still be written

$$\Delta Q = \frac{m^3}{\lambda}\left(\frac{2\delta\sqrt{\lambda}}{3\sqrt{\lambda}} - \frac{\delta m^2}{2m^2}\right) + Q_1. \tag{36}$$

Let us use subscripts A and B to respectively denote the schemes in Sections 5.1 and 5.2 respectively. The general arguments above show that

$$c_1 = \lambda\left(\frac{\delta_A m^2}{m^2} + 2\frac{\delta_A\sqrt{\lambda}}{\sqrt{\lambda}}\right) = \lambda\left(4\frac{\delta_B\sqrt{\lambda}}{\sqrt{\lambda}}\right) \tag{37}$$

$$c_2 = m^2\left(\frac{\delta_A m^2}{2m^2} - \frac{\delta_A\sqrt{\lambda}}{\sqrt{\lambda}}\right) = m^2\left(-\frac{\delta_B m^2}{2m^2} + \frac{\delta_B\sqrt{\lambda}}{\sqrt{\lambda}}\right). \tag{38}$$

Therefore the counterterm coefficients are related by

$$\frac{\delta_B\sqrt{\lambda}}{\sqrt{\lambda}} = \frac{\delta_A m^2}{4m^2} + \frac{\delta_A\sqrt{\lambda}}{2\sqrt{\lambda}} \quad \frac{\delta_B m^2}{m^2} = -\frac{\delta_A m^2}{2m^2} + \frac{3\delta_A\sqrt{\lambda}}{\sqrt{\lambda}}. \tag{39}$$

The resulting difference in the two loop tension corrections is

$$\Delta Q_B - \Delta Q_A = \frac{m^3}{\lambda}\left(\frac{2\delta_B\sqrt{\lambda}}{3\sqrt{\lambda}} - \frac{\delta_B m^2}{2m^2} - \frac{2\delta_A\sqrt{\lambda}}{3\sqrt{\lambda}} + \frac{\delta_A m^2}{2m^2}\right) = \frac{11m^3}{12\lambda}\left(\frac{\delta_A m^2}{m^2} - \frac{2\delta_A\sqrt{\lambda}}{\sqrt{\lambda}}\right). \tag{40}$$

In the case of the domain wall string in 2+1 dimensions [9]

$$\frac{\delta_A m^2}{m^2} = -\frac{9\ln(3)}{8\pi}\frac{\lambda}{m} \quad \frac{\delta_A\sqrt{\lambda}}{\sqrt{\lambda}} = \frac{9(\ln(3)-1)}{16\pi}\frac{\lambda}{m} \tag{41}$$

and so

$$\Delta Q_B - \Delta Q_A = \frac{33}{32\pi}(1 - 2\ln(3))m^2 \sim -0.392997m^2 \tag{42}$$

which is indeed the difference between the two results shown on Table IV of Ref. [10].

In the case of 3+1 dimensions [9]

$$\frac{\delta_A m^2}{m^2} = -\frac{9\lambda}{2\pi^2}\int_0^\infty dp \frac{p^2}{\omega_p(3m^2+4p^2)}$$
$$\frac{\delta_A\sqrt{\lambda}}{\sqrt{\lambda}} = \frac{9\lambda}{16\pi^2} + \frac{9\lambda}{8\pi^2}\int_0^\infty dp\, p^2\left(-\frac{1}{\omega_p^3} + \frac{2}{\omega_p(3m^2+4p^2)}\right) \tag{43}$$

and so

$$\Delta Q_B - \Delta Q_A = -\frac{33m^3}{32\pi^2} - \frac{33m^3}{4\pi^2}\int_0^\infty dp\, p^2\left(-\frac{1}{4\omega_p^3} + \frac{1}{\omega_p(3m^2+4p^2)}\right)$$
$$= \left(-\frac{99}{32\pi^2} + \frac{11\sqrt{3}}{32\pi}\right)m^3 \sim -0.123943m^3 \tag{44}$$

which is again consistent with the entries (The caption of Table IV is incorrect, as the renormalization conditions differ. Equation (44), which we have argued corresponds to the mismatch between the results with the two different renormalization conventions, is equal to the difference between the last two entries in the bottom row of the table.) in Table IV of Ref. [10].

6. Remarks

We have shown that the spectral methods, with dimensional regularization, of Refs. [4,10] yield the same domain wall tension as the Hamiltonian methods of Ref. [9] which employ a hard cutoff.

There were many potential problems with both approaches. For example, a hard cutoff in Ref. [13] yielded the wrong answer. In fact, the problem with the hard cutoff in Ref. [13] is that the vacuum and kink sectors were regularized separately and then there was an *ad hoc* matching between the two sectors, whereas in Ref. [9] the theory was regularized only once and the two sectors were treated with the same Hamiltonian. Similarly, one might have worried [14,15] that dimensional regularization would be ill-defined in a configuration that solves the classical field equations in an integral dimension.

However the agreement that we have found here puts both worries to rest. At least this is true for the present case, in which the divergences are only logarithmic and the field configuration is trivial along the directions that are regularized. It remains to be seen whether one method will have a problem in other applications without these features. Needless to say, we believe that such a comparison between the results of different methods will be essential for progress towards a treatment of more complicated but interesting solitons, such as the 't Hooft-Polyakov monopole or the gravitating kink [16,17].

Author Contributions

J.E.: Conceptualization, methodology, writing—original draft preparation, investigation; H.L.: Investigation, writing—reviewing and editing, validation. All authors have read and agreed to the published version of the manuscript.

Funding

This work was supported by the Higher Education and Science Committee of the Republic of Armenia (Research Project Nos. 24RL-1C047 and 24PostDoc/2-1C009).

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

References

1. Dashen, R.F.; Hasslacher, B.; Neveu, A. Nonperturbative methods and extended-hadron models in field theory. II. Two-dimensional models and extended hadrons. *Phys. Rev. D* **1974**, *10*, 4130. https://doi.org/10.1103/PhysRevD.10.4130.
2. Brezin, E.; Feng, S. Amplitude of the surface tension near the critical point. *Phys. Rev. B* **1984**, *29*, 472–475. https://doi.org/10.1103/PhysRevB.29.472.
3. Munster, G. Tunneling Amplitude and Surface Tension in ϕ^4-Theory. *Nucl. Phys. B* **1989**, *324*, 630–642. https://doi.org/10.1016/0550-3213(89)90524-5.
4. Rebhan, A.; van Nieuwenhuizen, P.; Wimmer, R. One loop surface tensions of (supersymmetric) kink domain walls from dimensional regularization. *New J. Phys.* **2002**, *4*, 31. https://doi.org/10.1088/1367-2630/4/1/331.
5. de Carvalho, C.A.; Marques, G.C.; da Silva, A.J.; et al. Domain Walls at Finite Temperature. *Nucl. Phys. B* **1986**, *265*, 45–64. https://doi.org/10.1016/0550-3213(86)90406-2.
6. de Carvalho, C.A.A. Thermal and quantum fluctuations around domain walls. *Phys. Rev. D* **2002** *65*, 065021 https://doi.org/10.1103/PhysRevD.65.065021.
7. Blanco-Pillado, J.J.; Jiménez-Aguilar, D.; Urrestilla, J.; Exciting the domain wall soliton. *J. Cosmol. Astropart. Phys.* **2021**, *2021*, 027. https://doi.org/10.1088/1475-7516/2021/01/027.
8. Blanco-Pillado, J.J.; Jiménez-Aguilar, D.; Queiruga, J.M.; et al. The dynamics of domain wall strings. *J. Cosmol. Astropart. Phys.* **2023**, *2023*, 011. https://doi.org/10.1088/1475-7516/2023/05/011.
9. Evslin, J.; Liu, H.; Zhang, B. The domain wall soliton's tension. *Eur. Phys. J. C* **2025**, *85*, 639. https://doi.org/10.1140/epjc/s10052-025-14383-8.
10. Graham, N.; Weigel, H. Quantum contribution to domain wall tension from spectral methods. *Phys. Rev. D* **2025**, *112*, 045003. https://doi.org/10.1103/9v1m-rwjb.
11. Cahill, K.E.; Comtet, A.; Glauber, R.J. Mass Formulas for Static Solitons. *Phys. Lett. B* **1976**, *64*, 283–285. https://doi.org/10.1016/0370-2693(76)90202-1.
12. Ogundipe, K.; Evslin, J.; Zhang, B.; et al. A (2+1)-dimensional domain wall at one-loop. *J. High Energy Phys.* **2024**, *2024*, 98. https://doi.org/10.1007/JHEP05(2024)098.
13. Rebhan, A.; van Nieuwenhuizen, P. No saturation of the quantum Bogomolnyi bound by two-dimensional $N = 1$ supersymmetric solitons. *Nucl. Phys. B* **1997**, *508*, 449–467. https://doi.org/10.1016/S0550-3213(97)00625-1.
14. Litvintsev, A.; van Nieuwenhuizen, P. Once more on the BPS bound for the SUSY kink. *arXiv* **2000**, arXiv:hep-th/0010051.
15. Parnachev, A.; Yaffe, L.G. One loop quantum energy densities of domain wall field configurations. *Phys. Rev. D* **2000**, *62*, 105034. https://doi.org/10.1103/PhysRevD.62.105034.
16. Zhong, Y. Revisit on two-dimensional self-gravitating kinks: superpotential formalism and linear stability. *J. High Energy Phys.* **2021**, *2021*, 118. https://doi.org/10.1007/JHEP04(2021)118.
17. Wang, H.; Zhong, Y.; Wang, Z. Rosen-Morse potential and gravitating kinks. *Phys. Lett. B* **2024**, *858*, 139071. https://doi.org/10.1016/j.physletb.2024.139071.