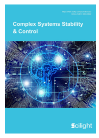
Complex Systems Stability & Control
https://www.sciltp.com/journals/cssc



Article

Fixed-Time Synchronization of Memristor-Based Stochastic BAM Neural Networks with Time-Varying Delays

Zuhao Li, Meidan Zhao and Abdujelil Abdurahman *

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China
* Correspondence: abdujelil@xju.edu.cn

How To Cite: Li, Z.; Zhao, M.; Abdurahman, A. Fixed-Time Synchronization of Memristor-Based Stochastic BAM Neural Networks with Time-Varying Delays. *Complex Systems Stability & Control* **2025**, *1*(1), 8.

Received: 9 October 2025
Revised: 10 November 2025
Accepted: 17 November 2025
Published: 3 December 2025

Abstract: This paper considers the fixed-time synchronization of memristor-based bidirectional associative memory neural networks with stochastic perturbations and time-varying delays via designing concise function based continuous controllers. First, under the Filippov set-valued mapping framework, the discontinuous stochastic memristor-based bidirectional associative memory neural networks model is transformed to stochastic differential inclusion. Then, two innovative controllers are designed to achieve fixed-time synchronization for the system: a nonlinear state feedback controller without linear terms, which significantly simplifies the design complexity, and an adaptive control scheme with an efficient update law, which automatically adjusts coupling strength of the control gain to save overall control cost. Furthermore, some sufficient criteria for the fixed-time synchronization in probability of considered system are established by using improved fixed-time stability results and some inequality techniques. Finally, the effectiveness of established theoretical results is demonstrated through two numerical examples.

Keywords: memristive neural network; fixed-time synchronization; adaptive control; time-varying delay; stochastic perturbation

1. Introduction

The memristor was initially proposed by Chua in 1971 based on the symmetry assumption of fundamental circuit theory [1], and its validity was later confirmed through real physical experiments by researchers at HP [2], demonstrating that memristors exhibit memory functionality, nanoscale dimensions, and lower power consumption compared to basic electronic component: resistor. Leveraging these advantages, researchers have used memristors to simulate the weights of neural networks (MNNs), which was done by resistors in the past, to model synaptic activities in neurons [3–6], thereby offering a promising alternative to traditional neural network architectures.

In 1988, Kosko introduced a significant advancement by extending single-layer unidirectional associative memory neural networks into a two-layer bidirectional architecture, now widely recognized as bidirectional associative memory (BAM) neural networks [7]. Owing to their practical utility in associative memory and pattern recognition, the dynamic behaviors of BAM neural networks have garnered extensive research attention [8–11]. In [9], the authors considered the stability of the single-inertial BAM neural networks with time-varying delays and external inputs by employing matrix measures and Halanay inequalities. [10] established sufficient criteria for the global exponential stability of both periodic discrete-time and continuous-time BAM neural network via using an extended recombination theory. Furthermore, in [11], exponential synchronization of the BAM neural networks with impulsive effects was investigated by leveraging linear matrix inequality (LMI).

It is noteworthy that the aforementioned studies mainly address the stability or synchronization of BAM neural networks over an infinite time horizon. However, to enhance the performance of systems in practical applications, it is imperative that the solutions of the considered systems converge to the equilibrium point at an optimal rate. This necessity has spurred significant advancements in the theory of finite-time (FNT) stability for neural networks over the past two decades, as evidenced by notable contributions in [12–14]. Nevertheless, a critical limitation of FNT stability lies in its dependence on the system's initial conditions for determining the settling time (ST), and correctly obtaining the initial value of system is often a challenging task to in some real-world scenarios. To overcome this issue, Polyakov introduced a fixed-time (FXT) stability concept in [15], which eliminates the reliance of ST on initial states of system. Owing to its vast potential for engineering applications, this theory has attracted considerable interest [16–19]. For instance, in [17], the FXT convergence and synchronization of memristive neural networks with impulsive effects were explored by developing novel FXT stability theory. In [18], the authors addressed the fixed and preassigned-time stabilization of delayed memristive neural network by employing the incomplete beta function and inequality technique. Under the framework of differential inclusions theory, [19] studied the FXT stabilization of discontinuous inertial networks with mixed time delays by introducing a new FXT stability lemma with more relaxed conditions for the time-delayed Filippov discontinuous systems.

It is important to mention that, however, most of the existing studies on FXT stabilization or synchronization of MB-NNs have primarily focused on specific types of networks without considering the effects of stochastic noise. As discussed in [20], random noise frequently influences dynamical behaviours of systems in both natural and societal contexts. For example, systems predominantly governed by stochastic disturbances find extensive applications across diverse domains, such as mechanical system infrastructure, smart grid architecture and cyber-physical systems [21,22]. Consequently, researchers have increasingly investigated the synchronization behavior of neural networks under stochastic influences, leading to significant advancements in complete, lag, projective, and finite-time synchronization for both delayed and non-delayed stochastic neural networks. In [23], a non-periodic event-triggered control method was employed to achieve mean square synchronization for neural networks with coexisting stochastic disturbances and time delays. In [24], the asymptotic behavior of stochastic Cohen-Grossberg neural networks with mixed time delays was analyzed through the application of stochastic functional differential equations. Ref. [25] first introduced the concept, FXT stability in probability by employing bounded function classes. This innovation represents a significant breakthrough in the stability analysis of stochastic systems.[26] investigated the finite/FXT synchronization of complex networks under stochastic disturbances. Nevertheless, in the existing research outcomes, results concerning FXT stability and synchronization of memristor-based bidirectional associative memory neural networks (MB-BAM-NNs) under the presence of stochastic disturbances are relatively scarce. Therefore, we need to conduct further research to establish a unified theoretical framework for ISS to estimate more accurate ST.

Since the vast majority of chaotic nonlinear systems hardly achieve spontaneous synchronization in practical operation, introducing a simple and easily implementable controller is a necessary solution, particularly under limited-cost requirements. Currently, a variety of methods have been developed for synchronization control of various nonlinear systems, including continuous and discontinuous control techniques such as linear feedback control [27], adaptive control [13], and power-law control [28], switching control [29] and intermittent control [30]. Among these, adaptive control is particularly notable due to its inherent robustness and dynamic adjustment capability under varying update rules. This approach is especially effective for strongly nonlinear systems with discontinuous right-hand sides [31–34]. For instance, in [32], adaptive control was employed to address the exponential synchronization of memristive BAM neural networks with mixed delays. Similarly, [33] combined the M-matrix method, stochastic analysis, and adaptive control to investigate both asymptotic and exponential synchronization in stochastic neutral-type neural networks subject to Markovian switching. In [35], the FXT synchronization of semi-Markovian switching discontinuous complex-valued dynamical networks with hybrid couplings and time-varying delays was considered via designing a type of complex adaptive controller that includes seven terms. Further, by introducing simpler adaptive controller, [36] studied the FXT synchronization of delayed neural networks with discontinuous activation functions. However, analysing the FXT synchronization of chaotic system via introducing a simple an effective adaptive controller within shorter ST while maintaining the synchronization performance of system is still an interesting topic.

In light of the aforementioned challenges and advancements, this paper delves into the problem of FXT synchronization of MB-BAM-NNs with stochastic disturbances and time-varying delays. The main highlights of this work are as follows. (1) For the first time, the FXT synchronization problem of MB-BAM-NNs with random perturbations and time-varying delays is under the framework of Filippov set-valued theory. (2) Two novel controllers are proposed to achieve the FXT synchronization of the considered system. First one is state feedback controller which can simplify the controller introduced in [16,17,37,38]. Second one is adaptive controller that automatically adjust control gains via a novel adaptive update law and is more efficient compared to the controllers used in [39,40]. (3) Some easily checkable criteria for FXT synchronization in probability are introduced and the upper bound of the ST is estimated more accusatively.

The rest of this article is organized as follows. Section 2 presents the MB-BAM-NNs model and the basic definitions and lemmas. Main FXT synchronization results are provided in Section 3. Section 4 offers a numerical example to verify the feasibility of the previous results. Finally, Section 5 presents the conclusions of the paper.

Notation 1. *Throughout this paper* $\mathcal{R}$ *and* $\mathcal{R}_+$ *stand for the set of all real numbers and set of all positive real numbers, respectively. The set of all* n *dimensional real vectors is denoted by* $\mathcal{R}^n$, $\mathcal{N} \triangleq \{1, 2, \ldots, n\}$, $\mathcal{M} \triangleq \{1, 2, \ldots, m\}$, $\overline{co}[-a, a]$ *denotes the convex closure formed* $-a$ *into* a, *sign*$(\cdot)$ *stands for the sign function and* $\mathbb{E}(\cdot)$ *represents mathematical expectation.*

2. Problem Description and Preliminaries

In this paper, we consider the following MB-BAM-NNs with stochastic perturbations:

$$\begin{cases} dx_i(s) = \Big(-\alpha_i x_i(s) + \sum_{j=1}^{m} a_{ij}(x_i(s)) f_j(y_j(s)) + \sum_{j=1}^{m} b_{ij}(x_i(s)) f_j(y_j(s-\tau(s))) + I_i\Big) ds \\ \qquad + \sigma_{1i}(x_i(s)) d\omega_1(s), \\ dy_j(s) = \Big(-\beta_j y_j(s) + \sum_{i=1}^{n} c_{ji}(y_j(s)) g_i(x_i(s)) + \sum_{i=1}^{n} d_{ji}(y_j(s)) g_i(x_i(s-\iota(s))) + \tilde{I}_j\Big) ds \\ \qquad + \sigma_{2j}(y_j(s)) d\omega_2(s), \end{cases} \tag{1}$$

where $i \in \mathcal{N}$, $j \in \mathcal{M}$, $x_i(s)$ and $y_j(s)$ respectively stand for the states of ith neuron in x-layer and jth neuron in y-layer neuron at time s; $\alpha_i, \beta_j > 0$ represents the self-inhibition rates; $a_{ij}(x_i(s))$, $b_{ij}(x_i(s))$, $c_{ji}(y_j(s))$ and $d_{ji}(y_j(s))$ represents the memristor-based connection weights, $f_j(\cdot)$ and $g_i(\cdot)$ denote the neuronal activation functions; $\tau(s)$ and $\iota(s)$ are time-varying delays satisfying $0 \le \tau(s) \le \tau$ and $0 \le \iota(s) \le \iota$, I_i and I_j are the external inputs; $\sigma_{1i} \in \mathcal{R}^n$ and $\sigma_{2j} \in \mathcal{R}^m$ denote noise intensity functions; $\omega_1 = (\omega_{11}, \omega_{12}, \ldots, \omega_{1n})^T$ and $\omega_2 = (\omega_{21}, \omega_{22}, \ldots, \omega_{2m})^T$ are Brownian motions defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ that $\mathbb{E}\{d\omega_1^2(s)\} = \mathbb{E}\{d\omega_2^2(s)\} = ds$. According to the switching property of memristor [12], the following discontinuous connection wights can be introduced for system (1)

$$a_{ij}(x_i(s)) = \begin{cases} a_{ij}^*, & |x_i(s)| < \daleth_i, \\ a_{ij}^{**}, & |x_i(s)| \ge \daleth_i, \end{cases} \qquad b_{ij}(x_i(s)) = \begin{cases} b_{ij}^*, & |x_i(s)| < \daleth_i, \\ b_{ij}^{**}, & |x_i(s)| \ge \daleth_i, \end{cases}$$

$$c_{ji}(y_j(s)) = \begin{cases} c_{ji}^*, & |y_j(s)| < \tilde{\daleth}_j, \\ c_{ji}^{**}, & |y_j(s)| \ge \tilde{\daleth}_j, \end{cases} \qquad d_{ji}(y_j(s)) = \begin{cases} d_{ji}^*, & |y_j(s)| < \tilde{\daleth}_j, \\ d_{ji}^{**}, & |y_j(s)| \ge \tilde{\daleth}_j, \end{cases}$$

where $i \in N, j \in M$; a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}, c_{ji}^*, c_{ji}^{**}, d_{ji}^*, d_{ji}^{**}, $\daleth_i$ and $\tilde{\daleth}_j$ are known scalar. Since the connection weights in MB-BAM-NNs have discontinuities, the solutions in this paper are in Filippov's framework [12].

In this paper, assume system (1) be the drive system, then its response system is given by

$$\begin{cases} d\bar{x}_i(s) = \Big(-\alpha_i \bar{x}_i(s) + \sum_{j=1}^{m} a_{ij}(\bar{x}_i(s)) f_j(\bar{y}_j(s)) + \sum_{j=1}^{m} b_{ij}(\bar{x}_i(s)) f_j(\bar{y}_j(s-\tau(s))) + I_i \\ \qquad + u_i(s)\Big) ds + \sigma_{1i}(\bar{x}_i(s)) d\omega_1(s), \\ d\bar{y}_j(s) = \Big(-\beta_j \bar{y}_j(s) + \sum_{i=1}^{n} c_{ji}(\bar{y}_j(s)) g_i(\bar{x}_i(s)) + \sum_{j=1}^{m} d_{ji}(\bar{y}_j(s)) g_i(\bar{x}_i(s-\iota(s))) + \tilde{I}_j \\ \qquad + v_j(s)\Big) ds + \sigma_{2j}(\bar{y}_j(s)) d\omega_2(s), \end{cases} \tag{2}$$

where $u_i(s)$ are $v_j(s)$ are the controllers to be designed.

For simplicity, we present the following notations $\bar{a}_{ij} = \max\{|a_{ij}^*|, |a_{ij}^{**}|\}$, $a_{ij}^+ = \max\{a_{ij}^*, a_{ij}^{**}\}$, $a_{ij}^- = \min\{a_{ij}^*, a_{ij}^{**}\}$, $\bar{b}_{ij} = \max\{|b_{ij}^*|, |b_{ij}^{**}|\}$, $b_{ij}^+ = \max\{b_{ij}^*, b_{ij}^{**}\}$, $b_{ij}^- = \min\{b_{ij}^*, b_{ij}^{**}\}$, $\bar{c}_{ji} = \max\{|c_{ji}^*|, |c_{ji}^{**}|\}$, $c_{ji}^+ = \max\{c_{ji}^*, c_{ji}^{**}\}$, $c_{ji}^- = \min\{c_{ji}^*, c_{ji}^{**}\}$, $\bar{d}_{ji} = \max\{|d_{ji}^*|, |d_{ji}^{**}|\}$, $d_{ji}^+ = \max\{d_{ji}^*, d_{ji}^{**}\}$ and $d_{ji}^- = \min\{d_{ji}^*, d_{ji}^{**}\}$.

Then it is not difficult to obtain that

$$\overline{co}[a_{ij}(x_i(s))] = \begin{cases} a_{ij}^{*}, & |x_i(s)| < \daleth_i, \\ [a_{ij}^{*}, a_{ij}^{**}], & |x_i(s)| = \daleth_i, \\ a_{ij}^{**}, & |x_i(s)| \geq \daleth_i. \end{cases} \quad \overline{co}[b_{ij}(x_i(s))] = \begin{cases} b_{ij}^{*}, & |x_i(s)| < \daleth_i, \\ [b_{ij}^{*}, b_{ij}^{**}], & |x_i(s)| = \daleth_i, \\ b_{ij}^{**}, & |x_i(s)| \geq \daleth_i. \end{cases}$$

$$\overline{co}[c_{ji}(y_j(s))] = \begin{cases} c_{ji}^{*}, & |y_j(s)| < \tilde{\daleth}_j, \\ [c_{ji}^{*}, c_{ji}^{**}], & |y_j(s)| = \tilde{\daleth}_j, \\ c_{ji}^{**}, & |y_j(s)| \geq \tilde{\daleth}_j. \end{cases} \quad \overline{co}[d_{ji}(y_j(s))] = \begin{cases} d_{ji}^{*}, & |y_j(s)| < \tilde{\daleth}_j, \\ [d_{ji}^{*}, d_{ji}^{**}], & |y_j(s)| = \tilde{\daleth}_j, \\ d_{ji}^{**}, & |y_j(s)| \geq \tilde{\daleth}_j. \end{cases}$$

Then, according to the stochastic differential inclusion theory [41,42], we can transform the discontinuous BAM neural networks (1) to the following inclusion form

$$\begin{cases} dx_i(s) \in \Big(-\alpha_i x_i(s) + \sum_{j=1}^{m} \overline{co}[a_{ij}(x_i(s))] f_j(y_j(s)) + \sum_{j=1}^{n} \overline{co}[b_{ij}(x_i(s))] f_j(y_j(s-\tau(s))) \\ \qquad + I_i\Big) ds + \sigma_{1i}(x_i(s)) d\omega_1(s), \\ dy_j(s) \in \Big(-\beta_j y_j(s) + \sum_{i=1}^{n} \overline{co}[c_{ji}(y_j(s))] g_i(x_i(s)) + \sum_{i=1}^{n} \overline{co}[d_{ji}(y_j(s))] g_i(x_i(s-\iota(s))) \\ \qquad + \tilde{I}_j\Big) ds + \sigma_{2j}(y_j(s)) d\omega_2(s), \end{cases}$$

according to the theory of differential inclusion theory and convex analysis method [5,41,42], there exist functions $\hat{a}(s) \in \overline{co}[a_{ij}(x_i(s))]$, $\hat{b}(s) \in \overline{co}[b_{ij}(x_i(s))]$, $\hat{c}(s) \in \overline{co}[c_{ji}(y_i(s))]$, and $\hat{d}(s) \in \overline{co}[d_{ji}(y_i(s))]$ such that

$$\begin{cases} dx_i(s) = \Big(-\alpha_i x_i(s) + \sum_{j=1}^{m} \hat{a}_{ij}(s) f_j(y_j(s)) + \sum_{j=1}^{n} \hat{b}_{ij}(s) f_j(y_j(s-\tau(s))) + I_i\Big) ds \\ \qquad + \sigma_{1i}(x_i(s)) d\omega_1(s), \\ dy_j(s) = \Big(-\beta_j y_j(s) + \sum_{i=1}^{n} \hat{c}_{ji}(s) g_i(x_i(s)) + \sum_{i=1}^{n} \hat{d}_{ji}(s) g_i(x_i(s-\iota(s))) + \tilde{I}_j\Big) ds \\ \qquad + \sigma_{2j}(y_j(s)) d\omega_2(s). \end{cases} \tag{3}$$

Similarly, we can obtain

$$\begin{cases} d\bar{x}_i(s) = \Big(-\alpha_i \bar{x}_i(s) + \sum_{j=1}^{m} \check{a}_{ij}(s) f_j(\bar{y}_j(s)) + \sum_{j=1}^{n} \check{b}_{ij}(s) f_j(\bar{y}_j(s-\tau(s))) + I_i \\ \qquad + u_i(s)\Big) ds + \sigma_{1i}(\bar{x}_i(s)) d\omega_1(s), \\ d\bar{y}_j(s) = \Big(-\beta_j \bar{y}_j(s) + \sum_{i=1}^{n} \check{c}_{ji}(s) g_i(\bar{x}_i(s)) + \sum_{i=1}^{n} \check{d}_{ji}(s) g_i(\bar{x}_i(s-\iota(s))) + \tilde{I}_j \\ \qquad + v_j(s)\Big) ds + \sigma_{2j}(\bar{y}_j(s)) d\omega_2(s), \end{cases} \tag{4}$$

where $\check{a}(s) \in \overline{co}[a_{ij}(\bar{x}_i(s))]$, $\check{b}(s) \in \overline{co}[b_{ij}(\bar{x}_i(s))]$, $\check{c}(s) \in \overline{co}[c_{ji}(\bar{y}_i(s))]$ and $\check{d}(s) \in \overline{co}[d_{ji}(\bar{y}_i(s))]$.

By defining synchronization errors as $e_i(s) = \bar{x}_i(s) - x_i(s)$ and $z_j(s) = \bar{y}_j(s) - y_j(s)$ $(i \in N, j \in M)$, we can obtain the error stochastic MB-BAM-NNs as

$$\begin{cases} de_i(s) \in \Big(-\alpha_i e_i(s) + \overline{co}[\bar{F}_j(s)] + u_i(s)\Big) ds + \bar{\sigma}_{1i}(e_i(s)) d\omega_1(s), \\ dz_j(s) \in \Big(-\beta_j z_j(s) + \overline{co}[\bar{G}_i(s)] + v_j(s)\Big) ds + \bar{\sigma}_{2j}(z_j(s)) d\omega_2(s), \end{cases} \tag{5}$$

where $\bar{\sigma}_{1i}(e_i(s)) d\omega_1(s) = \sigma_{1i}(\bar{x}_i(s)) d\omega_1(s) - \sigma_{1i}(x_i(s)) d\omega_1(s)$, $\bar{\sigma}_{2j}(z_j(s)) d\omega_2(s) = \sigma_{2j}(\bar{y}_j(s)) \times$

$d\omega_2(s) - \sigma_{2j}(y_j(s))d\omega_2(s)$,

$$\begin{aligned}\overline{co}[\bar{F}_j(s)] = &\sum_{j=1}^{m}\big(\overline{co}(a_{ij}(\bar{x}_i(s)))f_j(\bar{y}_j(s)) - \overline{co}(a_{ij}(x_i(s)))f_j(y_j(s))\\&+\overline{co}(b_{ij}(\bar{x}_i(s)))f_j(\bar{y}_j(s-\tau(s))) - \overline{co}(b_{ij}(x_i(s)))f_j(y_j(s-\tau(s)))\big),\\\overline{co}[\bar{G}_i(s)] = &\sum_{i=1}^{n}\big(\overline{co}(c_{ji}(\bar{x}_i((s)))g_i(\bar{x}_i(s)) - \overline{co}(c_{ji}(x_i(s)))g_i(x_i(s))\\&+\overline{co}(d_{ji}(\bar{x}_i(s)))g_i(\bar{x}_i(s-\iota(s))) - \overline{co}(d_{ji}(x_i(s)))g_i(x_i(s-\iota(s)))\big).\end{aligned}$$

Or, equivalently

$$\begin{cases}de_i(s) = \Big(-\alpha_i e_i(s) + F_j(s) + u_i(s)\Big)ds + \bar{\sigma}_{1i}(e_i(s))d\omega_1(s),\\dz_j(s) = \Big(-\beta_j z_j(s) + G_i(s) + v_j(s)\Big)ds + \bar{\sigma}_{2j}(z_j(s))d\omega_2(s),\end{cases}\tag{6}$$

where

$$F_j(s) = \sum_{j=1}^{m}\big(\check{a}_{ij}(s)f_j(\bar{y}_j(s)) - \hat{a}_{ij}(s)f_j(y_j(s)) + \check{b}_{ij}(s)f_j(\bar{y}_j(s-\tau(s))) - \hat{b}_{ij}(s)f_j(y_j(s-\tau(s)))\big),$$

$$G_i(s) = \sum_{i=1}^{n}\big(\check{c}_{ji}(s)g_i(\bar{x}_i(s)) - \hat{c}_{ji}(s)g_i(x_i(s)) + \check{d}_{ji}(s)g_i(\bar{x}_i(s-\iota(s))) - \hat{d}_{ji}(s)g_i(x_i(s-\iota(s)))\big).$$

Assumption 1. *[36] The activation function $f_j(\cdot)$ and $g_i(\cdot)$ are bounded Lipschitz continuous functions. That is, for $\forall\nu_1,\nu_2 \in \mathcal{R}$ and $\nu_1 \neq \nu_2$, there exist constants $\overline{L}_j^f$, $\overline{L}_i^g$ and positive real constants M_j, N_i such that*

$$|f_j(\nu_1) - f_j(\nu_2)| \le \overline{L}_j^f|\nu_1 - \nu_2|, \quad |f_j(\nu_1)| \le M_j,$$

$$|g_i(\nu_1) - g_i(\nu_2)| \le \overline{L}_i^g|\nu_1 - \nu_2|, \quad |g_i(\nu_1)| \le N_i.$$

Assumption 2. *[43] The noise intensity functions $\bar{\sigma}_{1i}(\cdot)$ and $\bar{\sigma}_{2j}(\cdot)$ satisfy the uniform Lipschitz continuity condition. That is, there exist two positive real numbers ϱ_{1i} and ϱ_{2j} such that*

$$\begin{aligned}trace[\bar{\sigma}_{1i}(e_i(s)\bar{\sigma}_{1i}(e_i(s)] \le \varrho_{1i}e_i^2(s),\\trace[\bar{\sigma}_{2j}(z_j(s)\bar{\sigma}_{2j}(z_j(s)] \le \varrho_{2j}z_j^2(s).\end{aligned}$$

Consider an n-dimensional stochastic differential equation

$$dx(s) = \xi(x(s))ds + \delta(x(s))d\omega(s),\tag{7}$$

where $x(0) = x_0$, ω is the scalar standard Brownian motion, $\xi(x) : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear function, and $\delta(x) : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ is the noise intensity function, with $\xi(0) = 0$ and $\delta(0) = 0$.

Definition 1. *[44] The zero solution $x = 0$ of system (7) is said to be globally FXT stable in probability, if the following statements are satisfied for all the initial states $x_0 \in \mathcal{R}^n$.*

(1) Finite-time attractiveness in probability: for any initial value $x_0(\neq 0) \in \mathcal{R}^n$, the first hitting time is finite almost surely, that is $Pro\{T(x_0,\omega) < \infty\} = 1$.

(2) Stability in probability: for every pair of $\zeta \in (0,1)$ and $\eta > 0$, there exists a $\delta = \delta(\zeta,\eta) > 0$ such that $Pro\{|x(s,s_0)| \le \eta,\ \forall s \ge s_0\} \ge 1 - \zeta$ for all $|e_0| \le \delta$.

(3) Mathematical expectation of ST function $T(x_0,\omega)$ is independent on the initial state x_0 of (7) and its upper bound is bounded by a positive constant T. That is, $\mathbb{E}(T(x_0,\omega)) \le T$ for all $x_0 \in \mathcal{R}^n$.

Let $V(x(s)) \in \mathcal{C}^2(\mathcal{R}^n,\mathcal{R}_+)$ is a real-valued Lyapunov function, then stochastic differential can be given by

Itô's lemma as

$$dV(x(s)) = \mathcal{L}V(x(s))dt + V_x(x(s))\delta(s)d\omega(s),$$

where $\mathcal{L}V(x(s)) = V_x(x(s),s)\xi(s) + \frac{1}{2}trace(\delta^T(s)V_{xx}(x(s),s)\delta(s))$.

Lemma 1. *[45] Suppose $V(x(s)) : \mathcal{R}^n \to \mathcal{R}$ is a C-regular function and the following differential inequality holds true*

$$\mathcal{L}V(x(s)) \le \lambda_1 V(x(s)) - \lambda_2 V^p(x(s)) - \lambda_3 V^q(x(s)), \quad x(s) \in \mathcal{R}^n \setminus \{0\},$$

then the zero equilibrium of the system (7) is FXT stable and its ST can be estimated as $E[T(x_0,\omega)] < T_{\max}$ with

$$T_{\max} \triangleq \begin{cases} T^1_{\max} = \frac{1}{\lambda_1\tau(1-p)} \ln\left(1 - \frac{\lambda_1}{\lambda_3}\left(\frac{\lambda_3}{\lambda_2}\right)^{\tau}\right), & \lambda_1 < 0, \\ T^2_{\max} = \frac{\pi}{(p-q)\lambda_3}\left(\frac{\lambda_3}{\lambda_2}\right)^{\tau} \csc(\tau\pi), & \lambda_1 = 0, \\ T^3_{\max} = \frac{\pi\csc(\tau\pi)}{\lambda_2(p-q)}\left(\frac{\lambda_2}{\lambda_3-\lambda_1}\right)^{1-\tau} I\left(\frac{\lambda_2}{\gamma},\tau,1-\tau\right) & \\ \qquad + \frac{\pi\csc(\tau\pi)}{\lambda_3(p-q)}\left(\frac{\lambda_3}{\lambda_2-\lambda_1}\right)^{\tau} I\left(\frac{\lambda_3}{\gamma},1-\tau,\tau\right), & 0 < \lambda_1 < \min\{\lambda_2,\lambda_3\}. \end{cases} \tag{8}$$

where $\lambda_2 > 0$, $\lambda_3 > 0$, $p > 1$, $0 < q < 1$, $\tau = \frac{1-q}{p-q}$, $\gamma = \lambda_2 + \lambda_3 - \lambda_1$,

Especially when $p + q = 2$, the ST can be more accurately estimated as $T(x_0,\omega) < \tilde{T}_{\max}$, where

$$\tilde{T}_{\max} \triangleq \begin{cases} \tilde{T}^4_{\max} = \frac{2}{p-1}\sqrt{\rho}\left(\frac{\pi}{2} + \arctan\left(\frac{\lambda_1}{\sqrt{\rho}}\right)\right), & -2\sqrt{\lambda_2\lambda_3} < \lambda_1 < 2\sqrt{\lambda_2\lambda_3}, \\ \tilde{T}^5_{\max} = \frac{2}{\lambda_1(p-1)}, & \lambda_1 = -2\sqrt{\lambda_2\lambda_3}, \\ \tilde{T}^6_{\max} = \frac{1}{(p-1)\sqrt{-\rho}} \ln\frac{\lambda_1+\sqrt{-\rho}}{\lambda_1-\sqrt{-\rho}}, & \lambda_1 < -2\sqrt{\lambda_2\lambda_3}, \end{cases} \tag{9}$$

where $\rho = 4\lambda_1\lambda_2 - \lambda_1^2$.

Lemma 2. *[38] Let $\varpi_i \ge 0, \varsigma > 1, 0 \le \kappa \le 1$, $i \in \mathcal{N}$, then*

$$\sum_{i=1}^{N}\varpi_i^{\varsigma} \ge N^{1-\varsigma}(\sum_{i=1}^{N}\varpi_i)^{\varsigma}, \quad \sum_{i=1}^{N}\varpi_i^{\kappa} \ge (\sum_{i=1}^{N}\varpi_i)^{\kappa}.$$

Lemma 3. *[5] If the Assumption 1 hold, then for any $x, y \in \mathcal{R}^n$, $i \in \mathcal{N}$ and $j \in \mathcal{M}$, the following inequalities hold:*

$$|\overline{co}(a_{ij}(y))f_j(y_j) - \overline{co}(a_{ij}(x))f_j(x_j)| \le \bar{a}_{ij}\bar{f}_j|y_j - x_j| + M_j|a^*_{ij} - a^{**}_{ij}|,$$
$$|\overline{co}(b_{ji}(y))g_i(y_i) - \overline{co}(b_{ji}(x))g_i(x_i)| \le \bar{b}_{ji}\bar{g}_i|y_i - x_i| + N_i|b^*_{ji} - b^{**}_{ji}|.$$

3. Main Results

In this section, we will analyse the FXT synchronization of MB-BAM-NNs with stochastic disturbances and time delays by introducing two types of controllers. First, we will introduce a following a state feedback controller

$$\begin{aligned} u_i(s) &= -k_{1i}\text{sign}(e_i(s)) - \text{sign}(e_i(s))(k_{2i}|e_i(s)|^{p_1} + \sum_{j=1}^{m} k_{3i}|e_j(s-\tau(s))|), \\ v_j(s) &= -r_{1j}\text{sign}(z_j(s)) - \text{sign}(z_j(s))(r_{2j}|z_j(s)|^{p_1} + \sum_{i=1}^{n} r_{3j}|z_i(s-\iota(s))|). \end{aligned} \tag{10}$$

where $k_{1i},\ k_{2i},\ k_{3i},\ r_{1j},\ r_{2j},\ r_{3j} > 0$ and $p_1 > 1$.

Furthermore, for the convenience of analysis, we introduce the following notations

$$\lambda_{11} = \max_i\{-\alpha_i + \frac{1}{2}\sum_{j=1}^{m}(\bar{a}_{ij}\bar{f}_j + \bar{a}_{ji}\bar{f}_i) + \frac{1}{2}\varrho_{1i}\},\ \lambda_{12} = \min_i\{n^{\frac{1-p_1}{2}}k_{2i}\},$$

$$\lambda_{21} = \min_j\{-\beta_j + \frac{1}{2}\sum_{i=1}^{n}(\bar{c}_{ji}\bar{g}_i + \bar{c}_{ij}\bar{g}_j) + \frac{1}{2}\varrho_{2j}\},\ \lambda_{22} = \min_j\{m^{\frac{1-p_1}{2}}r_{2j}\},$$

$$\lambda_{13} = \min_i\{k_{1i} - \sum_{j=1}^{m}(M_{1j}|a_{ij}^* - a_{ij}^{**}| + M_{2j}|b_{ij}^* - b_{ij}^{**}|)\},$$
$$\lambda_{23} = \max_j\{r_{1j} - \sum_{i=1}^{n}(N_{1i}|c_{ji}^* - c_{ji}^{**}| + N_{2i}|d_{ji}^* - d_{ji}^{**}|)\},$$
$$\lambda_1 = 2\max\{\lambda_{11},\ \lambda_{21}\},\ \lambda_2 = 2\min\{\lambda_{12},\ \lambda_{22}\},\ \lambda_3 = 2\min\{\lambda_{13},\ \lambda_{23}\}.$$

Theorem 1. *Suppose that Assumptions 1 and 2 hold true and the control gains k_{3i} and r_{3j} of the controller (10) satisfy the following inequality*

$$\sum_{j=1}^{m}\bar{b}_{ij}\bar{f}_j \le k_{3i},\quad \sum_{i=1}^{n}\bar{d}_{ji}\bar{g}_i \le r_{3j}\quad \textit{and}\quad \lambda_1 \le \min\{\lambda_2,\ \lambda_3\}, \tag{11}$$

then the drive-response networks (1) and (2) can be FXT synchronization via controller (10) and with the ST $T_{\max}$, where $T_{\max}$ is defined in Lemma 1.

Proof. Construct the following Lyapunov function

$$V(s) = V_1(s) + V_2(s),$$

in which

$$V_1(s) = \frac{1}{2}\sum_{i=1}^{n}e_i^2(s),\ V_2(s) = \frac{1}{2}\sum_{j=1}^{m}z_j^2(s).$$

For $V_1(s)$, from the definition of differential operator $\mathcal{L}V(s)$, we can obtain

$$\begin{aligned}\mathcal{L}V_1(s) =& \sum_{i=1}^{n}e_i(s)\Big(-\alpha_i e_i(s) - k_{1i}\textbf{sign}(e_i(s)) - k_{2i}\textbf{sign}(e_i(s))|e_i(s)|^{p_1}\\ &- k_{3i}\sum_{j=1}^{m}\text{sign}(e_i(s))|e_j(s-\tau(s))|\Big) + \sum_{i=1}^{n}e_i(s)F_j(s)\\ &+ \frac{1}{2}\sum_{i=1}^{n}trace(\bar{\sigma}_{1i}^T(s,e_i(s))\bar{\sigma}_{1i}(s,e_i(s))).\end{aligned} \tag{12}$$

Based on Assumption 1 and Lemma 3, one has

$$F_j(s) \le \sum_{j=1}^{m}\Big(\bar{a}_{ij}\bar{f}_j|e_j(s)| + M_{1j}|a_{ij}^* - a_{ij}^{**}| + \bar{b}_{ij}\bar{f}_j|e_j(s-\tau(s))| + M_{2j}|b_{ij}^* - b_{ij}^{**}|\Big). \tag{13}$$

Based on Assumption 2, one gets

$$\frac{1}{2}\sum_{i=1}^{n}trace(\bar{\sigma}_{1i}^T(s,e_i(s))\bar{\sigma}_{1i}(s,e_i(s))) \le \frac{1}{2}\sum_{i=1}^{n}\varrho_{1i}e_i^2(s). \tag{14}$$

Furthermore, it can be easily obtain that

$$\sum_{i=1}^{n}\sum_{j=1}^{m}|e_i(s)|\bar{a}_{ij}\bar{f}_j|e_j(s)| \le \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\bar{a}_{ij}\bar{f}_j + \bar{a}_{ji}\bar{f}_i\right)e_i^2(s). \tag{15}$$

Based on Lemma 1 and substituting (13)–(15) into (12), one has

$$\begin{aligned}
\mathcal{L}V_1(s) \leq & \sum_{i=1}^{n}\Big(- \alpha_i e_i^2(s) + \sum_{j=1}^{m} e_i(s)(\bar{a}_{ij}\bar{f}_j|e_j(s)| + \bar{b}_{ij}\bar{f}_j|e_i(s-\tau(s))| \\
& + M_{1j}|a_{ij}^{*} - a_{ij}^{**}| + M_{2j}|b_{ij}^{*} - b_{ij}^{**}|) - k_{1i}|e_i(s)| - k_{2i}|e_i(s)|^{p_1+1} \\
& - k_{3i}\sum_{j=1}^{m}|e_i(s)||e_j(s-\tau(s))|\Big) + \frac{1}{2}\sum_{i=1}^{n}\varrho_{1i}e_i^2(s) \\
\leq & \sum_{i=1}^{n}\Big(- \alpha_i + \frac{1}{2}\sum_{j=1}^{m}(\bar{a}_{ij}\bar{f}_j + \bar{a}_{ji}\bar{f}_i) + \frac{1}{2}\varrho_{1i}\Big)e_i^2(s) - k_{2i}\sum_{i=1}^{n}|e_i(s)|^{p_1+1} \\
& - \sum_{i=1}^{n}\Big(k_{1i} - \sum_{j=1}^{m}(M_{1j}|a_{ij}^{*} - a_{ij}^{**}| + M_{2j}|b_{ij}^{*} - b_{ij}^{**}|)\Big)|e_i(s)| \\
\leq & 2\lambda_{11}V_1(s) - 2\lambda_{12}V_1^{\frac{p_1+1}{2}}(s) - 2\lambda_{13}V_1^{\frac{1}{2}}(s).
\end{aligned} \tag{16}$$

Similarly, for $V_2(s)$, we obtain

$$\begin{aligned}
\mathcal{L}V_2(s) \leq & \sum_{j=1}^{m}\Big(- \beta_j z_j^2(s) + \sum_{i=1}^{n} z_j(s)(\bar{c}_{ji}\bar{g}_i|z_i(s)| + N_{1i}|c_{ji}^{*} - c_{ji}^{**}| \\
& + \bar{d}_{ji}\bar{g}_i|z_j(s-\iota(s))| + N_{2i}|d_{ji}^{*} - d_{ji}^{**}|) - r_{2j}|z_j(s)|^{p_1+1} \\
& - r_{1j}(s)|z_j(s)| - r_{3j}\sum_{i=1}^{n}|z_j(s)||z_i(s-\iota(s))|\Big) + \frac{1}{2}\sum_{j=1}^{m}\varrho_{2j}z_j^2(s) \\
\leq & \sum_{i=1}^{n}\Big(- \beta_j + \frac{1}{2}\sum_{i=1}^{n}(\bar{c}_{ji}\bar{g}_i + \bar{c}_{ji}\bar{g}_j) + \frac{1}{2}\varrho_{2j}\Big)z_j^2(s) - \sum_{i=1}^{n}\Big(r_{1i}(s) \\
& - \sum_{i=1}^{n}(N_{1i}|c_{ji}^{*} - c_{ji}^{**}| + N_{2i}|d_{ji}^{*} - d_{ji}^{**}|)\Big)|z_j(s)| - r_{2i}\sum_{j=1}^{m}|z_j(s)|^{p_1+1} \\
\leq & 2\lambda_{21}V_2(s) - 2\lambda_{22}V_2^{\frac{p_1+1}{2}}(s) - 2\lambda_{23}V_2^{\frac{1}{2}}(s).
\end{aligned} \tag{17}$$

Combining Equations (16) and (17), then

$$\mathcal{L}V(s) \leq \lambda_1 V(s) - \lambda_2 V^{\frac{p_1+1}{2}}(s) - \lambda_3 V^{\frac{1}{2}}(s). \tag{18}$$

Let $p = \frac{p_1+1}{2}$, $q = \frac{1}{2}$, in view of the condition $\lambda_1 \leq \min\{\lambda_2,\ \lambda_3\}$ in (11), we obtain from Lemma 1 that the drive-response MB-BAM-NNs (1) and (2) can achieve FXT synchronization in probability with the ST $T_{\max}$, where $T_{\max}$ is defined in (8). □

Corollary 1. *Assume that Assumptions 1–2, and inequality (11) hold true. If* $p_1 = 1.5$ *and* $\lambda_1 < \sqrt{\lambda_2\lambda_3}$*, then systems (1) and (2) can be FXT synchronization with the ST* $\tilde{T}_{\max}$*, where* $\tilde{T}_{\max}$ *is defined (9).*

When the time delays in systems (1) and (2) are removed, then it is reduced to the following form

$$\begin{cases}
dx_i(s) = \Big(- \alpha_i x_i(s) + \sum_{j=1}^{m} a_{ij}(x_i(s))f_j(y_j(s)) + I_i\Big)ds + \sigma_{1i}(s, x_i(s))d\omega_1(s), \\
dy_j(s) = \Big(- \beta_j y_j(s) + \sum_{i=1}^{n} c_{ji}(y_j(s))g_i(x_i(s)) + \tilde{I}_j\Big)ds + \sigma_{2j}(s, y_j(s))d\omega_2(s).
\end{cases} \tag{19}$$

Accordingly, the corresponding response system can be given as

$$\begin{cases} d\bar{x}_i(s) = \Big(-\alpha_i \bar{x}_i(s) + \sum_{j=1}^{m} a_{ij}(\bar{x}_i(s)) f_j(\bar{y}_j(s)) + u_i(s) + I_i \Big) ds \\ \qquad + \sigma_{1i}(s, \bar{x}_i(s)) d\omega_1(s), \\ d\bar{y}_j(s) = \Big(-\beta_j \bar{y}_j(s) + \sum_{i=1}^{n} c_{ji}(\bar{y}_j(s)) g_i(\bar{x}_i(s)) + v_j(s) + \tilde{I}_j \Big) ds \\ \qquad + \sigma_{2j}(s, \bar{y}_j(s)) d\omega_2(s). \end{cases} \tag{20}$$

For this case,we can introduce the following simplified controller

$$u_i(s) = -\delta_{1i}\text{sign}(e_i(s)) - \text{sign}(e_i(s))\delta_{2i}|e_i(s)|^{p_1},$$
$$v_j(s) = -\eta_{1j}\text{sign}(z_j(s)) - \text{sign}(z_j(s))\eta_{2j}|z_j(s)|^{p_1}.$$

Corollary 2. *Under the Assumptions 1–2 and conditions $\lambda_1 \le \min\{\lambda_2, \tilde{\lambda}_3\}$, the systems (19) and (20) driven by above controller can achieve FXT synchronization with a ST $T_{\max}$, where $\tilde{\lambda}_3 = 2\min\{\tilde{\lambda}_{13},\ \tilde{\lambda}_{23}\}$, $\tilde{\lambda}_{13} = \min_i\{k_{1i} - \sum_{j=1}^{m}(M_{1j}|a_{ij}^* - a_{ij}^{**}|)\}$ and $\tilde{\lambda}_{23} = \max_j\{r_{1j}(s) - \sum_{i=1}^{n}(N_{1i}|c_{ji}^* - c_{ji}^{**}|)\}$.*

It should be emphasized that in the presence of noise, delays and other uncertainties, traditional linear feedback controllers with a control gains hardly adapt to environmental changes. Thus, the required control gain is often much higher than what is needed in practice. In contrast, the adaptive feedback control architecture can effectively avoid high-gain control issues and adjust its structure or parameters. Especially, this approach remains effective for system parameter uncertainties. Therefore, we replace the state feedback controller (10) by using the following adaptive controller.

$$\bar{u}_i(s) = -\text{sign}(e_i(s))(m_{1i}(s) + m_{2i}|e_i(s)|^{p_2} + \sum_{j=1}^{m} m_{3i}|e_j(s-\tau(s))|), \tag{21}$$

$$\bar{v}_j(s) = -\text{sign}(z_j(s))(l_{1j}(s) + l_{2j}|z_j(s)|^{p_2} + \sum_{i=1}^{n} l_{3j}|z_i(s-\iota(s))|), \tag{22}$$

and the adaptive updating law $\dot{m}_{1i}(s)$, $\dot{l}_{1j}(s)$ satisfies

$$\dot{m}_{1i}(s) = \pi_i|e_i(s)| - m_{4i}\text{sign}(m_{1i}(s) - \phi_{1i})|m_{1i}(s) - \phi_{1i}|^{p_2} - m_{5i}\text{sign}(m_{1i}(s) - \phi_{1i}),$$
$$\dot{l}_{1j}(s) = \bar{\pi}_j|z_j(s)| - l_{4j}\text{sign}(l_{1j}(s) - \varphi_{1j})|l_{1j}(s) - \varphi_{1j}|^{p_2} - l_{5j}\text{sign}(l_{1j}(s) - \varphi_{1j}),$$

where $p_2 > 1, m_{2i},\ m_{3i},\ m_{4i},\ m_{5i},\ l_{2j},\ l_{3j},\ l_{4j},\ l_{5j},\ \pi_i,\ \bar{\pi}_j$ are positive constants, ϕ_{1i} and φ_{1j} is defined in Theorem 2.

In order to facilitate the subsequent calculation, we also introduce the following notations.

$$\bar{\lambda}_2 = 2\min\{\bar{\lambda}_{12}, \bar{\lambda}_{22}\},\ \bar{\lambda}_3 = 2\min\{\bar{\lambda}_{13}, \bar{\lambda}_{21}\},\ \bar{\lambda}_{13} = n^{\frac{1-p_2}{2}} \min_i\{m_{2i}, m_{4i}/\pi_i^{\frac{1-p_2}{2}}\},$$
$$\bar{\lambda}_{12} = \bar{q}_1 \min_i\{1, 1/\pi_i^{\frac{1}{2}}\},\ \bar{\lambda}_{22} = m^{\frac{1-p_2}{2}} \min_j\{l_{2j}, l_{4j}/\bar{\pi}_j^{\frac{1-p_2}{2}}\},\ \bar{\lambda}_{21} = \bar{q}_2 \min_j\{1, 1/\bar{\pi}_j^{\frac{1}{2}}\},$$
$$\bar{q}_1 = \min_i\{q_1, m_{5i}\},\ q_1 = \min_i\{\phi_{1i} - \sum_{j=1}^{m}(M_{1j}|a_{ij}^* - a_{ij}^{**}| + M_{2j}|b_{ij}^* - b_{ij}^{**}|)\},$$
$$\bar{q}_2 = \min_j\{q_2, l_{5j}\},\ q_2 = \min_j\{\varphi_{1j} - \sum_{i=1}^{n}(N_{1i}|c_{ji}^* - c_{ji}^{**}| + N_{2i}|d_{ji}^* - d_{ji}^{**}|)\}.$$

Theorem 2. *Suppose that Assumptions 1 and 2 hold true and the control gains m_{3i} and l_{3j} of the adaptive controllers (21) and (22) satisfy the following inequalities*

$$\bar{b}_{ij}\bar{f}_j \le m_{3i},\ \bar{d}_{ji}\bar{g}_i \le l_{3j},\ \ \phi_{1i} > \sum_{j=1}^{m}(M_{1j}|a_{ij}^* - a_{ij}^{**}| + M_{2j}|b_{ij}^* - b_{ij}^{**}|), \tag{23}$$

$$\lambda_1 \leq \min\{\bar{\lambda}_2, \bar{\lambda}_3\}, \quad \varphi_{1j} > \sum_{i=1}^{n}(N_{1i}|c_{ji}^{*} - c_{ji}^{**}| + N_{2i}|d_{ji}^{*} - d_{ji}^{**}|). \tag{24}$$

Then the drive-response networks (1) and (2) can achieve FXT synchronization via adaptive controller (21) and (22) with the ST $T_{\max}$, where $T_{\max}$ is defined in Lemma 1.

Proof. Construct the following Lyapunov function

$$V(s) = V_3(s) + V_4(s),$$

in which

$$\begin{aligned}
V_3(s) &= \frac{1}{2}\sum_{i=1}^{n}\Big(e_i^2(s) + \frac{1}{\pi_i}(m_{1i}(s) - \phi_{1i})^2\Big),\\
V_4(s) &= \frac{1}{2}\sum_{j=1}^{m}\Big(z_j^2(s) + \frac{1}{\bar{\pi}_j}(l_{1j}(s) - \varphi_{1j})^2\Big).
\end{aligned}$$

Then, by the Itôs formula, we calculate stochastic operator $\mathcal{L}V_3(s)$ of $V_3(s)$ along the trajectories of the system (7) as

$$\begin{aligned}
\mathcal{L}V_3(s) =& \sum_{i=1}^{n} e_i(s)\Big(- \alpha_i e_i(s) + F_j(s) - m_{1i}(s)\text{sign}(e_i(s))\\
& - m_{2i}\text{sign}(e_i(s))|e_i(s)|^{p_2} - \sum_{j=1}^{m} m_{3i}\text{sign}(e_i(s))|e_j(s-\tau(s))|\Big)\\
& + \sum_{i=1}^{n}\frac{1}{\pi_i}(m_{1i}(s) - \phi_i)\Big(\pi_i|e_i(s)| - m_{4i}\text{sign}(m_{1i}(s) - \phi_i)|m_{1i} - \phi_i|^{p_2}\\
& - m_{5i}\text{sign}(m_{1i}(s) - \phi_i)\Big) + \frac{1}{2}\sum_{i=1}^{n} trace(\bar{\sigma}_{1i}^{T}(s, e_i(s))\bar{\sigma}_{1i}(s, e_i(s)))\\
\leq& \sum_{i=1}^{n}\Big(- \alpha_i e_i^2(s) + \sum_{j=1}^{m} e_i(s)(\bar{a}_{ij}\bar{f}_j|e_j(s)| + \bar{b}_{ij}\bar{f}_j|e_j(s-\tau(s))|\\
& + M_{1j}|a_{ij}^{*} - a_{ij}^{**}| + M_{2j}|b_{ij}^{*} - b_{ij}^{**}|) - m_{1i}(s)|e_i(s)| - m_{2i}|e_i(s)|^{p_2+1}\\
& - \sum_{j=1}^{m} m_{3i}|e_i(s)||e_j(s-\tau(s))|\Big) + \sum_{i=1}^{n}\Big((m_{1i}(s) - \phi_{1i})|e_i(s)| - \frac{m_{4i}}{\pi_i}\times\\
& |m_{1i}(s) - \phi_{1i}|^{p_2+1} - \frac{m_{5i}}{\pi_i}|m_{1i}(s) - \phi_{1i}|\Big) + \frac{1}{2}\sum_{i=1}^{n}\varrho_{1i}e_i^2(s)\\
\leq& \sum_{i=1}^{n}\Big(- \alpha_i + \frac{1}{2}\sum_{j=1}^{m}(\bar{a}_{ij}\bar{f}_j + \bar{a}_{ji}\bar{f}_i) + \frac{1}{2}\varrho_{1i}\Big)e_i^2(s) - m_{2i}\sum_{i=1}^{n}|e_i(s)|^{p_2+1}\\
& - \sum_{i=1}^{n}\Big(\phi_{1i} - \sum_{j=1}^{m}(M_{1j}|a_{ij}^{*} - a_{ij}^{**}| + M_{2j}|b_{ij}^{*} - b_{ij}^{**}|)\Big)|e_i(s)|\\
& - \frac{m_{5i}}{\pi_i}\sum_{i=1}^{n}|m_{1i}(s) - \phi_{1i}| - \sum_{i=1}^{n}\frac{m_{4i}}{\pi_i}|m_{1i} - \phi_{1i}|^{p_2+1}.
\end{aligned}\tag{25}$$

According to Lemma 2, one has

$$
\begin{aligned}
&-\sum_{i=1}^{n}\Big(q_1|e_i(s)|+\frac{m_{5i}}{\pi_i}|m_{1i}(s)-\phi_{1i}|\Big)-\sum_{i=1}^{n}\Big(m_{2i}|e_i(s)|^{p_2+1}+\frac{m_{4i}}{\pi_i}|m_{1i}-\phi_{1i}|^{p_2+1}\Big)\\
&\quad\le-\bar{\lambda}_{12}\sum_{i=1}^{n}\Big(e_i^2(s)+\frac{1}{\pi_i}|m_{1i}(s)-\phi_{1i}|^2\Big)^{\frac{p_2+1}{2}}-\bar{\lambda}_{13}\sum_{i=1}^{n}\Big(e_i^2(s)+\frac{1}{\pi_i}|m_{1i}(s)-\phi_{1i}|^2\Big)^{\frac{1}{2}}\\
&\quad\le-2\bar{\lambda}_{12}V_3^{\frac{p_2+1}{2}}(s)-2\bar{\lambda}_{13}V_3^{\frac{1}{2}}(s).
\end{aligned}
\tag{26}
$$

Similarly, $\mathcal{L}V_4(s)$ can be estimated as

$$
\begin{aligned}
\mathcal{L}V_4(s)=&\sum_{j=1}^{m}\Big(-\beta_j z_j^2(s)+\sum_{i=1}^{n}z_j(s)(\bar{c}_{ji}g_i|z_i(s)|+\bar{d}_{ji}g_i|z_j(s-\iota(s))|\\
&+N_{1i}|c_{ji}^{*}-c_{ji}^{**}|+N_{2i}|d_{ji}^{*}-d_{ji}^{**}|)-l_{1j}(s)|z_j(s)|-l_{2j}|z_j(s)|^{p_2+1}\\
&-\sum_{i=1}^{n}l_{3j}|z_j(s)||z_i(s-\iota(s))|\Big)+\sum_{j=1}^{m}\Big((l_{1j}(s)-\varphi_{1j})|z_j(s)|-\frac{l_{4j}}{\bar{\pi}_j}|l_{1j}-\varphi_{1j}|^{p_2+1}\\
&-\frac{l_{5j}}{\bar{\pi}_j}|l_{1j}(s)-\varphi_{1j}|\Big)+\frac{1}{2}\sum_{j=1}^{m}\varrho_{2j}z_j^2(s)\\
\le&\sum_{j=1}^{m}\Big(-\beta_j+\frac{1}{2}\sum_{i=1}^{n}(\bar{c}_{ji}\bar{g}_i+\bar{c}_{ij}\bar{g}_j)+\frac{1}{2}\varrho_{2j}\Big)z_j^2(s)-\sum_{j=1}^{m}\frac{l_{5j}}{\bar{\pi}_j}|l_{1j}(s)-\varphi_{1j}|\\
&-\sum_{j=1}^{m}\Big(\varphi_{1j}-\sum_{i=1}^{n}(N_{1i}|c_{ji}^{*}-c_{ji}^{**}|+N_{2i}|d_{ji}^{*}-d_{ji}^{**}|)\Big)|z_j(s)|\\
&-\sum_{j=1}^{m}\frac{l_{4j}}{\bar{\pi}_j}|l_{1j}-\varphi_{1j}|^{p_2+1}-l_{2j}\sum_{j=1}^{m}|z_j(s)|^{p_2+1}.
\end{aligned}
\tag{27}
$$

According to Lemma 3, one has

$$
\begin{aligned}
&-\sum_{j=1}^{m}(q_2|z_j(s)|+\frac{l_{5j}}{\bar{\pi}_j}|l_{1j}(s)-\varphi_{1j}|)-\sum_{j=1}^{m}\big(l_{2j}|z_j(s)|^{p_1+1}+\frac{l_{4j}}{\bar{\pi}_j}|l_{1j}-\varphi_{1j}|^{p_2+1}\big)\\
&\quad\le-\bar{\lambda}_{21}\sum_{j=1}^{m}(z_j^2(s)+\frac{1}{\bar{\pi}_j}|l_{1j}(s)-\varphi_{1j}|^2)^{\frac{1}{2}}-\bar{\lambda}_{22}\sum_{j=1}^{m}(z_j^2(s)+\frac{1}{\bar{\pi}_j}|l_{1j}(s)-\varphi_{1j}|^2)^{\frac{p_2+1}{2}}\\
&\quad\le-2\bar{\lambda}_{21}V_4^{\frac{1}{2}}(s)-2\bar{\lambda}_{22}V_4^{\frac{p_2+1}{2}}(s).
\end{aligned}
\tag{28}
$$

By introducing the above inequalities (25)–(28) and by the definition of λ_1 defined in Theorem 1, we obtain

$$
\mathcal{L}V(s)=\mathcal{L}V_3(s)+\mathcal{L}V_4(s)\le\lambda_1 V(s)-\bar{\lambda}_2 V^{\frac{p_2+1}{2}}(s)-\bar{\lambda}_3 V^{\frac{1}{2}}(s).
\tag{29}
$$

Denoting $p=\frac{p_2+1}{2}$ and $q=\frac{1}{2}$, then from the inequality (27) we know that $\lambda_1\le\min\{\bar{\lambda}_2,\ \bar{\lambda}_3\}$. Therefore, based on Lemma 1, we know that the drive-response MB-BAM-NNs (1) and (2) can achieve FXT synchronization in probability with the ST $T_{\max}$, where $T_{\max}$ is defined (8). □

Corollary 3. *Assume that Assumptions 1–2, and inequality (23) and (24) hold true. If $p_2=1.5$ and $\lambda_1<\sqrt{\bar{\lambda}_2\bar{\lambda}_3}$, then systems (1) and (2) can be FXT synchronization with the ST $\tilde{T}_{\max}$ via adaptive controller (21) and (22), where $\tilde{T}_{\max}$ is defined (9).*

For the adaptive synchronization of systems (21) and (22), a simplified linear adaptive controller can be employed as follows

$$
\begin{aligned}
\bar{u}_i(s)=&-\pi_{2i}(s)\mathrm{sign}(e_i(s))-m_{2i}\mathrm{sign}(e_i(s))|e_i(s)|^{p_2},\\
\bar{p}_j(s)=&-\bar{\pi}_{1j}(s)\mathrm{sign}(z_j(s))-l_{2j}\mathrm{sign}(z_j(s))|z_j(s)|^{p_2}.
\end{aligned}
$$

Corollary 4. *Suppose that Assumptions 1 and 2 hold true and the tunable control parameters in adaptive controller (21) and (22) satisfy* $\lambda_1 \le \min\{\bar{\lambda}_2, \bar{\lambda}_3\}$, $\phi_{2i} > \sum_{j=1}^{m}(M_{1j}|a_{ij}^{*} - a_{ij}^{**}|)$, $\mu_1 \le \min\{\bar{\mu}_2, \bar{\mu}_3\}$, $\varphi_{2j} > \sum_{i=1}^{n}(N_{1i}|c_{ji}^{*} - c_{ji}^{**}|)$. *Then systems (21) and (22) driven by above controller will achieve FXT synchronization with a ST* $T_{\max}$, *where* $\bar{q}_1$ *and* $\bar{q}_2$ *in the definitions of* $\bar{\lambda}_2, \bar{\lambda}_3$ *are defined as* $\bar{q}_1 = \min_i\{\tilde{q}_1, m_{5i}\}, \tilde{q}_1 = \min_i\{\phi_{2i} - \sum_{j=1}^{m} M_{1j}|a_{ij}^{*} - a_{ij}^{**}|\}$, $\bar{q}_2 = \min_j\{\tilde{q}_2, l_{5j}\}$ *and* $\tilde{q}_2 = \min_j\{\varphi_{2j} - \sum_{i=1}^{n} N_{1i}|c_{ji}^{*} - c_{ji}^{**}|\}$.

Remark 1. *It is noteworthy that the two controllers we have designed, denoted as* $u_i(s)$ *and* $v_j(s)$*, are not only dependent on the current state of the* i*-th and* j*-th neurons, but also take into account the historical states of other neurons. This aligns with the actual conditions within neural systems, where synaptic transmission can be considered a stochastic process induced by the release of neurotransmitters and other probabilistic effects, thus introducing delays during the transmission and reception phases. This design approach not only mirrors the operational mechanisms of biological neural systems more closely but also enhances the controller's performance and robustness by accounting for the interactions between neurons and the effects of time delays.*

Remark 2. *In practical applications, the design of a simple and efficient control strategy for achieving synchronization across multiple systems is of utmost importance. To ensure FXT synchronization, previous works [16,17,37,38] have proposed controllers that incorporate a linear term* $ke_i(s)$ *along with three or four additional nonlinear terms . In contrast, the controller introduced in this paper significantly simplifies the controllers introduced in [16, 17, 37, 38] by eliminating the linear term, thereby reducing complexity while maintaining robustness and performance. Furthermore, when there is no delay in the original systems (1) and (2), the controller designed in Corollary 2 is equally applicable to the framework established in Theorem 1 [46,47], thereby broadening its scope of application.*

4. Numerical Simulations

In this section, we will provide two numerical examples to verify the feasibility of FXT synchronization obtained in the above sections.

Example 1. *For* $i, j = 2$*, consider the following MB-BAM-NNs with stochastic effects*

$$\begin{cases} \dot{x}_i(s) = \Big(-\alpha_i x_i(s) + \sum_{j=1}^{2} a_{ij}(x_i(s)) f_j(y_j(s)) + \sum_{j=1}^{2} b_{ij}(x_i(s)) f_j(y_j(s-\tau(s))) \\ \qquad + I_i \Big) ds + \sigma_{1i}(s, x_i(s)) d\omega_1(s), \\ \dot{y}_j(s) = \Big(-\beta_j y_j(s) + \sum_{i=1}^{2} c_{ji}(y_j(s)) g_i(x_i(s)) + \sum_{i=1}^{2} d_{ji}(y_j(s)) g_i(x_i(s-\iota(s))) \\ \qquad + \tilde{I}_j \Big) ds + \sigma_{2j}(s, y_j(s)) d\omega_2(s). \end{cases} \tag{30}$$

The memristor-based connection weights are given as

$$a_{11}(x_1(s)) = \begin{cases} 1.5, & |x_1(s)| < 0.5, \\ 1.2, & |x_1(s)| \ge 0.5, \end{cases} \qquad a_{12}(x_1(s)) = \begin{cases} -2, & |x_1(s)| < 0.5, \\ -1.2, & |x_1(s)| \ge 0.5, \end{cases}$$

$$a_{21}(x_1(s)) = \begin{cases} -0.95, & |x_2(s)| < 0.5, \\ 0.9, & |x_2(s)| \ge 0.5, \end{cases} \qquad a_{22}(x_1(s)) = \begin{cases} 0.8, & |x_2(s)| < 0.5, \\ 0.6, & |x_2(s)| \ge 0.5, \end{cases}$$

$$b_{11}(x_1(s)) = \begin{cases} -1, & |x_1(s)| < 0.5, \\ -0.8, & |x_1(s)| \ge 0.5, \end{cases} \qquad b_{12}(x_1(s)) = \begin{cases} 1.5, & |x_1(s)| < 0.5, \\ 0.7, & |x_1(s)| \ge 0.5, \end{cases}$$

$$b_{21}(x_2(s)) = \begin{cases} 0.6, & |x_2(s)| < 0.5, \\ 0.8, & |x_2(s)| \ge 0.5, \end{cases} \qquad b_{22}(x_2(s)) = \begin{cases} -1.3, & |x_2(s)| < 0.5, \\ -1.2, & |x_2(s)| \ge 0.5, \end{cases}$$

$$c_{11}(y_1(s)) = \begin{cases} 1, & |y_1(s)| < 0.5, \\ 1, & |y_1(s)| \ge 0.5, \end{cases} \qquad c_{12}(y_1(s)) = \begin{cases} 0.6, & |y_1(s)| < 0.5, \\ -1.2, & |y_1(s)| \ge 0.5, \end{cases}$$

$$c_{21}(y_1(s)) = \begin{cases} -1.1, & |y_2(s)| < 0.5, \\ -1.5, & |y_2(s)| \ge 0.5, \end{cases} \qquad c_{22}(y_1(s)) = \begin{cases} 1.8, & |y_2(s)| < 0.5, \\ 1.2, & |y_2(s)| \ge 0.5, \end{cases}$$

$$d_{11}(y_1(s)) = \begin{cases} -1.3, & |y_1(s)| < 0.5, \\ -1.5, & |y_1(s)| \ge 0.5, \end{cases} \qquad d_{12}(y_1(s)) = \begin{cases} -1.6, & |y_1(s)| < 0.5, \\ -0.8, & |y_1(s)| \ge 0.5, \end{cases}$$

$$d_{21}(y_2(s)) = \begin{cases} 0.4, & |y_2(s)| < 0.5, \\ 0.3, & |y_2(s)| \geq 0.5, \end{cases} \qquad d_{22}(y_2(s)) = \begin{cases} -2, & |y_2(s)| < 0.5, \\ -1.4, & |y_2(s)| \geq 0.5. \end{cases}$$

Choose the activation function $f_j(u) = tanh(u)$, $g_i(u) = \frac{|u+1|-|u-1|}{2}$, $\tau(s) = \frac{e^s}{0.3+e^s}$, $\iota(s) = \frac{e^s}{0.5+e^s}$, $I_i = \tilde{I}_j = 0$, $\omega_1(s), \omega_2(s)$ is a real valued Brown motion, and $\varrho_1 = \varrho_2 = 0.1$. Figure 1 shows the MATLAB simulations of system (30) with initial conditions $x(0) = (0.45,\ -0.8),\ y(0) = (-0.12,\ 0.25)$.

The corresponding slave system is described as

$$\begin{cases} \dot{\bar{x}}_i(s) = \Big(-\alpha_i \bar{x}_i(s) + \sum_{j=1}^{2} a_{ij}(\bar{x}_i(s)) f_j(\bar{y}_j(s)) + \sum_{j=1}^{2} b_{ij}(\bar{x}_i(s)) f_j(\bar{y}_j(s-\tau(s))) + I_i \\ \qquad + u_i(s) \Big) ds + \sigma_{1i}(s, \bar{x}_i(s)) d\omega_1(s), \\ \dot{\bar{y}}_j(s) = \Big(-\beta_j \bar{y}_j(s) + \sum_{i=1}^{2} c_{ji}(\bar{y}_j(s)) g_i(\bar{x}_i(s)) + \sum_{j=1}^{2} d_{ji}(\bar{y}_j(s)) g_i(\bar{x}_i(s-\iota(s))) + \tilde{I}_j \\ \qquad + v_j(s) \Big) ds + \sigma_{2j}(s, \bar{y}_j(s)) d\omega_2(s). \end{cases} \tag{31}$$

After simple calculation, we can be obtain $\bar{a}_{11} = 1.5,\ a^+_{11} = 1.5,\ a^-_{11} = 1.2,\ \bar{a}_{12} = 2,\ a^+_{12} = -1.2,\ a^-_{12} = -2,\ \bar{a}_{21} = 0.95,\ a^+_{21} = 0.9,\ a^-_{21} = -0.95,\ \bar{a}_{22} = 0.8,\ a^+_{22} = 0.8,\ a^-_{22} = 0.6,\ \bar{b}_{11} = 1,\ b^+_{11} = -0.8,\ b^-_{11} = -1,\ \bar{b}_{12} = 1.5,\ b^+_{12} = 1.5,\ b^-_{12} = 0.7,\ \bar{b}_{21} = 0.8,\ b^+_{21} = 0.8,\ b^-_{21} = 0.6,\ \bar{b}_{22} = 1.3,\ b^+_{22} = -1.2,\ b^-_{22} = -1.3,\ \bar{c}_{11} = 1,\ c^+_{11} = 1,\ c^-_{11} = 1,\ \bar{c}_{12} = 1.2,\ c^+_{12} = 0.6,\ c^-_{12} = -1.2,\ \bar{c}_{21} = 1.5,\ c^+_{21} = -1.1,\ c^-_{21} = -1.5,\ \bar{c}_{22} = 1.8,\ c^+_{22} = 1.8,\ c^-_{22} = 1.2.\ \bar{d}_{11} = 1.5,\ d^+_{11} = -1.3,\ d^-_{11} = -1.5,\ \bar{d}_{12} = 1.6,\ d^+_{12} = 0.8,\ d^-_{12} = -1.6,\ \bar{d}_{21} = 0.4,\ d^+_{21} = 0.4,\ d^-_{21} = 0.3,\ \bar{d}_{22} = 2,\ d^+_{22} = -1.4,\ d^-_{22} = -2$. First, we simulate the FXT synchronization of the systems (30) and (31) under the controller (10). Choosing $\bar{f}_j = \bar{g}_i = 1, \alpha_1 = 2.9,\ \alpha_2 = 2.2,\ \beta_1 = 2.45,\ \beta_2 = 3.25,\ k_{1i} = 2.9,\ r_{1j} = 2.3,\ k_{2i} = 1,\ k_{3i} = 2.9,\ r_{1j} = 2.3,\ r_{2j} = 1,\ r_{3j} = 3.2$ and $p_1 = 1.7$. Through calculation, $\lambda_{11} = \lambda_{21} = 0.175$, $\lambda_{12} = \lambda_{22} = \min_i\{2^{\frac{1-p_1}{2}} k_{2i}\} = 0.7846$, $\lambda_{13} = \max_i\{k_{1i} - \sum_{j=1}^{2}(M_{1j}|a^*_{ij} - a^{**}_{ij}| - M_{2j}|b^*_{ij} - b^{**}_{ij}|)\} = 0.55$, $\lambda_{23} = \max_j\{r_{1i}(s) - \sum_{i=1}^{2}(N_{1i}|c^*_{ji} - c^{**}_{ji}| - N_{2i}|d^*_{ji} - d^{**}_{ji}|)\} = 0.6$. According to Theorem 1, the master-slave systems (30) and (31) are synchronized with ST $T_1 = 4.1237$. Figure 2 shows the FXT synchronization in probability of the systems (30) and (31) under the feedback controller (10).

In addition, to validate the ST estimate given by Corollary 1, take $p_1 = 1.5$, the remaining parameters remain unchanged, then we get $\lambda_{12} = \lambda_{22} = 0.8409$. The corresponding numerical simulations is given in Figure 3, which confirms the theoretical results on the ST estimations given in this paper.

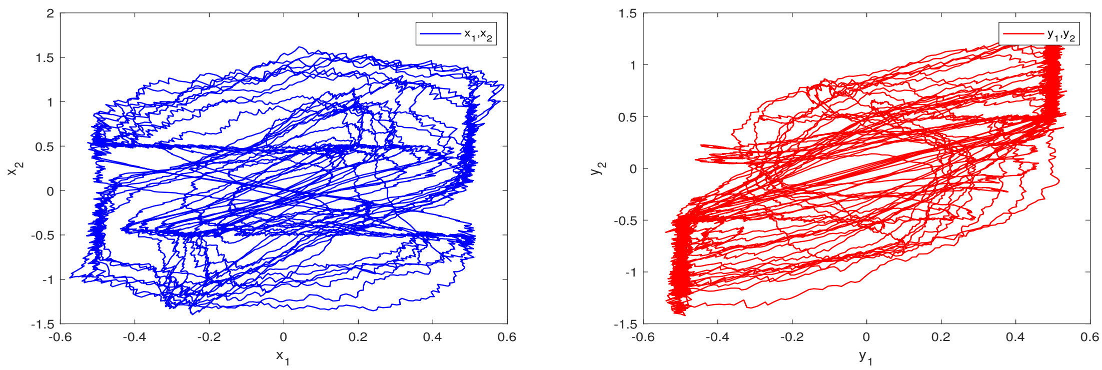


Figure 1. Chaos attractor of the system (30).

Example 2. *Consider the same system (30) and (31) with the adaptive controller (21) and (22), choosing* $m_{2i} = 1,\ m_{3i} = 2.9,\ l_{2j} = 1,\ l_{3j} = 3.2,\ \pi_i = \bar{\pi}_j = 0.8,\ m_{4i} = 0.9,\ m_{5i} = 1,\ l_{4j} = 0.9,\ l_{6j} = 1$ *and* $p_2 = 1.6$ *the other parameters can be chosen as the same as Example 1. Through simple calculation, then the inequalities (23) and (24) in Theorem 2 are also satisfied. According to Theorem 2, the systems (30) and (31) are FXT synchronized with ST* $T_2 = 7.6173$. *Figure 4 shows the probabilistic FXT synchronization of the systems (30) and (31) under the adaptive controller (21) and (22). Furthermore, to verify corollary 3, we select the parameter* $p_1 = 1.5$ *and the rest of the parameters are unchanged. After calculation* $\bar{\lambda}_{12} = \bar{\lambda}_{22} = 0.7333$, *the corresponding numerical simulations is provided Figure 5. Figure 6 shows the time evolution of adaptive control gains* $m_{1i}(s)$ *and* $l_{1j}(s)$.

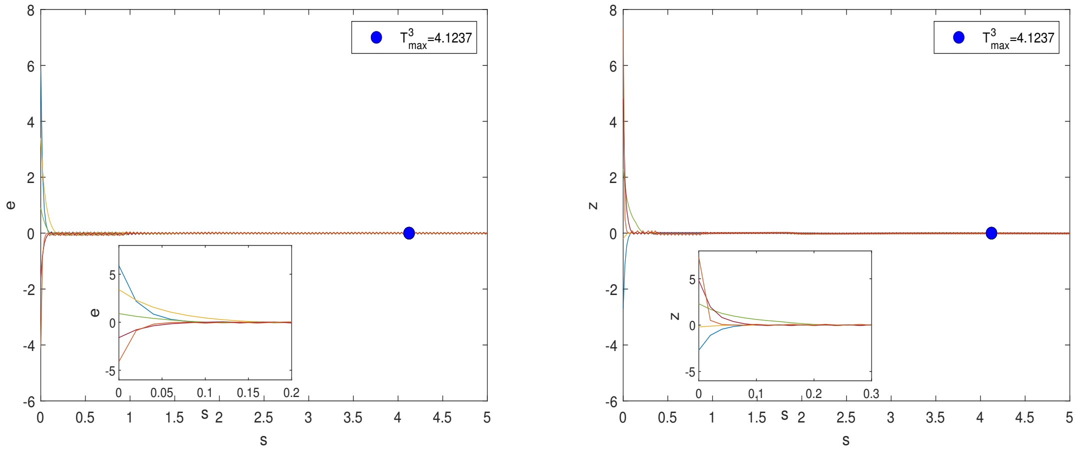


Figure 2. Synchronization error trajectories with state-feedback control ($p_1 = 1.7$).

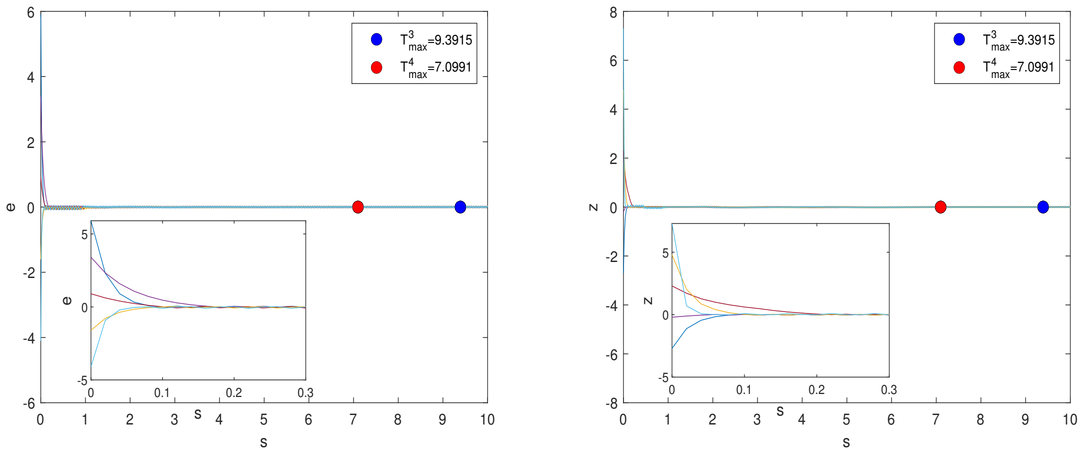


Figure 3. Synchronization error trajectories with state-feedback control ($p_1 = 1.5$).

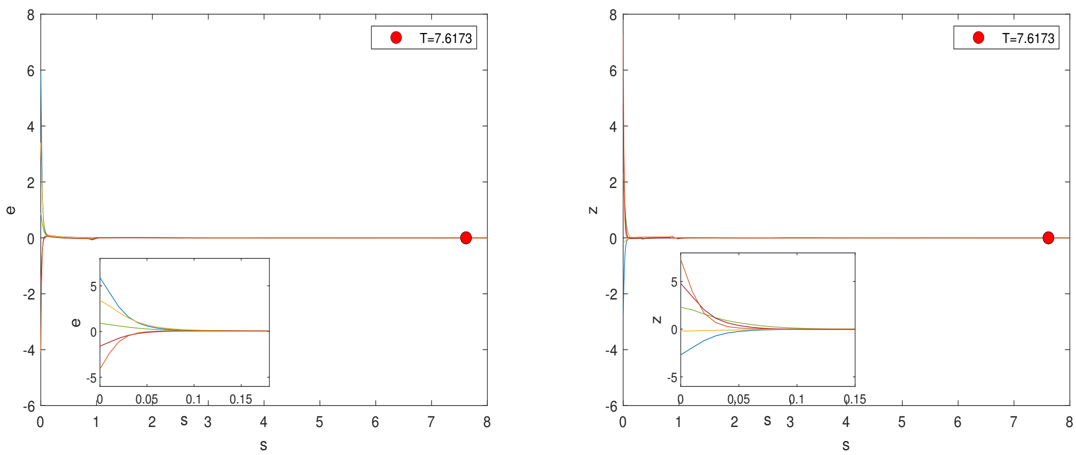


Figure 4. Synchronization error trajectories with adaptive feedback control ($p_2 = 1.6$).

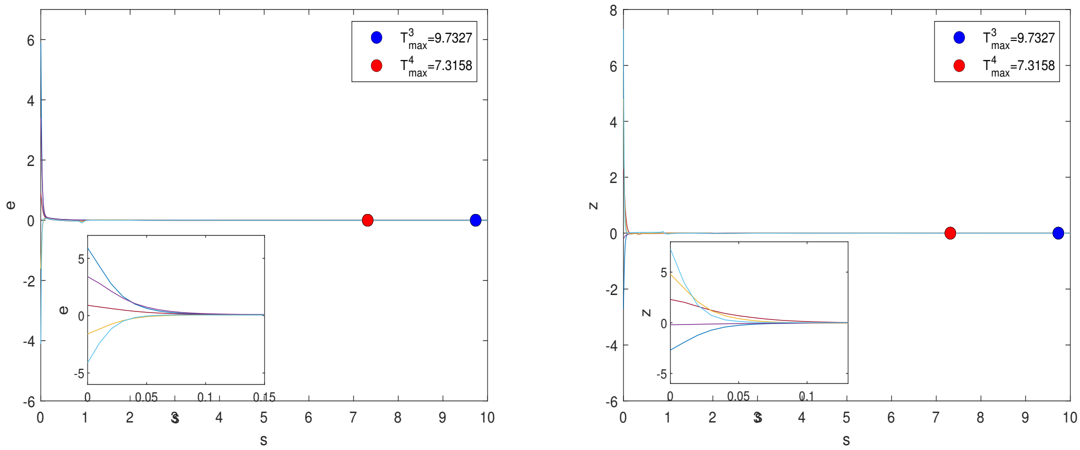


Figure 5. Synchronization error trajectories with adaptive feedback control $(p_2 = 1.5)$.

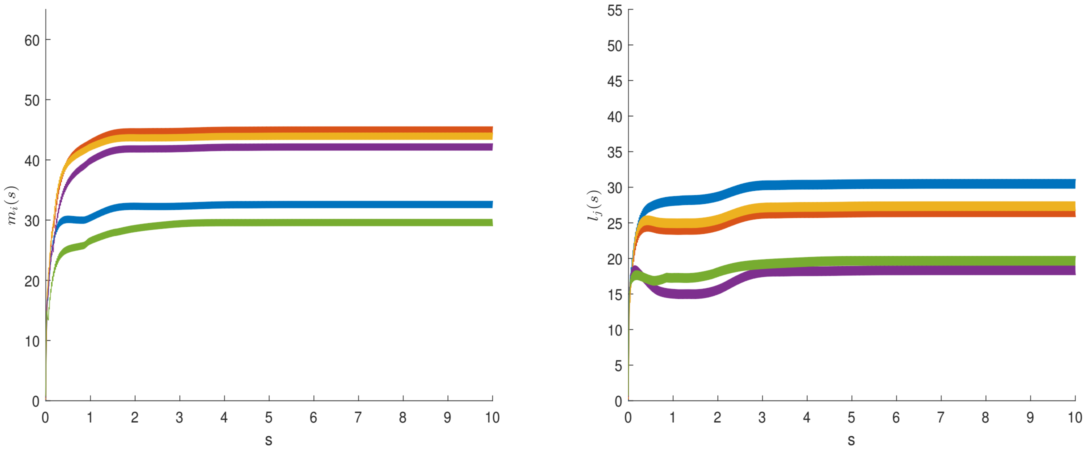


Figure 6. The evolution of the adaptive control gains $m_{1i}(s)$ and $l_{1j}(s)$ in Example 2.

5. Conclusions

In this study, FXT synchronization of stochastic MB-BAM-NNs has been investigated under the frameworks of stochastic differential inclusions theory. By introducing delayed state feedback controller and an adaptive controller, some sufficient criteria have been derived to ensure the FXT synchronization in probability of considered stochastic MB-BAM-NNs. Furthermore, the upper bounds of the ST have been precisely estimated, offering a clear understanding of the system's convergence behavior. Also, the feasibility of introduced FXT synchronization scheme is validated via two numerical examples. For future work, we aim to explore FXT synchronization of stochastic MB-BAM-NNs on time scales would unify continuous and discrete-time analyses, while investigating how time-scale calculus influences synchronization criteria and settling time estimation could provide new insights into convergence dynamics across temporal domains.

Author Contributions

Z.L.: writing—original draft; M.Z.: review and editing; A.A.: supervision, project administration and correspondence. All authors have read and agreed to the published version of the manuscript.

Funding

This work was supported by the National Natural Science Foundation of China (Grant no. 62266042) and the Outstanding Youth Program of Xinjiang, China (Grant no. 2022D01E10).

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

No data was used for the research described in the paper.

Conflicts of Interest

Given the role as Editorial Board Member, Abdujelil Abdurahman had no involvement in the peer review of this paper and had no access to information regarding its peer-review process. Full responsibility for the editorial process of this paper was delegated to another editor of the journal.

Use of AI and AI-Assisted Technologies

No AI tools were utilized for this paper.

References

1. Chua, L. Memristor-the missing circuit element. *IEEE Trans. Circuit Theory* **1971**, *18*, 507–519.
2. Williams, R.S. How we found the missing memristor. *IEEE Spectr.* **2008**, *45*, 28–35.
3. Corinto, F.; Ascoli, A. A boundary condition-based approach to the modeling of memristor nanostructures. *IEEE Trans. Circuits Syst. Regul. Pap.* **2012**, *59*, 2713–2726.
4. Wang, L.; He, H.; Zeng, Z. Global synchronization of fuzzy memristive neural networks with discrete and distributed delays. *IEEE Trans. Fuzzy Syst.* **2019**, *28*, 2022–2034.
5. Abdurahman, A.; Jiang, H.; Teng, Z. Finite-time synchronization for memristor-based neural networks with time-varying delays. *Neural Netw.* **2015**, *69*, 20–28.
6. Wang, L.; Wu, J.; Wang, X. Finite-time stabilization of memristive neural networks with time delays. *Neural Process. Lett.* **2021**, *53*, 299–318.
7. Kosko, B. Bidirectional associative memories. *IEEE Trans. Syst. Man Cybern.* **1988**, *18*, 49–60.
8. Gan, Q.; Li, L.; Yang, J.; et al. Improved results on fixed-/preassigned-time synchronization for memristive complex-valued neural networks. *IEEE Trans. Neural Netw. Learn. Syst.* **2022**, *33*, 5542–5556.
9. Cao, J.; Wan, Y. Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays. *Neural Netw.* **2014**, *53*, 165–172.
10. Zhang, Z.; Liu, K. Existence and global exponential stability of a periodic solution to interval general bidirectional associative memory (BAM) neural networks with multiple delays on time scales. *Neural Netw.* **2011**, *24*, 427–439.
11. Mathiyalagan, K.; Park, J.; Sakthivel, R. Synchronization for delayed memristive BAM neural networks using impulsive control with random nonlinearities. *Appl. Math. Comput.* **2015**, *259*, 967–979.
12. Abdurahman, A.; Jiang, H.; Teng, Z. Finite-time synchronization for fuzzy cellular neural networks with time-varying delays. *Fuzzy Sets Syst.* **2016**, *297*, 96–111.
13. Long, C.; Zhang, G.; Zeng, Z. Finite-time lag synchronization of inertial neural networks with mixed infinite time-varying delays and state-dependent switching. *Neurocomputing* **2021**, *433*, 50–58.
14. Wang, L.; Zhang, C.K. Exponential synchronization of memristor-based competitive neural networks with reaction-diffusions and infinite distributed delays. *IEEE Trans. Neural Netw. Learn. Syst.* **2022**, *35*, 745–758.
15. Polyakov, A. Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Trans. Autom. Control* **2011**, *57*, 2106–2110.
16. Xiao, J.; Zeng, Z.; Wen, S. Finite-/fixed-time synchronization of delayed coupled discontinuous neural networks with unified control schemes. *IEEE Trans. Neural Netw. Learn. Syst.* **2020**, *32*, 2535–2546.
17. Wang, D.; Li, L. Fixed-time synchronization of delayed memristive neural networks with impulsive effects via novel fixed-time stability theorem. *Neural Netw.* **2023**, *163*, 75–85.
18. Li, H.; Hu, C.; Zhang, G. Fixed-/Preassigned-time stabilization of delayed memristive neural networks. *Inf. Sci.* **2022**, *610*, 624–636.
19. Kong, F.; Zhu, Q.; Huang, T. Improved fixed-time stability lemma of discontinuous system and its application. *IEEE Trans. Circuits Syst. Regul. Pap.* **2021**, *69*, 835–846.
20. Song, Y.; Xu, W. Asymmetric lévy noise changed stability in a gene transcriptional regulatory system. *Chaos Solitons Fractals* **2021**, *151*, 111211.
21. Giebel, S.; Rainer, M. Stochastic processes adapted by neural networks with application to climate, energy, and finance. *Appl. Math. Comput.* **2011**, *218*, 1003–1007.
22. Arbi, A.; Tahri, N. Synchronization analysis of novel delayed dynamical Clifford-valued neural networks on timescales. *J. Algorithms Comput. Technol.* **2024**, *18*. https://doi.org/10.1177/17483026241241492

23. Gong, S.; Guo, Z.; Liu, M. Aperiodic event-triggered synchronization control for neural networks with stochastic perturbations and time delay. *IEEE Trans. Circuits Syst. II Express Briefs* **2022**, *70*, 1986–1990.
24. Wan, L.; Zhou, Q. Asymptotic behaviors of stochastic Cohen–Grossberg neural networks with mixed time-delays. *Appl. Math. Comput.* **2013**, *225*, 541–549.
25. Yu, J.; Yu, S.; Li, J. Fixed-time stability theorem of stochastic nonlinear systems. *Int. J. Control* **2018**, *92*, 2194–2200.
26. Liu, X.; Ho, D. Finite/fixed-time pinning synchronization of complex networks with stochastic disturbances. *IEEE Trans. Cybern.* **2019**, *49*, 2398–2403.
27. Wang, L.; Shen, Y.; Zhang, G. Synchronization of a class of switched neural networks with time-varying delays via nonlinear feedback control. *IEEE Trans. Cybern.* **2015**, *46*, 2300–2310.
28. Shi, T.; Hu, C.; Yu, J. Internal/Boundary Control-Based Fixed-Time Synchronization for Spatiotemporal Networks. *IEEE Trans. Cybern.* **2024**, *54*, 6693–6706.
29. Wan, P.; Sun, D.; Zhao, M. Finite-time and fixed-time anti-synchronization of Markovian neural networks with stochastic disturbances via switching control. *Neural Netw.* **2020**, *123*, 1–11.
30. Hu, X.; Wang, L.; Zhang, C.K. Fixed-time synchronization of fuzzy complex dynamical networks with reaction-diffusion terms via intermittent pinning control. *IEEE Trans. Fuzzy Syst.* **2024**, *32*, 2307–2317.
31. Kong, F.; Rajan, R. Finite-time and fixed-time synchronization control of discontinuous fuzzy Cohen-Grossberg neural networks with uncertain external perturbations and mixed time delays. *Fuzzy Sets Syst.* **2021**, *411*, 105–135.
32. Chen, C.; Li, L.; Peng, H. Adaptive synchronization of memristor-based BAM neural networks with mixed delays. *Appl. Math. Comput.* **2018**, *322*, 100–110.
33. Zhou, W.; Zhu, Q.; Shi, P. Adaptive synchronization for neutral-type neural networks with stochastic perturbation and Markovian switching parameters. *IEEE Trans. Cybern.* **2014**, *44*, 2848–2860.
34. Arbi, A.; Cao, J.; Alsaedi, A. Improved synchronization analysis of competitive neural networks with time-varying delays. *Nonlinear Anal. Model. Control* **2018**, *23*, 82–107.
35. Liu, S.; Wu, H.; Cao, J. Fixed-time synchronization for discontinuous delayed complex-valued networks with semi-Markovian switching and hybrid couplings via adaptive control. *Int. J. Adapt. Control Signal Process.* **2020**, *34*, 1359–1382.
36. Wang, L.; Zeng, Z.; Hu, J. Controller design for global fixed-time synchronization of delayed neural networks with discontinuous activations. *Neural Netw.* **2017**, *87*, 122–131.
37. Zhang, Y.; Zhuang, J.; Xia, Y. Fixed-time synchronization of the impulsive memristor-based neural networks. *Commun. Nonlinear Sci. Numer. Simul.* **2019**, *77*, 40–53.
38. Chen, C.; Li, L.; Peng, H. Fixed-time synchronization of inertial memristor-based neural networks with discrete delay. *Neural Netw.* **2019**, *109*, 81–89.
39. Zhou, L.; Lin, H.; Tan, F. Fixed/predefined-time synchronization of coupled memristor based neural networks with stochastic disturbance. *Chaos Solitons Fractals* **2023**, *173*, 113643.
40. Jiang, G.; Wang, L.; Hu, X. Semiglobal fixed/preassigned-time synchronization of stochastic neural networks with random delay via adaptive control. *Neurocomputing* **2024**, *593*, 127832.
41. Kisielewicz, M. *Stochastic Differential Inclusions and Applications*; Springer: New York, NY, USA, 2013.
42. Huang, J.; Han, Z.; Cai, X. Control of time-delayed linear differential inclusions with stochastic disturbance. *J. Frankl. Inst.* **2010**, *347*, 1895–1906.
43. Wang, Q.; Zhao, H.; Liu, A. Predefined-time synchronization of stochastic memristor-based bidirectional associative memory neural networks with time-varying delays. *IEEE Trans. Cogn. Dev. Syst.* **2021**, *14*, 1584–1593.
44. Abdurahman, A.; Abudusaimaiti, M.; Jiang, H. Fixed/predefined-time lag synchronization of complex-valued BAM neural networks with stochastic perturbations. *Appl. Math. Comput.* **2023**, *444*, 127811.
45. Abudusaimaiti, M.; Abdurahman, A.; Jiang, H. Fixed/predefined-time synchronization of fuzzy neural networks with stochastic perturbations. *Chaos Solitons Fractals* **2022**, *154*, 111596.
46. Bao, Y.; Zhang, Y.; Zhang, B. Fixed-time synchronization of coupled memristive neural networks via event-triggered control. *Appl. Math. Comput.* **2021**, *411*, 126542.
47. Ping, J.; Zhu, S.; Liu, X. Finite/fixed-time synchronization of memristive neural networks via event-triggered control. *Knowl.-Based Syst.* **2022**, *258*, 110013.