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New Stability Criterion for Interval Time-Varying Delay Systems

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Abstract: This paper mainly conducts an in-depth analysis for the stability of interval time-varying delay systems. Firstly, a new integral inequality is proposed to cover time-delay information of the system to the greatest extent. Secondly, with the idea of time-delay partitioning, the time-delay interval is divided into multiple non-uniform sub-intervals to consider the more detailed time-delay characteristics of the system. On this basis, Linear Matrix Inequality (LMI) is used to derive a relatively less conservative stability criterion than the conventional conclusion. Finally, the merits of the proposed stability criterion are verified via a numerical example.

Keywords: time-varying delay; integral inequality; time-delay partitioning method; stability

MSC Classification: 93D20; 34F05

1. Introduction

The time-delay factor often leads to the undesirable dynamic behaviour of the systems, thus negatively affecting the system's performance. The stability analysis of time-delay systems has attracted widespread attention since the 20th century [1–13]. For example, [1] proposed a new delay-dependent stability criterion for linear systems with time-varying delay. Ref. [13] developed sufficient conditions for the exponential stability of the constructed switched synchronization error delay system. The time-delay systems can be roughly divided into two categories: constant time-delay systems and time-varying delay systems. In view of the analytical complexity of time-varying delay systems, the stability research has attracted the attention of many researchers (see references [3,7–12] for detail). It is worth emphasizing that time-varying delay usually has specific interval properties in actual systems. Therefore, there is strong interest in studying interval time-varying delay systems (see references [7–9] for detail). The Lyapunov-Krasovskii Functional (LKF) method is an effective approach for the stability study of systems. Related researches mainly include the two parts: constructing an appropriate LKF and estimating the time derivative of LKF.

On one hand, scholars have already proposed a variety of methods to construct LKF, such as multiple integral LKF [14], augmented LKF [1,15], time-delay partitioning LKF [7–9], piecewise LKF [16], and time-delay product LKF [17], etc. Herein, the time-delay partitioning method of constructing an appropriate LKF has recently attracted much attention. For example, Ref. [8] divided the time-delay interval into multiple equidistant sub-intervals and proposed a stability criterion for interval time-varying delay systems. However, the stability criterion obtained by the non-uniform partitioning method of time-delay partitioning is more accurate and less conservative than that by the uniform partitioning method. For instance, Ref. [3] introduced a parameter $\alpha \in [0,1]$ to divide the time-delay interval $[0, h_1]$ into two unequal-length sub-intervals $[0, \alpha h_1]$ and $[\alpha h_1, h_1]$, to furtherly study the stability problem of interval time-varying delay systems, whose stability criterion has a certain flexibility. At the same time, ref. [9] proposed a new stability criterion for interval time-varying delay systems with $0 \le h_1 \le h(t) \le h_2$, where

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$h(t)$ is the time-varying delay. However, in [9], the researchers only uniformly divided interval $[0, h_1]$, while the interval $[h_1, h_2]$ was ignored. In response to the above problems, in view of the advantages of the non-uniformly partitioning method, a question naturally arises: Can a less conservative result be obtained by non-uniformly partitioning both intervals $[0, h_1]$ and $[h_1, h_2]$?

On the other hand, in the study of interval time-varying delay systems, scholars have proposed a variety of methods and techniques to estimate the derivatives of LKF, such as generalized model transformation [18], free weighting matrix [19], integral inequality method [20–27] and mutual convex method [28]. Herein, the integral inequality method is one of the essential methods. For example, [19] combined the parameterized model transformation method with the Leibniz-Newton formula to obtain free weighting matrix inequalities; Ref. [21] obtained a new integral inequality for quadratic terms by incorporating Moon inequality and the Leibniz-Newton formula; Ref. [24] proposed a new integral inequality using information about a double integral of the system state. Furthermore, various constructed integral inequalities have been widely applied in the stability analysis of systems. For instance, Ref. [20] used Jensen's inequality to propose an integral inequality to obtain results on absolute stability; in [15], seuret proposed an integral inequality based on Wirtinger to analyze time-varying delay systems and obtain relatively less conservative stability conclusions. In view of the broad applicability of Wirtinger's inequality and the idea of time-delay partitioning, another question is naturally raised: Can a new integral inequality be proposed to obtain more accurate results and avoid overly conservative results?

The two aforementioned issues constitute the motivations behind this work. Giving positive answers to them is our main contribution. The focus of this work is on the stability of interval time-varying delay systems by some novel technologies. Specifically, in comparison with already- obtained results, the main novelties of this work are listed as follows:

(1) By importing flexible parameters, we adjust the non-uniform partitioning of time-delay interval $[0, h_1]$ to the time-delay intervals $[0, h_1]$ and $[h_1, h_2]$, for which a detailed designed rule is given.

(2) For the widely application, it is necessary to extend the existed Wirtinger's integral inequality. Fortunately, this work successfully develops an optimized version of Wirtinger's integral inequality integrated with the idea of time-delay partitioning.

(3) A novel stability criterion for interval time-varying delay systems is proposed based on innovative Wirtinger's integral inequality and time-delay partitioning strategy, which shows that the proposed Wirtinger's integral inequality can be adapted to any interval switching strategy in stability analysis.

Notations: *R^n represents the n-dimensional column vector, and $R^{m\times n}$ denotes the set of all real matrices in rows m and columns n. The superscripts T and -1 stand for the transpose and inverse of the matrix, respectively. $P>0 (P\ge 0, P<0, P\le 0)$ means P is a real symmetric positive definite (semi-positive definite, negative definite, semi-negative definite) matrix, I_n is the n-dimensional identity matrix, $*$ always denotes the symmetric block in one symmetric matrix, and $Sym\{X\} = X + X^T$.*

2. Preliminary

Consider the following differential system with a time-varying delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) \\ x(t) = \phi(t), \quad t\in[-h_2, 0], \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $\phi(t)\in R^n$ is the initial value, $A, B\in R^{n\times n}$ are constant matrices, and the time-varying delay $h(t)$ satisfies

$$0 \le h_1 \le h(t) \le h_2. \tag{2}$$

Lemma 1 Error! Reference source not found.. *For any vector ζ_1, ζ_2, constant matrices M_1, M_2, symmetric matrix X, and scalars $\alpha\ge 0, \beta\ge 0$, if $\begin{bmatrix} M_1 & X \\ * & M_2 \end{bmatrix} \ge 0$ and $\alpha+\beta=1$ are satisfied, then*

$$-\frac{1}{\alpha}\zeta_1^T M_1\zeta_1 - \frac{1}{\beta}\zeta_2^T M_2\zeta_2 \le -\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}^T \begin{bmatrix} M_1 & X\\ * & M_2\end{bmatrix}\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}. \tag{3}$$

Lemma 2 Error! Reference source not found.. *Given* $\forall W > 0 \in R^{n\times n}$ *, for any differentiable vector function* $x:[a,b]\to R^n$ *, there is*

$$-(b-a)\int_a^b \dot{x}^T(s)W\dot{x}(s)ds \le \begin{bmatrix} x(b)-x(a)\\ x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)ds\end{bmatrix}^T \begin{bmatrix} -W & 0\\ * & -3W\end{bmatrix}\begin{bmatrix} x(b)-x(a)\\ x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)ds\end{bmatrix}. \tag{4}$$

Lemma 3. *Given* $\forall W > 0 \in R^{n\times n}$ *, a positive integer* N *, for any differentiable vector function* $x:[a,b]\to R^n$ *, there is*

$$-(b-a)\int_a^b \dot{x}^T(s)W\dot{x}(s)ds \le N\left[\eta_1^T,\ \eta_2^T,\ \cdots,\ \eta_N^T\right]\begin{bmatrix}\tilde{W} & 0 & 0 & 0\\ * & \tilde{W} & 0 & 0\\ * & * & \ddots & 0\\ * & * & * & \tilde{W}\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_2\\ \vdots\\ \eta_N\end{bmatrix}. \tag{5}$$

where

$$\eta_i = \begin{bmatrix} x(a+\frac{i(b-a)}{N}) - x(a+\frac{(i-1)(b-a)}{N})\\ x(a+\frac{i(b-a)}{N}) + x(a+\frac{(i-1)(b-a)}{N}) - \frac{2N}{b-a}\int_{a+\frac{(i-1)(b-a)}{N}}^{a+\frac{i(b-a)}{N}} x(s)ds\end{bmatrix}, (i=1,2,\cdots,N), \tilde{W} = \begin{bmatrix}-W & 0\\ * & -3W\end{bmatrix}.$$

Proof: Divide interval $b-a$ into equal parts N, then interval $[a,b]$ is expressed as follows

$$[a,b] = \bigcup_{i=1}^{N}\left[a+\frac{(i-1)(b-a)}{N}, a+\frac{i(b-a)}{N}\right].$$

According to Lemma 2, the integral on each small interval above is estimated, then

$$\begin{aligned}
-\frac{b-a}{N}\int_a^{a+\frac{(b-a)}{N}} \dot{x}^T(s)W\dot{x}(s)ds &\le \eta_1^T\tilde{W}\eta_1,\\
-\frac{b-a}{N}\int_{a+\frac{(b-a)}{N}}^{a+\frac{2(b-a)}{N}} \dot{x}^T(s)W\dot{x}(s)ds &\le \eta_2^T\tilde{W}\eta_2,\\
-\frac{b-a}{N}\int_{a+\frac{2(b-a)}{N}}^{a+\frac{3(b-a)}{N}} \dot{x}^T(s)W\dot{x}(s)ds &\le \eta_3^T\tilde{W}\eta_3,\\
&\vdots\\
-\frac{b-a}{N}\int_{a+\frac{(N-1)(b-a)}{N}}^{b} \dot{x}^T(s)W\dot{x}(s)ds &\le \eta_N^T\tilde{W}\eta_N.
\end{aligned}$$

From the additivity of the integration interval, we know

$$\begin{aligned}
-\frac{b-a}{N}\int_a^b \dot{x}^T(s)W\dot{x}(s)ds &\le \eta_1^T\tilde{W}\eta_1 + \eta_2^T\tilde{W}\eta_2 + \cdots + \eta_N^T\tilde{W}\eta_N\\
&= \left[\eta_1^T,\ \eta_2^T,\ \cdots,\ \eta_N^T\right]\begin{bmatrix}\tilde{W} & 0 & 0 & 0\\ * & \tilde{W} & 0 & 0\\ * & * & \ddots & 0\\ * & * & * & \tilde{W}\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_2\\ \vdots\\ \eta_N\end{bmatrix}.
\end{aligned}$$

The conclusion is proved. □

Remark 1. *The integral inequality is a core tool for analyzing the stability of time-delay systems when using LKF method. In the process of deriving the time derivative of LKF, integral inequalities are required to handle integral terms containing time-delay terms (*e.g.*, term* $\int_{t-h(t)}^{t} x^T(s)Px(s)ds$ *). If an integral inequality overestimates the upper bound of the integral term, it will result in overly strict derived stability criteria (*i.e.*, high conservatism)—*

a actually stable system may be misjudged as unstable, or the allowable time-delay range may be severely underestimated. Naturally, a highly accurate integral inequality can significantly reduce conservatism.

Remark 2. *The idea of time-delay partitioning promotes the generation of Lemma 3, whose primary purpose is to increase the amount of information in the inequality and obtain a more accurate integral bound.*

3. Stability Criterion

Theorem 1. *For given scalars* $h_1 > 0$, $h_2 > 0$ *and positive integers* m_1, m_2, *the interval time-varying delay system (1) subject to (2) is asymptotically stable if there exist matrices* $P > 0 \in R^{(m_1+m_2+1)n\times(m_1+m_2+1)n}$, $S_k, Z_k > 0 \in R^{n\times n}, X_{k1}, X_{k2}, X_{k3}, X_{k4} \in R^{n\times n} (k = 0,1,\cdots,m_2-1)$, $Q_i, M_i > 0 \in R^{n\times n}\left(i = 0,1,\cdots,m_1-1\right)$, *and* $G > 0 \in R^{n\times n}$ *such that the following linear matrix inequalities hold for* $k = 0,1,\cdots,m_2-1$:

$$\varpi_k = sym\left\{\Pi_1^T P \Pi_2\right\} + \Pi_3 + \Pi_4 + \Pi_5 + \Pi_{6,k} + \Pi_{7,k} + \Pi_{8,k} < 0, \qquad k = 0,1,\cdots m_2-1, \tag{6}$$

$$\begin{bmatrix} Z_k & 0 & X_{k1} & X_{k2} \\ * & 3Z_k & X_{k3} & X_{k4} \\ * & * & Z_k & 0 \\ * & * & * & 3Z_k \end{bmatrix} > 0, \qquad k = 0,1,\cdots,m_2-1, \tag{7}$$

where

$$\begin{aligned}
\Pi_1 =& \Big[e_1^T,\ (\rho_1-\rho_0)\delta_1 e_{m_1+m_2+3}^T,\ (\rho_2-\rho_1)\delta_1 e_{m_1+m_2+4}^T,\ (\rho_3-\rho_2)\delta_1 e_{m_1+m_2+5}^T,\ \cdots,\ (\rho_{m_1-1}-\rho_{m_1-2})\delta_1 e_{2m_1+m_2+1}^T,\\
& (\rho_{m_1}-\rho_{m_1-1})\delta_1 e_{2m_1+m_2+2}^T,\ (L_1-L_0)\delta_2 e_{2m_1+m_2+3}^T,\ (L_2-L_1)\delta_2 e_{2m_1+m_2+4}^T,\ (L_3-L_2)\delta_2 e_{2m_1+m_2+5}^T,\ \cdots,\\
& (L_{m_2-1}-L_{m_2-2})\delta_2 e_{2m_1+2m_2+1}^T,\ (L_{m_2}-L_{m_2-1})\delta_2 e_{2m_1+m_2+2}^T\Big]^T,\\
\Pi_2 =& \Big[e_0^T,\ e_1^T-e_2^T,\ e_2^T-e_3^T,\ e_3^T-e_4^T,\ \cdots,\ e_{m_1-1}^T-e_{m_1}^T,\ e_{m_1}^T-e_{m_1+1}^T,\ e_{m_1+1}^T-e_{m_1+2}^T,\ e_{m_1+2}^T-e_{m_1+3}^T,\ e_{m_1+3}^T-e_{m_1+4}^T,\\
& \cdots,\ e_{m_1+m_2-1}^T-e_{m_1+m_2}^T,\ e_{m_1+m_2}^T-e_{m_1+m_2+1}^T\Big]^T,\\
\Pi_3 =& \sum_{i=1}^{m_1}(e_i^T Q_{i-1}e_i - e_{i+1}^T Q_{i-1}e_{i+1}) + \sum_{j=m_1+1}^{m_1+m_2}(e_j^T S_{j-m_1-1}e_j - e_{j+1}^T S_{j-m_1-1}e_{j+1}),\\
\Pi_4 =& \sum_{i=1}^{m_1}(\rho_i-\rho_{i-1})^2\delta_1^2 e_0^T M_{i-1}e_0 + \sum_{j=1}^{m_2}(L_j-L_{j-1})^2\delta_2^2 e_0^T Z_{j-1}e_0,\\
\Pi_5 =& -\sum_{i=1}^{m_1}\begin{bmatrix} e_i-e_{i+1} \\ e_i+e_{i+1}-2e_{m_1+m_2+2+i}\end{bmatrix}^T \begin{bmatrix} M_{i-1} & 0 \\ * & 3M_{i-1}\end{bmatrix}\begin{bmatrix} e_i-e_{i+1} \\ e_i+e_{i+1}-2e_{m_1+m_2+2+i}\end{bmatrix},\\
\Pi_{6,k} =& -\sum_{\substack{j=m_1+1\\(j\neq m_1+1+k)}}^{m_1+m_2}\begin{bmatrix} e_j-e_{j+1} \\ e_j+e_{j+1}-2e_{m_1+m_2+2+j}\end{bmatrix}^T \begin{bmatrix} Z_{j-m_1-1} & 0 \\ * & 3Z_{j-m_1-1}\end{bmatrix}\begin{bmatrix} e_j-e_{j+1} \\ e_j+e_{j+1}-2e_{m_1+m_2+2+j}\end{bmatrix},\\
\Pi_{7,k} =& (L_{k+1}-L_k)^2\delta_2^T e_0^T G e_0 - \begin{bmatrix} e_{m_1+k+1}-e_{m_1+k+2} \\ e_{m_1+k+1}+e_{m_1+k+2}-2e_{2m_1+m_2+k+3}\end{bmatrix}^T \begin{bmatrix} G & 0 \\ * & 3G\end{bmatrix}\begin{bmatrix} e_{m_1+k+1}-e_{m_1+k+2} \\ e_{m_1+k+1}+e_{m_1+k+2}-2e_{2m_1+m_2+k+3}\end{bmatrix},\\
\Pi_{8,k} =& -\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2} \\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}^T \begin{bmatrix} Z_k & 0 \\ * & 3Z_k\end{bmatrix}\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2} \\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}\\
& + sym\left\{-\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2} \\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}^T \begin{bmatrix} X_{k1} & X_{k2} \\ X_{k3} & X_{k4}\end{bmatrix}\begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2} \\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix}\right\}\\
& - \begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2} \\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix}^T \begin{bmatrix} Z_k & 0 \\ * & 3Z_k\end{bmatrix}\begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2} \\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
e_0 &= Ae_1 + Be_{m_1+m_2+2},\\
e_i &= [0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(2m_1+2m_2+4-i)n}],\ i = 1,2,\ldots,2m_1+2m_2+4.
\end{aligned}$$

Proof. Non-uniformly divide the intervals $\left[0, h_1\right]$ and $\left[h_1, h_2\right]$ into segments m_1 and m_2, respectively. The specific divisions are as follows

$$\begin{aligned}
\left[0, h_1\right] &= \bigcup\nolimits_{i=0}^{m_1-1}\left[\rho_i\delta_1, \rho_{i+1}\delta_1\right], \quad 0 = \rho_0 < \rho_1 < \cdots < \rho_{m_1-1} < \rho_{m_1} = 1,\\
\left[h_1, h_2\right] &= \bigcup\nolimits_{k=0}^{m_2-1}\left[h_1 + L_k\delta_2, h_1 + L_{k+1}\delta_2\right], \quad 0 = L_0 < L_1 < \cdots < L_{m_2-1} < L_{m_2} = 1,
\end{aligned}$$

where $\delta_1 = h_1$, $\delta_2 = h_2 - h_1$. Thus, for any $t \geq 0$, there exists an integer $k \in \{0, 1, 2 \cdots, m_2 - 2, m_2 - 1\}$ such that $h(t) \in [h_1 + L_k\delta_2, h_1 + L_{k+1}\delta_2]$. Then we introduce the following LKF candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{8}$$

where

$$\begin{aligned}
&V_1(t) = \eta^T(t)P\eta(t),\\
&V_2(t) = \sum_{i=1}^{m_1}\int_{t-\rho_i\delta_1}^{t-\rho_{i-1}\delta_1} x^T(s)Q_{i-1}x(s)ds + \sum_{j=1}^{m_2}\int_{t-h_1-L_j\delta_2}^{t-h_1-L_{j-1}\delta_2} x^T(s)S_{j-1}x(s)ds,\\
&V_3(t) = \sum_{i=1}^{m_1}(\rho_i - \rho_{i-1})\delta_1\int_{-\rho_i\delta_1}^{-\rho_{i-1}\delta_1}\int_{t+u}^{t}\dot{x}^T(s)M_{i-1}\dot{x}(s)dsdu + \sum_{j=1}^{m_2}(L_j - L_{j-1})\delta_2\int_{-h_1-L_j\delta_2}^{-h_1-L_{j-1}\delta_2}\int_{t+u}^{t}\dot{x}^T(s)Z_{j-1}\dot{x}(s)dsdu,\\
&V_4(t) = (L_{k+1} - L_k)\delta_2\int_{-h_1-L_{k+1}\delta_2}^{-h_1-L_k\delta_2}\int_{t+u}^{t}\dot{x}^T(s)G\dot{x}(s)dsdu, \quad k = 0, 1, \cdots, m_2 - 1,
\end{aligned}$$

where $\eta(t) = \left[x^T(t),\ r_1^T(t),\ r_2^T(t)\right]^T$,

$$r_1(t) = \left[\int_{t-\rho_1\delta_1}^{t-\rho_0\delta_1} x^T(s)ds,\ \int_{t-\rho_2\delta_1}^{t-\rho_1\delta_1} x^T(s)ds,\ \int_{t-\rho_3\delta_1}^{t-\rho_2\delta_1} x^T(s)ds,\ \cdots\cdots \int_{t-\rho_{m_1-1}\delta_1}^{t-\rho_{m_1-2}\delta_1} x^T(s)ds,\ \int_{t-\rho_{m_1}\delta_1}^{t-\rho_{m_1-1}\delta_1} x^T(s)ds\right]^T,$$

$$r_2(t) = \left[\int_{t-h_1-L_1\delta_2}^{t-h_1-L_0\delta_2} x^T(s)ds,\ \int_{t-h_1-L_2\delta_2}^{t-h_1-L_1\delta_2} x^T(s)ds,\ \cdots\cdots,\ \int_{t-h_1-L_{m_2-1}\delta_2}^{t-h_1-L_{m_2-2}\delta_2} x^T(s)ds,\ \int_{t-h_1-L_{m_2}\delta_2}^{t-h_1-L_{m_2-1}\delta_2} x^T(s)ds\right]^T.$$

Taking the derivative of $V(t)$, yields

$$\begin{aligned}
&\dot{V}_1(t) = \xi_1^T(t)(sym\{\Pi_1^T P\Pi_2\})\xi_1(t),\\
&\dot{V}_2(t) = \xi_1^T(t)\Pi_3\xi_1(t),\\
&\dot{V}_3(t) = \xi_1^T(t)\Pi_4\xi_1(t) + \Lambda_1 + \Lambda_2,\\
&\dot{V}_4(t) = (L_{k+1} - L_k)^2\delta_2^2\dot{x}^T(s)G\dot{x}(s) - (L_{k+1} - L_k)\delta_2\int_{t-h_1-L_{k+1}\delta_2}^{t-h_1-L_k\delta_2}\dot{x}^T(s)G\dot{x}(s)ds,
\end{aligned}$$

where $\Lambda_1, \Lambda_2, \xi_1(t)$ are as follows

$$\begin{aligned}
&\Lambda_1 = -\sum_{i=1}^{m_1}(\rho_i - \rho_{i-1})\delta_1\int_{t-\rho_i\delta_1}^{t-\rho_{i-1}\delta_1}\dot{x}^T(s)M_{i-1}\dot{x}(s)ds,\\
&\Lambda_2 = -\sum_{j=1}^{m_2}(L_j - L_{j-1})\delta_2\int_{t-h_1-L_j\delta_2}^{t-h_1-L_{j-1}\delta_2}\dot{x}^T(s)Z_{j-1}\dot{x}(s)ds,\\
&\xi_1(t) = \Big[x^T(t),\ x^T(t-\rho_1\delta_1),\ x^T(t-\rho_2\delta_1),\ x^T(t-\rho_3\delta_1),\ \cdots,\ x^T(t-\rho_{m_1-1}\delta_1),\ x^T(t-h_1),\ x^T(t-h_1-L_1\delta_2),\\
&\quad x^T(t-h_1-L_2\delta_2),\ x^T(t-h_1-L_3\delta_2),\ \cdots,\ x^T(t-h_1-L_{m_2-1}\delta_2),\ x^T(t-h_2),\ x^T(t-h(t)),\\
&\quad \frac{1}{(\rho_1-\rho_0)\delta_1}\int_{t-\rho_1\delta_1}^{t-\rho_0\delta_1} x^T(s)ds,\ \frac{1}{(\rho_2-\rho_1)\delta_1}\int_{t-\rho_2\delta_1}^{t-\rho_1\delta_1} x^T(s)ds,\ \frac{1}{(\rho_3-\rho_2)\delta_1}\int_{t-\rho_3\delta_1}^{t-\rho_2\delta_1} x^T(s)ds,\ \cdots,\\
&\quad \frac{1}{(\rho_{m_1-1}-\rho_{m_1-2})\delta_1}\int_{t-\rho_{m_1-1}\delta_1}^{t-\rho_{m_1-2}\delta_1} x^T(s)ds,\ \frac{1}{(\rho_{m_1}-\rho_{m_1-1})\delta_1}\int_{t-\rho_{m_1}\delta_1}^{t-\rho_{m_1-1}\delta_1} x^T(s)ds,\\
&\quad \frac{1}{(L_1-L_0)\delta_2}\int_{t-h_1-L_1\delta_2}^{t-h_1-L_0\delta_2} x^T(s)ds,\ \frac{1}{(L_2-L_1)\delta_2}\int_{t-h_1-L_2\delta_2}^{t-h_1-L_1\delta_2} x^T(s)ds,\ \frac{1}{(L_3-L_2)\delta_2}\int_{t-h_1-L_3\delta_2}^{t-h_1-L_2\delta_2} x^T(s)ds,\ \cdots,\\
&\quad \frac{1}{(L_{m_2-1}-L_{m_2-2})\delta_2}\int_{t-h_1-L_{m_2-1}\delta_2}^{t-h_1-L_{m_2-2}\delta_2} x^T(s)ds,\ \frac{1}{(L_{m_2}-L_{m_2-1})\delta_2}\int_{t-h_1-L_{m_2}\delta_2}^{t-h_1-L_{m_2-1}\delta_2} x^T(s)ds,\\
&\quad \frac{1}{(h_1+L_{k+1}\delta_2-h(t))}\int_{t-h_1-L_{k+1}\delta_2}^{t-h(t)} x^T(s)ds,\ \frac{1}{(h(t)-h_1-L_k\delta_2)}\int_{t-h(t)}^{t-h_1-L_k\delta_2} x^T(s)ds\Big]^T.
\end{aligned}$$

Using Lemma 2, estimate the integral terms in $\Lambda_1, \Lambda_2, \dot{V}_4(t)$ above. The results are as follows

$$\begin{aligned}
\Lambda_1 &\leq \xi_1^T(t)\Pi_5\xi_1(t),\\
\Lambda_2 &\leq \xi_1^T(t)\Pi_{6,k}\xi_1(t) - \Lambda_3,\\
\dot{V}_4(t) &\leq \xi_1^T(t)\Pi_{7,k}\xi_1(t),
\end{aligned}$$

where $\Lambda_3 = -(L_{k+1} - L_k)\delta_2\int_{t-h_1-L_{k+1}\delta_2}^{t-h_1-L_k\delta_2}\dot{x}^T(s)Z_k\dot{x}(s)ds$. Then, under the condition (7), use Lemma 1 and Lemma 2 to process item Λ_3 as follows

$$\begin{aligned}\Lambda_3 &= -(L_{k+1}-L_k)\delta_2\int_{t-h_1-L_{k+1}\delta_2}^{t-h(t)}\dot{x}^T(s)Z_k\dot{x}(s)ds-(L_{k+1}-L_k)\delta_2\int_{t-h(t)}^{t-h_1-L_k\delta_2}\dot{x}^T(s)Z_k\dot{x}(s)ds\\ &\le \xi_1^T(t)\left[-\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2}\\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}^T\begin{bmatrix} Z_k & 0\\ * & 3Z_k\end{bmatrix}\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2}\\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}\right.\\ &\quad+sym\left\{-\begin{bmatrix} e_{m_1+m_2+2}-e_{m_1+k+2}\\ e_{m_1+m_2+2}+e_{m_1+k+2}-2e_{2m_1+2m_2+3}\end{bmatrix}^T\begin{bmatrix} X_{k1} & X_{k2}\\ X_{k3} & X_{k4}\end{bmatrix}\begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2}\\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix}\right\}\\ &\quad\left.-\begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2}\\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix}^T\begin{bmatrix} Z_k & 0\\ * & 3Z_k\end{bmatrix}\begin{bmatrix} e_{m_1+k+1}-e_{m_1+m_2+2}\\ e_{m_1+k+1}-e_{m_1+m_2+2}-2e_{2m_1+2m_2+4}\end{bmatrix}\right]\xi_1(t)\\ &= \xi_1^T(t)\Pi_{8,k}\xi_1(t).\end{aligned}$$

To sum up, there are

$$\begin{aligned}\dot{V}(t) &\le \xi_1^T(t)\left[sym\left\{\Pi_1^T P\Pi_2\right\}+\Pi_3+\Pi_4+\Pi_5+\Pi_{6,k}+\Pi_{7,k}+\Pi_{8,k}\right]\xi_1(t)\\ &= \xi_1^T(t)\varpi_k\xi_1(t). \qquad k=0,1,\cdots,m_2-1\end{aligned}$$

Suppose Equation (6) holds, then $\xi_1^T(t)\varpi_k\xi_1(t)<0$ holds and $\dot{V}(t)<0$. Therefore, system (1) is asymptotically stable. That is, the conclusion is proved. □

Remark 3. *Theorem 1 provides a new stability criterion for interval time-varying delay systems. This criterion is demonstrated by constructing the LKF of time-delay partitioning and applying methods such as Wirtinger integral inequality and mutual convex inequality.*

Remark 4. *Theorem 1 non-uniformly divides the intervals $[0,h_1]$ and $[h_1,h_2]$. This partitioning strategy not only helps to retain more time-delay information in LKF but also considers the particular case of uniform partitioning, thus giving a more comprehensive range of applicability.*

We obtained a new stability criterion based on Lemma 3 ($N=2$). The specific results are as follows:

Theorem 2. *For given scalars $h_1>0$, $h_2>0$ and positive integers m_1, m_2, the interval time-varying delay system (1) subject to (2) is asymptotically stable if there exist matrices $P>0\in R^{(m_1+m_2+1)n\times(m_1+m_2+1)n}$, $S_k,Z_k>0\in R^{n\times n},Y_{kj}\in R^{n\times n}(k=0,1,\cdots,m_2-1\ \ j=1,2,\cdots 16)$, $Q_i,M_i>0\in R^{n\times n}\left(i=0,1,\cdots,m_1-1\right)$, and $G>0\in R^{n\times n}$ such that the following linear matrix inequalities hold for $k=0,1,\cdots,m_2-1$:*

$$\tau_k = sym\left\{\Xi_1^T P\Xi_2\right\}+\Xi_3+\Xi_4+\Xi_5+\Xi_{6,k}+\Xi_{7,k}+\Xi_{8,k}<0, \qquad k=0,1,\cdots,m_2-1, \tag{9}$$

$$\begin{bmatrix} 2Z_k & 0 & 0 & 0 & Y_{k1} & Y_{k2} & Y_{k3} & Y_{k4}\\ * & 6Z_k & 0 & 0 & Y_{k5} & Y_{k6} & Y_{k7} & Y_{k8}\\ * & * & 2Z_k & 0 & Y_{k9} & Y_{k10} & Y_{k11} & Y_{k12}\\ * & * & * & 6Z_k & Y_{k13} & Y_{k14} & Y_{k15} & Y_{k16}\\ * & * & * & * & 2Z_k & 0 & 0 & 0\\ * & * & * & * & * & 6Z_k & 0 & 0\\ * & * & * & * & * & * & 2Z_k & 0\\ * & * & * & * & * & * & * & 6Z_k\end{bmatrix}>0, \qquad k=0,1,\cdots,m_2-1, \tag{10}$$

where

$$\tilde{e}_0 = A\tilde{e}_1 + B\tilde{e}_{2m_1+2m_2+4},$$

$$\tilde{e}_i = [0_{n\times(i-1)n} \;\; I_n \;\; 0_{n\times(4m_1+4m_2+8-i)n}],\ i = 1,2,\ldots,4m_1+4m_2+8,$$

$$\begin{aligned}\Xi_1 = \Big[& \tilde{e}_1^T,\ \frac{(\rho_1-\rho_0)\delta_1}{2}(\tilde{e}_{2m_1+2m_2+5}^T + \tilde{e}_{2m_1+2m_2+6}^T),\ \frac{(\rho_2-\rho_1)\delta_1}{2}(\tilde{e}_{2m_1+2m_2+7}^T + \tilde{e}_{2m_1+2m_2+8}^T),\\ & \frac{(\rho_3-\rho_2)\delta_1}{2}(\tilde{e}_{2m_1+2m_2+9}^T + \tilde{e}_{2m_1+2m_2+10}^T),\ \cdots,\\ & \frac{(\rho_{m_1-1}-\rho_{m_1-2})\delta_1}{2}(\tilde{e}_{4m_1+2m_2+1}^T + \tilde{e}_{4m_1+2m_2+2}^T),\ \frac{(\rho_{m_1}-\rho_{m_1-1})\delta_1}{2}(\tilde{e}_{4m_1+2m_2+3}^T + \tilde{e}_{4m_1+2m_2+4}^T),\\ & \frac{(L_1-L_0)\delta_2}{2}(\tilde{e}_{4m_1+2m_2+5}^T + \tilde{e}_{4m_1+2m_2+6}^T),\ \frac{(L_2-L_1)\delta_2}{2}(\tilde{e}_{4m_1+2m_2+7}^T + \tilde{e}_{4m_1+2m_2+8}^T),\\ & \frac{(L_3-L_2)\delta_2}{2}(\tilde{e}_{4m_1+2m_2+9}^T + \tilde{e}_{4m_1+2m_2+10}^T),\ \cdots,\\ & \frac{(L_{m_2-1}-L_{m_2-2})\delta_2}{2}(\tilde{e}_{4m_1+4m_2+1}^T + \tilde{e}_{4m_1+4m_2+2}^T),\ \frac{(L_{m_2}-L_{m_2-1})\delta_2}{2}(\tilde{e}_{4m_1+4m_2+3}^T + \tilde{e}_{4m_1+4m_2+4}^T)\Big],\end{aligned}$$

$$\begin{aligned}\Xi_2 = \Big[& \tilde{e}_0^T,\ \tilde{e}_1^T-\tilde{e}_3^T,\ \tilde{e}_3^T-\tilde{e}_5^T,\ \tilde{e}_5^T-\tilde{e}_7^T,\ \cdots,\ \tilde{e}_{2m_1-3}^T-\tilde{e}_{2m_1-1}^T,\ \tilde{e}_{2m_1-1}^T-\tilde{e}_{2m_1+1}^T,\\ & \tilde{e}_{2m_1+1}^T-\tilde{e}_{2m_1+3}^T,\ \tilde{e}_{2m_1+3}^T-\tilde{e}_{2m_1+5}^T,\ \tilde{e}_{2m_1+5}^T-\tilde{e}_{2m_1+7}^T,\ \cdots,\ \tilde{e}_{2m_1+2m_2-3}^T-\tilde{e}_{2m_1+2m_2-1}^T,\ \tilde{e}_{2m_1+2m_2-1}^T-\tilde{e}_{2m_1+2m_2+1}^T\Big],\end{aligned}$$

$$\Xi_3 = \sum_{i=1}^{m_1}(\tilde{e}_{2i-1}^T Q_{i-1}\tilde{e}_{2i-1} - \tilde{e}_{2i+1}^T Q_{i-1}\tilde{e}_{2i+1}) + \sum_{j=1}^{m_2}(\tilde{e}_{2m_1+2j-1}^T S_{j-1}\tilde{e}_{2m_1+2j-1} - \tilde{e}_{2m_1+2j+1}^T S_{j-1}\tilde{e}_{2m_1+2j+1}),$$

$$\Xi_4 = \sum_{i=1}^{m_1}(\rho_i-\rho_{i-1})^2\delta_1^2\tilde{e}_0^T M_{i-1}\tilde{e}_0 + \sum_{j=1}^{m_2}(L_j-L_{j-1})^2\delta_2^2\tilde{e}_0^T Z_{j-1}\tilde{e}_0,$$

$$\Xi_5 = -\sum_{i=1}^{m_1}\begin{bmatrix}\tilde{e}_{2i}-\tilde{e}_{2i+1}\\ \tilde{e}_{2i}+\tilde{e}_{2i+1}-2\tilde{e}_{2m_1+2m_2+2i+3}\\ \tilde{e}_{2i-1}-\tilde{e}_{2i}\\ \tilde{e}_{2i-1}+\tilde{e}_{2i}-2\tilde{e}_{2m_1+2m_2+2i+4}\end{bmatrix}^T \begin{bmatrix}2M_{i-1} & 0 & 0 & 0\\ * & 6M_{i-1} & 0 & 0\\ * & * & 2M_{i-1} & 0\\ * & * & * & 6M_{i-1}\end{bmatrix} \begin{bmatrix}\tilde{e}_{2i}-\tilde{e}_{2i+1}\\ \tilde{e}_{2i}+\tilde{e}_{2i+1}-2\tilde{e}_{2m_1+2m_2+2i+3}\\ \tilde{e}_{2i-1}-\tilde{e}_{2i}\\ \tilde{e}_{2i-1}+\tilde{e}_{2i}-2\tilde{e}_{2m_1+2m_2+2i+4}\end{bmatrix},$$

$$\begin{aligned}\Xi_{6,k} = -\sum_{\substack{j=m_1+1\\ j\neq m_1+k+1}}^{m_1+m_2} & \begin{bmatrix}\tilde{e}_{2j}-\tilde{e}_{2j+1}\\ \tilde{e}_{2j}+\tilde{e}_{2j+1}-2\tilde{e}_{2m_1+2m_2+2j+3}\\ \tilde{e}_{2j-1}-\tilde{e}_{2j}\\ \tilde{e}_{2j-1}+\tilde{e}_{2j}-2\tilde{e}_{2m_1+2m_2+2j+4}\end{bmatrix}^T \begin{bmatrix}2Z_{j-m_1-1} & 0 & 0 & 0\\ * & 6Z_{j-m_1-1} & 0 & 0\\ * & * & 2Z_{j-m_1-1} & 0\\ * & * & * & 6Z_{j-m_1-1}\end{bmatrix}\\ & \times\begin{bmatrix}\tilde{e}_{2j}-\tilde{e}_{2j+1}\\ \tilde{e}_{2j}+\tilde{e}_{2j+1}-2\tilde{e}_{2m_1+2m_2+2j+3}\\ \tilde{e}_{2j-1}-\tilde{e}_{2j}\\ \tilde{e}_{2j-1}+\tilde{e}_{2j}-2\tilde{e}_{2m_1+2m_2+2j+4}\end{bmatrix},\end{aligned}$$

$$\begin{aligned}\Xi_{7,k} = & (L_{k+1}-L_k)^2\delta_2^2\tilde{e}_0^T G\tilde{e}_0\\ & - \begin{bmatrix}\tilde{e}_{2m_1+2k+2}-\tilde{e}_{2m_1+2k+3}\\ \tilde{e}_{2m_1+2k+2}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+2m_2+2k+5}\\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2k+2}\\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2k+2}-2\tilde{e}_{4m_1+2m_2+2k+6}\end{bmatrix}^T \begin{bmatrix}2G & 0 & 0 & 0\\ * & 6G & 0 & 0\\ * & * & 2G & 0\\ * & * & * & 6G\end{bmatrix} \begin{bmatrix}\tilde{e}_{2m_1+2k+2}-\tilde{e}_{2m_1+2k+3}\\ \tilde{e}_{2m_1+2k+2}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+2m_2+2k+5}\\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2k+2}\\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2k+2}-2\tilde{e}_{4m_1+2m_2+2k+6}\end{bmatrix},\end{aligned}$$

$$\begin{aligned}\Xi_{8,k} = & -\begin{bmatrix}\tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3}\\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5}\\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3}\\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6}\end{bmatrix}^T \begin{bmatrix}2Z_k & 0 & 0 & 0\\ * & 6Z_k & 0 & 0\\ * & * & 2Z_k & 0\\ * & * & * & 6Z_k\end{bmatrix} \begin{bmatrix}\tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3}\\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5}\\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3}\\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6}\end{bmatrix}\\ & - sym\left\{\begin{bmatrix}\tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3}\\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5}\\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3}\\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6}\end{bmatrix}^T \begin{bmatrix}Y_{k1} & Y_{k2} & Y_{k3} & Y_{k4}\\ Y_{k5} & Y_{k6} & Y_{k7} & Y_{k8}\\ Y_{k9} & Y_{k10} & Y_{k11} & Y_{k12}\\ Y_{k13} & Y_{k14} & Y_{k15} & Y_{k16}\end{bmatrix} \begin{bmatrix}\tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4}\\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7}\\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2}\\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8}\end{bmatrix}\right\}\\ & - \begin{bmatrix}\tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4}\\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7}\\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2}\\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8}\end{bmatrix}^T \begin{bmatrix}2Z_k & 0 & 0 & 0\\ * & 6Z_k & 0 & 0\\ * & * & 2Z_k & 0\\ * & * & * & 6Z_k\end{bmatrix} \begin{bmatrix}\tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4}\\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7}\\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2}\\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8}\end{bmatrix}.\end{aligned}$$

Proof: This theorem uses the same LKF as the previous Theorem 1. Then, execute the derivative operation on $V(t)$ and obtain the following results

$$\dot{V}_1(t)=\xi_2^T(t)(sym\left\{\Xi_1^T P\Xi_2\right\})\xi_2(t),$$
$$\dot{V}_2(t)=\xi_2^T(t)\Xi_3\xi_2(t),$$
$$\dot{V}_3(t)=\xi_2^T(t)\Xi_4\xi_2(t)+\Lambda_1+\Lambda_2,$$
$$\dot{V}_4(t)=(L_{k+1}-L_k)^2\delta_2^2\dot{x}^T(s)G\dot{x}(s)-(L_{k+1}-L_k)\delta_2\int_{t-h_1-L_{k+1}\delta_2}^{t-h_1-L_k\delta_2}\dot{x}^T(s)G\dot{x}(s)ds,$$

where Λ_1,Λ_2 are consistent with the above,

$$\begin{aligned}\xi_2(t)=\Big[&x^T(t),\ x^T(t-\frac{(\rho_0+\rho_1)\delta_1}{2}),\ x^T(t-\rho_1\delta_1),\ x^T(t-\frac{(\rho_1+\rho_2)\delta_1}{2}),\ x^T(t-\rho_2\delta_1),\ x^T(t-\frac{(\rho_2+\rho_3)\delta_1}{2}),\\
&x^T(t-\rho_3\delta_1),\ x^T(t-\frac{(\rho_3+\rho_4)\delta_1}{2}),\ \cdots,\ x^T(t-\rho_{m_1-2}\delta_1),\ x^T(t-\frac{(\rho_{m_1-2}+\rho_{m_1-1})\delta_1}{2}),\ x^T(t-\rho_{m_1-1}\delta_1),\\
&x^T(t-\frac{(\rho_{m_1-1}+\rho_{m_1})\delta_1}{2}),\ x^T(t-h_1),\ x^T(t-h_1-\frac{(L_0+L_1)\delta_2}{2}),\ x^T(t-h_1-L_1\delta_2),\\
&x^T(t-h_1-\frac{(L_1+L_2)\delta_2}{2}),\ x^T(t-h_1-L_2\delta_2),\ x^T(t-h_1-\frac{(L_2+L_3)\delta_2}{2}),\ x^T(t-h_1-L_3\delta_2),\ \cdots,\\
&x^T(t-h_1-L_{m_2-2}\delta_2),\ x^T(t-h_1-\frac{(L_{m_2-2}+L_{m_2-1})\delta_2}{2}),\ x^T(t-h_1-L_{m_2-1}\delta_2),\ x^T(t-h_1-\frac{(L_{m_2-1}+L_{m_2})\delta_2}{2}),\\
&x^T(t-h_2),\ x^T(t-\frac{h_1+L_k\delta_2+h(t)}{2}),\ x^T(t-\frac{h_1+L_{k+1}\delta_2+h(t)}{2}),\ x^T(t-h(t)),\\
&\frac{2}{(\rho_1-\rho_0)\delta_1}\int_{t-\rho_1\delta_1}^{t-\frac{(\rho_0+\rho_1)\delta_1}{2}}x^T(s)ds,\ \frac{2}{(\rho_1-\rho_0)\delta_1}\int_{t-\frac{(\rho_0+\rho_1)\delta_1}{2}}^{t-\rho_0\delta_1}x^T(s)ds,\ \frac{2}{(\rho_2-\rho_1)\delta_1}\int_{t-\rho_2\delta_1}^{t-\frac{(\rho_1+\rho_2)\delta_1}{2}}x^T(s)ds,\\
&\frac{2}{(\rho_2-\rho_1)\delta_1}\int_{t-\frac{(\rho_1+\rho_2)\delta_1}{2}}^{t-\rho_1\delta_1}x^T(s)ds,\ \frac{2}{(\rho_3-\rho_2)\delta_1}\int_{t-\rho_3\delta_1}^{t-\frac{(\rho_2+\rho_3)\delta_1}{2}}x^T(s)ds,\ \frac{2}{(\rho_3-\rho_2)\delta_1}\int_{t-\frac{(\rho_2+\rho_3)\delta_1}{2}}^{t-\rho_2\delta_1}x^T(s)ds,\ \cdots,\\
&\frac{2}{(\rho_{m_1-1}-\rho_{m_1-2})\delta_1}\int_{t-\rho_{m_1-1}\delta_1}^{t-\frac{(\rho_{m_1-2}+\rho_{m_1-1})\delta_1}{2}}x^T(s)ds,\ \frac{2}{(\rho_{m_1-1}-\rho_{m_1-2})\delta_1}\int_{t-\frac{(\rho_{m_1-2}+\rho_{m_1-1})\delta_1}{2}}^{t-\rho_{m_1-2}\delta_1}x^T(s)ds,\\
&\frac{2}{(\rho_{m_1}-\rho_{m_1-1})\delta_1}\int_{t-\rho_{m_1}\delta_1}^{t-\frac{(\rho_{m_1-1}+\rho_{m_1})\delta_1}{2}}x^T(s)ds,\ \frac{2}{(\rho_{m_1}-\rho_{m_1-1})\delta_1}\int_{t-\frac{(\rho_{m_1-1}+\rho_{m_1})\delta_1}{2}}^{t-\rho_{m_1-1}\delta_1}x^T(s)ds,\\
&\frac{2}{(L_1-L_0)\delta_2}\int_{t-h_1-L_1\delta_2}^{t-h_1-\frac{(L_0+L_1)\delta_2}{2}}x^T(s)ds,\ \frac{2}{(L_1-L_0)\delta_2}\int_{t-h_1-\frac{(L_0+L_1)\delta_2}{2}}^{t-h_1-L_0\delta_2}x^T(s)ds,\ \frac{2}{(L_2-L_1)\delta_2}\int_{t-h_1-L_2\delta_2}^{t-h_1-\frac{(L_1+L_2)\delta_2}{2}}x^T(s)ds,\\
&\frac{2}{(L_2-L_1)\delta_2}\int_{t-h_1-\frac{(L_1+L_2)\delta_2}{2}}^{t-h_1-L_1\delta_2}x^T(s)ds,\ \frac{2}{(L_3-L_2)\delta_2}\int_{t-h_1-L_3\delta_2}^{t-h_1-\frac{(L_2+L_3)\delta_2}{2}}x^T(s)ds,\ \frac{2}{(L_3-L_2)\delta_2}\int_{t-h_1-\frac{(L_2+L_3)\delta_2}{2}}^{t-h_1-L_2\delta_2}x^T(s)ds,\\
&\cdots,\ \frac{2}{(L_{m_2-1}-L_{m_2-2})\delta_2}\int_{t-h_1-L_{m_2-1}\delta_2}^{t-h_1-\frac{(L_{m_2-2}+L_{m_2-1})\delta_2}{2}}x^T(s)ds,\ \frac{2}{(L_{m_2-1}-L_{m_2-2})\delta_2}\int_{t-h_1-\frac{(L_{m_2-2}+L_{m_2-1})\delta_2}{2}}^{t-h_1-L_{m_2-2}\delta_2}x^T(s)ds,\\
&\frac{2}{(L_{m_2}-L_{m_2-1})\delta_2}\int_{t-h_1-L_{m_2}\delta_2}^{t-h_1-\frac{(L_{m_2-1}+L_{m_2})\delta_2}{2}}x^T(s)ds,\frac{2}{(L_{m_2}-L_{m_2-1})\delta_2}\int_{t-h_1-\frac{(L_{m_2-1}+L_{m_2})\delta_2}{2}}^{t-h_1-L_{m_2-1}\delta_2}x^T(s)ds,\\
&\frac{2}{(h_1+L_{k+1}\delta_2-h(t))}\int_{t-h_1-L_{k+1}\delta_2}^{t-\frac{h_1+L_{k+1}\delta_2+h(t)}{2}}x^T(s)ds,\ \frac{2}{(h_1+L_{k+1}\delta_2-h(t))}\int_{t-\frac{h_1+L_{k+1}\delta_2+h(t)}{2}}^{t-h(t)}x^T(s)ds,\\
&\frac{2}{(h(t)-h_1-L_k\delta_2)}\int_{t-h(t)}^{t-\frac{h_1+L_k\delta_2+h(t)}{2}}x^T(s)ds,\ \frac{2}{(h(t)-h_1-L_k\delta_2)}\int_{t-\frac{h_1+L_k\delta_2+h(t)}{2}}^{t-h_1-L_k\delta_2}x^T(s)ds\Big]^T.\end{aligned}$$

Using Lemma 3 ($N=2$), estimate the integral terms in $\Lambda_1,\Lambda_2,\dot{V}_4(t)$ above. The results are as follows

$$\Lambda_1\le\xi_2^T(t)\Xi_5\xi_2(t),$$
$$\Lambda_2\le\xi_2^T(t)\Xi_{6,k}\xi_2(t)-\Lambda_3,$$
$$\dot{V}_4(t)\le\xi_2^T(t)\Xi_{7,k}\xi_2(t).$$

Then, under the condition (10), use Lemma 1 and Lemma 3 ($N=2$) to process item Λ_3 as follows

$$\Lambda_3 = -(L_{k+1}-L_k)\delta_2\int_{t-h_1-L_{k+1}\delta_2}^{t-h(t)} \dot{x}^T(s)Z_k\dot{x}(s)ds - (L_{k+1}-L_k)\delta_2\int_{t-h(t)}^{t-h_1-L_k\delta_2} \dot{x}^T(s)Z_k\dot{x}(s)ds$$
$$\le \xi_2^T(t)\left[-\begin{bmatrix} \tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3} \\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5} \\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3} \\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6} \end{bmatrix}^T \begin{bmatrix} 2Z_k & 0 & 0 & 0 \\ * & 6Z_k & 0 & 0 \\ * & * & 2Z_k & 0 \\ * & * & * & 6Z_k \end{bmatrix} \begin{bmatrix} \tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3} \\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5} \\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3} \\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6} \end{bmatrix}\right.$$
$$+sym\left\{-\begin{bmatrix} \tilde{e}_{2m_1+2m_2+3}-\tilde{e}_{2m_1+2k+3} \\ \tilde{e}_{2m_1+2m_2+3}+\tilde{e}_{2m_1+2k+3}-2\tilde{e}_{4m_1+4m_2+5} \\ \tilde{e}_{2m_1+2m_2+4}-\tilde{e}_{2m_1+2m_2+3} \\ \tilde{e}_{2m_1+2m_2+4}+\tilde{e}_{2m_1+2m_2+3}-2\tilde{e}_{4m_1+4m_2+6} \end{bmatrix}^T \begin{bmatrix} Y_{k1} & Y_{k2} & Y_{k3} & Y_{k4} \\ Y_{k5} & Y_{k6} & Y_{k7} & Y_{k8} \\ Y_{k9} & Y_{k10} & Y_{k11} & Y_{k12} \\ Y_{k13} & Y_{k14} & Y_{k15} & Y_{k16} \end{bmatrix} \begin{bmatrix} \tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4} \\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7} \\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2} \\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8} \end{bmatrix}\right\}$$
$$\left.-\begin{bmatrix} \tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4} \\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7} \\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2} \\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8} \end{bmatrix}^T \begin{bmatrix} 2Z_k & 0 & 0 & 0 \\ * & 6Z_k & 0 & 0 \\ * & * & 2Z_k & 0 \\ * & * & * & 6Z_k \end{bmatrix} \begin{bmatrix} \tilde{e}_{2m_1+2m_2+2}-\tilde{e}_{2m_1+2m_2+4} \\ \tilde{e}_{2m_1+2m_2+2}+\tilde{e}_{2m_1+2m_2+4}-2\tilde{e}_{4m_1+4m_2+7} \\ \tilde{e}_{2m_1+2k+1}-\tilde{e}_{2m_1+2m_2+2} \\ \tilde{e}_{2m_1+2k+1}+\tilde{e}_{2m_1+2m_2+2}-2\tilde{e}_{4m_1+4m_2+8} \end{bmatrix}\right]\xi_2(t)$$
$$= \xi_2^T(t)\Xi_{8,k}\xi_2(t).$$

To sum up, there are

$$\begin{aligned} \dot{V}(t) &\le \xi_2^T(t)\left[sym\left\{\Xi_1^T P \Xi_2\right\} + \Xi_3 + \Xi_4 + \Xi_5 + \Xi_{6,k} + \Xi_{7,k} + \Xi_{8,k}\right]\xi_2(t) \\ &= \xi_2^T(t)\tau_k\xi_2(t). \qquad k = 0,1,\cdots,m_2-1 \end{aligned}$$

Suppose equation (9) holds, then $\xi_2^T(t)\tau_k\xi_2(t) < 0$ holds and $\dot{V}(t) < 0$. Therefore, system (1) is asymptotically stable. That is, the conclusion is proved. □

Remark 5. *Unlike Theorem 1 based on Lemma 2, Theorem 2 obtains the corresponding stability criterion based on Lemma 3.*

4. Numerical Example

In this section, a commonly used numerical example is applied to illustrate the advantages of the stability criterion.

Consider system (1) with the following parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

The goal here is to calculate the upper bound h_2 of $h(t)$ given h_1, m_1, m_2, where m_1 and m_2 represent the number of partition segments of intervals $[0, h_1]$ and $[h_1, h_2]$, and h_1 is the lower bound of delay $h(t)$.

Through Theorems 1 and 2, the upper bound h_2 of $h(t)$ is calculated and listed in Table 1 for detail. In order to highlight the superiority of the time-delay interval partitioning method, Table 1 not only gives the value of the upper bound h_2 in the case without any partitioning ($m_1 = 1, m_2 = 1$) but also gives the value of h_2 in the case of time-delay partitioning ($m_1 = 2, m_2 = 2$). When $m_1 = 2, m_2 = 2$, the partitioning entirely depends on the values of ρ_1, L_1: if $\rho_1 = 0.5, L_1 = 0.5$, it is a uniform two-partitioning; if $\rho_1 \ne 0.5, L_1 \ne 0.5$, it is a non-uniform two-partitioning (Parameters ρ_1 and L_1 are introduced in the partitioning of intervals $[0, h_1]$ and $[h_1, h_2]$, respectively). The upper bound h_2 of $h(t)$ in cases $\rho_1 = 0.5, L_1 = 0.5$ and $\rho_1 = 0.6, L_1 = 0.3$ is given in Table 1, which is enough to highlight the superiority of the method proposed in this article compared with uniform partitioning.

Regarding Table 1, the effect of non-uniform partitioning outperforms that of uniform case, and the effect of uniform partitioning outperforms that of no partitioning in Theorem 1. This conclusion holds for all the different tested lower bounds of delay h_1; the results of Theorem 2 also conform to this pattern.

Table 1. Upper bound h_2 of $h(t)$ under different h_1.

h_1	1.0	2.0	3.0	4.0	5.0
Theorem 1 $(m_1 = 1, m_2 = 1)$	2.1521	2.7785	3.4964	4.2939	5.1372
Theorem 1 $(\rho_1 = 0.5, L_1 = 0.5)$	2.3019	2.8060	3.5052	4.3025	5.1476
Theorem 1 $(\rho_1 = 0.6, L_1 = 0.3)$	2.3343	2.9226	3.6319	4.3948	5.1971
Theorem 2 $(m_1 = 1, m_2 = 1)$	2.2277	2.7977	3.4985	4.2939	5.1371
Theorem 2 $(\rho_1 = 0.5, L_1 = 0.5)$	2.3413	2.8066	3.5052	4.3025	5.1476
Theorem 2 $(\rho_1 = 0.6, L_1 = 0.3)$	2.3733	2.9289	3.6328	4.3948	5.1970

When $m_1 = 2, m_2 = 2$, Table 1 only presents the results of the upper bound h_2 for the two situations $\rho_1 = 0.5, L_1 = 0.5$ and $\rho_1 = 0.6, L_1 = 0.3$. In order to ensure reliability, it is necessary to increase the experimental sample size appropriately. Furthermore, we calculate the values of the upper bound h_2 under 81 different situations corresponding to $\rho_1 = 0.1, 0.2, \cdots 0.9$ and $L_1 = 0.1, 0.2, \cdots 0.9$ and draw the figures of the changes in h_2 (Due to the setting of the figure size, the horizontal axis labels in Figures 1–3 are abbreviated, which is consistent with Figure 4). Figure 1 shows the changes in the difference of h_2 under the two-partitioning situation of Theorem 1 and that under the without any partitioning situation of Theorem 1, i.e., $\Delta h_2 =$ h_2 under the two-partitioning of Theorem 1 minus h_2 without any partitioning of Theorem 1. The positive values represent the increase in the upper bound, and the negative ones represent the decrease in the upper bound. Similarly, Figure 2 shows the changes in the difference of h_2 under the two-partitioning situation of Theorem 1 and that under the uniform two-partitioning situation of Theorem 1, that is, Δh_2 = h_2 under the two-partitioning situation of Theorem 1 minus h_2 under the uniform two-partitioning situation of Theorem 1. Figure 3 shows the changes in the difference of h_2 under the two-partitioning situation of Theorem 2 and that under the two-partitioning situation of Theorem 1, that is, Δh_2 = h_2 under the two-partitioning situation of Theorem 2 minus h_2 under the two-partitioning situation of Theorem 1.

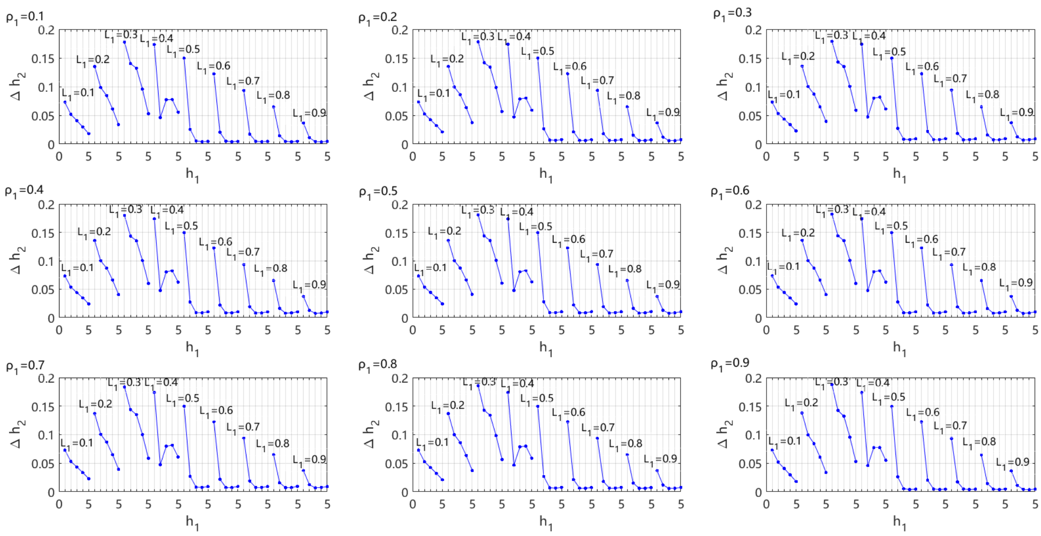

Figure 1. The changes in the difference of h_2 under the two-partitioning situation of Theorem 1 and that without any partitioning situation of Theorem 1.

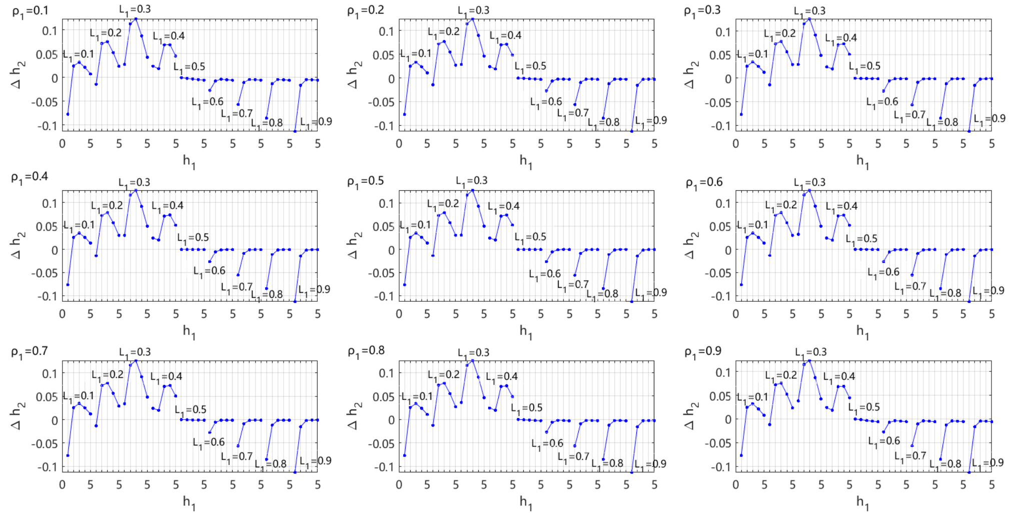

Figure 2. The changes in the difference of h_2 under the two-partitioning situation of Theorem 1 and that under the uniform two-partitioning situation of Theorem 1.

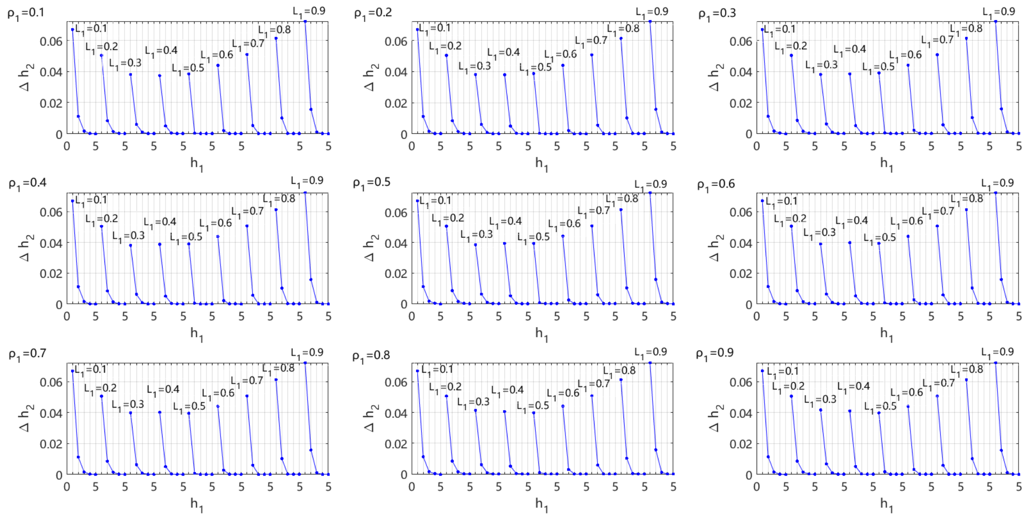

Figure 3. The changes in the difference of h_2 under the two-partitioning situation of Theorem 2 and that under the two-partitioning situation of Theorem 1.

Remark 6. *For Figures 1–3, it is worth noting that the figure contains nine sub-figures based on different values of* ρ_1*, and each sub-figure contains nine curves with values 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9 of* L_1*. Taking the first subfigure as an example, its specific meaning is as follows: Given the division* $\rho_1 = 0.1$ *of* $[0, h_1]$*, the partitioning of* $[h_1, h_2]$ *adopts 9 different cases, and these 9 partitioning cases correspond to the 9 curves in the figure, respectively. Each curve reflects the variation trend of the study increment under 5 different step sizes when partitioning* $\rho_1 = 0.1$ *of* $[0, h_1]$ *and a specific partitioning of* $[h_1, h_2]$ *are fixed, where the specific values of 5 step sizes are listed in Table 1.*

It can be seen from Figure 1 that the upper bound h_2 in the case of two-partitioning in Theorem 1 is generally larger than the upper bound h_2 without any partitioning, which is enough to show the superiority of the time-delay partitioning method. Observing Figure 1 from the perspective of ρ_1, it can be inferred that, the value range of ρ_1 making the upper bound h_2 relatively large is $0.5 < \rho_1 < 0.7$. As can be seen from Figure 1, the changing trends of h_2 in the nine sub-figures are roughly the same. In order to show the value range of L_1 more clearly, sub-figure ($\rho_1 = 0.6$) in Figure 1 is enlarged, as shown in Figure 4. It can be seen from Figure 4 that, the value range of L_1 making the upper bound h_2 relatively large is $0.2 < L_1 < 0.4$.

From Figure 2, the following conclusions can be drawn: (1) When $\rho_1 = 0.5, L_1 = 0.5$, the change of the upper bound h_2 is zero; (2) For any ρ_1, when $L_1 = 0.5$, the change of the upper bound h_2 is close to zero; (3) For any ρ_1, when $L_1 < 0.5$, the larger L_1 is, the more pronounced the advantage of non-uniform partitioning is, compared with uniform partitioning; (4) For any ρ_1, when $L_1 > 0.5$, the larger L_1 is, the more pronounced the advantage of uniform partitioning is, compared with non-uniform partitioning. Figure 3 shows that the changes in the upper bound h_2 are all positive, which means that compared with Theorem 1, Theorem 2 can provide a less conservative stability criterion.

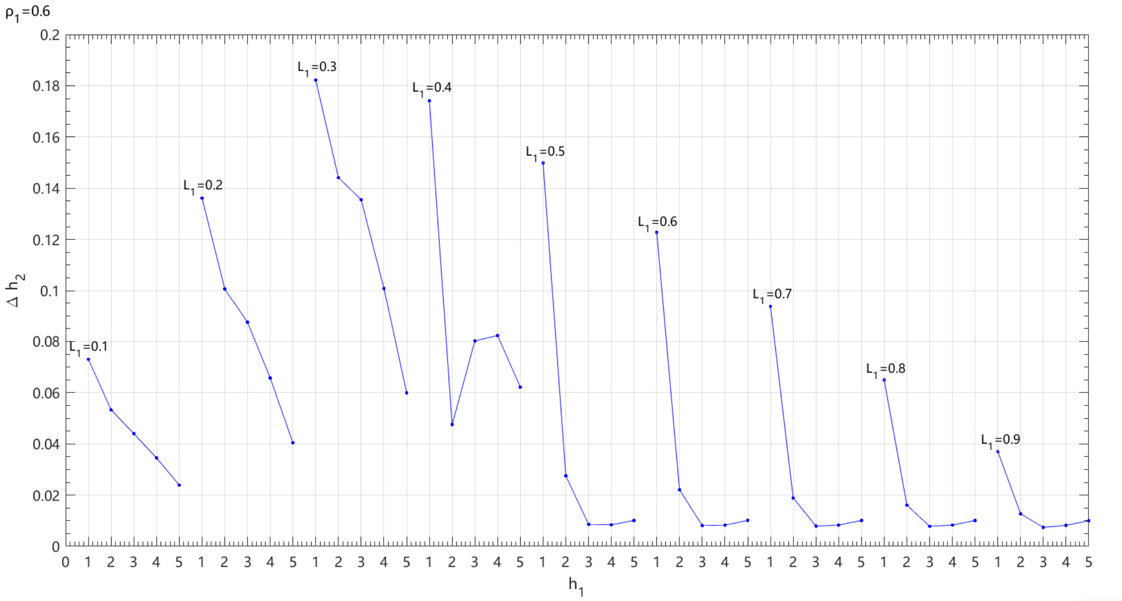

Figure 4. The changes in the difference of h_2 under the two-partitioning situation of Theorem 1 ($\rho_1 = 0.6$) and that without any partitioning situation of Theorem 1.

5. Conclusions

Aiming at the conservative problem of integral inequality and the construction problem of LKF in the stability study of interval time-varying delay systems, this paper proposes a new integral inequality based on the idea of time-delay partitioning and a new interval partitioning strategy based on the non-uniform interval partitioning method. Subsequently, combined with the proposed new integral inequality and interval partitioning strategy, a new stability criterion for interval time-varying delay systems, which is flexible and less conservative, is obtained. Finally, the conservatism of the proposed method is verified through numerical simulations: the conservatism reduction degree of the proposed method can be intuitively reflected by comparing the maximum allowable delay ranges under different strategies.

Essentially, the potential conservatism of the proposed method mainly stems from two aspects: the fixed granularity of the interval partitioning strategy (e.g., the presetted number of partitions), which cannot fully adapt to the dynamic change characteristics of delay and the inherent approximation of integral inequalities when handling integrals containing delay terms. Although its accuracy is superior to that of traditional inequalities, a slight looseness remains in the estimation.

Regarding the computational complexity, the stability criteria have been converted into an LMI solution form, and its complexity is mainly jointly determined by the system state dimension and the LMI matrix scale. Among these, the matrix scale is directly related to the number of partitions (the more partitions there are, the higher the matrix dimension), but this influence is dominated by the inherent complexity of the LMI solution. Therefore, elaborating on the overall complexity solely based on the introduced variable parameters (such as partition-related parameters) has limited practical significance.

Regarding future research, it is planned to carry out in-depth work in the following aspects:

(1) To further improve the accuracy of stability criterion, we will explore the application of optimization algorithms such as Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) in determining the optimal values of parameters ρ_i and L_i.

(2) We will extend the proposed method to more complex system types, such as nonlinear time-varying delay

systems and stochastic delay systems.

(3) Combine practical engineering scenarios, such as power systems and mechanical vibration systems, to carry out application research on the proposed stability criterion.

Author Contributions

T.F.: Methodology, Writing-original draft; Y.Y.: Software, visualization, supervision; C.Z.: Conceptualization, writing—review & editing; L.F.: Methodology, validation, formal analysis, Writing—review & editing. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest

The authors declare no conflict of interest.

Use of AI and AI-assisted Technologies

No AI tools were utilized for this paper.

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