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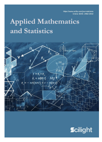

Article

Event-Triggered Control Based Fixed/Preassigned-Time Synchronization of Memristive BAM Neural Networks with Mixed-Time Delays

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Abstract: This paper focuses on fixed-time synchronization (FTS) and preassigned-time synchronization (PTS) of bidirectional associative memory memristive neural networks (BAMMNNs) with mixed-time delays via event-triggered control (ETC). Firstly, by using Lyapunov stability theory, fixed/preassigned-time stability lemmas and inequality techniques, results on FTS and PTS of BAMMNNs are derived. Secondly, compared to asymptotic synchronization and finite-time synchronization, the FTS and PTS studied here achieve faster convergence speeds and more precise settling times. Thirdly, the model incorporates state switching, time-varying and distributed delays; specifically, the time delays do not require differentiability, which enhances the generality of the results. Additionally, a segmented ETC strategy is designed to suit the dual-layer structure of BAMMNNs, where control actions are executed based on set triggering conditions, thus significantly reducing information transmission power consumption. Finally, a numerical simulation example is provided to verify the correctness of the results.

Keywords: event-triggered control; fixed-time synchronization; preassigned-time synchronization; memristive neural networks; BAM neural networks; mixed time delays

1. Introduction

Neural networks are computational models inspired by the structure and function of the human nervous system. They consist of numerous interconnected nodes, similar to how neurons interact in the brain. While traditional neural networks can map inputs to outputs, they don't fully capture the bidirectional associations between them. In 1988, Kosko proposed BAM neural networks [1], which addressed this limitation. BAM neural networks feature a dual-layer architecture that employs non-linear mapping to effectively encode the associative connections between the input and output layers. This unique structure facilitates bidirectional retrieval of information, enhancing the model's ability to process and recognize complex patterns. As a result, BAM networks are widely applied in various domains, including robotics [2], control systems [3], and pattern recognition [4], etc.

On the other hand, the memristor was first proposed by Chua [5], and its properties, such as memory, nonlinear response, and variable resistance, have made it an ideal material for recording and updating synaptic weights. Since both BAM neural networks and memristors exhibit characteristics similar to biological synapses, their combination enables the construction of neural network models that closely emulate the operation of the biological brain, referred to as bidirectional associative memory memristive neural networks (BAMMNNs) [6].

Neural networks often exhibit response delays due to their inherent structure and operation methods. Wang et al. [7] investigated the fixed-time synchronization of discontinuous complex-valued BAM neural networks with time delays, while Lan et al. [8] studied global exponential synchronization of BAM networks with multiple time-varying delay intervals, these researches only focused on time-varying delays. However, real-world applications often involve delays from various sources and types. Therefore, distributed delays are additionally considered in this work, which enhances the model's adaptability and predictive capability. As a result, the study of BAMMNNs with mixed time delays is a promising direction.

Modern control systems consistently face challenges posed by resource constraints. Previous studies have made significant progress in the control of BAMMNNs. For example, In 2023, Yang et al. [6] derived several sufficient conditions for achieving fixed-time synchronization (FTS) of BAMMNNs by designing specialized feedback control strategies. Similarly, Chen et al. [9] developed effective complex-valued feedback control laws with bounded gains to realize both FTS and preassigned-time synchronization (PTS). In addition, Yan et al. [10] proposed non-chattering quantized state feedback and pinning control schemes to ensure FTS. However, these approaches predominantly rely on conventional feedback control frameworks, which necessitate continuous system state monitoring and inherently lead to substantial resource consumption. In contrast, the event-triggered control (ETC) strategy adopted in this study determines optimal sampling and control instants through well-designed triggering rules. It executes control actions only when specific conditions are met, thereby significantly improving resource utilization. Its characteristics have garnered significant scholarly interest and yielded excellent results across diverse fields [11–14]. However, the current research on applying ETC to BAMMNNs remains relatively limited, suggesting a noteworthy research space in this field that warrants further investigation.

As is well known, synchronization research plays a crucial role not only in optimizing neural network performance but also in its interdisciplinary applications, resulting in substantial findings [6,8–12,15–17]. Among these, [8,15–17] have studied traditional asymptotic synchronization, where time approaches positive infinity, which often fails to meet practical requirements. In contrast, finite-time synchronization can achieve synchronization within a finite period, making it more valuable and extensively studied by scholars [13,18].

Unfortunately, the settling time of finite-time synchronization is influenced by initial conditions. To address this issue, [19] introduced the concept of fixed-time stabilization, where the system achieves stabilization within a fixed time regardless of the initial conditions, thus demonstrating stronger stability. In recent years, significant advancements in fixed-time stabilization and FTS have been reported across various models [10,12–14,20,21].

Moreover, this paper also incorporates PTS, in which the synchronization convergence time can be explicitly specified by the designer according to practical requirements, independent of the initial states or system parameters. PTS is highly compatible with the ETC strategy, as ETC requires events to be triggered at specific time instants. By guaranteeing that these events occur precisely at the predetermined moments, PTS enables a more accurate simulation of the behaviouraland responsive patterns observed in biological neural systems. Based on the above, our goal is to investigate the FTS and PTS issues of BAMMNNs with mixed-time delays under the ETC strategy. The main innovations are presented as follows:

(1) Unlike existing studies on BAM neural networks that focus solely on time-varying delays [8,22,23], this paper introduces mixed delays that incorporate both time-varying and distributed delays. This approach enables a more comprehensive simulation of complex dynamics across multiple time scales, with the advantage that these delays do not require differentiability, thus enhancing the generality of the findings.

(2) The FTS and PTS discussed in this paper present distinct advantages over asymptotic synchronization and finite-time synchronization [8,13,18,22]. In this case, the settling time is independent of initial conditions, resulting in enhanced convergence speed and timing precision. Consequently, FTS and PTS emerge as superior alternatives for modern system design.

(3) Compared to the feedback control methods used in [6, 9, 10], which have the drawback of requiring continuous data transmission, this study proposes an ETC strategy specifically designed for the unique structure of BAM neural networks. The controller is triggered and executes control operations only under specific conditions, effectively addressing this drawback and conserving resources.

The following outlines the organization of this paper. Section 2 presents the model and preliminary work for mixed-time delays BAMMNNs. Section 3 provides the main results on FTS and PTS of BAMMNNs. Numerical simulations in Section 4 verify the effectiveness of these theoretical results. Finally, Section 5 presents the conclusions.

Notations: In this paper, all solutions to the systems are Filippov's solutions [1]. $\mathbb{R}^n$ represents an n-dimensional Euclidean space; $\mathbb{N}$ denotes the set of natural numbers; $\mathrm{co}[-\theta,\theta]$ denotes the convex hull of $\{-\theta,\theta\}$; $a_{vn}^{\min}=\min\{a_{vn}^{\flat},a_{vn}^{\flat\flat}\}$, $a_{vn}^{\max}=\max\{a_{vn}^{\flat},a_{vn}^{\flat\flat}\}$. $a_{vn}=\frac{1}{2}(a_{vn}^{\min}+a_{vn}^{\max})$, $\breve{a}_{vn}=\frac{1}{2}(a_{vn}^{\max}-a_{vn}^{\min})$, $\overline{a}_{vn}=\max\{|a_{vn}^{\flat}|,|a_{vn}^{\flat\flat}|\}$, $\overline{b}_{vn}=\max\{|b_{vn}^{\flat}|,|b_{vn}^{\flat\flat}|\}$, $b_{vn}^{\min}=\min\{b_{vn}^{\flat},b_{vn}^{\flat\flat}\}$, $b_{vn}^{\max}=\max\{b_{vn}^{\flat},b_{vn}^{\flat\flat}\}$. $b_{vn}=\frac{1}{2}(b_{vn}^{\min}+b_{vn}^{\max})$, $\breve{b}_{vn}=\frac{1}{2}(b_{vn}^{\max}-b_{vn}^{\min})$, $\overline{h}_{vn}=\max\{|h_{vn}^{\flat}|,|h_{vn}^{\flat\flat}|\}$, $h_{vn}^{\min}=\min\{h_{vn}^{\flat},h_{vn}^{\flat\flat}\}$, $h_{vn}^{\max}=\max\{h_{vn}^{\flat},h_{vn}^{\flat\flat}\}$, $h_{vn}=\frac{1}{2}(h_{vn}^{\min}+h_{vn}^{\max})$, $\breve{h}_{vn}=\frac{1}{2}(h_{vn}^{\max}-h_{vn}^{\min})$, $\overline{c}_{nv}=\max\{|c_{nv}^{\flat}|,|c_{nv}^{\flat\flat}|\}$, $c_{nv}^{\min}=\min\{c_{nv}^{\flat},c_{nv}^{\flat\flat}\}$, $c_{nv}^{\max}=\max\{c_{nv}^{\flat},c_{nv}^{\flat\flat}\}$, $c_{nv}=\frac{1}{2}(c_{nv}^{\min}+c_{nv}^{\max})$, $\breve{c}_{nv}=\frac{1}{2}(c_{nv}^{\max}-c_{nv}^{\min})$, $\overline{d}_{nv}=\max\{|d_{nv}^{\flat}|,|d_{nv}^{\flat\flat}|\}$, $d_{nv}^{\min}=\min\{d_{nv}^{\flat},d_{nv}^{\flat\flat}\}$, $d_{nv}^{\max}=\max\{d_{nv}^{\flat},d_{nv}^{\flat\flat}\}$, $d_{nv}=\frac{1}{2}(d_{nv}^{\min}+d_{nv}^{\max})$, $\breve{d}_{nv}=\frac{1}{2}(d_{nv}^{\max}-d_{nv}^{\min})$, $\overline{k}_{nv}=\max\{|k_{nv}^{\flat}|,|k_{nv}^{\flat\flat}|\}$, $k_{nv}^{\min}=\min\{k_{nv}^{\flat},k_{nv}^{\flat\flat}\}$, $k_{nv}^{\max}=\max\{k_{nv}^{\flat},k_{nv}^{\flat\flat}\}$. $k_{nv}=\frac{1}{2}(k_{nv}^{\min}+k_{nv}^{\max})$, $\breve{k}_{nv}=\frac{1}{2}(k_{nv}^{\max}-k_{nv}^{\min})$, the above $v\in\mathcal{W}=\{1,2,\ldots,w\}$ and $n\in\mathcal{M}=\{1,2,\ldots,m\}$.

2. Preliminaries

Consider the BAMMNNs model with mixed delays:

$$\begin{cases} \dot{x}_v(t) = -\alpha_v x_v(t) + \sum_{n=1}^{w} a_{vn}(x_v(t)) f_n(y_n(t)) \\ \qquad + \sum_{n=1}^{w} b_{vn}(x_v(t)) f_n(y_n(t-z(t))) \\ \qquad + \sum_{n=1}^{w} h_{vn}(x_v(t)) \int_{t-\xi(t)}^{t} f_n(y_n(s))\,\mathrm{d}s + I_v, \\ x_v(\sigma) = \varphi(\sigma), \quad -z \le \sigma \le 0, \\ \dot{y}_n(t) = -\beta_n y_n(t) + \sum_{v=1}^{m} c_{nv}(y_n(t)) g_v(x_v(t)) \\ \qquad + \sum_{v=1}^{m} d_{nv}(y_n(t)) g_v(x_v(t-r(t))) \\ \qquad + \sum_{v=1}^{m} k_{nv}(y_n(t)) \int_{t-\gamma(t)}^{t} g_v(x_v(s))\,\mathrm{d}s + J_n, \\ y_n(\sigma) = \psi(\sigma), \quad -r \le \sigma \le 0, \end{cases} \tag{1}$$

above $t \ge 0$, $v \in \mathcal{W}$, $n \in \mathcal{M}$; $\varphi(\sigma)$, $\psi(\sigma)$ serve as the initial values; $\alpha_v, \beta_n > 0$ are self-feedback weights; $x_v(t)$ and $y_n(t)$ respectively represent the neuron state of v th and n th; $f_n(\cdot)$ and $g_v(\cdot)$ are nonlinear activation function, $z(\cdot)$, $r(\cdot)$ are discrete time-varying delays, where $0 \le z(\cdot) \le z$, $0 \le r(\cdot) \le r$; $\xi(t)$ and $\gamma(t)$ are distributed time delays, where $0 \le \xi(\cdot) \le \xi$, $0 \le \gamma(\cdot) \le \gamma$, z, r, ξ and γ are constants; I_v, J_n are external input. $a_{vn}(x_v(t)), b_{vn}(x_v(t)), h_{vn}(x_v(t)), c_{nv}(y_n(t)), d_{nv}(y_n(t))$ and $k_{nv}(y_n(t))$ serve as feedback connection weights, satisfying the following switching:

$$a_{vn}(x_v(t)) = \begin{cases} a_{vn}^{\flat}, & |x_v(t)| \le \daleth_v, \\ a_{vn}^{\flat\flat}, & |x_v(t)| > \daleth_v, \end{cases}$$

$$b_{vn}(x_v(t)) = \begin{cases} b_{vn}^{\flat}, & |x_v(t)| \le \daleth_v, \\ b_{vn}^{\flat\flat}, & |x_v(t)| > \daleth_v, \end{cases}$$

$$h_{vn}(x_v(t)) = \begin{cases} h_{vn}^{\flat}, & |x_v(t)| \le \daleth_v, \\ h_{vn}^{\flat\flat}, & |x_v(t)| > \daleth_v, \end{cases}$$

$$c_{nv}(y_n(t)) = \begin{cases} c_{nv}^{\flat}, & |y_n(t)| \le \daleth'_n, \\ c_{nv}^{\flat\flat}, & |y_n(t)| > \daleth'_n, \end{cases}$$

$$d_{nv}(y_n(t)) = \begin{cases} d_{nv}^{\flat}, & |y_n(t)| \le \daleth'_n, \\ d_{nv}^{\flat\flat}, & |y_n(t)| > \daleth'_n, \end{cases}$$

$$k_{nv}(y_n(t)) = \begin{cases} k_{nv}^{\flat}, & |y_n(t)| \le \daleth'_n, \\ k_{nv}^{\flat\flat}, & |y_n(t)| > \daleth'_n, \end{cases}$$

where $\daleth_v > 0, \daleth'_n > 0$, and $a_{vn}^{\flat}, a_{vn}^{\flat\flat}, b_{vn}^{\flat}, b_{vn}^{\flat\flat}, h_{vn}^{\flat}, h_{vn}^{\flat\flat}, c_{nv}^{\flat}, c_{nv}^{\flat\flat}, d_{nv}^{\flat}, d_{nv}^{\flat\flat}, k_{nv}^{\flat}, k_{nv}^{\flat\flat}$ are known constant numbers, $v \in \mathcal{W}, n \in \mathcal{M}$.

Remark 1. *Unlike [8,22,23], which only consider single time-varying delays, our model additionally incorporates distributed delays, making it applicable to more complex real-world scenarios. Furthermore, our delays do not require differentiability, which enhances the generality of our results.*

Let BAMMNNs (1) as the driving system, then the response system of (1) is

$$\begin{cases} \dot{\tilde{x}}_v(t) = -\alpha_v \tilde{x}_v(t) + \sum_{n=1}^{w} a_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t)) \\ \quad + \sum_{n=1}^{w} b_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t-z(t))) \\ \quad + \sum_{n=1}^{w} h_{vn}(\tilde{x}_v(t)) \int_{t-\xi(t)}^{t} f_n(\tilde{y}_n(s))\mathrm{d}s + I_v \\ \quad + u_v(t), \\ \tilde{x}_v(\sigma) = \tilde{\varphi}(\sigma), \quad -z \le \sigma \le 0, \\ \dot{\tilde{y}}_n(t) = -\beta_n \tilde{y}_n(t) + \sum_{v=1}^{m} c_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t)) \\ \quad + \sum_{v=1}^{m} d_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t-r(t))) \\ \quad + \sum_{v=1}^{m} k_{nv}(\tilde{y}_n(t)) \int_{t-\gamma(t)}^{t} g_v(\tilde{x}_v(s))\mathrm{d}s + J_n \\ \quad + v_n(t), \\ \tilde{y}_n(\sigma) = \tilde{\psi}(\sigma), \quad -r \le \sigma \le 0, \end{cases} \tag{2}$$

where $u_v(t)$ and $v_n(t)$ are controllers; $\tilde{\varphi}(\sigma)$ and $\tilde{\psi}(\sigma)$ are initial conditions of BAMMNNs (2).

According to the identifier set by Nations, we can convert BAMMNNs (1) into the following form:

$$\begin{cases} \dot{x}_v(t) \in -\alpha_v x_v(t) + \sum_{n=1}^{w} (a_{vn} + co[-\breve{a}_{vn}, \breve{a}_{vn}]) f_n(y_n(t)) \\ \quad + \sum_{n=1}^{w} (b_{vn} + co[-\breve{b}_{vn}, \breve{b}_{vn}]) f_n(y_n(t-z(t))) \\ \quad + \sum_{n=1}^{w} (h_{vn} + co[-\breve{h}_{vn}, \breve{h}_{vn}]) \\ \quad \times \int_{t-\xi(t)}^{t} f_n(y_n(s))\mathrm{d}s + I_v, \\ x_v(\sigma) = \varphi(\sigma), \quad -z \le \sigma \le 0, \\ \dot{y}_n(t) \in -\beta_n y_n(t) + \sum_{v=1}^{m} (c_{nv} + co[-\breve{c}_{nv}, \breve{c}_{nv}]) g_v(x_v(t)) \\ \quad + \sum_{v=1}^{m} (d_{nv} + co[-\breve{d}_{nv}, \breve{d}_{nv}]) g_v(x_v(t-r(t))) \\ \quad + \sum_{v=1}^{m} (k_{nv} + co[-\breve{k}_{nv}, \breve{k}_{nv}]) \\ \quad \times \int_{t-\gamma(t)}^{t} g_v(x_v(s))\mathrm{d}s + J_n, \\ y_n(\sigma) = \psi(\sigma), \quad -r \le \sigma \le 0, \end{cases} \tag{3}$$

By employing differential inclusion theory [24], convex analysis methods, and measurable selection theory, we can obtain there exist functions $\sigma_{vn}^a(t)$, $\sigma_{vn}^b(t)$, $\sigma_{vn}^h(t)$, $\sigma_{nv}^c(t)$, $\sigma_{nv}^d(t)$ and $\sigma_{nv}^k(t) \in co[-1, 1]$, such that

$$
\begin{cases}
\dot{x}_v(t) = -\alpha_v x_v(t) + \sum_{n=1}^{w}(a_{vn} + \sigma_{vn}^{a}(t)\breve{a}_{vn})f_n(y_n(t)) \\
\qquad + \sum_{n=1}^{w}(b_{vn} + \sigma_{vn}^{b}(t)\breve{b}_{vn})f_n(y_n(t-z(t))) \\
\qquad + \sum_{n=1}^{w}(h_{vn} + \sigma_{vn}^{h}(t)\breve{h}_{vn})\int_{t-\xi(t)}^{t} f_n(y_n(s))\mathrm{d}s + I_v, \\
x_v(\sigma) = \varphi(\sigma), \quad -z \le \sigma \le 0, \\
\dot{y}_n(t) = -\beta_n y_n(t) + \sum_{v=1}^{m}(c_{nv} + \sigma_{nv}^{c}(t)\breve{c}_{nv})g_v(x_v(t)) \\
\qquad + \sum_{v=1}^{m}(d_{nv} + \sigma_{nv}^{d}(t)\breve{d}_{nv})g_v(x_v(t-r(t))) \\
\qquad + \sum_{v=1}^{m}(k_{nv} + \sigma_{nv}^{k}(t)\breve{k}_{nv})\int_{t-\gamma(t)}^{t} g_v(x_v(s))\mathrm{d}s + J_n, \\
y_n(\sigma) = \psi(\sigma), \quad -r \le \sigma \le 0.
\end{cases}
$$

In the same way, there exist $\tilde{\sigma}_{vn}^{a}(t)$, $\tilde{\sigma}_{vn}^{b}(t)$, $\tilde{\sigma}_{vn}^{h}(t)$, $\tilde{\sigma}_{nv}^{c}(t)$, $\tilde{\sigma}_{nv}^{d}(t)$, $\tilde{\sigma}_{nv}^{k}(t) \in co[-1,1]$ and controller $u_v^*(t) \in co[u_v(t)]$, $v_n^*(t) \in co[v_n(t)]$ such that

$$
\begin{cases}
\dot{\tilde{x}}_v(t) = -\alpha_v \tilde{x}_v(t) + \sum_{n=1}^{w}(a_{vn} + \tilde{\sigma}_{vn}^{a}(t)\breve{a}_{vn})f_n(\tilde{y}_n(t)) \\
\qquad + \sum_{n=1}^{w}(b_{vn} + \tilde{\sigma}_{vn}^{b}(t)\breve{b}_{vn})f_n(\tilde{y}_n(t-z(t))) \\
\qquad + \sum_{n=1}^{w}(h_{vn} + \tilde{\sigma}_{vn}^{h}(t)\breve{h}_{vn})\int_{t-\xi(t)}^{t} f_n(\tilde{y}_n(s))\mathrm{d}s \\
\qquad + I_v + u_v^*(t), \\
\tilde{x}_v(\sigma) = \tilde{\varphi}(\sigma), \quad -z \le \sigma \le 0, \\
\dot{\tilde{y}}_n(t) = -\beta_v \tilde{y}_n(t) + \sum_{v=1}^{m}(c_{nv} + \tilde{\sigma}_{nv}^{c}(t)\breve{c}_{nv})g_v(\tilde{x}_v(t)) \\
\qquad + \sum_{v=1}^{m}(d_{nv} + \tilde{\sigma}_{nv}^{d}(t)\breve{d}_{nv})g_v(\tilde{x}_n(t-r(t))) \\
\qquad + \sum_{v=1}^{m}(k_{nv} + \tilde{\sigma}_{nv}^{k}(t)\breve{k}_{nv})\int_{t-\gamma(t)}^{t} g_v(\tilde{x}_v(s))\mathrm{d}s \\
\qquad + J_n + v_n^*(t), \\
\tilde{y}_n(\sigma) = \tilde{\psi}(\sigma), \quad -r \le \sigma \le 0,
\end{cases}
\tag{4}
$$

Now, we consider the following synchronization error:

$$
\begin{cases}
e_v^x(t) = \tilde{x}_v(t) - x_v(t), & v \in \mathcal{W} \\
e_v^x(\sigma) = \tilde{\varphi}_v(\sigma) - \varphi_v(\sigma), & -z \le \sigma \le 0, \\
e_n^y(t) = \tilde{y}_n(t) - y_n(t), & n \in \mathcal{M} \\
e_n^y(\sigma) = \tilde{\psi}_n(\sigma) - \psi_n(\sigma), & -r \le \sigma \le 0.
\end{cases}
\tag{5}
$$

Subsequently, according to (4) and (5), the error system of them can be expressed as

$$\begin{cases} \dot{e}_v^x(t) = -\alpha_v e_v^x(t) + \sum_{n=1}^{w} a_{vn}(t) H_n(e_n^y(t)) \\ \quad + \sum_{n=1}^{w} b_{vn}(t) H_n(e_n^y(t-z(t))) \\ \quad + \sum_{n=1}^{w} h_{vn}(t) \int_{t-\xi(t)}^{t} H_n(e_n^y(s)) \mathrm{d}s \\ \quad + \sum_{n=1}^{w} (\tilde{\sigma}_{vn}^a(t) - \sigma_{vn}^a(t)) \breve{a}_{vn} f_n(y_n(t)) \\ \quad + \sum_{n=1}^{w} (\tilde{\sigma}_{vn}^b(t) - \sigma_{vn}^b(t)) \breve{b}_{vn} f_n(y_n(t-z(t))) \\ \quad + \sum_{n=1}^{w} (\tilde{\sigma}_{vn}^h(t) - \sigma_{vn}^h(t)) \breve{h}_{vn} \int_{t-\xi(t)}^{t} f_n(y_n(s)) \mathrm{d}s \\ \quad + u_v^*(t), \\ \dot{e}_n^y(t) = -\beta_n e_n^y(t) + \sum_{v=1}^{m} c_{nv}(t) W_v(e_v^x(t)) \\ \quad + \sum_{v=1}^{m} d_{nv}(t) W_v(e_v^x(t-r(t))) \\ \quad + \sum_{v=1}^{m} k_{nv}(t) \int_{t-\gamma(t)}^{t} W_v(e_v^x(s)) \mathrm{d}s \\ \quad + \sum_{v=1}^{m} (\tilde{\sigma}_{nv}^c(t) - \sigma_{nv}^c(t)) \breve{c}_{nv} g_v(x_v(t)) \\ \quad + \sum_{v=1}^{m} (\tilde{\sigma}_{nv}^d(t) - \sigma_{nv}^d(t)) \breve{d}_{nv} g_v(x_v(t-r(t))) \\ \quad + \sum_{v=1}^{m} (\tilde{\sigma}_{nv}^k(t) - \sigma_{nv}^k(t)) \breve{k}_{nv} \int_{t-\gamma(t)}^{t} g_v(x_v(s)) \mathrm{d}s \\ \quad + v_n^*(t) \end{cases} \tag{6}$$

where $a_{vn}(t) = a_{vn} + \tilde{\sigma}_{vn}^a(t)\breve{a}_{vn}$, $b_{vn}(t) = b_{vn} + \tilde{\sigma}_{vn}^b(t)\breve{b}_{vn}$, $h_{vn}(t) = h_{vn} + \tilde{\sigma}_{vn}^h(t)\breve{h}_{vn}$, $c_{nv}(t) = c_{nv} + \tilde{\sigma}_{nv}^c(t)\breve{c}_{nv}$, $d_{nv}(t) = d_{nv} + \tilde{\sigma}_{nv}^d(t)\breve{d}_{nv}$, $k_{nv}(t) = k_{nv} + \tilde{\sigma}_{nv}^k(t)\breve{k}_{nv}$, $H_n(e_n^y(t)) = f_n(\tilde{y}_n(t)) - f_n(y_n(t))$, $W_v(e_v^x(t)) = g_v(\tilde{x}_v(t)) - g_v(x_v(t))$.

Assumption 1. *The activation functions $f_n(\cdot)$ and $g_v(\cdot)$ are Lipschitz continuous and bounded, and for $\rho_1 \neq \rho_2$ ($\forall \rho_1, \rho_2 \in \mathbb{R}$), there exists positive constants ℓ_n, j_v, F_n and G_v, such that*

$$|f_n(\rho_1) - f_n(\rho_2)| \leq \ell_n |\rho_1 - \rho_2|,\ |f_n(\rho_1)| \leq F_n,$$
$$|g_v(\rho_1) - g_v(\rho_2)| \leq j_v |\rho_1 - \rho_2|,\ |g_v(\rho_1)| \leq G_v.$$

Therefore, we can infer that

$$|H_n(e_n^y(t))| \leq \ell_n |e_n^y(t)|,\ |H_n(e_n^y(t))| \leq 2F_n,$$
$$|W_v(e_v^x(t))| \leq j_v |e_v^x(t)|,\ |W_v(e_v^x(t))| \leq 2G_v.$$

Definition 1. *[13] For any $e_0 = e(0) \in \mathbb{R}^n$, let the settling time function be $T(e_0) = \{t' : e(t) = 0, \forall t > t'\}$, if there exists $T_{max} > 0$, satisfying $T(e(0)) \leq T_{max}$ and $\lim_{t \to T_{max}} \|e(s)\| = 0$, then the system (1) and (2) are achieved FTS and T_{max} is called settling-time.*

Definition 2. *[25] If there exists a preassigned constant $T_p > 0$, such that the function $T(e(0)) \leq T_p$ for any $e(0) \in \mathbb{R}^n$, and if $\lim_{t \to T_p} \|e(s)\| = 0$ holds, then the system (1) and (2) are achieved PTS and T_p is called* *preassigned-time.*

Lemma 1. *[26] Let $\pi_v \geq 0, (v = 1, 2, \ldots, w), 0 < \epsilon_1 \leq 1$, and $\epsilon_2 \geq 1$; one has*

$$\sum_{v=1}^{w} \pi_v^{\epsilon_1} \geq (\sum_{v=1}^{w} \pi_v)^{\epsilon_1}, \sum_{v=1}^{w} \pi_v^{\epsilon_2} \geq w^{1-\epsilon_2} (\sum_{v=1}^{w} \pi_v)^{\epsilon_2}.$$

By following a similar proof method as in [21], we establish the following FTS lemma:

Lemma 2. *[21] Function $V(\cdot) : \mathbb{R}^n \to [0, +\infty)$ is continuous, positive definite and radially unbounded, $e(t) = 0 \Leftrightarrow V(e(t)) = 0$, if any solution of (6) satisfies*

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} \leq \lambda V(t) - \varepsilon V^{\rho}(t) - \eta V^{\delta}(t), \tag{7}$$

where $\varepsilon, \eta > 0$, $\lambda < \min\{\varepsilon, \eta\}$, $0 < \rho < 1$, $\delta > 1$, BAMMNNs (1) and (2) achieve FTS and settling-time is

$$T_{max} \triangleq \begin{cases} \frac{1}{\varepsilon(1-\rho)} + \frac{\left[(\varepsilon-\lambda)^{\frac{1}{\delta}} + \eta^{\frac{1}{\delta}}\right]^{1-\delta}}{2^{1-\delta}(\delta-1)\eta^{\frac{1}{\delta}}}, & \lambda < 0, \\ \frac{1}{\varepsilon(1-\rho)} + \frac{\left[\varepsilon^{\frac{1}{\delta}} + \eta^{\frac{1}{\delta}}\right]^{1-\delta}}{2^{1-\delta}(\delta-1)\eta^{\frac{1}{\delta}}}, & \lambda = 0, \\ \frac{1}{(\varepsilon-\lambda)(1-\rho)} + \frac{\left[\varepsilon^{\frac{1}{\delta}} + (\eta-\lambda)^{\frac{1}{\delta}}\right]^{1-\delta}}{2^{1-\delta}(\delta-1)(\eta-\lambda)^{\frac{1}{\delta}}}, & \lambda > 0. \end{cases}$$

Lemma 3. *Function $V(\cdot) : \mathbb{R}^n \to [0, +\infty)$ is continuous, positive definite and radially unbounded, $e(t) = 0 \Leftrightarrow V(e(t)) = 0$, if any solution of (6) satisfies*

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} \leq \frac{T_{max}}{T_p}(\lambda V(t) - \varepsilon V^{\rho}(t) - \eta V^{\delta}(t)), \tag{8}$$

BAMMNNs (1) and (2) achieve PTS, where T_p is the preassigned-time, and all other parameters are the same as in Lemma 2.

Remark 2. *Based on recent research findings [21], we considered the special case where the constant term in the inequality involving the derivative of the Lyapunov function ($\dot{V}(t)$) is zero. From this, we established a lemma for FTS, referred to as Lemma 2, and subsequently developed a new PTS lemma, denoted as Lemma 3. The settling-time results obtained here are more comprehensive compared to earlier studies [27,28].*

3. Main Results

3.1. FTS of BAMMNNs (1) and (2)

Based on the dual-layer structure of BAMMNNs, we design the following two controllers.

$$\begin{cases} u_{1v}^*(t) = p_{1v}e_v^x(t_l) - p_{2v}\phi_v(t_l) - p_{3v}\phi_v(t_l)|e_v^x(t_l)|^{2\rho-1} \\ \qquad - p_{4v}\phi_v(t_l|e_v^x(t_l)|^{2\delta-1}, \\ v_{1n}^*(t) = q_{1n}e_n^y(t_l) - q_{2n}\phi_n^*(t_l) - q_{3n}\phi_n^*(t_l)|e_n^y(t_l)|^{2\rho-1} \\ \qquad - q_{4n}\phi_n^*(t_l)|e_n^y(t_l)|^{2\delta-1}, \end{cases} \tag{9}$$

where $v \in \mathcal{W}$ and $n \in \mathcal{M}$, $\frac{1}{2} \leq \rho < 1$, $\delta > 1$, p_{1v}, p_{2v}, q_{1n} and q_{2n} are defined as constants, p_{3v}, p_{4v}, q_{3n} and q_{4n} represent suitable positive constants. For $t \in [t_l, t_{l+1})$, $\phi_v(t) \in co[sign(e_v^x(t))]$ and for $t \in [t_{l^*}, t_{l^*+1})$, $\phi_n^*(t) \in co[sign(e_n^y(t))]$.

Let

$$\begin{aligned} \mathcal{U}_{1v}(t) =& p_{1v}e_v^x(t) - p_{2v}\phi_v(t) - p_{3v}\phi_v(t)|e_v^x(t)|^{2\rho-1} \\ & - p_{4v}\phi_v(t)|e_v^x(t)|^{2\delta-1}, \\ \mathcal{V}_{1n}(t) =& q_{1n}e_n^y(t) - q_{2n}\phi_n^*(t) - q_{3n}\phi_n^*(t)|e_n^y(t)|^{2\rho-1} \\ & - q_{4n}\phi_n^*(t)|e_n^y(t)|^{2\delta-1}. \end{aligned}$$

The measurement error is

$$\begin{cases} \mathbb{E}_{1v}(t) = \mathcal{U}_{1v}(t) - u_{1v}^*(t), \\ \mathbb{E}_{1n}^*(t) = \mathcal{V}_{1n}(t) - v_{1n}^*(t), \end{cases}$$

then, we designed the ETC into the following two stages:

$$\begin{cases} t_{l+1} = \{t \mid t > t_l, |\mathbb{E}_{1v}(t)| \geq \zeta_v |e_v^x(t)| + \jmath_v p_{3v} |e_v^x(t)|^{2\rho-1} \\ \qquad + \omega_v p_{4v} |e_v^x(t)|^{2\delta-1} + \tau_v (1-\iota_v)^t\} \\ t_{l^*+1} = \{t \mid t > t_{l^*}, |\mathbb{E}_{1n}^*(t)| \geq \zeta_n^* |e_n^y(t)| + \jmath_n^* q_{3n} |e_n^y(t)|^{2\rho-1} \\ \qquad + \omega_n^* q_{4n} |e_n^y(t)|^{2\delta-1} + \tau_n^* (1-\iota_n^*)^t\}, \end{cases} \tag{10}$$

where $\jmath_v, \omega_v, \iota_v, \jmath_v^*, \omega_v^*, \iota_v^* \in (0,1)$, $\zeta_v, \tau_v, \zeta_v^*, \tau_v^* > 0$, $t_l(l = 1, 2, \ldots, v \in \mathbb{N})$ and $t_{l^*}(l^* = 1, 2, \ldots, n \in \mathbb{N})$ respectively represent the l th and l^* th triggering instants.

Let

$$p_{2v} \geq \Sigma_{n=1}^{w} 2(\breve{a}_{vn} + \breve{b}_{vn} + \breve{h}_{vn}\xi + \bar{b}_{vn} + \bar{h}_{vn}\xi)F_n + \tau_v, \tag{11}$$

$$q_{2n} \geq \Sigma_{v=1}^{m} 2(\breve{c}_{nv} + \breve{d}_{nv} + \breve{k}_{nv}\gamma + \bar{d}_{nv} + \bar{k}_{nv}\gamma)G_v + \tau_n^*, \tag{12}$$

$$\lambda = 2\max\{\lambda_1, \lambda_2\}, \tag{13}$$

$$\varepsilon = \min\{min_v\{p_{3v} - \jmath_v p_{3v}\}, min_n\{q_{3n} - \jmath_n^* q_{3n}\}\}2^{\rho}, \tag{14}$$

$$\eta = \min\{\eta_1, \eta_1\}2^{\delta}, \tag{15}$$

in which $\lambda_1 = \max_v\{-\alpha_v + p_{1v} + \zeta_v + \Sigma_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)\}$, $\lambda_2 = \max_n\{-\beta_n + q_{1n} + \zeta_n^* + \Sigma_{v=1}^{m}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{nv}\ell_n^2)\}$, $\eta_1 = min_v\{p_{4v} - \omega_v p_{4v}\}m^{1-\delta}$ and $\eta_2 = min_n\{q_{4n} - \omega_n^* q_{4n}\}w^{1-\delta}$.

Next, the main results of FTS are presented.

Theorem 1. *Under Assumption 1, Lemma 2, and ETC* (9) *and* (10)*, if* (11)–(15) *hold, BAMMNNs* (1) *and* (2) *get FTS and the settling-time is* T_{max}.

Proof. Now, we construct the Lyapunov function:

$$V(t) = V_1(t) + V_2(t), \tag{16}$$

in which

$$V_1(t) = \frac{1}{2}\sum_{v=1}^{m}(e_v^x(t))^2,$$

$$V_2(t) = \frac{1}{2}\sum_{n=1}^{w}(e_n^y(t))^2.$$

For $V_1(t)$, we provide its derivative as follows:

$$\begin{aligned} \dot{V}_1(t) = & \sum_{v=1}^{m} e_v^x(t)\{-\alpha_v e_v^x(t) + \sum_{n=1}^{w} a_{vn}(t)H_n(e_n^y(t)) \\ & + \sum_{n=1}^{w} b_{vn}(t)H_n(e_n^y(t-z(t))) \\ & + \sum_{n=1}^{w} h_{vn}(t)\int_{t-\xi(t)}^{t} H_n(e_n^y(s))\mathrm{d}s \\ & + \sum_{n=1}^{w}(\tilde{\sigma}_{vn}^a(t) - \sigma_{vn}^a(t))\breve{a}_{vn}f_n(y_n(t)) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{b}(t)-\sigma_{vn}^{b}(t))\breve{b}_{vn}f_n(y_n(t-z(t)))\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{h}(t)-\sigma_{vn}^{h}(t))\breve{h}_{vn}\int_{t-\xi(t)}^{t}f_n(y_n(s))\mathrm{d}s\\
&+u_{1v}^{*}(t)\}\\
\le&\sum_{v=1}^{m}\{-\alpha_v(e_v^x(t))^2+\sum_{n=1}^{w}\bar{a}_{vn}|e_v^x(t)|H_n(e_n^y(t))\\
&+\sum_{n=1}^{w}\bar{b}_{vn}|e_v^x(t)|H_n(e_n^y(t-z(t)))\\
&+\sum_{n=1}^{w}\bar{h}_{vn}|e_v^x(t)|\int_{t-\xi(t)}^{t}H_n(e_n^y(s))\mathrm{d}s\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{a}(t)-\sigma_{vn}^{a}(t))\breve{a}_{vn}|e_v^x(t)|f_n(y_n(t))\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{b}(t)-\sigma_{vn}^{b}(t))\breve{b}_{vn}|e_v^x(t)|f_n(y_n(t-z(t)))\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{h}(t)-\sigma_{vn}^{h}(t))\breve{h}_{vn}|e_v^x(t)|int_{t-\xi(t)}^{t}f_n(y_n(s))\mathrm{d}s\\
&+\phi_v(t)u_{1v}^{*}(t)e_v^x(t)\}.
\end{aligned}
\tag{17}
$$

Based on condition $\tilde{\sigma}_{vn}^{a}(t),\sigma_{vn}^{a}(t),\tilde{\sigma}_{vn}^{b}(t),\sigma_{vn}^{b}(t),\tilde{\sigma}_{vn}^{h}(t),\sigma_{vn}^{h}(t)\in co[-1,1]$ and Assumption 1, we know that

$$
\begin{aligned}
&\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{a}(t)-\sigma_{vn}^{a}(t))\breve{a}_{vn}f_n(y_n(t))\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{b}(t)-\sigma_{vn}^{b}(t))\breve{b}_{vn}f_n(y_n(t-z(t)))\\
&+\sum_{n=1}^{w}(\tilde{\sigma}_{vn}^{h}(t)-\sigma_{vn}^{h}(t))\breve{h}_{vn}\int_{t-\xi(t)}^{t}f_n(y_n(s))\mathrm{d}s\\
&\le\sum_{n=1}^{w}2(\breve{a}_{vn}+\breve{b}_{vn}+\breve{h}_{vn}\xi)F_n,
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
\sum_{n=1}^{w}\bar{a}_{vn}|e_v^x(t)|H_n(e_n^y(t))\le&\sum_{n=1}^{w}\bar{a}_{vn}|e_v^x(t)|\ell_n e_n^y(t)\\
\le&\sum_{n=1}^{w}\frac{1}{2}\bar{a}_{vn}(e_v^x(t))^2\\
&+\sum_{n=1}^{w}\frac{1}{2}\bar{a}_{vn}\ell_n^2(e_n^y(t))^2,
\end{aligned}
\tag{19}
$$

and

$$
\sum_{n=1}^{w}\bar{b}_{vn}|e_v^x(t)|H_n(e_n^y(t-z(t)))\le\sum_{n=1}^{w}2\bar{b}_{vn}F_n|e_v^x(t)|,
\tag{20}
$$

similarly, one can provide

$$
\sum_{n=1}^{w}\bar{h}_{vn}|e_v^x(t)|\int_{t-\xi(t)}^{t}H_n(e_n^y(s))\mathrm{d}s\le\sum_{n=1}^{w}2\bar{h}_{vn}F_n\xi|e_v^x(t)|.
\tag{21}
$$

Subsequently, (18) and (21) are substituted into (17), such that

$$\begin{aligned}\dot{V}_1(t) \leq & \sum_{v=1}^{m}\{-\alpha_v(e_v^x(t))^2 + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)(e_v^x(t))^2 \\ & + \sum_{n=1}^{w} 2\bar{b}_{vn}F_n|e_v^x(t)| + \sum_{n=1}^{w} 2(\breve{a}_{vn} + \breve{b}_{vn} + \breve{h}_{vn}\xi \\ & + \bar{h}_{vn}\xi)F_n|e_v^x(t)| + \phi_v(t)e_v^x(t)(\mathcal{U}_{1v}(t) - \mathbb{E}_{1v}(t))\} \\ \leq & \sum_{v=1}^{m}\{[-\alpha_v + p_{1v} + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)](e_v^x(t))^2 \\ & + [-p_{2v} + \tau_v + \sum_{n=1}^{w} 2(\breve{a}_{vn} + \breve{b}_{vn} + \breve{h}_{vn}\xi + \bar{b}_{vn} + \bar{h}_{vn}\xi)F_n] \\ & \times |e_v^x(t)| - (p_{3v} - \jmath_v p_{3v})|e_v^x(t)|^{2\rho} - (p_{4v} - \omega_v p_{4v})|e_v^x(t)|^{2\delta} \\ & + |e_v^x(t)|[|\mathbb{E}_{1v}(t)| - \zeta_v|e_v^x(t)| - \jmath_v p_{3v}|e_v^x(t)|^{2\rho-1} \\ & - \omega_v p_{4v}|e_v^x(t)|^{2\delta-1} - \tau_v(1-\iota_v)^t]\}.\end{aligned} \tag{22}$$

From (11) and ETC (9) and (10), there is

$$\begin{aligned}\dot{V}_1(t) \leq & \sum_{v=1}^{m}\{[-\alpha_v + p_{1v} + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)](e_v^x(t))^2 \\ & - (p_{3v} - \jmath_v p_{3v})|e_v^x(t)|^{2\rho} - (p_{4v} - \omega_v p_{4v})|e_v^x(t)|^{2\delta}\} \\ \leq & \sum_{v=1}^{m}[-\alpha_v + p_{1v} + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)](e_v^x(t))^2 \\ & - min_v\{p_{3v} - \jmath_v p_{3v}\}[\sum_{v=1}^{m}(e_v^x(t))^2]^{\rho} \\ & - min_v\{p_{4v} - \omega_v p_{4v}\}[\sum_{v=1}^{m}(e_v^x(t))^2]^{\delta} \\ \leq & 2[-\alpha_v + p_{1v} + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)]V_1(t) \\ & - min_v\{p_{3v} - \jmath_v p_{3v}\}2^{\rho}V_1^{\rho}(t) \\ & - min_v\{p_{4v} - \omega_v p_{4v}\}m^{1-\delta}2^{\delta}V_1^{\delta}(t),\end{aligned} \tag{23}$$

same as above, we have

$$\begin{aligned}\dot{V}_2(t) \leq & 2[-\beta_n + q_{1n} + \zeta_n^* + \sum_{v=1}^{m}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{nv}\ell_n^2)]V_2(t) \\ & - min_n\{q_{3n} - \jmath_n^* q_{3n}\}2^{\rho}V_2^{\rho}(t) \\ & - min_v\{q_{4n} - \omega_n^* q_{4n}\}w^{1-\delta}2^{\delta}V_2^{\delta}(t).\end{aligned} \tag{24}$$

According to (15)–(17) and Lemma 2, it combines $V_1(t)$ and $V_2(t)$ such that:

$$\begin{aligned}\dot{V}(t) = & \dot{V}_1(t) + \dot{V}_2(t) \\ \leq & \lambda V(t) - \varepsilon V_1^{\rho}(t) - \eta V_1^{\delta}(t) - \varepsilon V_2^{\rho}(t) - \eta V_2^{\delta}(t) \\ \leq & \lambda V(t) - \varepsilon[V_1(t) + V_2(t)]^{\rho} - \eta[V_1(t) + V_2(t)]^{\delta} \\ \leq & \lambda V(t) - \varepsilon V^{\rho}(t) - \eta V^{\delta}(t),\end{aligned} \tag{25}$$

where $\lambda < \min\{\varepsilon, \eta\}$, $\varepsilon, \eta > 0$.

From Definition 1 and Lemma 2, we get BAMMNNs (1) and (2) achieve FTS at settling-time T_{max}. The proof is finished. □

Theorem 2. *The error system* (6) *does not have Zeno-behaviour with ETC* (9) *and* (10).

Proof. When $t \in [t_l, t_{l+1}), l = 1, 2, \ldots,$

$$\begin{aligned}\frac{|\mathrm{d}\mathbb{E}_{1v}(t)|}{\mathrm{d}t} \leq & \frac{|\mathrm{d}\mathcal{U}_{1v}(t)|}{\mathrm{d}t} \\ \leq & [p_{1v} + (2\rho - 1)p_{3v}|e_v^x(t)|^{2\rho-2} \\ & + (2\delta - 1)p_{4v}|e_v^x(t)|^{2\delta-2}]|\frac{\mathrm{d}e_v^x(t)}{\mathrm{d}t}|.\end{aligned} \tag{26}$$

From (6) and Assumption 1, one gets

$$\begin{aligned}|\frac{\mathrm{d}e_v^x(t)}{\mathrm{d}t}| \leq & \alpha_v|e_v^x(t)| + \sum_{n=1}^{w} \bar{a}_{vn} H_n(e_n^y(t)) \\ & + \sum_{n=1}^{w} \bar{b}_{vn} H_n(e_n^y(t - z(t))) \\ & + \sum_{n=1}^{w} \bar{h}_{vn} \int_{t-\xi(t)}^{t} H_n(e_n^y(s))\mathrm{d}s + \sum_{n=1}^{w} 2(\breve{a}_{vn} \\ & + \breve{b}_{vn} + \breve{h}_{vn}\xi + \bar{b}_{vn})F_n + |u_{1v}^*(t)| \\ \leq & \alpha_v|e_v^x(t)| + \sum_{n=1}^{w} 2\bar{a}_{vn}F_n + \sum_{n=1}^{w} 2\bar{b}_{vn}F_n \\ & + \sum_{n=1}^{w} 2\bar{h}_{vn}\xi F_n + \sum_{n=1}^{w} 2(\breve{a}_{vn} + \breve{b}_{vn} \\ & + \breve{h}_{vn}\xi + \bar{b}_{vn})F_n + |u_{1v}^*(t)|.\end{aligned} \tag{27}$$

Due to $\frac{\mathrm{d}\mathcal{V}(t)}{\mathrm{d}t} \leq 0$, thus $|e_v^x(t)| \leq V(0)$, then

$$\begin{aligned}|\frac{\mathrm{d}e_v^x(t)}{\mathrm{d}t}| \leq & \alpha_v V(0) + \sum_{n=1}^{w} 2(\bar{a}_{vn} + \bar{b}_{vn} + \bar{h}_{vn}\xi \\ & + \breve{a}_{vn} + \breve{b}_{vn} + \breve{h}_{vn}\xi)F_n + |u_{1v}^*(t)| \\ = & \mathfrak{R}_v(t_l) > 0.\end{aligned} \tag{28}$$

Let $\mathit{\Psi}_v = \max_{t\in[t_l,t_{l+1})}[p_{1v} + (2\rho-1)p_{3v}|e_v^x(t)|^{2\rho-2} + (2\delta-1)p_{4v}|e_v^x(t)|^{2\delta-2}]$ and by using (26)–(28), one obtains

$$\frac{|\mathrm{d}\mathbb{E}_{1v}(t)|}{\mathrm{d}t} \leq \mathit{\Psi}_v \mathfrak{R}_v(t_l). \tag{29}$$

Because $|\mathbb{E}_{1v}(t_l)| = 0$, then

$$|\mathbb{E}_{1v}(t)| \leq \int_{t_l}^{t} \mathit{\Psi}_v \mathfrak{R}_v(t_l)\mathrm{d}s = \mathit{\Psi}_v \mathfrak{R}_v(t_l)(t - t_l). \tag{30}$$

From ETC (10), one has

$$\begin{aligned}|\mathbb{E}_{1v}(t_{l+1})| \geq & \zeta_v|e_v^x(t_{l+1})| + \jmath_v p_{3v}|e_v^x(t_{l+1})|^{2\rho-1} \\ & + \omega_v p_{4v}|e_v^x(t_{l+1})|^{2\delta-1} + \tau_v(1 - \iota_v)^{t_{l+1}} \\ \geq & \tau_v(1 - \iota_v)^{t_{l+1}} > 0.\end{aligned} \tag{31}$$

From (32) and (33), one derives

$$t_{l+1} - t_l \geq \frac{\tau_v(1 - \iota_v)^{t_{l+1}}}{\mathit{\Psi}_v \mathfrak{R}_v(t_l)} > 0. \tag{32}$$

Similarly, when $t \in [t_{l^*}, t_{l^*+1}), l^* = 1, 2, \ldots,$ it can be proven that

$$t_{l^*+1} - t_{l^*} \geq \frac{\tau_n^*(1 - \iota_n^*)^{t_{l^*+1}}}{\Psi_n^* \mathfrak{R}_n^*(t_{l^*})} > 0. \tag{33}$$

□

Therefore, the internal execution time satisfies $t_{l+1} - t_l > 0$ and $t_{l^*+1} - t_{l^*} > 0$, and the error system (6) does not exhibit Zeno behaviour.

The proof is finished.

In the following, we proceed to investigate the FTS of BAMMNNs (1) and (2) under the special case where $\xi(t) = \gamma(t) = 0$.

If $f_v(0) = g_n(0) = 0, \xi(t) = \gamma(t) = 0$ and the error system (6) of BAMMNNs (1) and (2) exhibit prolonged oscillations or chaotic behaviour, the synchronization model of (1) and (2) is

$$\begin{cases} \dot{x}_v(t) = - \alpha_v x_v(t) + \sum_{n=1}^{w} a_{vn}(x_v(t)) f_n(y_n(t)) \\ \qquad\qquad + \sum_{n=1}^{w} b_{vn}(x_v(t)) f_n(y_n(t - z(t))) + I_v, \\ x_v(\sigma) = \varphi(\sigma), \quad -z \leq \sigma \leq 0, \\ \dot{y}_n(t) = - \beta_n y_n(t) + \sum_{v=1}^{m} c_{nv}(y_n(t)) g_v(x_v(t)) \\ \qquad\qquad + \sum_{v=1}^{m} d_{nv}(y_n(t)) g_v(y_n(t - r(t))) + J_n, \\ y_n(\sigma) = \psi(\sigma), \quad -r \leq \sigma \leq 0, \end{cases} \tag{34}$$

and

$$\begin{cases} \dot{\tilde{x}}_v(t) = - \alpha_v \tilde{x}_v(t) + \sum_{n=1}^{w} a_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t)) \\ \qquad\qquad + \sum_{n=1}^{w} b_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t - z(t))) \\ \qquad\qquad + I_v + \mathfrak{U}_{1v}(t), \\ \tilde{x}_v(\sigma) = \tilde{\varphi}(\sigma), \quad -z \leq \sigma \leq 0, \\ \dot{\tilde{y}}_n(t) = - \beta_n \tilde{y}_n(t) + \sum_{v=1}^{m} c_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t)) \\ \qquad\qquad + \sum_{v=1}^{m} d_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t - r(t))) \\ \qquad\qquad + J_n + \mathfrak{V}_{1n}(t), \\ \tilde{y}_n(\sigma) = \tilde{\psi}(\sigma), \quad -r \leq \sigma \leq 0, \end{cases} \tag{35}$$

in which $\mathfrak{U}_{1v}(t)$ $(v \in \mathcal{W})$ and $\mathfrak{V}_{1n}(t)$ $(n \in \mathcal{M})$ are

$$\begin{cases} \mathfrak{U}_{1v}(t) = \tilde{p}_{1v} x_v(t_l) - \tilde{p}_{2v} \phi_v(t_l) - \tilde{p}_{3v} \phi_v(t_l) |x_v(t_l)|^{2\tilde{\rho}-1} \\ \qquad\qquad - \tilde{p}_{4v} \phi_v(t_l) |x_v(t_l)|^{2\tilde{\delta}-1}, \\ \mathfrak{V}_{1n}(t) = \tilde{q}_{1n} y_n(t_l) - \tilde{q}_{2n} \phi_n(t_l) - \tilde{q}_{3n} \phi_n(t_l) |y_n(t_l)|^{2\tilde{\rho}-1} \\ \qquad\qquad - \tilde{q}_{4n} \phi_n(t_l) |y_n(t_l)|^{2\tilde{\delta}-1}, \end{cases} \tag{36}$$

where $\tilde{p}_{1v}, \tilde{p}_{2v}, \tilde{q}_{1n}$ and $\tilde{q}_{2n}$ are constants, $\tilde{p}_{3v}, \tilde{p}_{4v}, \tilde{q}_{3n}$ and $\tilde{q}_{4n}$ are positive constants.

For $t \in [t_l, t_{l+1})$ or $t \in [t_{l^*}, t_{l^*+1})$, let

$$\begin{aligned}\mathfrak{U}^*_{1v}(t) =&\tilde{p}_{1v}x_v(t) - \tilde{p}_{2v}\phi_v(t) - \tilde{p}_{3v}\phi_v(t)|x_v(t)|^{2\tilde{\rho}-1}\\ &- \tilde{p}_{4v}\phi_v(t)|x_v(t)|^{2\tilde{\delta}-1},\\ \mathfrak{V}^*_{1n}(t) =&\tilde{q}_{1n}y_n(t) - \tilde{q}_{2n}\phi_n(t) - \tilde{q}_{3n}\phi_n(t)|y_n(t)|^{2\tilde{\rho}-1}\\ &- \tilde{q}_{4n}\phi_n(t)|y_n(t)|^{2\tilde{\delta}-1}.\end{aligned}$$

The measurement error is

$$\begin{cases}\mathcal{E}_{1v}(t) =\mathfrak{U}^*_{1v}(t) - \mathfrak{U}_{1v}(t)\\ \mathcal{E}^*_{1n}(t) =\mathfrak{V}^*_{1n}(t) - \mathfrak{V}_{1n}(t).\end{cases}$$

and event-triggering is

$$\begin{cases}t_{l+1} =\{t \mid t > t_l, |\mathcal{E}_{1v}(t)| \geq \tilde{\zeta}_v|x_v(t)| + \tilde{\jmath}_v\tilde{p}_{3v}|x_v(t)|^{2\tilde{\rho}-1}\\ \qquad + \tilde{\omega}_v\tilde{p}_{4v}|x_v(t)|^{2\tilde{\delta}-1}+\tilde{\tau}_v(1-\tilde{\iota}_v)^t\}\\ t_{l^*+1} =\{t \mid t > t_{l^*}, |\mathcal{E}^*_{1n}(t)| \geq \tilde{\zeta}^*_n|y_n(t)| + \tilde{\jmath}^*_n\tilde{q}_{3n}|y_n(t)|^{2\tilde{\rho}-1}\\ \qquad + \tilde{\omega}^*_n\tilde{q}_{4n}|y_n(t)|^{2\tilde{\delta}-1} + \tilde{\tau}^*_n(1-\tilde{\iota}^*_n)^t\},\end{cases}. \tag{37}$$

where $\tilde{\jmath}_v, \tilde{\omega}_v, \tilde{\iota}_v, \tilde{\jmath}^*_n, \tilde{\omega}^*_n, \tilde{\iota}^*_n \in (0,1)$, $\tilde{\zeta}_v, \tilde{\tau}_v, \tilde{\zeta}^*_n, \tilde{\tau}^*_n > 0$, $t_l(l = 1,2,\ldots,v \in \mathbb{N})$ and $t_{l^*}(l^* = 1,2,\ldots,n \in \mathbb{N})$ respectively represent the l th and l^* th triggering instants.

Let

$$\tilde{p}_{2v} \geq \Sigma^w_{n=1}2(\breve{a}_{vn} + \breve{b}_{vn} + \bar{b}_{vn})F_n + \tilde{\tau}_v, \tag{38}$$
$$\tilde{q}_{2n} \geq \Sigma^w_{n=1}2(\breve{c}_{nv} + \breve{d}_{nv} + \bar{d}_{nv})G_v + \tilde{\tau}^*_n, \tag{39}$$
$$\tilde{\lambda} = 2\max\{\tilde{\lambda}_1, \tilde{\lambda}_2\}, \tag{40}$$
$$\tilde{\varepsilon} = \min\{min_v\{\tilde{p}_{3v} - \tilde{\jmath}_v\tilde{p}_{3v}\}, \min_n\{\tilde{q}_{3n} - \tilde{\jmath}^*_n\tilde{q}_{3n}\}\}2^{\tilde{\rho}}, \tag{41}$$
$$\tilde{\eta} = \min\{\tilde{\eta}_1, \tilde{\eta}_2\}2^{\tilde{\delta}}. \tag{42}$$

in which $\tilde{\lambda}_1 = \max_v\{-\tilde{\alpha}_v + \tilde{p}_{1v} + \tilde{\zeta}_v + \Sigma^w_{n=1}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j^2_v)\}$, $\tilde{\lambda}_2 = \max_n\{-\tilde{\beta}_n + \tilde{q}_{1n} + \tilde{\zeta}^*_n + \Sigma^m_{v=1}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{vn}\ell^2_n)\}$, $\tilde{\eta}_1 = min_v\{\tilde{p}_{4v} - \tilde{\omega}_v\tilde{p}_{4v}\}m^{1-\tilde{\delta}}$ and $\tilde{\eta}_2 = min_n\{\tilde{q}_{4n} - \tilde{\omega}^*_n\tilde{q}_{4n}\}w^{1-\tilde{\delta}}$

Based on the above content, we have the following Corollary 1 related to Theorem 1.

Corollary 1. *Under Assumption 1, Lemma 2, ETC* (36) *and* (37)*, if* (38)–(42)*,* $\xi(t) = \gamma(t) = 0$ *hold, BAMMNNs* (34) *and* (35) *achieve FTS, and the settling-time is* T_{max}*.*

3.2. PTS of BAMMNNs (1) *and* (2)

Next, we study the PTS results of BAMMNNs (1) and (2), and the controllers are designed as

$$\begin{cases}u^*_{2v}(t) =p'_{1v}e^x_v(t_l) - p_{2v}\phi_v(t_l) - p'_{3v}\phi_v(t_l)|e^x_v(t_l)|^{2\rho-1}\\ \qquad - p'_{4v}\phi_v(t_l)|e^x_v(t_l)|^{2\delta-1},\\ v^*_{2n}(t) =q'_{1n}e^y_n(t_{l^*}) - q_{2n}\phi^*_v(t_{l^*}) - q'_{3n}\phi^*_n(t_{l^*})|e^y_n(t_{l^*})|^{2\rho-1}\\ \qquad - q'_{4n}\phi^*_n(t_{l^*})|e^y_n(t_{l^*})|^{2\delta-1},\end{cases} \tag{43}$$

where $p'_{1v} = \frac{T_{max}}{T_p}[-\alpha_v + p_{1v} + \zeta_v + \Sigma^w_{n=1}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j^2_v)] + \alpha_v - \zeta_v - \Sigma^w_{n=1}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j^2_v)$, $q'_{1n} = \frac{T_{max}}{T_p}[-\beta_n + q_{1n} + \zeta^*_n + \Sigma^m_{v=1}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{vn}\ell^2_n)] + \beta_n - \zeta^*_n - \Sigma^m_{v=1}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{vn}\ell^2_n)$, $p'_{3v} = \frac{T_{max}}{T_p}p_{3v}$, $p'_{4v} = \frac{T_{max}}{T_p}p_{4v}$, $q'_{3n} = \frac{T_{max}}{T_p}q_{3n}$, $q'_{4n} = \frac{T_{max}}{T_p}q_{4n}$, $v \in \mathcal{W}$, $n \in \mathcal{M}$, $\frac{1}{2} \leq \rho < 1$, $\delta > 1$, p_{1v}, p_{2v}, q_{1n} and q_{2n} are constants, p_{3v}, p_{4v}, q_{3n} and q_{4n} are positive constants. For $t \in [t_l, t_{l+1})$, $\phi_v(t_l) \in co[sign(e^x_v(t))]$ and for $t \in [t_{l^*}, t_{l^*+1})$, $\phi^*_n(t_{l^*}) \in co[sign(e^y_n(t))]$. T_{max} is given by Theorem 1 and T_P is preassigned-time.

Let

$$\begin{aligned}\mathcal{U}_{2v}^*(t) =& p_{1v}' e_v^x(t) - p_{2v}\phi_v(t) - p_{3v}'\phi_v(t)|e_v(t)|^{2\rho-1} \\ &- p_{4v}'\phi_v(t)|e_v(t)|^{2\delta-1}, \\ \mathcal{V}_{2n}^*(t) =& q_{1n}' e_n^y(t) - q_{2n}\phi_n^*(t) - q_{3n}'\phi_n^*(t)|e_n^y(t)|^{2\rho-1} \\ &- q_{4n}'\phi_n^*(t)|e_n^y(t)|^{2\delta-1}.\end{aligned}$$

The measurement error is

$$\begin{cases}\mathbb{E}_{2v}(t) = \mathcal{U}_{2v}^*(t) - u_{2v}^*(t) \\ \mathbb{E}_{2n}^*(t) = \mathcal{V}_{2n}^*(t) - v_{2n}^*(t),\end{cases}$$

then, we designed the event-triggering into the following two stages:

$$\begin{cases}t_{l+1} = \{t \mid t > t_l, |\mathbb{E}_{2v}(t)| \geq \zeta_v|e_v^x(t)| + \jmath_v p_{3v}'|e_v^x(t)|^{2\rho-1} \\ \qquad + \omega_v p_{4v}'|e_v^x(t)|^{2\delta-1} + \tau_v(1-\iota_v)^t\} \\ t_{l^*+1} = \{t \mid t > t_{l^*}, |\mathbb{E}_{2n}^*(t)| \geq \zeta_n^*|e_n^y(t)| + \jmath_n^* q_{3n}'|e_n^y(t)|^{2\rho-1} \\ \qquad + \omega_n^* q_{4n}'|e_n^y(t)|^{2\delta-1} + \tau_n^*(1-\iota_n^*)^t\},\end{cases} \tag{44}$$

where $\jmath_v, \omega_v, \iota_v, \jmath_v^*, \omega_v^*, \iota_v^* \in (0,1)$, $\zeta_v, \tau_v, \zeta_v^*, \tau_v^* > 0$, $t_l(l = 1, 2, \ldots, v \in \mathbb{N})$ and $t_{l^*}(l^* = 1, 2, \ldots, n \in \mathbb{N})$ respectively represent the l th and l^* th triggering instants.

Let $p_{2v}, q_{2n}, \lambda, \varepsilon, \eta$ satisfy conditions (11)–(15), respectively.

Theorem 3. *Under Assumption 1, Lemma 3, and ETC (43)–(44), if (11)–(15) hold, BAMMNNs (1) and (2) get PTS, the settling-time is T_{max} and the preassigned-time is T_p.*

Proof. By establishing a Lyapunov function identical to Theorem 1 and providing a similar proof, we obtain

$$\begin{aligned}\dot{V}_1(t) \leq& \sum_{v=1}^{m}[-\alpha_v + p_{1v}' + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}\jmath_v^2)] \\ &\times (e_v^x(t))^2 + \sum_{v=1}^{m}[-p_{2v} + \tau_v + \sum_{n=1}^{w} 2(\breve{a}_{vn} + \breve{b}_{vn} \\ &+ \breve{h}_{vn}\xi + \bar{b}_{vn} + \bar{h}_{vn}\xi)F_n]|e_v^x(t)| \\ &- \sum_{v=1}^{m}(p_{3v}' - \jmath_v p_{3v}')|e_v^x(t)|^{2\rho} \\ &- \sum_{v=1}^{m}(p_{4v}' - \omega_v p_{4v}')|e_v^x(t)|^{2\delta} \\ &+ |e_v^x(t)|[|\mathbb{E}_{2v}(t)| - \zeta_v|e_v^x(t)| - \jmath_v p_{3v}'|e_v^x(t)|^{2\rho-1} \\ &- \omega_v p_{4v}'|e_v^x(t)|^{2\delta-1} - \tau_v(1-\iota_v)^t] \\ \leq& \frac{T_{max}}{T_p}\{2[-\alpha_v + p_{1v} + \zeta_v + \sum_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}\jmath_v^2)] \\ &\times V_1(t) - min_v\{p_{3v} - \jmath_v p_{3v}\}2^\rho V_1^\rho(t) \\ &- min_v\{p_{4v} - \omega_v p_{4v}\}m^{1-\delta}2^\delta V_1^\delta(t)\},\end{aligned} \tag{45}$$

the same can be obtained

$$\begin{aligned}\dot{V}_2(t) \leq& \frac{T_{max}}{T_p}\{2[-\beta_n + q_{1n} + \zeta_n^* + \sum_{v=1}^{m}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{vn}\ell_n^2)]V_2(t) \\ &- min_n\{q_{3n} - \jmath_n^* q_{3n}\}2^\rho V_2^\rho(t) \\ &- min_v\{q_{4n} - \omega_n^* q_{4n}\}w^{1-\delta}2^\delta V_2^\delta(t)\}.\end{aligned} \tag{46}$$

According to (13)–(15), and Lemma 2, it combines $V_1(t)$ and $V_2(t)$ such that:

$$\begin{aligned}\dot{V}(t) =&\dot{V}_1(t) + \dot{V}_2(t)\\ \leq&\frac{T_{max}}{T_p}[\lambda V(t) - \varepsilon V^{\rho}(t) - \eta V^{\delta}(t)],\end{aligned} \tag{47}$$

where $\lambda < \min\{\varepsilon, \eta\}, \varepsilon, \eta > 0$.

From Definition 2 and Lemma 3, we get BAMMNNs (1) and (2) achieve PTS at preassigned-time T_p. The proof is finished. □

Remark 3. *The main system can avoid Zeno behaviour under ETC* (43) *and* (44)*, with a proof similar to Theorem 2, which is omitted here.*

In the following, we proceed to investigate the PTS of BAMMNNs (1) and (2) under the special case where $\xi(t) = \gamma(t) = 0$.

When $f_v(0) = g_n(0) = 0$ and $\xi(t) = \gamma(t) = 0$, the synchronization model of (1) and (2) becomes

$$\begin{cases}\dot{x}_v(t) =&- \alpha_v x_v(t) + \sum\limits_{n=1}^{w} a_{vn}(x_v(t)) f_n(y_n(t))\\ &+ \sum\limits_{n=1}^{w} b_{vn}(x_v(t)) f_n(y_n(t - z(t)))\\ &+ I_v,\\ x_v(\sigma) =&\varphi(\sigma), \quad -z \leq \sigma \leq 0,\\ \dot{y}_n(t) =&- \beta_n y_n(t) + \sum\limits_{v=1}^{m} c_{nv}(y_n(t)) g_v(x_v(t))\\ &+ \sum\limits_{v=1}^{m} d_{nv}(y_n(t)) g_v(y_n(t - r(t)))\\ &+ J_n,\\ y_n(\sigma) =&\psi(\sigma), \quad -r \leq \sigma \leq 0,\end{cases} \tag{48}$$

$$\begin{cases}\dot{\tilde{x}}_v(t) =&- \alpha_v \tilde{x}_v(t) + \sum\limits_{n=1}^{w} a_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t))\\ &+ \sum\limits_{n=1}^{w} b_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t - z(t)))\\ &+ I_v + \mathfrak{U}_{2v}(t),\\ \tilde{x}_v(\sigma) =&\tilde{\varphi}(\sigma), \quad -z \leq \sigma \leq 0,\\ \dot{\tilde{y}}_n(t) =&- \beta_n \tilde{y}_n(t) + \sum\limits_{v=1}^{m} c_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t))\\ &+ \sum\limits_{v=1}^{m} d_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t - r(t)))\\ &+ J_n + \mathfrak{V}_{2n}(t),\\ \tilde{y}_n(\sigma) =&\tilde{\psi}(\sigma), \quad -r \leq \sigma \leq 0,\end{cases} \tag{49}$$

in which $\mathfrak{U}_{2v}(t)$ $(v \in \mathcal{W})$ and $\mathfrak{V}_{2n}(t)$ $(n \in \mathcal{M})$ are

$$\begin{cases}\mathfrak{U}_{2v}(t) =&\tilde{p}'_{1v} x_v(t_l) - \tilde{p}_{2v}\phi_v(t_l) - \tilde{p}'_{3v}\phi_v(t_l)|x_v(t_l)|^{2\tilde{\rho}-1}\\ &- \tilde{p}'_{4v}\phi_v(t_l)|x_v(t_l)|^{2\tilde{\delta}-1},\\ \mathfrak{V}_{2n}(t) =&\tilde{q}'_{1n} y_n(t_l) - \tilde{q}_{2n}\phi_n(t_l) - \tilde{q}'_{3n}\phi_n(t_l)|y_n(t_l)|^{2\tilde{\rho}-1}\\ &- \tilde{q}'_{4n}\phi_n(t_l)|y_n(t_l)|^{2\tilde{\delta}-1},\end{cases} \tag{50}$$

where $\tilde{p}'_{1v} = \frac{T_{max}}{T_p}[-\tilde{\alpha}_v + \tilde{p}_{1v} + \tilde{\zeta}_v + \Sigma_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)] + \tilde{\alpha}_v - \tilde{\zeta}_v - \Sigma_{n=1}^{w}(\frac{1}{2}\bar{a}_{vn} + \frac{1}{2}\bar{a}_{vn}j_v^2)$, $\tilde{q}'_{1n} = \frac{T_{max}}{T_p}[-\tilde{\beta}_n + \tilde{q}_{1n} + \tilde{\zeta}_n^* + \Sigma_{v=1}^{m}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{nv}\ell_n^2)] + \tilde{\beta}_n - \tilde{\zeta}_n^* - \Sigma_{v=1}^{m}(\frac{1}{2}\bar{c}_{nv} + \frac{1}{2}\bar{c}_{nv}\ell_n^2)$, $\tilde{p}'_{3v} = \frac{T_{max}}{T_p}\tilde{p}_{3v}$, $\tilde{p}'_{4v} = \frac{T_{max}}{T_p}\tilde{p}_{4v}$, $\tilde{q}'_{3n} = \frac{T_{max}}{T_p}\tilde{q}_{3n}$, $\tilde{q}'_{4n} = \frac{T_{max}}{T_p}\tilde{q}_{4n}$, $v \in \mathcal{W}$, $n \in \mathcal{M}$, $\frac{1}{2} \le \rho < 1$, $\delta > 1$, $\tilde{p}_{1v}, \tilde{p}_{2v}, \tilde{q}_{1n}$ and $\tilde{q}_{2n}$ are defined as constants, $\tilde{p}_{3v}, \tilde{p}_{4v}, \tilde{q}_{3n}$ and q_{4n} represent suitable positive constants.

For $t \in [t_l, t_{l+1})$ or $t \in [t_{l^*}, t_{l^*+1})$, let

$$\begin{aligned}\mathfrak{U}_{2v}^*(t) =& \tilde{p}'_{1v}x_v - \tilde{p}_{2v}\phi_v(t) - \tilde{p}'_{3v}\phi_v(t)|x_v(t)|^{2\tilde{\rho}-1} \\ & - \tilde{p}'_{4v}\phi_v(t)|x_v(t)|^{2\tilde{\delta}-1}, \\ \mathfrak{V}_{2n}^*(t) =& \tilde{q}'_{1n}y_n(t) - \tilde{q}_{2n}\phi_n(t) - \tilde{q}'_{3n}\phi_n(t)|y_n(t)|^{2\tilde{\rho}-1} \\ & - \tilde{q}'_{4n}\phi_n(t)|y_n(t)|^{2\tilde{\delta}-1}.\end{aligned}$$

The measurement error is

$$\begin{cases}\mathcal{E}_{1v}(t) = \mathfrak{U}_{2v}^*(t) - \mathfrak{U}_{2v}(t), \\ \mathcal{E}_{1n}^*(t) = \mathfrak{V}_{2n}^*(t) - \mathfrak{V}_{2n}(t),\end{cases}$$

and event-triggering is

$$\begin{cases}t_{l+1} = \{t \mid t > t_l, |\mathcal{E}_{2v}(t)| \ge \tilde{\zeta}_v|x_v(t)| + \tilde{\jmath}_v\tilde{p}'_{3v}|x_v(t)|^{2\tilde{\rho}-1} \\ \qquad + \tilde{\omega}_v\tilde{p}'_{4v}|x_v(t)|^{2\tilde{\delta}-1} + \tilde{\tau}_v(1-\tilde{\iota}_v)^t\}, \\ t_{l^*+1} = \{t \mid t > t_{l^*}, |\mathcal{E}_{2n}^*(t)| \ge \tilde{\zeta}_n^*|y_n(t)| + \tilde{\jmath}_n^*\tilde{q}'_{3n}|y_n(t)|^{2\tilde{\rho}-1} \\ \qquad + \tilde{\omega}_n^*\tilde{q}'_{4n}|y_n(t)|^{2\tilde{\delta}-1} + \tilde{\tau}_n^*(1-\tilde{\iota}_n^*)^t\},\end{cases} \tag{51}$$

where $\tilde{\jmath}_v, \tilde{\omega}_v, \tilde{\iota}_v, \tilde{\jmath}_n^*, \tilde{\omega}_n^*, \tilde{\iota}_n^* \in (0,1)$, $\tilde{\zeta}_v, \tilde{\tau}_v, \tilde{\zeta}_n^*, \tilde{\tau}_n^* > 0$, $t_l(l = 1, 2, \ldots, v \in \mathbb{N})$ and $t_{l^*}(l^* = 1, 2, \ldots, n \in \mathbb{N})$ respectively represent the l th and l^* th triggering instants.

Let $\tilde{p}_{2v}, \tilde{q}_{2n}, \tilde{\lambda}, \tilde{\varepsilon}, \tilde{\eta}$ satisfy conditions (38)–(42), respectively.

Based on the above content, we have the following Corollary 2 related to Theorem 3.

Corollary 2. *Under Assumption 1, Lemma 3,* $f_v(0) = 0$, $g_n(0) = 0$ *and ETC* (50) *and* (51), *if* (38)–(42) *and* $\xi(t) = \gamma(t) = 0$ *hold, BAMMNNs* (48) *and* (49) *achieve PTS, and the preassigned-time is* T_p.

Remark 4. *This paper designs a segmented ETC strategy specifically tailored for the dual-layer structure of BAMMNNs. In contrast to the continuous feedback control methods used in [6, 9, 10], our ETC operations are executed only when the system state changes beyond a certain threshold, thereby significantly enhancing resource utilization efficiency.*

Remark 5. *While previous work has primarily focused on finite-time synchronization and FTS of MNNs [13], the exploration of PTS results remains limited. Unlike traditional asymptotic and finite-time synchronization approaches, which rely on initial conditions, the proposed FTS and PTS strategies achieve synchronization within predetermined time frames independent of initial states. This feature is particularly advantageous in real-world applications, where initial conditions are often unknown or difficult to measure, making the proposed strategies more practical and robust.*

4. Numerical Example

In this section, a numerical simulation example is employed to demonstrate the results of FTS and PTS.

Example 1. *Consider the following two-dimensional driving system and response system are:*

$$\begin{cases} \dot{x}_v(t) = -\alpha_v x_v(t) + \sum_{n=1}^{2} a_{vn}(x_v(t)) f_n(y_n(t)) \\ \qquad + \sum_{n=1}^{2} b_{vn}(x_v(t)) f_n(y_n(t-z(t))) \\ \qquad + \sum_{n=1}^{2} h_{vn}(x_v(t)) \int_{t-\xi(t)}^{t} f_n(y_n(s))\mathrm{d}s + I_v, \\ x_v(\sigma) = \varphi(\sigma), \quad -z \le \sigma \le 0, \\ \dot{y}_n(t) = -\beta_n y_n(t) + \sum_{v=1}^{2} c_{nv}(y_n(t)) g_v(x_v(t)) \\ \qquad + \sum_{v=1}^{2} d_{nv}(y_n(t)) g_v(x_v(t-r(t))) \\ \qquad + \sum_{v=1}^{2} k_{nv}(y_n(t)) \int_{t-\gamma(t)}^{t} g_v(x_v(s))\mathrm{d}s + J_n, \\ y_n(\sigma) = \psi(\sigma), \quad -r \le \sigma \le 0, \end{cases} \tag{52}$$

and

$$\begin{cases} \dot{\tilde{x}}_v(t) = -\alpha_v \tilde{x}_v(t) + \sum_{n=1}^{2} a_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t)) \\ \qquad + \sum_{n=1}^{2} b_{vn}(\tilde{x}_v(t)) f_n(\tilde{y}_n(t-z(t))) \\ \qquad + \sum_{n=1}^{2} h_{vn}(\tilde{x}_v(t)) \int_{t-\xi(t)}^{t} f_n(\tilde{y}_n(s))\mathrm{d}s \\ \qquad + I_v + u_{1v}^*(t), \\ \tilde{x}_v(\sigma) = \tilde{\varphi}(\sigma), \quad -z \le \sigma \le 0, \\ \dot{\tilde{y}}_n(\sigma) = -\beta_v \tilde{y}_n(t) + \sum_{v=1}^{2} c_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t)) \\ \qquad + \sum_{v=1}^{2} d_{nv}(\tilde{y}_n(t)) g_v(\tilde{x}_v(t-r(t))) \\ \qquad + \sum_{v=1}^{2} k_{nv}(\tilde{y}_n(t)) \int_{t-\gamma(t)}^{t} g_v(\tilde{x}_v(s))\mathrm{d}s \\ \qquad + J_n + v_{1v}^*(t), \\ \tilde{y}_n(\sigma) = \tilde{\psi}(\sigma), \quad -r \le \sigma \le 0, \end{cases} \tag{53}$$

in which $v, n = 1, 2$*,* $t \ge 0$*, and the parameters in* (52) *and* (53) *are as follows:* $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0.1$*,* $I_1 = I_2 = J_1 = J_2 = 0$*,*

$$a_{11}(\hbar_1(t)) = \begin{cases} -0.06, & |\hbar_1(t)| \le 1, \\ -0.05, & |\hbar_1(t)| > 1, \end{cases} \quad a_{12}(\hbar_1(t)) = \begin{cases} -4, & |\hbar_1(t)| \le 1, \\ -3.8, & |\hbar_1(t)| > 1, \end{cases}$$

$$a_{21}(\hbar_2(t)) = \begin{cases} 7.4, & |\hbar_2(t)| \le 1, \\ 7.6, & |\hbar_2(t)| > 1, \end{cases} \quad a_{22}(\hbar_2(t)) = \begin{cases} 2, & |\hbar_2(t)| \le 1, \\ 1, & |\hbar_2(t)| > 1, \end{cases}$$

$$h_{11}(\hbar_1(t)) = \begin{cases} 4.1, & |\hbar_1(t)| \le 1, \\ 3.9, & |\hbar_1(t)| > 1, \end{cases} \quad h_{12}(\hbar_1(t)) = \begin{cases} -0.1, & |\hbar_1(t)| \le 1, \\ -0.2, & |\hbar_1(t)| > 1, \end{cases}$$

$$
\begin{aligned}
&h_{21}(\hbar_2(t)) = \begin{cases} 0.1, & |\hbar_2(t)| \le 1, \\ 0.2, & |\hbar_2(t)| > 1, \end{cases} \qquad h_{22}(\hbar_2(t)) = \begin{cases} -3.2, & |\hbar_2(t)| \le 1, \\ -3, & |\hbar_2(t)| > 1, \end{cases} \\
&c_{11}(\varpi_1(t)) = \begin{cases} 4.8, & |\varpi_1(t)| \le 1, \\ 4.6, & |\varpi_1(t)| > 1, \end{cases} \qquad c_{12}(\varpi_1(t)) = \begin{cases} -2, & |\varpi_1(t)| \le 1, \\ -1, & |\varpi_1(t)| > 1, \end{cases} \\
&c_{21}(\varpi_2(t)) = \begin{cases} 5, & |\varpi_2(t)| \le 1, \\ 5.2, & |\varpi_2(t)| > 1, \end{cases} \qquad c_{22}(\varpi_2(t)) = \begin{cases} -0.1, & |\varpi_2(t)| \le 1, \\ -0.2, & |\varpi_2(t)| > 1, \end{cases} \\
&k_{11}(\varpi_1(t)) = \begin{cases} 1.1, & |\varpi_1(t)| \le 1, \\ 1.2, & |\varpi_1(t)| > 1, \end{cases} \qquad k_{12}(\varpi_1(t)) = \begin{cases} -1.2, & |\varpi_1(t)| \le 1, \\ -1.18, & |\varpi_1(t)| > 1, \end{cases} \\
&k_{21}(\varpi_2(t)) = \begin{cases} -1.1, & |\varpi_2(t)| \le 1, \\ -1.2, & |\varpi_2(t)| > 1, \end{cases} \qquad k_{22}(\varpi_2(t)) = \begin{cases} 1.2, & |\varpi_2(t)| \le 1, \\ 1.18, & |\varpi_2(t)| > 1, \end{cases} \\
&b_{11}(\hbar_1(t)) = 1, b_{12}(\hbar_1(t)) = -2, b_{21}(\hbar_2(t)) = -4, b_{22}(\hbar_2(t)) = -1.5, \\
&d_{11}(\varpi_1(t)) = -5.9, d_{12}(\varpi_1(t)) = -1.5, d_{21}(\varpi_2(t)) = 6, d_{22}(\varpi_2(t)) = 2.5.
\end{aligned}
$$

where $\hbar$ and ϖ respectively represent x and y, therefore, it is clear that $\bar{a}_{11} = 0.06,\ \breve{a}_{11} = 0.01,\ \bar{a}_{12} = 4,\ \breve{a}_{12} = 0.1,\ \bar{a}_{21} = 7.6,\ \breve{a}_{21} = 0.1,\ \bar{a}_{22} = 2,\ \breve{a}_{22} = 0.5,\ \bar{b}_{11} = 1,\ \breve{b}_{11} = 0, \bar{b}_{12} = 2,\ \breve{b}_{12} = 0,\ \bar{b}_{21} = 4,\ \breve{b}_{21} = 0,\ \bar{b}_{22} = 1.5,\ \breve{b}_{22} = 0,\ \bar{h}_{11} = 4.1,\ \breve{h}_{11} = 0.1, \bar{h}_{12} = 0.2,\ \breve{h}_{12} = 0.05,\ \bar{h}_{21} = 0.2,\ \breve{h}_{21} = 0.05,\ \bar{h}_{22} = 3.2,\ \breve{h}_{22} = 0.1,\ \bar{c}_{11} = 4.8,\ \breve{c}_{11} = 0.1,\ \bar{c}_{12} = 2,\ \breve{c}_{12} = 0.5,\ \bar{c}_{21} = 5.2,\ \breve{c}_{21} = 0.1,\ \bar{c}_{22} = 0.2,\ \breve{c}_{22} = 0.05,\ \bar{d}_{11} = 5.9,\ \breve{d}_{11} = 0,\ \bar{d}_{12} = 1.5,\ \breve{d}_{12} = 0,\ \bar{d}_{21} = 6,\ \breve{d}_{21} = 0,\ \bar{d}_{22} = 2.5,\ \breve{d}_{22} = 0, \bar{k}_{11} = 1.2,\ \breve{k}_{11} = 0.05,\ \bar{k}_{12} = 1.2,\ \breve{k}_{12} = 0.01,\ \bar{k}_{21} = 1.2,\ \breve{k}_{21} = 0.05,\ \bar{k}_{22} = 1.2,\ \breve{k}_{22} = 0.01,$

Define the activation function as $f_n(\cdot) = g_v(\cdot) = \tanh(\cdot)$, the time-varying delays $z(t) = r(t) = 0.5 - 0.2\sin(t)$ and distributed delays $\xi(t) = \gamma(t) = 0.3 - 0.2cos(t)$. According to Assumption 1, we know that $F_1 = F_2 = G_1 = G_2 = 1$. The phase trajectory diagrams of BAMMNNs (52) are presented Figures 1 and 2. Figures 3 and 4 illustrate the state trajectories and synchronization errors on x-layer and y-layer for (52) and (53) without ETC. From conditions (11) and (12), we choose $p_{21} = 7.4, p_{22} = 13.5, q_{21} = 17.1$ and $q_{22} = 18.5$, then we set the other parameters to be $p_{11} = -30, p_{12} = -30, q_{11} = -40, q_{12} = -40, p_{31} = p_{32} = q_{31} = q_{32} = 1, p_{41} = p_{42} = q_{41} = q_{42} = 0.6,\ \rho = 0.7,\ \delta = 3,\ \zeta_1 = 14, \zeta_2 = 15, \zeta_1^* = 14, \zeta_2^* = 15,\ \jmath_1 = \jmath_2 = \jmath_1^* = \jmath_2^* = 0.3,\ \omega_1 = \omega_2 = \omega_1^* = \omega_2^* = 0.4,\ \tau_1 = \tau_2 = \tau_1^* = \tau_2^* = 0.5,\ \iota_1 = \iota_2 = \iota_1^* = \iota_2^* = 0.8$. Based on (13)–(15), a simple calculation yields $\lambda = -7.6, \varepsilon = 1.137, \eta = 0.72$. According to Lemma 2, it can be concluded that $T_{max} = 3.184$.

Under the initial condition $x_0 = [-0.6, 2.5]$, $\tilde{x}_0 = [0.6 - 2.8]$, $y_0 = [-3.5, -0.5]$, $\tilde{y}_0 = [-0.3, -0.2]$, Figures 5 and 6 illustrate the trajectories of states and synchronization errors on x-layers and y-layers of BAMMNNs (52) and (53) under ETC (9) and (10), the results show that the main system achieves FTS within T_{max}. Additionally, we randomly selected 20 initial values, and Figure 7 presents the synchronization errors $e^x(t)$ and $e^y(t)$ for models (52) and (53) under ETC (9) and (10), it is evident that varying the initial values does not influence the FTS results.

With all other parameters identical to FTS, based on ETC (43) and (44), we computed $p'_{11} = -32.67, p'_{12} = -31.026, q'_{11} = -44.89, p'_{12} = -45, p'_{31} = p'_{32} = q'_{31} = q'_{32} = 1.27,\ p'_{41} = p'_{42} = q'_{41} = q'_{42} = 0.762$, and additionally selected preassigned-time $T_p = 2.5 < T_{max} = 3.184$.

Figure 8 depicts the synchronization errors $e^x(t)$ and $e^y(t)$ stabilized under the influence of ETC (43) and (44), indicating that the main system achieves PTS when $T_p = 3.8$. Finally, in Figures 9 and 10, we demonstrate the time transmission intervals under ETC (9), (10), (43) and (44), thus, we conclude that the ETC strategy presented in this paper is effective in reducing controller workload time and thereby minimizing transmission power consumption.

Remark 6. *The integration of BAMMNN models with ETC strategies provides an effective approach to addressing complex system modelling and control challenges. However, current research in this field still shows gaps. Future studies will focus on refining ETC strategies and exploring advanced BAMMNN models, which hold significant scientific implications and promising application potential.*

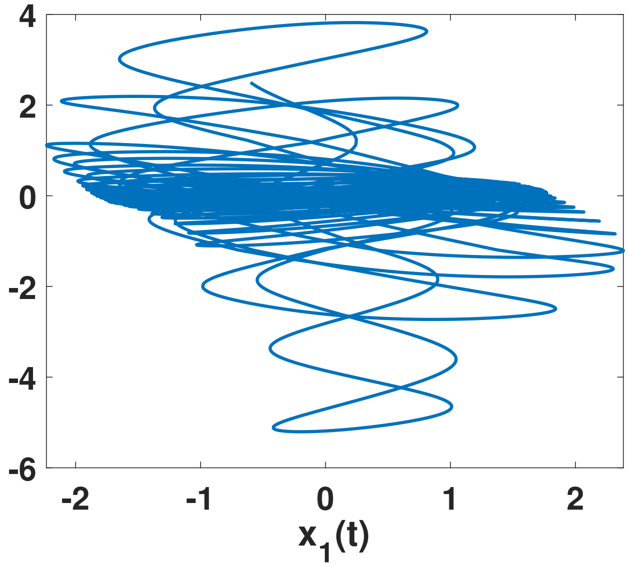

Figure 1. Phase trajectory diagram on x-layer without ETC.

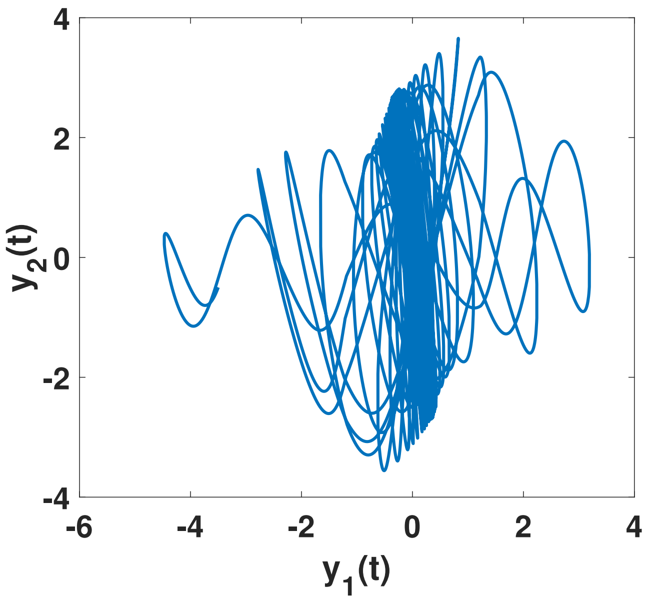

Figure 2. Phase trajectory diagram on y-layer without ETC.

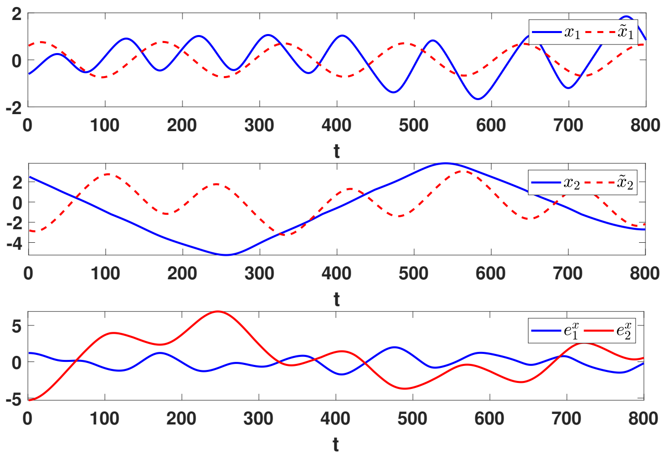

Figure 3. State trajectories and synchronization errors of the x-layer without ETC.

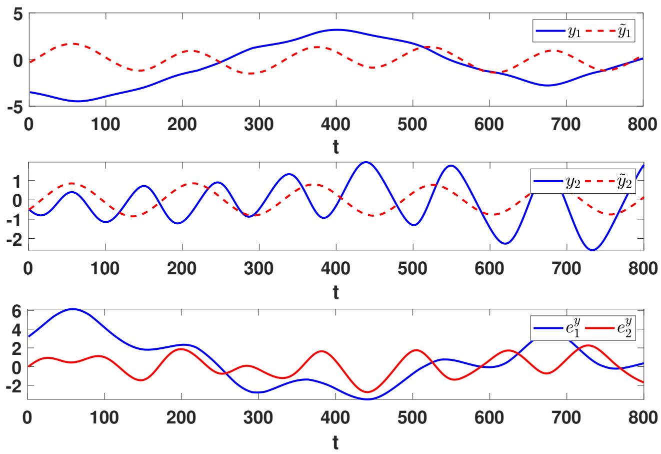

Figure 4. State trajectories and synchronization errors of the y-layer without ETC.

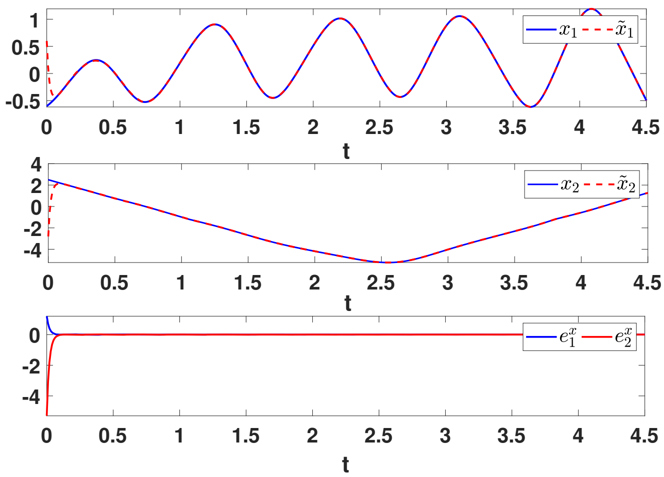

Figure 5. The state trajectories and synchronization errors of the x-layer under ETC (9) and (10) within T_{max}.

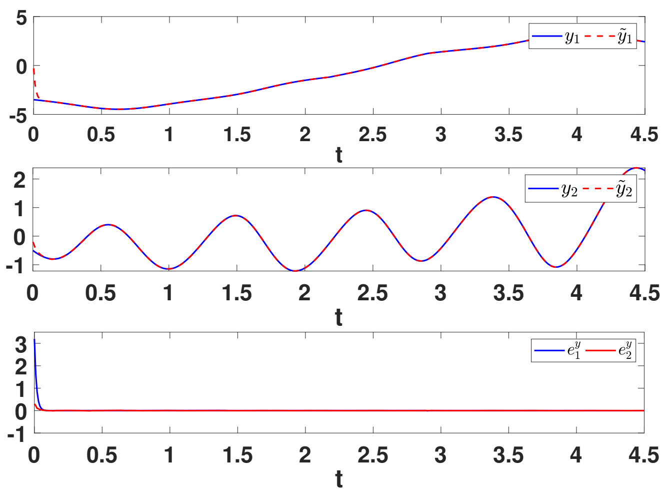

Figure 6. The state trajectories and synchronization errors of the y-layer under ETC (9) and (10) within T_{max}.

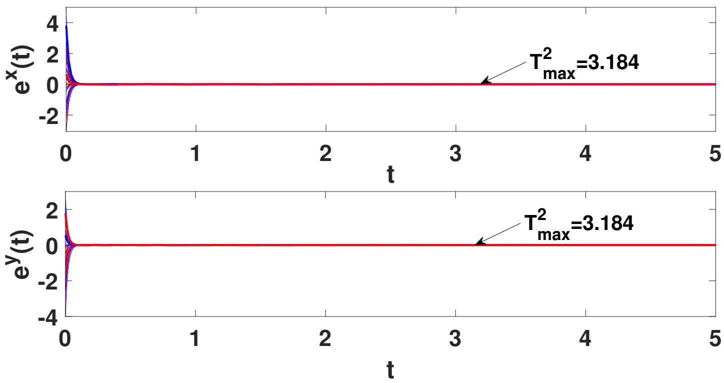

Figure 7. The stabilization of $e^x(t)$ and $e^y(t)$ of BAMMNNs (52) and (53) under ETC (9) and (10) within $T_{max} = 3.184$.

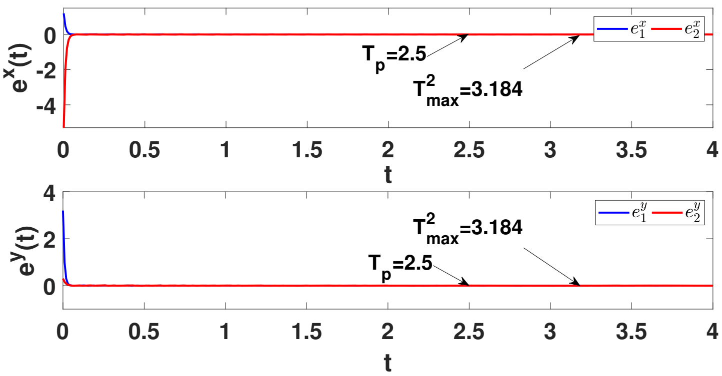

Figure 8. PTS of BAMMNNs (52) and (53) under ETC (43) and (44) when $T_p = 2.5$.

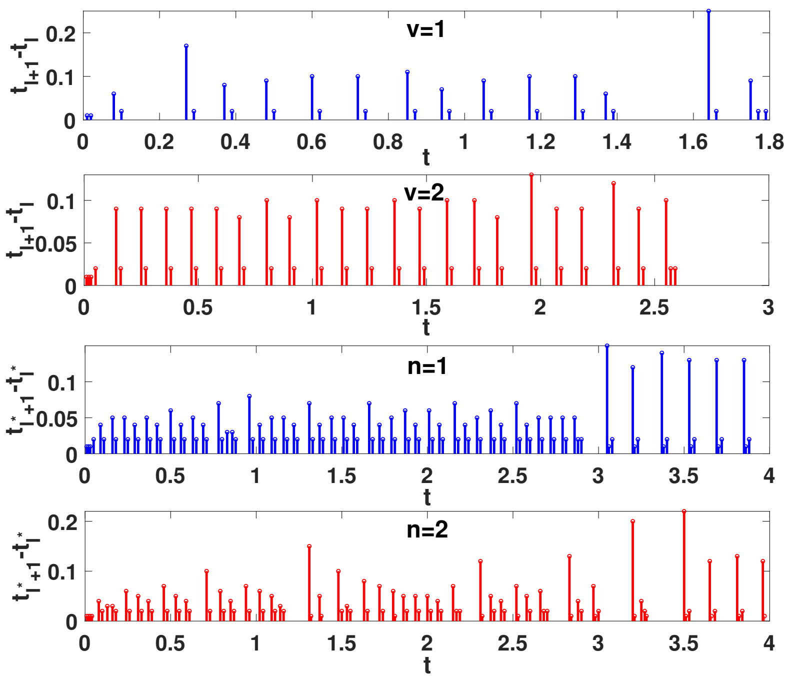

Figure 9. The time transmission intervals of ETC (9) and (10).

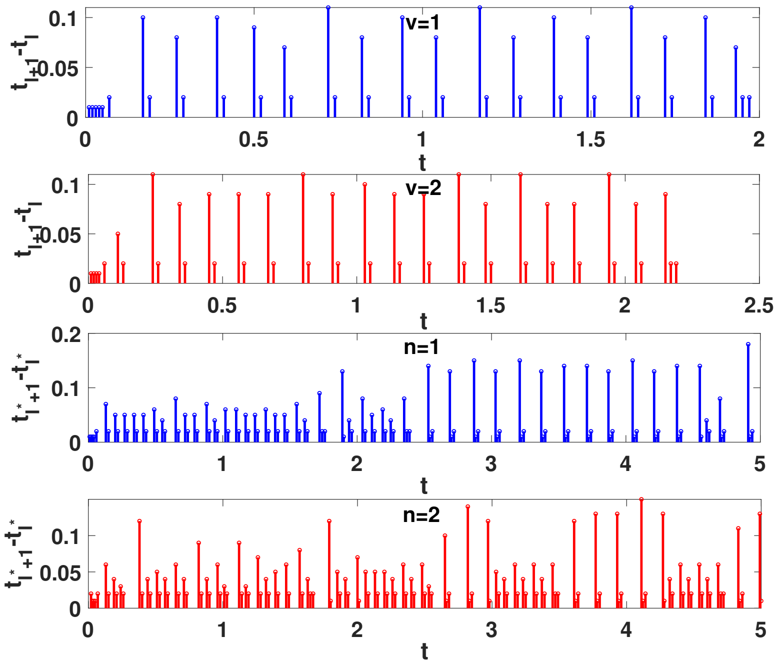

Figure 10. The time transmission intervals of ETC (43) and (44).

5. Conclusions

This paper examined the FTS and PTS of BAMMNNs that incorporated both time-varying delays and distributed delays. Differential inclusion theory, measurable selection theory, and convex analysis methods were utilised to reformulate the dynamics of discontinuous systems. By employing an improved fixed-time stability lemma, more precise and comprehensive settling times were achieved, and corresponding preassigned-time stability lemmas were developed. In addition, considering the dual-layer structure of BAMMNNs, a piecewise ETC strategy was proposed to optimise common feedback control challenges, particularly the excessive information transmission power consumption associated with continuous control actions. Notably, the introduction of mixed delays enhanced the model's applicability, and the FTS and PTS results were independent of initial condition constraints.

The integration of BAMMNNs with event-triggered control ETC provides an intelligent and efficient solution framework for various complex systems. To our knowledge, complex numbers and quaternions play a crucial role in neural information processing, particularly in modelling phase-sensitive signals and multidimensional spatial transformations. In future work, we will expand on this research by exploring the application of ETC in relation to complex domains and quaternions.

Author Contributions

J.G.: Software, Writing-original draft.; G.Z.: Supervision, conceptualization, methodology, writing-review & editing; J.H.: Methodology, writing-review & editing; G.C.: Writing-review & editing. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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