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Article

Torus-Event-Based Sliding Mode Control for Networked Interval Type-2 Fuzzy Systems Under Deception Attacks

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Abstract: In this paper, the security control problem is investigated for the discrete networked interval type-2 (IT2) fuzzy system under limited communication resources. A torus-event-triggering protocol is developed via two thresholds to regulate the transmission of data, ensuring avoiding the transmission of abnormal data. The deception attacks considered are assumed to have the ability of injecting false information into the data transmitted between sensor and controller. By constructing the new membership functions, a security sliding mode controller is proposed and the theoretical analysis proves that the stochastic stability of the closed-loop system and the reachability of the prescribed sliding surface can be guaranteed. Finally, an illustrative numerical example is proposed to demonstrate the effectiveness of the proposed control strategy.

Keywords: interval type-2 (IT2) fuzzy systems; sliding mode control (SMC); cyber attacks; torus-event-triggering

1. Introduction

Since originally put forward by L.A.Zedeh [1] in 1965, the study of systems has so far achieved significant progress over the last few decades, from perspectives of both theoretical research and practical applications. Nowadays, the fuzzy theory could find extensive utilization in such as system control, pattern recognition, signal processing and decision analysis, see [2–5] for instance. It should be mentioned that proposed in 1980s, the well known T-S fuzzy model (which was named after Takagi and Sugeno) is capable of globally approximating nonlinear systems in terms of a series of linear submodels with certain appropriately selected fuzzy weights. Moreover, such an approximation can achieve arbitrary precision over any convex compact set, see, e. g. [6, 7]. It should be pointed out that, nevertheless, the traditional T-S fuzzy model approach cannot be adopted to characterize nonlinear systems subject to structure variations and parameter uncertainties, due to the difficulty in designing exact membership functions (MFs). Accordingly, it gives the rise to the study on interval type-2 (IT2) T-S fuzzy model that has the ability to use bounded membership functions to describe the parameter uncertainties [8–11]. However, such special membership functions will inevitably bring difficulties for the performance analysis as well as controller design. Up to now, quite a few control strategies have been presented, including but not limited to, state feedback control [12, 13], output feedback control [14, 15], and sliding mode control (SMC) [16–18], etc.

In modern industrial engineering, a large part of systems can be described as cyber-physical systems (CPS), such as smart grid [19] and internet of vehicles (IoV) [20, 21]. One of the key features of CPSs is the integration of computation module and physical processes via networks such as internet, which makes it very vulnerable to cyber-threats. Among them, the cyber-attacks are recognized as the main menace [22–25] and therefore has attracted considerable attention from either the aspect of attack detection [26] or the perspective of security control [18, 27–29]. There have been so far several attack forms that are frequently applied in practical engineering, among which the most two popular are the so-called denial-of-service (DOS) attacks and deception attacks. Note that the deception attack could manipulate the targeted systems by injecting false data into original ones while keeping stealthy, and it thus is recognized as one of the most powerful and applicable attack forms. As such, recently, in the context of deception attacks, the corresponding security control issue for fuzzy systems has aroused the interest of many researchers [30, 31]. Specifically, an adaptive torus-event-based $\mathcal{H}_\infty$ fuzzy control approach was proposed in [30] for the fuzzy system, when the system state contains the false information injected by the attackers. When facing both actuator failure and deception attacks, an event-triggered dynamic output feedback controller was designed in reference [31] for T-S fuzzy system.

Recently, a special type of communication technique (event-triggering mechanism) has gained growing interest that could help alleviate the burden on communication resource. In practical engineering, it is usually very difficult to achieve a balance between the communication cost and the control/estiamtion performances, owing to the fact that better performance always demands higher cost [32]. In the context of networked control system, the issue on how to reduce the communication frequency while maintaining satisfactory performance has become a hot spot. An applicable technique is to introduce protocols to regulate the process of information transmission [33–35], as a special case of which the event-triggering scheme has become more and more popular. Such a scheme schedules the data transmission by comparing certain system-related values with the validating conditions to seek the triggering instants (the time steps the data are permitted to be sent) [36–38]. It should be emphasized that, however, all the aforementioned even-triggering mechanisms are designed on basis of the unilateral triggering, which means that the data will be sent once the triggering condition is validated, regardless of the data quality. Nevertheless, the real-world systems under operation are always confronting unusual situations such as measurement outliers and malicious attacks, which makes the data abnormal and obviously, not suitable for usage. Accordingly, a two-side event-triggering scheme (called torus-event-triggering mechanism) was developed in [39], which has the ability to mitigate the effects resulting from the abnormal data. The main idea behind such a mechanism lies in that the triggering indices of a normally operating system should reside within a certain range characterized by a two-side inequality condition, and neither too large nor too small values should trigger the condition. Up to now, however, the research on security control of fuzzy system in combination with the torus-event-triggering mechanism has been far from enough.

It is widely acknowledged that the sliding mode control (SMC) method is an effective algorithm with strong robustness, which has been found extensive successful application in practical engineering. With the help of its strong robustness, the SMC technique is utilized in this paper to design a security controller for the IT2 fuzzy system based on torus-event-triggering scheme and subject to deception attacks. The main contributions of this paper are summarized as follows: 1) The torus-event-triggering mechanism and deception attacks are considered simultaneously, where the two-side event-triggering mechanism could effectively help reduce the influence of abnormal measurement data caused by attacks. 2) A security sliding mode controller for the new IT2 fuzzy system subject to deception attacks is proposed, ensuring the stochastic stability of the closed-loop system and the reachability of the prescribed sliding surface.

Notation: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\|x\|$ describes the Euclidean norm of a vector x. A^T represents the transpose of A. I denotes the identity matrix of compatible dimension. The notation $X \geqslant Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). $\mathrm{diag}\{F_1, F_2, \cdots\}$ stands for a block-diagonal matrix whose diagonal blocks are given by $F_1, F_2, \cdots$. The symbol $*$ in a matrix means that the corresponding term of the matrix can be obtained by symmetric property. $\mathrm{tr}[A]$ means the trace of matrix A and sgn represents the sign function.

2. Problem Formulation

2.1. System Description

Consider the following r-rule IT2 fuzzy systems:

Rule i: IF $\omega_1(k)$ is M_1^i and... and $\omega_p(k)$ is M_p^i, THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) \\ y(k) &= C_i x(k) \end{aligned} \tag{1}$$

where $\omega_j(k), (j = 1, 2, \cdots, p)$ represents the jth premise variable, $\omega(k) = [\omega_1(k), \cdots, \omega_p(k)]$ and p is the number of the fuzzy sets; $M_j^i, (j = 1, 2, \cdots, p; i = 1, 2, \cdots, r)$ represents the jth fuzzy set in the ith IF-THEN rule and r is the number of the fuzzy rules; A_i, B_i and C_i are known system matrices with appropriate dimensions; $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^q$ stand for the system state, control input and output; The firing strength of the ith fuzzy rule is defined as follows:

$$D_i(\omega(k)) = [\underline{h}_i(\omega(k)), \overline{h}_i(\omega(k)] \tag{2}$$

with the lower and upper membership functions

$$\underline{h}_i(\omega(k)) = \prod_{j=1}^{p} \underline{\mu}_{M_j^i}(\omega_j(k)) \geqslant 0$$
$$\overline{h}_i(\omega(k)) = \prod_{j=1}^{p} \overline{\mu}_{M_j^i}(\omega_j(k)) \geqslant 0$$

where $\underline{\mu}_{M_j^i}(\omega_j(k)) \in [0,1]$ and $\overline{\mu}_{M_j^i}(\omega_j(k)) \in [0,1]$ denote the lower and upper membership grades of $\omega_j(k)$ in fuzzy set M_j^i, which satisfy $0 \leqslant \underline{\mu}_{M_j^i}(\omega_j(k)) \leqslant \overline{\mu}_{M_j^i}(\omega_j(k)) \leqslant 1$.

Then the overall IT2 fuzzy system can be formulated by

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} h_i(\omega(k))[A_i x(k) + B_i u(k)] \\ y(k) &= \sum_{i=1}^{r} h_i(\omega(k)) C_i x(k) \end{aligned} \tag{3}$$

with the MFs $h_i(\omega(k))$ satisfying $\sum_{i=1}^{r} h_i(\omega(k)) = 1$, and

$$h_i(\omega(k)) = \underline{h}_i(\omega(k))\underline{\upsilon}_i(\omega(k)) + \overline{h}_i(\omega(k))\overline{\upsilon}_i(\omega(k))$$
$$\underline{\upsilon}_i(\omega(k)), \overline{\upsilon}_i(\omega(k)) \in [0,1], \underline{\upsilon}_i(\omega(k)) + \overline{\upsilon}_i(\omega(k)) = 1$$

2.2. Torus-event-triggering mechanism and deception attacks

For the purpose of relieving the communication burden, torus-event-triggering mechanism is utilized to reduce the communication frequency. Suppose that the releasing instants $0 \leqslant \gamma_0 < \gamma_1 < \cdots < \gamma_\iota < \cdots$ is obtained iteratively by

$$\gamma_{\iota+1} = \min_{k > \gamma_\iota} \{k \in \mathbb{N}^+ | \delta_1 y^{\mathrm{T}}(k)\Omega_k y(k) \leqslant e^{\mathrm{T}}(k)\Omega_k e(k) \leqslant \delta_2 y^{\mathrm{T}}(k)\Omega_k y(k)\} \tag{4}$$

where $e(k) = y(k) - y(\gamma_\iota)$, $y(k)$ is the current measurement and $y(\gamma_\iota)$ is the measurement at the latest triggering instant. $\Omega_k > 0$ stands for the triggering threshold matrix. δ_1, δ_2 are the lower and upper bounds of the torus events.

Then the triggered measurement signals will be send to the controller, and the zero-order holder is applied to hold the signals until a new measurement is achieved. Suppose the available signals without cyber attacks for controller are

$$\bar{y}(k) = y(\gamma_\iota), k \in [\gamma_\iota, \gamma_{\iota+1}) \tag{5}$$

When the triggered measurement $y(\gamma_\iota)$ is transmitted to the controller via an open network, it may be attacked by the adversary. In this paper, deception attacks are considered, which can be described as follows:

$$\hat{y}(k) = \bar{y}(k) + \beta(k)\varpi(k), k \in [\gamma_\iota, \gamma_{\iota+1}) \tag{6}$$

where $\beta(k) \in \{0,1\}$ is a stochastic variable with the expectation

$$\mathbb{E}\{\beta(k)\} = \bar{\beta} \tag{7}$$

and $\bar{\beta} \in (0,1]$.

Assume that there is a known matrix W, and the deception attack $\xi(k)$ satisfies $\| \varpi(k) \| \leqslant \| Wx(k) \|$.

The purpose of this article is to design a desired security IT2 fuzzy controller to attain the stochastic stability of the system (3) under the torus-event-based communication protocol and deception attack.

Definition 1[40]: The IT2 fuzzy system (3) is stochastically stable for any initial condition $x(0) \in \mathbb{R}^n$ if

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \| x(k) \|^2 \right\} < \infty \tag{8}$$

Remark 1. *In this paper, the torus-event-triggering mechanism is adopted with the purpose of mitigating the effects from abnormal data. It is known that the traditional event-triggering scheme is always designed to send the data only when the difference between the sent data and sampling data becomes sufficiently large (characterized by the trig-*

gering threshold). The fact behind such an idea is that data with decided changes are more worthy of being transmitted. However, in practice, many cases such as measurement outliers and cyber attacks would make the data abnormal and inappropriate for subsequent usage. Under the framework of conventional triggering scheme, these abnormal data could easily validate the triggering condition as they always change abruptly and obviously. The traditional one-side triggering mechanism cannot properly handle such abnormal situations. Accordingly, in this paper, due to the existence of deception attacks, we apply the two-side event-triggering scheme (i.e., torus-event-triggering mechanism) to mitigate the effects resulting from the abnormal data.

3. Main results

3.1. Sliding Mode Controller Design

Choose the following sliding function:

$$s(k) = Hy(k) \tag{9}$$

where $H \in \mathbb{R}^{m\times q}$ is a given matrix.

According to the torus-event-triggering mechanism and deception attacks, it is obvious that only corrupted discrete data $\hat{y}(k)$ can be received by controller. Then the sliding mode controller for the IT2 fuzzy systems when $k \in [\gamma_\iota, \gamma_{\iota+1})$ is designed as follows:

Rule j: IF $\omega_1(\gamma_\iota)$ is M_1^i and... and $\omega_p(\gamma_\iota)$ is M_p^i, THEN

$$u(k) = K_j\hat{y}(k) - \breve{K} \parallel \hat{y}(k) \parallel sgn(\hat{s}(k)) \tag{10}$$

where $K_j, (j = 1, 2, \cdots, \lambda)$ is the jth fuzzy gain and λ represents the number of the fuzzy rules for the fuzzy sliding mode controller. $\hat{s}(k)$ is acquired by replacing $y(k)$ in (9) with $\hat{y}(k)$. $\breve{K}$ is a positive scalar. Thus the overall fuzzy sliding mode controller can be written as:

$$u(k) = \sum_{j=1}^{\lambda} \xi_j(\omega(\gamma_\iota))K_j\hat{y}(k) - \breve{K} \parallel \hat{y}(k) \parallel sgn(\hat{s}(k)) \tag{11}$$

where $\xi_i(\omega(\gamma_\iota))$ is the new membership function constructed for the controller:

$$\xi_j(\omega(\gamma_\iota)) = \underline{\xi}_j(\omega(\gamma_\iota))\underline{\varsigma}_j(\omega(k)) + \overline{\xi}_j(\omega(k))\overline{\varsigma}_j(\omega(k))$$

with $\underline{\varsigma}_j(\omega(\gamma_\iota)), \overline{\varsigma}_j(\omega(\gamma_\iota)) \in [0,1], \underline{\varsigma}_j(\omega(\gamma_\iota)) + \overline{\varsigma}_j(\omega(\gamma_\iota)) = 1$ and $\sum_{j=1}^{\lambda} \xi_j(\omega(\gamma_\iota)) = 1$.

For simplicity, $h_i(\omega(k))$, $\xi_j(\omega(\gamma_\iota))$ and $h_l(\omega(k))$ are denoted as h_i, ξ_j and h_l. Substituting the fuzzy sliding mode controller (11) and (6) into the IT2 fuzzy system (3), we can acquire the following closed-loop IT2 fuzzy system for $k \in [\gamma_\iota, \gamma_{\iota+1})$:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l[(A_i + B_iK_jC_l)x(k) - B_iK_je(k) + B_iK_j\beta(k)\xi(k) - B_iZ(k)] \tag{12}$$

with $Z(k) = \breve{K} \parallel \hat{y}(k) \parallel sgn(\hat{s}(k))$.

3.2. Stochastic stability analysis

Theorem 1: If there exist matrices $P > 0$, scalar $\eta_1 > 0$, K_j satisfying

$$B_i^{\mathrm{T}}PB_i \leqslant \eta_1 I \tag{13}$$

$$\phi_{ijl} < 0 \tag{14}$$

with

$$\phi_{ijl} = \begin{bmatrix} \phi_{ijl}^{11} & * \\ \phi_{ijl}^{21} & \phi_{ijl}^{22} \end{bmatrix}$$

$$\begin{aligned}\phi_{ijl}^{11} =& (\rho_1\delta_1 - \rho_2\delta_2)C_l^{\mathrm{T}}C_l + 4m\breve{K}^2\eta_1C_l^{\mathrm{T}}C_l + 4(A_i + B_iK_jC_l)^{\mathrm{T}}P(A_i + B_iK_jC_l) \\ &+ 4\bar{\beta}(1-\bar{\beta})(B_iK_jW)^{\mathrm{T}}PB_iK_jW + 2\bar{\beta}(1-\bar{\beta})m\breve{K}^2\eta_1W^{\mathrm{T}}W - P\end{aligned}$$

$$\phi_{ijl}^{21} = -4K_j^{\mathrm{T}} B_i^{\mathrm{T}} P(A_i + B_i K_j C_l)$$

$$\phi_{ijl}^{22} = 4K_j^{\mathrm{T}} B_i^{\mathrm{T}} P B_i K_j + 4m\breve{K}^2 \eta_1 I - \rho_1 I + \rho_2 I$$

for $i = 1,2,\cdots,r$, $j = 1,2,\cdots\lambda$ and $l = 1,2,\cdots,r$, the closed-loop IT2 fuzzy system (12) is stochastically stable.
Proof: Choose the following Lyapunov function:

$$V_1(k) = x^{\mathrm{T}}(k)Px(k) \tag{15}$$

It follows that

$$\mathbb{E}\{\Delta V_1(k) \mid x(k)\} = \mathbb{E}\{(V_1(k+1) \mid x(k)) - V_1(k)\} = \mathbb{E}\{x^{\mathrm{T}}(k+1)Px(k+1) \mid x(k)\} - x^{\mathrm{T}}(k)Px(k) \tag{16}$$

Noting that $\mathbb{E}\{\beta^2(k)\} = \bar{\beta}(1-\bar{\beta})$, For $k \in [\gamma_\iota, \gamma_{\iota+1})$, from the expressions (12), one has

$$\begin{aligned}
&\mathbb{E}\{x^{\mathrm{T}}(k+1)Px(k+1) \mid x(k)\} = \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l[(A_i + B_iK_jC_l)x(k) \\
&- B_iK_je(k) + B_iK_j\beta(k)\xi(k) - B_iZ(k)]\}^{\mathrm{T}} \times P\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l[(A_i + B_iK_jC_l)x(k) \\
&- B_iK_je(k) + B_iK_j\beta(k)\xi(k) - B_iZ(k)] \mid x(k)\} \leqslant \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l[(A_i + B_iK_jC_l)x(k) \\
&- B_iK_je(k) + B_iK_j\beta(k)\xi(k) - B_iZ(k)]^{\mathrm{T}} \times P[(A_i + B_iK_jC_l)x(k) - B_iK_je(k) \\
&+ B_iK_j\beta(k)\xi(k) - B_iZ(k)] \mid x(k)\} \leqslant \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l 2[(A_i + B_iK_jC_l)x(k) \\
&- B_iK_je(k) + B_iK_j\beta(k)\xi(k)]^{\mathrm{T}} \times P[(A_i + B_iK_jC_l)x(k) - B_iK_je(k) \\
&+ B_iK_j\beta(k)\xi(k)] \mid x(k)\} + \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l 2[B_iZ(k)]^{\mathrm{T}} P \times B_iZ(k) \mid x(k)\} \\
&\leqslant \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l\{4[(A_i + B_iK_jC_l)x(k) - B_iK_je(k)]^{\mathrm{T}} P[(A_i + B_iK_jC_l)x(k) \\
&- B_iK_je(k)] + 4\bar{\beta}(1-\bar{\beta})[B_iK_j\xi(k)]^{\mathrm{T}} \times PB_iK_j\xi(k)\} + \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l \\
&\times 2[B_iZ(k)]^{\mathrm{T}} PB_iZ(k) \mid x(k)\}
\end{aligned} \tag{17}$$

By using $\| \xi(k) \| \leqslant \| Wx(k) \|$, we have

$$[B_iK_j\xi(k)]^{\mathrm{T}} PB_iK_j\xi(k) \leqslant [B_iK_jWx(k)]^{\mathrm{T}} PB_iK_jWx(k) \tag{18}$$

On the other hand, under condition (13), it has

$$\begin{aligned}
&\mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l Z^{\mathrm{T}}(k)B_i^{\mathrm{T}}PB_iZ(k) \mid x(k)\} \leqslant \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l m\breve{K}^2\eta_1 \hat{y}^{\mathrm{T}}(k)\hat{y}(k) \mid x(k)\} \\
&= \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l m\breve{K}^2\eta_1 [C_lx(k) - e(k) + \beta(k)\xi(k)]^{\mathrm{T}}[C_lx(k) - e(k) + \beta(k)\xi(k)] \mid x(k)\} \\
&\leqslant \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l 2m\breve{K}^2\eta_1\{[C_lx(k) - e(k)]^{\mathrm{T}} \times [C_lx(k) - e(k)] + \bar{\beta}(1-\bar{\beta})\xi^{\mathrm{T}}(k)\xi(k)\} \\
&\leqslant \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l 2m\breve{K}^2\eta_1\{[C_lx(k) - e(k)]^{\mathrm{T}} \times [C_lx(k) - e(k)] + \bar{\beta}(1-\bar{\beta})[Wx(k)]^{\mathrm{T}}Wx(k)\} \\
&\leqslant \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} h_i\xi_j h_l\{4m\breve{K}^2\eta_1[C_lx(k)]^{\mathrm{T}}C_lx(k) + 4m\breve{K}^2\eta_1 e^{\mathrm{T}}(k)e(k) + 2m\breve{K}^2\eta_1\bar{\beta}(1-\bar{\beta})[Wx(k)]^{\mathrm{T}}Wx(k)\}
\end{aligned} \tag{19}$$

By introducing the torus-event-triggering mechanism conditions, for $k \in [\gamma_\iota, \gamma_{\iota+1})$, it has

$$\begin{aligned}&\delta_1 y^{\mathrm{T}}(k)y(k)-e^{\mathrm{T}}(k)e(k)\geqslant 0\\&e^{\mathrm{T}}(k)e(k)-\delta_2 y^{\mathrm{T}}(k)y(k)\geqslant 0\end{aligned}\tag{20}$$

Combining (16)-(20), we can obtain

$$\begin{aligned}&\mathbb{E}\{\Delta V_1(k)\mid x(k)\}\leqslant\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r}h_i\xi_j h_l\{4[(A_i+B_iK_jC_l)x(k)\\&-B_iK_je(k)]^{\mathrm{T}}P[(A_i+B_iK_jC_l)x(k)-B_iK_je(k)]+4\bar{\beta}(1-\bar{\beta})[B_iK_jWx(k)]^{\mathrm{T}}PB_iK_jWx(k)\\&+4m\breve{K}^2\eta_1[C_lx(k)]^{\mathrm{T}}C_lx(k)+4m\breve{K}^2\eta_1e^{\mathrm{T}}(k)e(k)+2\bar{\beta}(1-\bar{\beta})m\breve{K}^2\eta_1[Wx(k)]^{\mathrm{T}}Wx(k)\\&+\rho_1[\delta_1[C_lx(k)]^{\mathrm{T}}C_lx(k)-e^{\mathrm{T}}(k)e(k)]+\rho_2[e^{\mathrm{T}}(k)e(k)-\delta_2[C_lx(k)]^{\mathrm{T}}C_lx(k)]-x^{\mathrm{T}}(k)Px(k)\}\\&=\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r}h_i\xi_jh_l\theta^{\mathrm{T}}(k)\phi_{ijl}\theta(k)\end{aligned}\tag{21}$$

where $\theta(k)=[x^{\mathrm{T}}(k)\ e^{\mathrm{T}}(k)]^{\mathrm{T}}$.
Under the condition (14), it has

$$\mathbb{E}\{\Delta V_1(k)\mid x(k)\}\leqslant\theta^{\mathrm{T}}(k)\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r}h_i\xi_jh_l\phi_{ijl}\theta(k)<0\tag{22}$$

Therefore, there exists a sufficiently small scalar κ such that

$$\mathbb{E}\{\Delta V_1(k)\mid x(k)\}\leqslant-\kappa\parallel x(k)\parallel^2\tag{23}$$

Taking expectation for both sides of (23), it has

$$\mathbb{E}\{\Delta V_1(k)\}\leqslant-\kappa\mathbb{E}\{\parallel x(k)\parallel^2\}\tag{24}$$

By adding up both sides of (24) from $k=0$ to $k=d$ for any $d>0$, we have

$$\mathbb{E}\{V_1(d+1)\}-\mathbb{E}\{V_1(0)\}\leqslant-\kappa\mathbb{E}\{\sum_{k=0}^{d}\parallel x(k)\parallel^2\}\tag{25}$$

Furthermore, it has

$$\mathbb{E}\{\sum_{k=0}^{d}\parallel x(k)\parallel^2\}\leqslant\frac{\mathbb{E}\{V_1(0)\}}{\kappa}<\infty\tag{26}$$

Let $d\to\infty$, we finally obtain

$$\mathbb{E}\{\sum_{k=0}^{\infty}\parallel x(k)\parallel^2\}<\infty\tag{27}$$

This completes the proof.

3.3. Reachability analysis

Theorem 2: If there exist matrices $Q>0$, scalar $\eta_2>0$, K_j satisfying

$$B_i^{\mathrm{T}}C_m^{\mathrm{T}}H^{\mathrm{T}}QHC_mB_i\leqslant\eta_2 I\tag{28}$$

$$\vec{\phi}_{ijlm}<0\tag{29}$$

with

$$\vec{\phi}_{ijlm}=\begin{bmatrix}\vec{\phi}_{ijlm}^{11}&*\\\vec{\phi}_{ijlm}^{21}&\vec{\phi}_{ijlm}^{22}\end{bmatrix}$$

$$\begin{aligned}\vec{\phi}_{ijlm}^{11}=&\phi_{ijl}^{11}+4(A_i+B_iK_jC_l)^{\mathrm{T}}C_m^{\mathrm{T}}H^{\mathrm{T}}QHC_m\times(A_i+B_iK_jC_l)+4\bar{\beta}(1-\bar{\beta})(B_iK_jW)^{\mathrm{T}}\\&\times C_m^{\mathrm{T}}H^{\mathrm{T}}QHC_mB_iK_jW+4m\breve{K}^2\eta_2C_l^{\mathrm{T}}C_l\end{aligned}$$

$$\vec{\phi}_{ijlm}^{21} = \phi_{ijl}^{21} + 8K_j^{\mathrm{T}} B_i^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m (A_i + B_i K_j R)$$

$$\vec{\phi}_{ijlm}^{22} = \phi_{ijl}^{22} + 4K_j^{\mathrm{T}} B_i^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m B_i K_j + 4m\breve{K}^2 \eta_2 I$$

for $i = 1, 2, \cdots r, j = 1, 2, \cdots \lambda$, Then the states of closed-loop IT2 fuzzy system (12) can be driven into the domain:

$$\Re = \{s(k) ||| s(k) || \leqslant \sqrt{\frac{g(k)}{\lambda_{min}\{Q\}}}\} \tag{30}$$

with $g(k) = 2\bar{\beta}(1-\bar{\beta})m\breve{K}^2\eta_2 \| W^{\mathrm{T}} W \| \| x(k) \|^2$.

Proof: Choose the following Lyapunov function:

$$V_2(k) = V_1(k) + s^{\mathrm{T}}(k) Q s(k) \tag{31}$$

Then we have

$$\mathbb{E}\{\Delta V_2(k) \mid x(k)\} = \mathbb{E}\{\Delta V_1(k) \mid x(k)\} + \mathbb{E}\{s^{\mathrm{T}}(k+1) Q s(k+1) \mid x(k)\} - s^{\mathrm{T}}(k) Q s(k) \tag{32}$$

Denote $\tilde{h}_m = h_m(\omega(k+1))$. From the sliding function (9), we have

$$\begin{aligned} s(k+1) &= \sum_{m=1}^{r} h_m(\omega(k+1)) H C_m x(k+1) = \sum_{m=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l H C_m [(A_i \\ &\quad + B_i K_j C_l) x(k) - B_i K_j e(k) + B_i K_j \beta(k) \xi(k) - B_i Z(k)] \end{aligned} \tag{33}$$

For $k \in [\gamma_\iota, \gamma_{\iota+1})$, it has

$$\begin{aligned} &\mathbb{E}\{s^{\mathrm{T}}(k+1) Q s(k+1) \mid x(k)\} \leqslant \sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \{4[(A_i + B_i K_j C_l) x(k) \\ &- B_i K_j e(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m [(A_i + B_i K_j C_l) x(k) - B_i K_j e(k)] + 4\bar{\beta}(1-\bar{\beta})[B_i K_j \xi(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} \\ &\times Q H C_m B_i K_j \xi(k)\} + \mathbb{E}\{\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \times 2[B_i Z(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m B_i Z(k) \mid x(k)\} \end{aligned} \tag{34}$$

Following the similar methods in (18) and (19), we can easily derive

$$\begin{aligned} &\mathbb{E}\{s^{\mathrm{T}}(k+1) Q s(k+1) \mid x(k)\} \leqslant \sum_{m=1}^{r}\sum_{i=1}^{\lambda}\sum_{j=1}^{r}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \{4[(A_i + B_i K_j C_l) x(k) \\ &- B_i K_j e(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m [(A_i + B_i K_j C_l) x(k) - B_i K_j e(k)] + 4\bar{\beta}(1-\bar{\beta})[B_i K_j W x(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} \\ &\times Q H C_m B_i K_j W x(k) + 4m\breve{K}^2 \eta_2 e^{\mathrm{T}}(k) e(k) + 4m\breve{K}^2 \eta_2 [C_l x(k)]^{\mathrm{T}} C_l x(k) \\ &+ 2\bar{\beta}(1-\bar{\beta}) m\breve{K}^2 \eta_2 x^{\mathrm{T}}(k) W^{\mathrm{T}} W x(k)\} \end{aligned} \tag{35}$$

Moreover, combining (32) and (35), one obtains

$$\begin{aligned} &\mathbb{E}\{\Delta V_2(k) \mid x(k)\} \leqslant \mathbb{E}\{\Delta V_1(k) \mid x(k)\} + \sum_{m=1}^{r}\sum_{i=1}^{\lambda}\sum_{j=1}^{r}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \{4[(A_i + B_i K_j C_l) x(k) \\ &+ B_i K_j e(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m [(A_i + B_i K_j C_l) x(k) + B_i K_j e(k)] + 4\bar{\beta}(1-\bar{\beta})[B_i K_j W x(k)]^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} \\ &\times Q H C_m B_i K_j W x(k) + 4m\breve{K}^2 \eta_2 e^{\mathrm{T}}(k) e(k) + 4m\breve{K}^2 \eta_2 [C_l x(k)]^{\mathrm{T}} C_l x(k)\} \\ &+ 2\bar{\beta}(1-\bar{\beta}) m\breve{K}^2 \eta_2 x^{\mathrm{T}}(k) W^{\mathrm{T}} W x(k) - s^{\mathrm{T}}(k) Q s(k) \end{aligned} \tag{36}$$

Under condition (28), it follows from (36) that

$$\begin{aligned} \mathbb{E}\{\Delta V_2(k) \mid x(k)\} &\leqslant \sum_{m=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \theta^{\mathrm{T}}(k) \vec{\phi}_{ijl} \theta(k) - [s^{\mathrm{T}}(k) Q s(k) - 2\bar{\beta}(1-\bar{\beta}) m\breve{K}^2 \eta_2 x^{\mathrm{T}}(k) W^{\mathrm{T}} W x(k)] \\ &\leqslant \sum_{m=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{\lambda}\sum_{l=1}^{r} \tilde{h}_m h_i \xi_j h_l \theta^{\mathrm{T}}(k) \vec{\phi}_{ijl} \theta(k) - [\lambda_{min}\{Q\} \| s(k) \|^2 - g(k)] \end{aligned} \tag{37}$$

where $g(k)$ is described in Theorem 2. If $s(k) \geqslant \sqrt{\frac{g(k)}{\lambda_{min}\{Q\}}}$, we can conducted from (29) that

$$\mathbb{E}\{\Delta V_2(k) \mid x(k)\} < 0 \tag{38}$$

Taking expectations on both sides of the inequalities (38), it has

$$\mathbb{E}\{\Delta V_2(k)\} < 0 \tag{39}$$

which means that the closed-loop IT2 fuzzy system (12) can be driven into the domain $\Re$.

3.4. Solving algorithm

Theorem 3: If there exist matrices $\breve{P} > 0$, $\breve{Q} > 0$, positive scalar $\breve{\eta}_1$, $\breve{\eta}_2$, satisfying

$$\begin{bmatrix} -\breve{\eta}_1 I & B_i^{\mathrm{T}} \\ B_i & -P \end{bmatrix} < 0, \begin{bmatrix} -\breve{\eta}_2 I & (HC_m B_i)^{\mathrm{T}} \\ HC_m B_i & -Q \end{bmatrix} < 0 \tag{40}$$

$$\Phi_{ijlm} < 0 \tag{41}$$

with

$$\Phi_{ijlm} = \begin{bmatrix} \Phi^{11} & * & * & * & * \\ \Phi^{21} & \Phi^{22} & * & * & * \\ \Phi^{31} & 0 & -\breve{\eta}_1 I & * & * \\ \Phi^{41} & 0 & 0 & \Phi^{44} & * \\ \Phi^{51} & 0 & 0 & 0 & -\breve{\eta}_2 I \end{bmatrix}$$

$$\Phi^{11} = \begin{bmatrix} \begin{matrix} 2\bar{\beta}(1-\bar{\beta})m\breve{K}^2\eta_1 W^{\mathrm{T}} W \\ +(\rho_1\delta_1 - \rho_2\delta_2)C_l^{\mathrm{T}} C_l - P \end{matrix} & 0 \\ 0 & \begin{matrix} 4m\breve{K}^2(\eta_1+\eta_2)I \\ -(\rho_1-\rho_2)I \end{matrix} \end{bmatrix}$$

$$\Phi^{21} = \begin{bmatrix} 2(A_i + B_i K_j C_l) & 2B_i K_j \\ 2\sqrt{\bar{\beta}(1-\bar{\beta})} B_i K_j W & 0 \end{bmatrix}$$

$$\Phi^{22} = diag\{-\breve{P}, -\breve{P}\}$$

$$\Phi^{31} = \begin{bmatrix} 2\sqrt{m}\breve{K} C_l & 0 \end{bmatrix}$$

$$\Phi^{41} = \begin{bmatrix} 2HC_m(A_i + B_i K_j C_l) & 2HC_m B_i K_j \\ 2\sqrt{\bar{\beta}(1-\bar{\beta})} HC_m B_i K_j W & 0 \end{bmatrix}$$

$$\Phi^{44} = diag\{-\breve{Q}, -\breve{Q}\}$$

$$\Phi^{51} = \begin{bmatrix} 2\sqrt{m}\breve{K} C_l & 0 \end{bmatrix}$$

then the closed-loop IT2 fuzzy system is stochastically stable, and the system states will be driven into the domain $\Re$ simultaneously.

Proof: We just need to prove that the conditions (13),(14),(28),(29) can be ensured by conditions (40),(41).

Letting $\breve{P} = P^{-1}$ and $\breve{Q} = Q^{-1}$, we can obtain (13) and (28) by using (40) via the Schur complement. Condition (14) in Theorem 1 is ensured by condition (29) Theorem 2, for condition (29) implies that

$$\begin{aligned} \vec{\phi}_{ijlm} =& \phi_{ijl} + \begin{bmatrix} G_{ijlm} & 0 \\ 0 & 4m\breve{K}^2\eta_2 \end{bmatrix} \\ &+ \begin{bmatrix} [HC_m(A_i + B_i K_j C_l)]^{\mathrm{T}} \\ [HC_m B_i K_j]^{\mathrm{T}} \end{bmatrix} 4Q \\ &\times \begin{bmatrix} HC_m(A_i + B_i K_j C_l) & HC_m B_i K_j \end{bmatrix} < 0 \end{aligned} \tag{42}$$

where $G_{ijlm} = 4\bar{\beta}(1-\bar{\beta})(B_i K_j W)^{\mathrm{T}} C_m^{\mathrm{T}} H^{\mathrm{T}} Q H C_m B_i K_j W + 4m\breve{K}^2\eta_2 C_l^{\mathrm{T}} C_l$. Since $Q > 0$, $\phi_{ijl} < 0$ is ensured. Moreover, by applying Schur complement to (41), condition (29) is ensured.

We use CCL algorithm to solve the feasibility problem (40-41), it can be converted into the minimization problem:

$$\begin{aligned} &min\ tr(P\check{P}) \\ &s.t(40),(41)and \begin{bmatrix} P & I \\ I & \check{P} \end{bmatrix} \geqslant 0 \end{aligned} \tag{43}$$

where $\check{P} = P^{-1}$.

Remark 2. *So far, the addressed SMC problem has been studied for IT2 fuzzy system subject to deception attack by utilizing the torus-event-triggering mechanism. The solvability of the investigated problem has been cast into the feasibility of certain linear matrix inequalities. The required SMC law can be designed by solving the proposed optimization problem. In comparison to those existing literature where SMC method is used, this paper tries to combine the torus-event-triggering mechanism with SMC technique, to help enhance the robustness of the system against malicious attacks. The main merits of our proposed algorithm in comparison to those existing ones can be highlighted as follows: 1) the torus-event-triggering mechanism adopted in this paper could effectively mitigate the influence of the abnormal data resulting from deception attacks; and 2) the sliding mode control conception is utilized in our paper, which makes the designed controller has a strong robustness against to both external disturbances and internal uncertainties.*

4. Simulation

In this part, the effectiveness of the proposed control law will be verified by a mass-spring-damper system. Consider the following mass-spring-damper system

$$m\ddot{\mathbf{x}}(t) + c\dot{\mathbf{x}}(t) + \hat{\kappa}\mathbf{x}(t) + \hat{\kappa}a^2\mathbf{x}^2(t)\mathbf{x}(t) = u(t) \tag{44}$$

Let $x(t) = [x_1(t)\ x_2(t)] = [\mathbf{x}(t)\ \dot{\mathbf{x}}(t)]$ and $g(\hat{\kappa},t) = \dfrac{-\hat{\kappa} - \hat{\kappa}a^2x_1^2(t)}{m}$, system (44) can be represented as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ g(\hat{\kappa},t) & \frac{-c}{m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) \tag{45}$$

The system parameters are given as $a = 0.3\ \mathrm{m}^{-1}, m = 1\ \mathrm{kg}, c = 2\ \mathrm{N.m/s}$. Assume that $\hat{\kappa} \in [\hat{\kappa}_1\ \hat{\kappa}_2]$ with $\hat{\kappa}_1 = 5N/m, \hat{\kappa}_2 = 8N/m$. Under the condition $x_1(t) \in [-1,1]$, the nonlinear function $g(\hat{\kappa},t) = \dfrac{-\hat{\kappa} - \hat{\kappa}a^2x_1^2(t)}{m}$ satisfies $g_{\min} \leqslant g(\hat{\kappa},t) \leqslant g_{\max}$, where $g_{\min} = \dfrac{-\hat{\kappa}_2 - \hat{\kappa}_2a^2}{m}$, $g_{\max} = \dfrac{-\hat{\kappa}_1}{m}$. The membership functions can be calculated by the following functions:

$$\begin{cases} g(\hat{\kappa},t) = h_1(x_1(t))g_{\min} + h_2(x_1(t))g_{\max} \\ h_1(x_1(t)) + h_2(x_1(t)) = 1 \end{cases}$$

Then we obtain the membership functions

$$\begin{aligned} h_1(x_1(t)) &= \frac{g(\hat{\kappa},t) - g_{\max}}{g_{\min} - g_{\max}} \\ h_2(x_1(t)) &= \frac{g_{\min} - g(\hat{\kappa},t)}{g_{\min} - g_{\max}} \end{aligned}$$

The lower and upper membership functions are defined as

$$\begin{aligned} \underline{h}_1(x_1(t)) &= \frac{g_{\max} + \frac{\hat{\kappa}_1 + \hat{\kappa}_1a^2x_1^2(t)}{m}}{g_{\max} - g_{\min}} \\ \overline{h}_1(x_1(t)) &= \frac{g_{\max} + \frac{\hat{\kappa}_2 + \hat{\kappa}_2a^2x_1^2(t)}{m}}{g_{\max} - g_{\min}} \\ \underline{h}_2(x_1(t)) &= \frac{-g_{\min} + \frac{-\hat{\kappa}_2 - \hat{\kappa}_2a^2x_1^2(t)}{m}}{g_{\max} - g_{\min}} \\ \overline{h}_2(x_1(t)) &= \frac{-g_{\min} + \frac{-\hat{\kappa}_1 - \hat{\kappa}_1a^2x_1^2(t)}{m}}{g_{\max} - g_{\min}} \end{aligned}$$

Consider the variable $\hat{\kappa}_1 \leqslant \hat{\kappa} \leqslant \hat{\kappa}_2$, the system (45) can be approximated via the following two T-S fuzzy rules:

Rule 1: If $g(\hat{\kappa},t)$ is 'Small', Then

$$\dot{x}(t) = A_1 x(t) + B_1 u(t)$$
$$y(t) = C_1 x(t)$$

Rule 2: If $g(\hat{x}, t)$ is 'Big', Then

$$\dot{x}(t) = A_2 x(t) + B_2 u(t)$$
$$y(t) = C_2 x(t)$$

with

$$A_1 = \begin{bmatrix} 0 & 1 \\ g_{\min} & \frac{-c}{m} \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ g_{\max} & \frac{-c}{m} \end{bmatrix}$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

Set the sampling period as 0.2s, we obtain the following matrices:

$$A_1 = \begin{bmatrix} 0.8510 & 0.1554 \\ -1.3555 & 0.5401 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.9135 & 0.1594 \\ -0.7971 & 0.5947 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0.0171 \\ 0.1554 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.0173 \\ 0.1594 \end{bmatrix}$$

and we set that $C_1 = C_2 = [1\ 0]$. Then the overall IT2 fuzzy system can be expressed as

$$x(k+1) = \sum_{i=1}^{2} h_i(x_1(k))[A_i x(k) + B_i u(k)]$$
$$y(k) = \sum_{i=1}^{2} h_i(x_1(k)) C_i x(k)$$

Note that here we adopted zero-order hold (ZOH) method to transform a continuous-time system into a discrete-time one. The principle of the ZOH method is based on the assumption that the continuous-time input signal remains constant within each sampling interval. This means that the value of the continuous-time signal is held constant until the next sampling instant occurs.

The lower and upper membership functions of the controller are chosen as $\underline{h}_i(\hat{y}(k))$ and $\overline{h}_i(\hat{y}(k))$ for $i = 1, 2$. In this example, the membership functions of system and controller $h_i(x_1(k))$, $\xi_i(\hat{y}(k))$ are determined by the parameter $\overline{\upsilon}_1 = \sin^2(x_1(k))$, $\underline{\upsilon}_1 = 1 - \overline{\upsilon}_1$, $\overline{\varsigma}_1 = 0.5$ and $\underline{\varsigma}_1 = 1 - \overline{\varsigma}_1$.

The triggering parameters are given as $\delta_1 = 0.1$, $\delta_2 = 1.5$, $\Omega_k = 0.5I$ and we set the attack probability $\bar{\beta} = 0.3$. The sliding surface parameter is $H = [1\ 10]$ and the deception attacks signal is designed as $\varpi(k) = \sin(x_1(k))$. By using Theorem 3, the controller gains are calculated as $K_1 = K_2 = [3.2309\ \ -1.9086]$.

Assuming that the initial states are $x(0) = [-1\ 5]^{\mathrm{T}}$, Figure 1 shows the triggering instants under the torus-event-triggering mechanism. The occurrences of the deception attacks are depicted in Figure 2, where $\beta(k) = 1$ represents that the deception attacks occur. As is shown in Figure 3–4, the control law proposed in this paper can ensure the stability of the IT2 fuzzy system under the deception attacks and the sliding variable $s(k)$ is shown in Figure 5.

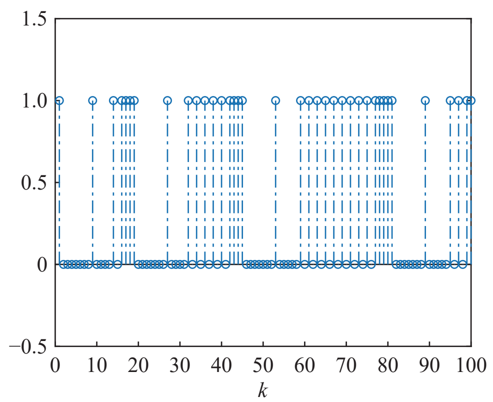

Figure 1. Triggering instants under Torus-Event-Triggering Mechanism.

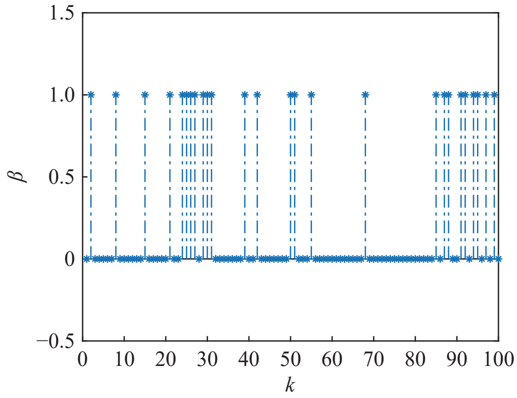

Figure 2. The deception attacks.

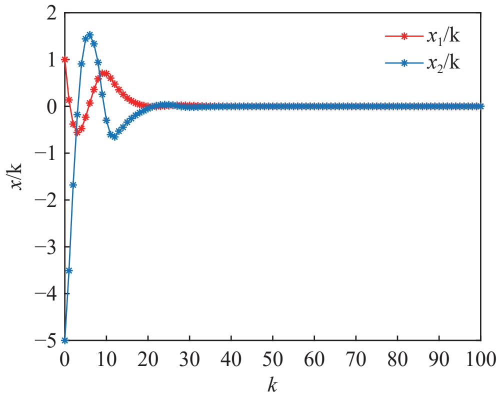

Figure 3. The curve of the system state $x(k)$.

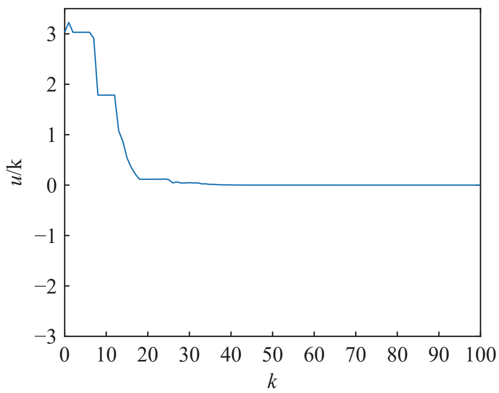

Figure 4. The curve of the control signal $u(k)$.

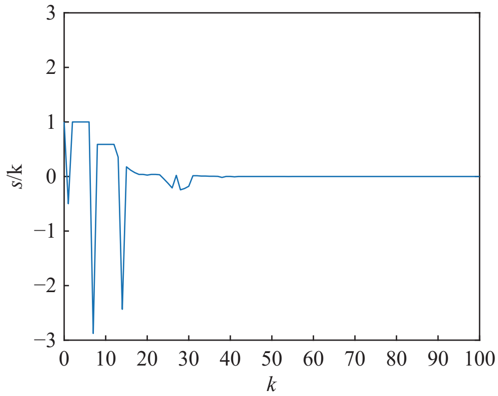

Figure 5. The curve of the sliding variable $s(k)$.

5. Conclusion

This paper has designed the security controller based on SMC technique for IT2 T-S fuzzy system under deception attacks. The torus-event-triggering scheme has been used to decrease the frequency of data transmission and overcome the shortcomings of conventional unilateral-event- triggering strategy. A sliding mode controller has been proposed to ensure the stochastic stability of the closed-loop system and the reachability of the prescribed sliding surface. The effectiveness of the proposed method has been illustrated by a simulation example. In the future work, we will focus on the sliding mode control for different systems subject to different attacks.

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